Bulk viscosity of a hot QCD medium in a strong magnetic field within the relaxation-time approximation

Manu Kurian* and Vinod Chandra†
Indian Institute of Technology Gandhinagar, Gandhinagar 382355, Gujarat, India

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The bulk viscosity of hot QCD medium has been obtained in the presence of strong magnetic field. The present investigation involves the estimation of the quark damping rate and subsequently the thermal relaxation time for quarks in the presence of magnetic field while realizing the hot QCD medium as an effective Grand-canonical ensemble of effective gluons and quarks antiquarks. The dominant process in the strong field limit is $1 \to 2$ ($g \to q\bar{q}$), which contributes to the bulk viscosity in the most significant way. Further, setting up the linearized transport equation in the framework of an effective kinetic theory with hot QCD medium effects and employing the relaxation time approximation, the bulk viscosity has been estimated in lowest Landau level and beyond. The temperature dependence of the ratio of the bulk viscosity to entropy density indicates its rising behavior near the transition temperature.

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I. INTRODUCTION

Relativistic heavy-ion collision experiments (RHIC) set the platform for the creation and study of quark-gluon plasma (QGP) as a near-perfect fluid [1,2]. Recent investigations on the QGP suggest the presence of extremely high magnetic field in the early stages of the collisions (especially in the noncentral asymmetric collisions) [3–6]. In this context, a deeper understanding of various aspects of the QGP in the strong magnetic field is the prime focus of the current research on the physics of the RHIC. In particular, chiral magnetic effect [7–9] and chiral vortical effects [10–12] gained much attention in the QGP community. More recently, the discovery of global Λ-hyperon polarization in noncentral RHIC [13,14] opened up a new direction in the study of the QGP in the presence of strong magnetic field.

Recall that the quark-antiquark pair production and fusion processes are kinematically possible in the presence of the strong magnetic field [15,16] via $1 \to 2$ processes that dominate over $2 \to 2$ scattering processes while estimating the transport coefficients. This could be understood in terms of the fact that the rate is proportional to coupling constant α_s in the case of the former, whereas in that of the binary processes, it is proportional to α_s^2 [17]. The magnetic field effects enter in the quark-antiquark degrees of freedom through the Landau levels. The strong magnetic field restricts the calculation to the $(1+1)$-dimensional ground state, i.e., lowest Landau level (LLL) [18,19] (the dimensional reduction). On the other hand, the electrically chargeless gluons are not directly coupled to the magnetic field through the dispersion relation. However, the gluonic dynamics in the presence of magnetic field can be affected through the quark loop while defining the gluon vertex through the self-energy where the quark/antiquark loop contributes.

The quantitative study of the transport coefficients in the hot QCD medium is required for the estimation of the experimental observables like transverse momentum spectra and collective flow of the QGP within the dissipative relativistic hydrodynamic framework. In particular, extremely low viscosity to entropy ratio indicates the larger elliptic flow observed in RHIC. Besides providing the basis for understanding the probes of QGP, the transport coefficients give insights to the electromagnetic response of the medium. Recently, a number of ALICE results have shown the relevance of transport processes in the RHIC [20–22]. Since the strong magnetic field is generated in the noncentral asymmetric HIC, the dissipative magnetohydrodynamics describes the transport process of the medium. This sets the strong motivation for the estimation of transport coefficients of the QGP in the presence of the strong magnetic field.

There have been several attempts to estimate the transport coefficients of the hot QCD medium in the strong magnetic field [23–27]. In a very recent work, Fukushima and Hidaka [28] estimated the longitudinal conductivity in the magnetic field beyond LLL approximation by solving the kinetic equation, considering the scattering amplitude of synchrotron radiation and the pair annihilation processes. The authors have numerically shown that the contribution from LLL is the dominant one.

The goal of the present investigations is to estimate the temperature dependence of the thermal relaxation time and thereby the effective bulk viscosity while encoding the hot

*manu.kurian@iitgn.ac.in
†vchandra@iitgn.ac.in

QCD medium effects in strong magnetic field background through an effective quasiparticle model. The current analysis has been done with relativistic semiclassical transport theory, in which microscopic particle interactions bridge to macroscopic transport phenomena of the thermodynamic system. The kinetic theory approach is followed within the linear response analysis of the transport equation in which magnetic field enters through the propagator (matrix element in collision integral) and momentum distribution functions of the quarks and antiquarks. Note that another equivalent approach to investigate the transport coefficients of the hot QCD in the magnetic field background is the hard thermal loop effective theory (HTL) [29,30]. We are following the former one here.

