Emulating a ΛCDM-like expansion on the phantom brane

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In [Phys. Rev. D **80**, 123003 (2009)] Schmidt suggested that dynamical dark energy (DDE) propagating on the phantom brane could mimick ΛCDM. Schmidt went on to derive a phenomenological expression for $\rho_{\rm DE}$ which could achieve this. We demonstrate that while Schmidt's central premise is correct, the expression for $\rho_{\rm DE}$ derived by [Schmidt, Phys. Rev. D **80**, 123003 (2009)] is flawed. We derive the correct expression for $\rho_{\rm DE}$ which leads to ΛCDM-like expansion on the phantom brane. We also show that DDE on the brane can be associated with a quintessence field and derive a closed form expression for its potential $V(\phi)$. Interestingly the α-attractor based potential $V(\phi) \propto \coth^2 \lambda\phi$ makes braneworld expansion resemble ΛCDM. However the two models can easily be distinguished on the basis of density perturbations which grow at different rates on the braneworld and in ΛCDM.

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I. INTRODUCTION

Cosmological expansion appears to be speeding up. The source of cosmic acceleration may be a novel constituent called dark energy (DE) which violates the strong energy condition $\rho + 3p \geq 0$. An alternative to this scenario rests on the possibility that general relativity (GR) inadequately describes late-time cosmic expansion and needs to be supplanted by a modified theory of gravity. Of the various DE models suggested in the literature [1] the cosmological constant Λ occupies a special place since its equation of state $p = -\rho$ is manifestly Lorentz invariant [2,3]. Λ, when taken together with cold dark matter (CDM), constitutes ΛCDM cosmology. The ΛCDM universe appears to agree remarkably well with a slew of cosmological observations [4]. Yet some data sets [5,6] also appear to support a phantom universe possessing a strongly negative equation of state (EOS) of dark energy (DE), $w < -1$ [7]. While current data sets are unable to unambiguously differentiate between these orthogonal models, high quality data expected from future DE experiments are likely to do so.

It is well known that a phantom universe is plagued by instabilities which render the simplest versions of this scenario untenable [8]. For this reason considerable interest has been roused by modified gravity models in which the EOS is an *effective* quantity and therefore its becoming phantomlike is not associated with underlying instabilities. To this class of models belongs the phantom brane. Originally proposed in [9,10] the phantom brane has an effective equation of state of dark energy which is *phantomlike*, i.e., $w_{\rm eff} < -1$. The expansion rate on the phantom brane is given by [9]

$$h(x) \equiv \frac{H(x)}{H_0} = \sqrt{\Omega_{0m}x^3 + \Omega_\sigma + \Omega_\ell} - \sqrt{\Omega_\ell},$$
$$x \equiv (1+z) = a_0/a, \tag{1}$$

where Ω_σ describes the brane tension while Ω_ℓ depends upon the ratio between the five-dimensional (M_p) and four-dimensional Plank mass (m_p)

$$\Omega_\ell = \frac{1}{\ell^2 H_0^2} \quad \text{where } \ell = \frac{2m_p^2}{M_p^3}. \tag{2}$$

Since $h(x=1) = 1$ the constants in (1) are related through the constraint equation

$$\Omega_\sigma = 1 - \Omega_{0m} + 2\sqrt{\Omega_\ell}. \tag{3}$$

Note that in the limit $\Omega_\ell \to 0$ (or $\ell \to \infty$), (1) describes Friedmann-Robertson-Walker (FRW) expansion in general relativity (GR). As its name suggests, the phantom brane has an effective equation of state

$$w_{\rm eff}(x) = \frac{(2x/3)d\ln H/dx - 1}{1 - (H_0/H)^2\Omega_{m0}x^3}, \qquad x = 1 + z, \tag{4}$$

whose value becomes phantomlike, $w_{\rm eff} < -1$, at the present epoch. It is interesting that the phantom brane does not possess any of the singularities which usually afflict conventional phantom models and agrees very well with observations [11].

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In [12] Schmidt suggested the intriguing possibility that the presence of dynamical dark energy (DDE) on the brane might give rise to ΛCDM-like expansion at late times. In this paper we demonstrate that while Schmidt's original conjecture is correct, his expression for DDE is flawed. In Sec. II, we revisit Schmidt's formalism and derive the correct expression for DDE. In Sec. III, we also show how a Quintessence field propagating on the brane can give rise to ΛCDM-like expansion. We summarize our results in Sec. IV with useful discussions.

II. DARK ENERGY ON THE BRANE

It is instructive to generalize braneworld expansion in (1) to

$$h(x) = \sqrt{\Omega_{0m}x^3 + \Omega_{\rm DE}(x) + \Omega_\ell} - \sqrt{\Omega_\ell}, \tag{5}$$

where the constant brane tension Ω_σ in (1) has been replaced by the dynamical quantity $\Omega_{\rm DE}(x) \equiv \rho_{\rm DE}(x)/\rho_{cr,0}$. The critical density at the present epoch is given by $\rho_{cr,0} = 3m_p^2 H_0^2$. Accordingly (3) becomes

$$\Omega_{\rm DE}(x=1) = 1 - \Omega_{0m} + 2\sqrt{\Omega_\ell}. \tag{6}$$

Next we demand that brane expansion in (5) coincide with that in the ΛCDM model

$$h_{\Lambda\rm CDM}(x) = \sqrt{\Omega_{0m}x^3 + \Omega_\Lambda}. \tag{7}$$

Equating (5) and (7) one easily gets

$$\Omega_{\rm DE}(x) = \Omega_\Lambda + 2\sqrt{\Omega_\ell}\sqrt{\Omega_{0m}x^3 + \Omega_\Lambda} = \Omega_\Lambda + 2h\sqrt{\Omega_\ell}, \tag{8}$$

which reduces to $\Omega_{\rm DE}(x) = \Omega_\Lambda$ when $\Omega_\ell = 0$.

Surprisingly the expression for $\Omega_{\rm DE}(x)$ in (8) differs from that in [12], namely

$$\Omega_{\rm DE}^{\rm Schmidt}(x) = \Omega_\Lambda + 2\Omega_\ell\left[\sqrt{(\Omega_{0m}/\Omega_\ell)x^3 + 1} - 1\right], \tag{9}$$

(see Eq. (2.4) of [12]). Indeed, even a cursory comparison of (9) and our expression (8) reveals that the two expressions for $\Omega_{\rm DE}$ are very different. (Note that Ω_ℓ in our notation coincides with Ω_{rc} in [12].) Clearly (8) satisfies the present epoch constraint (6) whereas (9) fails to do so, since

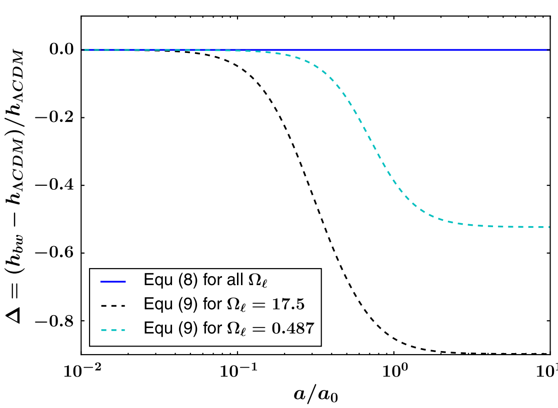
$$\Omega_{\rm DE}^{\rm Schmidt}(x=1) = \Omega_\Lambda + 2\Omega_\ell[\sqrt{(\Omega_{0m}/\Omega_\ell) + 1} - 1]. \tag{10}$$

Figure 1(a) shows the fractional difference, Δ, between the expansion rate in ΛCDM and in the two braneworld models, [12] and ours. In both cases h_{bw} is given by (5) with $\Omega_{\rm DE}$ determined from (9) in [12] and from (8) in our model.

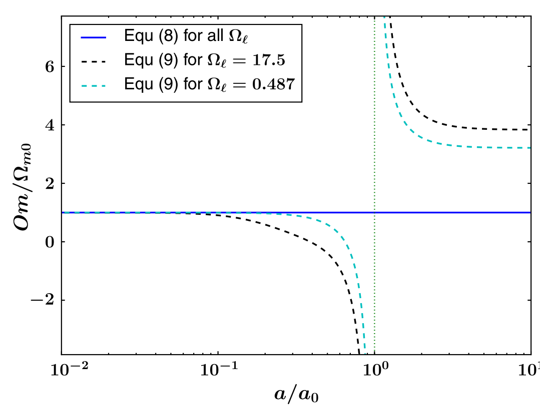
Figure 1(a) clearly demonstrates that while $\Delta = 0$ in our model (as required), $\Delta \neq 0$ in Schmidt's model (9). The possibility of an error in (9) is further supported by an analysis of the Om diagnostic [13]

$$Om(x) = \frac{h^2(x) - 1}{x^3 - 1}, \qquad x = 1 + z. \tag{11}$$

It is well known that $Om = \Omega_{0m}$ only in ΛCDM [13]. In other DE models $Om \neq \Omega_{0m}$ and in dynamical DE models