Hot QCD medium effects encrypted as the equation of sate (EoS) dependence on the transport coefficients within effective linear transport theory are well understood [31–37]. In [38], the authors have recently estimated the EoS/medium dependence on the longitudinal electrical conductivity for the $1 \rightarrow 2$ processes in the strong magnetic field background. In the present work, we followed the effective fugacity quasiparticle model (EQPM), proposed in [39,40] and extended in the case of the strong magnetic in Ref. [38]. The first step towards the evaluation of the bulk viscosity is the quark damping rate $\Gamma_{\rm eff}$ in the strong field limit that leads to the thermal relaxation time $\tau_{\rm eff}$, followed by the estimation of the bulk viscosity $\zeta_{\rm eff}$ in the presence of magnetic field by setting up an effective linearized transport equation. This has been done not only in LLL but also with the higher Landau level (HLL) corrections.

The paper is organized as follows. Section II deals with the mathematical formalism for the estimation of the effective thermal relaxation time and the bulk viscosity along with the description of hot QCD effective coupling constant with HLL corrections. Section III constitutes the predictions on the bulk viscosity and the related discussions. Finally, in Sec. IV, the conclusion and outlook of the work are presented.

II. EFFECTIVE DESCRIPTION OF THERMAL RELAXATION AND BULK VISCOSITY IN STRONG MAGNETIC FIELD

The Green-Kubo formula is employed to estimate the bulk viscosity of the medium both in the presence and the absence of the strong magnetic field background in the studies [24,41–43]. In this work, we are adopting the kinetic theory approach for the analytical calculation of $\zeta_{\rm eff}$ in the strong magnetic field, in which we need to start from the relativistic transport equation. The strong magnetic field limit, $T^2 \ll eB$, has been considered for computing various quantities under consideration in LLL. The contributions from higher Landau levels are negligible (proportional to $e^{-\frac{\sqrt{eB}}{T}}$) in the regime. Now, for the weaker magnetic fields, going beyond LLL might help in understanding the impact of the magnitude of the field on the transport coefficients. A full computation in the weak field domain also requires computation of the quark/antiquark propagators under the same approximation and is beyond the scope of the present work. The formalism for the estimation of effective bulk viscosity includes the quasiparticle modeling of the system followed by the estimation of the thermal relaxation time of the process.

A. EQPM in the strong magnetic field

EQPM describes the hot QCD medium effects with temperature dependent effective fugacities—quasigluon and quasiquark/antiquark fugacities, z_g and z_q, respectively 44]]. Various quasiparticle models encode the medium effects, viz., effective masses with Polyakov loop [45], Nambu-Jona-Lasinio (NJL) and Polyakov-loop-extended Nambu-Jona-Lasinio-based quasiparticle models [46], self-consistent and single parameter quasiparticle models [47], and recently proposed quasiparticle models based on the Gribov-Zwanziger quantization, leading to a nontrivial IR-improved dispersion relation in terms of the Gribov parameter [48–50]. EQPM encodes the medium effects as EoS dependence of the distribution functions enters through the effective fugacities.

Here, we consider the recent $(2+1)$ flavor lattice QCD EoS (LEoS) [51] and three-loop HTL perturbative (HTLpt) EOS [52,53]. The three-loop HTLpt EOS has recently been computed by N. Haque *et al.*, which is very close to the recent lattice results [54,55]. These EoS have been carefully embedded in z_q and z_q for both isotropic and to anisotropic hot QCD medium [56,57]. z_q and z_q have complicated temperature dependence as discussed in Ref. [58].