(a) deviation from ΛCDM expansion



(b) Om diagnostic

FIG. 1. Left panel: The fractional difference, Δ, in the expansion rate of ΛCDM and the two braneworld models (8) and (9) is shown for different values of Ω_ℓ. As expected $\Delta = 0$ for (8), implying that the braneworld (8) and ΛCDM have the same expansion rate. However $\Delta \neq 0$ for the braneworld in (9) indicating that the expansion rate in this braneworld does not mimic ΛCDM. Right panel: This panel shows the Om diagnostic for the two braneworld models (8) and (9). We find that $Om/\Omega_{0m} = 1$ in (8) which is a reflection of the fact that the expansion rate in (8) is the same as that in ΛCDM. However $Om/\Omega_{0m} \neq 1$ in the braneworld in (9) which implies that braneworld expansion in this model does not mimic ΛCDM (as claimed). Note that Ω_ℓ in our notation coincides with Ω_{rc} in [12]. In this figure we have set the parameters to the same values as were used in [12] for illustration.

Om can also be time dependent. Figure 1(b) (right panel) shows the ratio Om/Ω_{0m} for our model (8) and for (9) from [12]. We find that $Om/\Omega_{0m} = 1$ in our model but Om is strongly time dependent for (9). We therefore conclude that the derivation of (9) in [12] is incorrect.

The equation of state (EOS) of the dark energy, defined as $w_{\rm DE} \equiv p_{\rm DE}/\rho_{\rm DE}$, can be calculated using the relation

$$\dot{\rho}_{\rm DE} = -3H\rho_{\rm DE}(1 + w_{\rm DE}), \tag{12}$$

and the expression of $\Omega_{\rm DE}$ in (8) as

$$w_{\rm DE} = -1 + \frac{\Omega_{0m}x^3}{\Omega_{\rm DE}}\sqrt{\frac{\Omega_\ell}{\Omega_{0m}x^3 + \Omega_\Lambda}} = -1 + \frac{\Omega_{0m}x^3\sqrt{\Omega_\ell}}{h\Omega_{\rm DE}}. \tag{13}$$

On the other hand, if we assume the incorrect expression for dark energy given in [12], the expression for $w_{\rm DE}$ is coming out to be

$$w_{\rm DE}^{\rm Schmidt} = -1 + \frac{\Omega_{0m}x^3}{\Omega_{\rm DE}^{\rm Schmidt}}\sqrt{\frac{\Omega_\ell}{\Omega_{0m}x^3 + \Omega_\ell}}, \tag{14}$$

which itself is of course fallacious ($\Omega_{\rm DE}^{\rm Schmidt}$ is given by (9)).

The solid curves in Fig. 2 show the evolution of the correct equation of state, $w_{\rm DE}$, given in (13), for two values of Ω_ℓ which were used in [12] for illustration. The early matter domination and late dark energy domination asymptotes are $w_{\rm DE} = -1/2$ and -1 respectively. In Fig. 2, the dashed curves represent the evolution of the incorrect expression for $w_{\rm DE}$, given in (14), for the same two values of Ω_ℓ. Since the plots corresponding to the incorrect expression for $w_{\rm DE}$, given in (14), exactly match with the right panel of Fig. 1 of [12], we conclude that the error (9), committed in [12] was not just a simple typo and also carried along in Fig. 1 of that paper. But this error does not probably plague rest of that paper since only the expansion rate (which is trivially same as ΛCDM) remains important, not the explicit expression for $\Omega_{\rm DE}$ causing the expansion. The parameter Ω_ℓ in this "mimicry model," based on braneworld framework, is constrained as $\Omega_\ell \lesssim 0.25$ at 2σ using growth rate observations [14]. Note that, since this braneworld model mimics the background expansion of ΛCDM model, the EOS of the *effective dark energy*, $w_{\rm eff} = -1$ always.

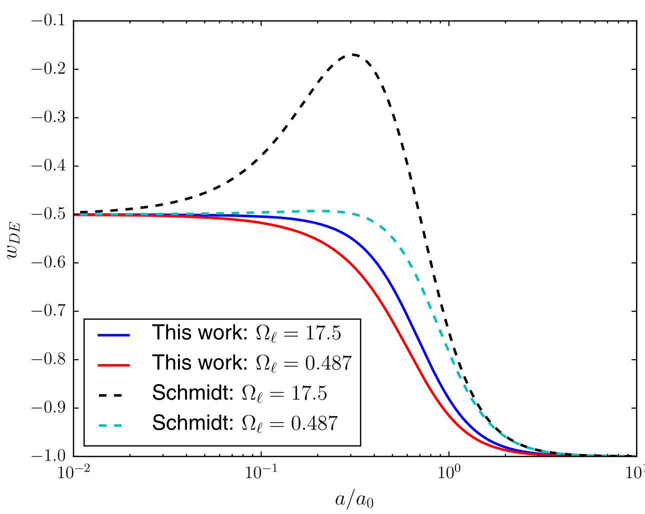


FIG. 2. The evolution of the correct expression for $w_{\rm DE}$, given by (13), is plotted with solid curves. The dashed curves represent the incorrect expression for $w_{\rm DE}$ given by (14), resulting from assuming the incorrect expression for $\Omega_{\rm DE}$ in (9). For comparison, we set the parameters to the same values that were chosen in [12] for illustration purposes. The incorrect plots (dashed curves) match with the corresponding curves in [12] (see right panel of Fig. 1 in that paper). So we believe that the error (9), committed in [12], was not just a simple typo and also carried along in Fig 1 of that paper.

III. QUINTESSENCE ON THE BRANE

In this section we derive the precise form of the Quintessence potential, $V(\phi)$, which gives rise to ΛCDM-like expansion on the brane. Consequently we replace $\Omega_{\rm DE}(z)$ in (5) and (8) by Ω_ϕ, with the result that the expansion history becomes

$$h_\phi(x) = \sqrt{\Omega_{0m}x^3 + \Omega_\phi(x) + \Omega_\ell} - \sqrt{\Omega_\ell}, \qquad x = 1 + z, \tag{15}$$

where $\Omega_\phi \equiv \rho_\phi/\rho_{cr,0}$. The energy density ($\rho_\phi$) and pressure ($p_\phi$) of the scalar field are given by,

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{16}$$

Using (15), (16) and the equations of motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \tag{17}$$

one finds

$$\frac{\phi'^2}{\rho_{cr,0}} = \frac{2}{3xH_0^2}\left(\frac{h'}{h}\right)\left(1 + \frac{\sqrt{\Omega_\ell}}{h}\right) - \frac{\Omega_{0m}x}{H^2}, \tag{18}$$

and

$$\frac{V(x)}{\rho_{cr,0}} = h^2 - \frac{\Omega_{0m}x^3}{2} + 2h\sqrt{\Omega_\ell} - \frac{xh'(h + \sqrt{\Omega_\ell})}{3}. \tag{19}$$

Here prime denotes differentiation with respect to x (or z). Note that (18) and (19) reduce to the usual equations for the scalar field in the GR limit, $\Omega_\ell \to 0$.

In order to determine $V(\phi)$ one needs to solve (18) and substitute the resulting expression for $h(\phi)$ in (19). This process can be simplified by noting that $h(x)$ in this mimicry model is given by the ΛCDM expression (7). Consequently (18) becomes

$$\frac{\phi'^2}{\rho_{cr,0}} = \frac{\Omega_{0m}\sqrt{\Omega_\ell}}{H_0^2}\left(\frac{x}{h^3}\right). \tag{20}$$

We choose the negative square root in (20) so that ϕ rolls towards more positive values (*ie* $\dot{\phi} > 0$). Consequently the evolution of ϕ is determined by

$$\phi' = -\left(m_p\sqrt{3\Omega_{0m}\sqrt{\Omega_\ell}}\right)\sqrt{\frac{x}{h^3}}, \tag{21}$$

where $h(x)$ is given by (7). In this case (19) reduces to

$$\frac{V(x)}{\rho_{cr,0}} = \Omega_\Lambda + \sqrt{\Omega_\ell}\left(\frac{3h^2+\Omega_\Lambda}{2h}\right). \tag{22}$$

Next we look for the solutions to (21) and (22) for the following important limiting cases.