We have extended the EQPM in the presence of magnetic field ($\vec{B} = B\hat{z}$) [38] in which the quasiquark/antiquark distribution function is given as

$$\bar{f}_q^l = \frac{z_q \exp\left(-\beta\sqrt{p_z^2 + m^2 + 2l|q_f eB|}\right)}{1 + z_q \exp\left(-\beta\sqrt{p_z^2 + m^2 + 2l|q_f eB|}\right)}, \quad (1)$$

where $E_p^l = \sqrt{p_z^2 + m^2 + 2l|q_f eB|}$ is the Landau energy eigenvalue and $q_f e$ is the fractional charge of quarks. $l = 0, 1, 2, \ldots$ is the order of the energy levels. Since the dispersion relation of the electrically neutral gluon remains intact in the strong magnetic field background, the quasigluon distribution function remains as

$$\bar{f}_g = \frac{z_g \exp(-\beta|\vec{p}|)}{1 + z_g \exp(-\beta|\vec{p}|)}. \quad (2)$$

We are working in units where $k_B = 1$, $c = 1$, $\hbar = 1$ and hence $\beta = \frac{1}{T}$. The parton distribution functions leads to the dispersion relations

$$\omega_q^l = \sqrt{p_z^2 + m^2 + 2l|q_f eB|} + T^2 \partial_T \ln(z_q), \tag{3}$$

and

$$\omega_g = |\vec{p}| + T^2 \partial_T \ln(z_g). \tag{4}$$

The physical significance of the effective fugacity comes in the second term of dispersion relations Eqs. (3) and (4), which corresponds to the collective excitation of quasipartons. Effects of the magnetic field are entering into the system through the dispersion relations and the Debye screening mass [59].

1. Debye mass and effective coupling in the strong magnetic field with HLL corrections

The EQPM is based on charge renormalization in the hot QCD medium, whereas the effective mass model is motivated from the mass renormalization of QCD [60]. Realization of this charge renormalization could be related to the estimation of Debye mass from semiclassical transport theory. There are several investigations on the screening masses of the QGP as a function of the magnetic field [61–63]. Employing EQPM, we can compute the screening mass as [38,60],

$$m_D^2 = -4\pi\alpha_s \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{d}{d\vec{p}} (2N_c \bar{f}_g + N_f(\bar{f}_q^l + \bar{f}_{\bar{q}}^l)), \tag{5}$$

where $\bar{f}_q^l$ and $\bar{f}_g$ are the quasiparton distribution function as defined in Eqs. (1) and (2), and $\alpha_s(T)$ is the running coupling constant at finite temperature taken from two-loop QCD gauge coupling constants [64]. Including the effects of HLLs in the presence of the strong magnetic field $\vec{B} = B\hat{z}$, m_D for quarks and antiquarks becomes

$$m_D^2 = \frac{4\alpha_s}{T} \frac{|q_f eB|}{\pi} \int_0^\infty \sum_{l=0}^\infty dp_z (2 - \delta_{l0}) \bar{f}_q^l (1 - \bar{f}_q^l), \tag{6}$$

in which the integration phase factor due to dimensional reduction in the strong field [19,65,66] can be represented as

$$\int \frac{d^3p}{(2\pi)^3} \rightarrow \frac{|q_f eB|}{2\pi} \sum_{l=0}^\infty \int \frac{dp_z}{2\pi} (2 - \delta_{0l}). \tag{7}$$

After performing the momentum integral Eq. (5) using Eq. (1) we obtain

$$(m_D^2/\alpha_s) = \frac{24T^2}{\pi} \mathrm{PolyLog}[2, z_g] + \frac{12|q_f eB|}{\pi} \left(\frac{z_q}{1+z_q}\right) + \frac{8}{T} \frac{|q_f eB|}{\pi} \int_0^\infty \sum_{l=1}^\infty dp_z \bar{f}_q^l (1 - \bar{f}_q^l). \tag{8}$$

We have plotted the ratio of Debye mass to running coupling constant ratio at $|eB| = 0.3$ GeV^2 as a function of temperature for different Landau levels in Fig. 1. For the chosen temperature range we are focusing up to $l = 3$ Landau level. The contribution from HLLs beyond $l = 3$ is negligible for the given temperature range. Since the occupation in HLLs is exponentially suppressed by $\exp -(\frac{|eB|}{T})$, the effect of HLLs is significant for higher temperature ranges. For ideal EoS $z_{q,g} = 1$ (ultrarelativistic noninteracting quarks and gluons), the definition of Debye mass can be rewritten as

$$(m_D^2)_{\mathrm{Ideal}} = 4\pi\alpha_s(T) \left[T^2 + \frac{3|q_f eB|}{2\pi^2} + \frac{2}{T} \frac{|q_f eB|}{\pi^2} \times \int_0^\infty \sum_{l=1}^\infty dp_z \bar{n}_q^l (1 - \bar{n}_q^l) \right], \tag{9}$$