(i) GR. Substituting $\Omega_\ell \to 0$ in (21) and (22) one easily gets $\phi = \text{constant}$ and $V/\rho_{cr,0} = \Omega_\Lambda$, as expected.

(ii) Early times. For $1 \ll x \ll 10^3$, $h \simeq \sqrt{\Omega_{0m}x^3}$ so that

$$\frac{\phi}{m_p} \approx \frac{4}{\sqrt{3}}\left(\frac{\Omega_\ell}{\Omega_{0m}}\right)^{1/4} x^{-3/4} \approx \frac{4}{\sqrt{3}}\frac{\Omega_\ell^{1/4}}{\sqrt{h}} \tag{23}$$

where the constant of integration is chosen such that the scalar field rolls from zero initially, $\phi(x \gg 1) = 0$. One also finds

$$\frac{V}{\rho_{cr,0}} \approx \Omega_\Lambda + \frac{3}{2}\sqrt{\Omega_\ell}h \approx \Omega_\Lambda + \frac{8\Omega_\ell}{(\phi/m_p)^2} \propto \frac{1}{\phi^2}. \tag{24}$$

(iii) Late times. For $x \ll 1$ one has $h \to \sqrt{\Omega_\Lambda}$ with the result that

$$\phi \simeq -\frac{2}{\sqrt{3}}m_p\sqrt{\frac{\Omega_{0m}}{\Omega_\Lambda}\left(\frac{\Omega_\ell}{\Omega_\Lambda}\right)^{1/4}} x^{3/2} + \phi_1, \tag{25}$$

where $\phi_1 = \phi(x \to 0)$. It is easy to show that $\dot{\phi}^2 \propto x^3 \ll 1$ and

$$\frac{V}{\rho_{cr,0}} \approx \Omega_\Lambda + 2\sqrt{\Omega_\ell\Omega_\Lambda} = \text{constant}. \tag{26}$$

It is interesting that $V(\phi)$ in (24) and (26) has precisely the same asymptotic form as the potential $V = V_0\coth^2(\lambda\phi/m_p)$. Accordingly we determine $V(\phi)$ in terms of the following ansatz.[1]

[1] A companion potential to (27) which gives a somewhat better approximation to ΛCDM is $V(\phi)/\rho_{cr,0} = \Omega_\Lambda + 2\sqrt{\Omega_\ell\Omega_\Lambda}\coth(\frac{\lambda\phi}{m_p})^2$.

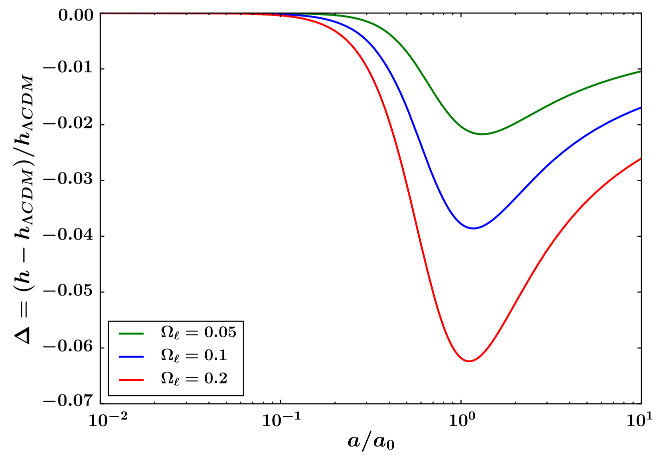


FIG. 3. The fractional difference between the expansion rate on the brane (15) and that in the ΛCDM model is shown for the ansatz potential (27).

$$\Omega_{0V} \equiv \frac{V(\phi)}{\rho_{cr,0}} = A\coth^2\left(\frac{\lambda\phi}{m_p}\right),$$

$$\text{where } A = \Omega_\Lambda + 2\sqrt{\Omega_\Lambda\Omega_\ell} \quad \text{and} \quad \lambda = \sqrt{\frac{A}{8\Omega_\ell}}. \tag{27}$$

As demonstrated in Fig. 3, a scalar field propagating on the brane under the influence of the potential (27) reproduces ΛCDM-like expansion to an accuracy of ≤7% for $\Omega_\ell \leq 0.2$. This figure was generated by solving the equation of motion of the scalar field (17) with H given by (15) and $\Omega_\phi = \Omega_{0V} + \Omega_{0,KE}$ where Ω_{0V} defined in (27) and $\Omega_{0,KE} = \frac{1}{2}\dot{\phi}^2/\rho_{cr,0}$. Note that, the potential (27) belongs to the class of potentials—$V(\phi) \propto \coth^p(\lambda\phi)$—which are based on α-attractor family of potentials [15]. This set of potentials possesses the same early time tracking feature of the *inverse power law* potentials [16,17] and the former has been comprehensively studied in [18] in the context of dark energy.

But one can do even better. Below we reconstruct the *exact form* of $V(\phi)$ which allows the brane to mimic ΛCDM-like expansion precisely.

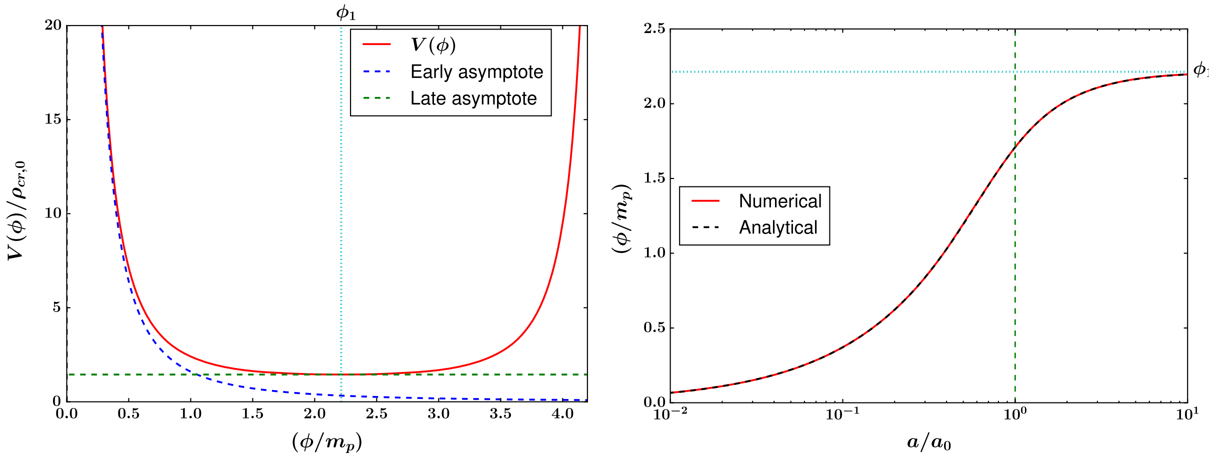
A. Exact form for $V(\phi)$

Integrating (21), one obtains the following exact solution[2] for ϕ

[2] The exact solution for ϕ can also be written as follows

$$\phi = -\frac{2}{\sqrt{3}}m_p\sqrt{\frac{\Omega_{0m}}{\Omega_\Lambda}\sqrt{\frac{\Omega_\ell}{\Omega_\Lambda}}}x^{3/2}{}_2F_1\left(\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{\Omega_{0m}x^3}{\Omega_\Lambda}\right) + \phi_1,$$

where ${}_2F_1(a,b;c;\mu)$ is the Gauss hypergeometric function and ϕ_1 is given in (31).



(a) (b)

FIG. 4. (a) The potential (33) is shown (red curve) for the braneworld parameter $\Omega_\ell = 0.2$. The early and late time asymptotic behaviour of the potential is shown by blue and green dashed curves respectively. (b) The numerically obtained value for ϕ (red) is compared with the analytical expression (dashed black), given by (28). Note that the numerical results match the analytical expression exactly. This panel demonstrates that, commencing from $\phi = 0$, the scalar field asymptotically rolls up to a finite value $\phi \to \phi_1$ as $x = a_0/a \to 0$. (ϕ_1 is shown by the dotted horizontal cyan line). Note that the potential has a minimum at ϕ_1, which has been shown by the vertical dotted cyan line in the left panel. The scalar field rolls to that minimum very slowly and settles there in the infinite future.

$$\phi = CF\left(\sin^{-1}\left(\frac{\Omega_\Lambda^{1/4}}{\sqrt{h}}\right)\Big| -1\right), \tag{28}$$

where C is a constant (having dimensions of mass) given by

$$C = \frac{4}{3}\sqrt{\frac{\rho_{cr,0}}{H_0^2}\sqrt{\frac{\Omega_\ell}{\Omega_\Lambda}}} = \frac{4}{\sqrt{3}}\left(\frac{\Omega_\ell}{\Omega_\Lambda}\right)^{1/4} m_p, \tag{29}$$

and $F(\zeta|m)$ is an elliptic integral of the first kind, defined as

$$F(\zeta|m) = \int_0^\zeta \frac{d\theta}{\sqrt{1 - m\sin^2(\theta)}}. \tag{30}$$