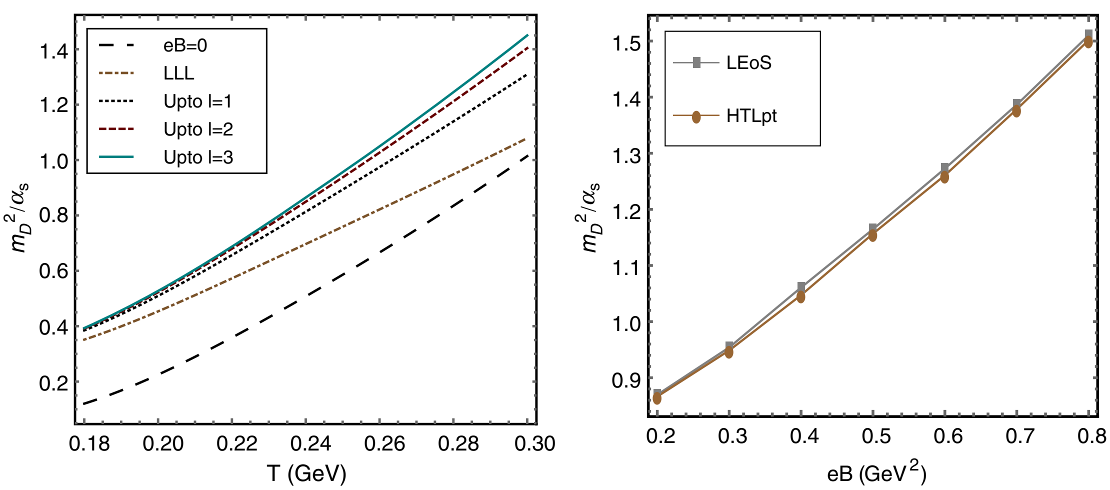

FIG. 1. (Left panel) Temperature behavior of the ratio of Debye mass to coupling constant for different Landau levels at $|eB| = 0.3$ GeV^2. (Right panel) Behavior of m_D^2/α_s at $T = 0.25$ GeV^2 with different magnetic field.

with $\bar{n}_q^l = \frac{1}{\exp(\beta E_p^l)+1}$. From Eqs. (8) and (9), including HLLs we can define the effective running coupling constant $\alpha_{\rm eff}^l(T, z_q, z_g, |eB|)$ so that

$$m_D^2 = \frac{\alpha_{\rm eff}^l}{\alpha_s} m_{D\,\rm Ideal}^2. \tag{10}$$

Therefore,

$$m_D^2 = 4\pi\alpha_{\rm eff}^l(T, z_q, z_g)\left[T^2 + \frac{3|q_f eB|}{2\pi^2} + \frac{2}{T}\frac{|q_f eB|}{\pi^2}\int_0^\infty \sum_{l=1}^{\infty} dp_z \bar{n}_q^l(1-\bar{n}_q^l)\right]. \tag{11}$$

Now, $\alpha_{\rm eff}^l$ can be expressed as

$$\frac{\alpha_{\rm eff}^l}{\alpha_s} = \frac{\frac{6T^2}{\pi^2}\mathrm{PolyLog}[2, z_g] + \frac{3|q_f eB|}{\pi^2}\frac{z_q}{(1+z_q)}}{(T^2 + \frac{3|q_f eB|}{2\pi^2} + h(T, |eB|))} + \frac{\frac{2}{T}\frac{|q_f eB|}{\pi^2}\int_0^\infty \sum_{l=1}^\infty dp_z \bar{f}_q^l(1-\bar{f}_q^l)}{(T^2 + \frac{3|q_f eB|}{2\pi^2} + h(T, |eB|))}, \tag{12}$$

where $h(T, |eB|) = \frac{2}{T}\frac{|q_f eB|}{\pi^2}\int_0^\infty \sum_{l=1}^\infty dp_z \bar{n}_q^l(1-\bar{n}_q^l)$.