In obtaining (28) we have chosen the constant of integration such that $\phi(x \gg 1) = 0$. It is worth noting that starting from $\phi = 0$ initially (when $x \gg 1$), the scalar field rolls up to the following asymptotic value in the infinite future ($x \to 0$)

$$\phi_1 \equiv \phi(x \to 0) = CK(-1), \tag{31}$$

where $K(-1) = \Gamma(\frac{1}{4})^2/(4\sqrt{2\pi}) \approx 1.31$. The complete elliptic integral of the first kind is defined as $K(m) = F(\frac{\pi}{2}|m)$.

Inverting equation (28) one can express the expansion rate h in terms of ϕ as follows

$$h(\phi) = \frac{\sqrt{\Omega_\Lambda}}{[\mathrm{sn}(\frac{\phi}{C}| -1)]^2}, \tag{32}$$

where $\mathrm{sn}((\phi/C)| -1)$ is one of the Jacobi elliptic functions.[3] Next, by inserting the expression for $h(\phi)$ from (32) into (22), one easily gets the *exact form* for the reconstructed potential as

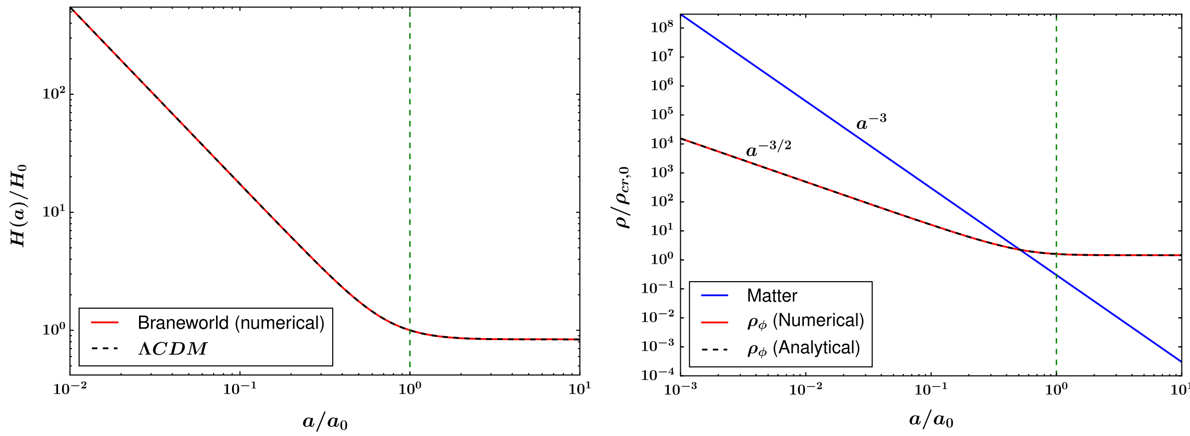
$$\frac{V(\phi)}{\rho_{cr,0}} = \Omega_\Lambda + \frac{1}{2}\sqrt{\Omega_\Lambda\Omega_\ell}\left[\frac{3}{\nu^2} + \nu^2\right] \quad \text{where}$$
$$\nu = \mathrm{sn}\left(\frac{\phi}{C}\Big| -1\right). \tag{33}$$

Using the properties of the concerned special functions, one can show that both (28) and (33) possess the correct limiting values given by (24) and (26) respectively.

The reconstructed potential in (33) is periodic in ϕ and its relevant part is plotted in Figure 4(a) (red curve) for $\Omega_\ell = 0.2$. The early and late time asymptotes, given by (24) and (26), are shown by the blue and green dashed curves respectively. Starting from its initial value (set at $\phi = 0$) the scalar field ϕ rolls up to ϕ_1, given in (31), in the infinite future ($x \to 0$). This is illustrated in Fig. 4(b) for $\Omega_\ell = 0.2$. The potential has a minimum at ϕ_1, as shown in Fig. 4(a) by the vertical dotted cyan line. The scalar field rolls to that minimum very slowly in the infinite future ($x \to 0$).