For LLL quarks Eq. (12) reduced to

$$\frac{\alpha_{\rm eff}^0}{\alpha_s} = \frac{\left(\frac{6T^2}{\pi^2}\mathrm{PolyLog}[2, z_g] + \frac{3|q_f eB|}{\pi^2}\frac{z_q}{(1+z_q)}\right)}{(T^2 + \frac{3|q_f eB|}{2\pi^2})}. \tag{13}$$

The temperature behavior of $\frac{\alpha_{\rm eff}^l}{\alpha_s}$ with HLL corrections is depicted in Fig. 2. As expected, asymptotically the ratio approaches unity. Dominant contribution of $\alpha_{\rm eff}^l$ comes from the LLL, where the HLLs give the higher order corrections.

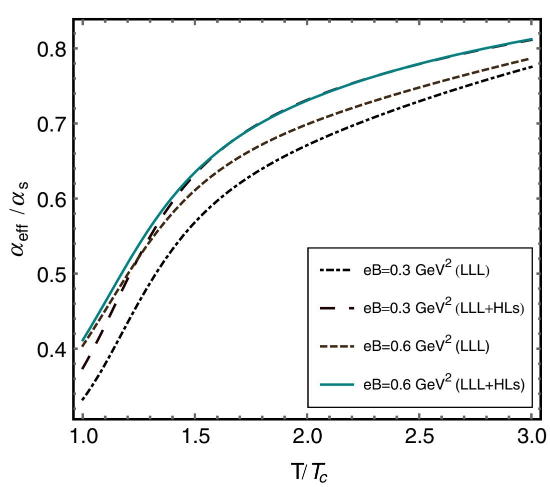

FIG. 2. The effective coupling constant in the strong magnetic field with HLL corrections.

More interestingly, including HLLs $\frac{\alpha_{\rm eff}^l}{\alpha_s}$ are almost identical for $|eB| = 0.3$ GeV^2 and $|eB| = 0.6$ GeV^2, which implies the weaker dependence of the strength of magnetic field on $\frac{\alpha_{\rm eff}^l}{\alpha_s}$. The ratio is showing a small but quantitative change with the HLL corrections. Hence, these corrections are significant in the estimation of the bulk viscosity in the strong field.

B. Thermal relaxation in strong magnetic field

The microscopic interactions, which are the dynamical inputs of the bulk viscosity, are incorporated through the thermal relaxation time ($\tau_{\rm eff}$). The focus of this work is on the dominant $1 \to 2$ processes (gluon to quark-antiquark pair). The relaxation time, $\tau_{\rm eff}$, can be defined from the relativistic transport equation of quasiparton distribution functions for the process $k \to p + p'$ in strong magnetic field $\vec{B} = B\hat{z}$ as

$$\frac{df_q^l}{dt} = C(f_q^l) = -\frac{\delta f_q^l}{\tau_{\rm eff}}. \tag{14}$$

The quantity δf_q^l is the nonequilibrium part of the distribution function of quasiquark/antiquark,

$$f_q^l(p_z) = \bar{f}_q^l + \delta f_q^l, \tag{15}$$

and given by

$$\delta f_q^l = \beta \bar{f}_q^l(p_z)(1-\bar{f}_q^l(p_z))\chi_q(p_z), \tag{16}$$

where $\chi(p_z)$ is the response function (primed notation for antiquark). Here, $C(f_q^l)$ is the collision integral that quantifies the rate of change of the distribution function. In strong magnetic field background, the collision integral for $1 \to 2$ processes has the following form [38],

$$C(f_q^l) = \alpha_{\rm eff}^l C_2 m^2 \int_{-\infty}^{\infty} \frac{dp_z'}{\omega_p^l \omega_{p'}^l}\beta \bar{f}_q^l(E_{p'}^l)\bar{f}_q^l(E_p^l) \times (1 + \bar{f}_g(E_p^l + E_{p'}^l)(\chi_q(p_z') - \chi_q(p_z)), \tag{17}$$

where C_2 is the Casimir factor and $\alpha_{\rm eff}^l$ is the effective coupling constant that encoded the EoS dependence. ω_p^l is the single quark energy as defined in Eq. (3). The response χ for quark and antiquark in the strong magnetic field has opposite sign (since their charges are opposite). This implies that $\chi_q(p_z')$ is an odd function as described in [23] within LLL approximation. Since the Landau levels enter as $E^l = \sqrt{p_z^2 + m^2 + 2l|eB|}$ in the dispersion relations and distribution functions, the odd nature of $\chi(p_z')$ is completely independent on the order of LL. Hence we have