Figures 4(b) and 5 show that numerical simulations carried out using the potential (33) lead to *precisely* ΛCDM-like expansion. Figure 5(b) demonstrates that the

[3]If $u = F(\sin^{-1}(\nu)|m)$, then the inverse $\nu = \mathrm{sn}(u|m)$ is a Jacobi elliptic function.



(a) (b)

FIG. 5. The *left panel* shows that the expansion rate obtained by numerically integrating the reconstructed potential (33) coincides with the expansion rate of the ΛCDM model. The red curve in the *right panel* demonstrates that the potential (33) possesses an early time tracking feature which is identical to that of the inverse power law potential [16,17], $V \propto 1/\phi^2$. This leads to $w_\phi \simeq -1/2$ so that $\rho_\phi \propto a^{-3/2}$ during the matter dominated epoch. The black dashed curve overlaid on the red curve demonstrates that the analytical expression for dark energy, given by (8), *exactly matches* the numerical result obtained by integrating (33).

potential (33) possesses the same tracking feature as the inverse power law potential with alike large basin of attraction at early times, even within the braneworld framework. Therefore, the scalar field can mimick the expansion of a ΛCDM universe while rolling on the potential (33), without requiring fine-tuned initial conditions.

It is interesting that although the braneworld and ΛCDM have exactly the same expansion history, the two models can be easily distinguished on the basis of structure formation, since linearized density perturbations grow at different rates in the two models.[4] This has been illustrated in Fig. 6; also see Fig. 2 of [12].

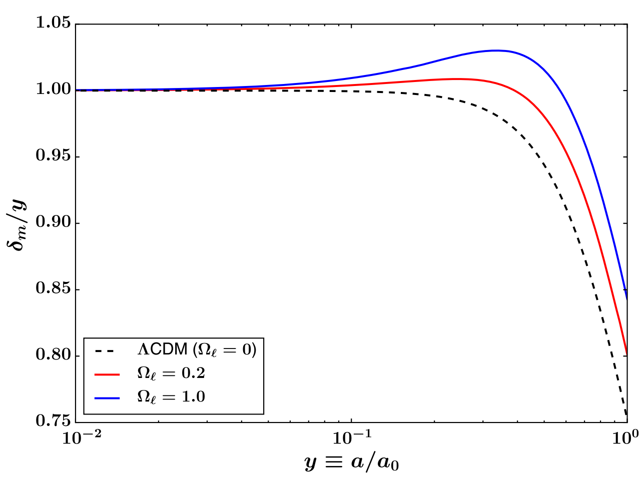


FIG. 6. Late time growth of linearized matter perturbations on the brane. Perturbation growth was determined assuming the quasistatic approximation [19]. Note that for $\Omega_\ell \to 0$ one recovers ΛCDM. This figure illustrates that although the braneworld with dark energy defined by (8) has exactly the same expansion rate as ΛCDM, gravitational clustering in the two models proceeds at very different rates; also see [12].

IV. DISCUSSION

In this paper we have derived an expression for the dark energy density which, when residing on the phantom brane, causes the brane to expand like a ΛCDM universe. We have also shown how DE can be related to a scalar field and derived a precise form for the scalar field potential $V(\phi)$. Interestingly, the potential possesses the same early time tracking feature as that of an inverse power law potential and the former can be well approximated by a α-attractor potential. We have thus demonstrated that a scalar field propagating on the phantom-brane can make the latter mimic the expansion of ΛCDM model.

It may be appropriate to note in this connection that braneworld expansion can mimic ΛCDM even in the complete absence of dynamical dark energy on the brane. As shown in [20,21] such a scenario of "cosmic mimicry" [20] can arise in either of the following cases:

(i) The brane tension is large and there is a large cosmological constant associated with the bulk fifth dimension [20]. (The present treatment assumed that there was no Λ-term associated with the bulk.)

[4] Since the quintessence dark energy does not cluster on the brane in usual setup, the perturbation of the quintessential field can be ignored. Therefore, one can assume the *quasistatic* approximation [19] for calculating the growth of matter perturbation in late times on the phantom brane.

(ii) The brane violates Z_2 symmetry with respect to the bulk [21]. In this case a small Λ-term on the brane is induced by a slight asymmetry in values of the fundamental constants in the bulk.

Our present paper extends this previous work by constructing an entirely different scenario for cosmic mimicry.

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