$$C(f_q^l) = -\chi_q(p_z)\alpha_{\rm eff}^l C_2 m^2 \beta \times \int_{-\infty}^{\infty} \frac{dp'_z}{\omega_p^l \omega_{p'}^l} \bar{f}_q^l(E_{p'}^l)\bar{f}_q^l(E_p^l)(1+\bar{f}_g(E_p^l+E_{p'}^l)). \tag{18}$$

Thermal relaxation time $\tau_{\rm eff}$, which is the inverse of the quark damping rate $\Gamma_{\rm eff}$, can be obtained from Eqs. (14), (16), and (18) as

$$\tau_{\rm eff}^{-1} \equiv \Gamma_{\rm eff} = \frac{\alpha_{\rm eff}^l C_2 m^2}{\omega_p(1-\bar{f}_q^l)} \int \frac{dp'_z}{\omega_{p'}^l} \bar{f}_q^l(E_{p'}^l)(1+\bar{f}_g(E_p^l+E_{p'}^l)). \tag{19}$$

Being motivated by the recent work of Refs. [17,23], we constrained our calculation in the regime in which the dominant contribution comes from the quarks of the momentum of order T. Hence, the energy of quarks $E_q \sim T$ and this makes the gluon energy $E_q + E_{\bar{q}} \sim T$, where $E_{\bar{q}}$ is the quark energy. Hence, we have $p'_z \ll T$ or $\frac{p'_z}{T} \sim 0$ [17,23]. Solving the integral in Eq. (19) within these assumptions gives the logarithmic factor. Finally, we obtain the momentum dependent thermal relaxation time $\tau_{\rm eff}(p_z, z_{q/g}, |eB|)$ from the extended EQPM as

$$\tau_{\rm eff}^{-1} = \frac{2\alpha_{\rm eff}^l C_2 m^2}{\omega_p^l(1-\bar{f}_q^l)} \frac{z_q}{(z_q+1)} (1+\bar{f}_g(E_p^l)) \ln(T/m). \tag{20}$$

The impact of the hot QCD medium effects on the relaxation time can be explored by comparing it with the case where the hot QCD/QGP is described as the free ultrarelativistic gas of quarks and gluons, as done in [23]. This could be described by choosing $z_{g/q} = 1$, and in that case, the relaxation time reduces to

$$\tau_{\rm ideal}^{-1} = \frac{\alpha_s C_2 m^2}{E_p^l(1-\bar{n}_q^l)} (1+\bar{n}_g(E_p^l)) \ln(T/m), \tag{21}$$

with $\bar{n}_q^l = \frac{1}{(e^{\beta E_p^l}+1)}$ and $\bar{n}_g = \frac{1}{(e^{\beta E_p^l}-1)}$ for ideal fermions and bosons, respectively.

Since the dominant charge carriers have momenta in the order of T, we are employing $\langle p_z \rangle = T$ for the comparison of $\tau_{\rm eff}$ with $\tau_{\rm ideal}$ to investigate the EoS dependence. Note that the momentum dependence of the relaxation time is significant in the estimation of bulk viscosity. Therefore, while computing the bulk viscosity, the momentum dependent thermal relaxation time as defined in Eq. (20) is employed. Here, we plotted the temperature variation of $\frac{\tau_{\rm eff}^{-1}}{\tau_{\rm ideal}^{-1}}$ with $\langle p_z \rangle = T$ for the ground state quarks ($l = 0$) at $|eB| = 0.3\ {\rm GeV}^2$ and $|eB| = 0.9\ {\rm GeV}^2$ in Fig. 3. Hot medium effects are identical for the system under consideration irrespective of the magnitude of the magnetic field. EoS effects in relaxation time are embedded in Eq. (19) through the quasiparton distribution function and the effective coupling defined in Eq. (12). Since $\alpha_{\rm eff}^l$ is lower than α_s at the lower temperature, the $\tau_{\rm eff}^{-1}$ to $\tau_{\rm ideal}^{-1}$ ratio gives lower value in that temperature range.

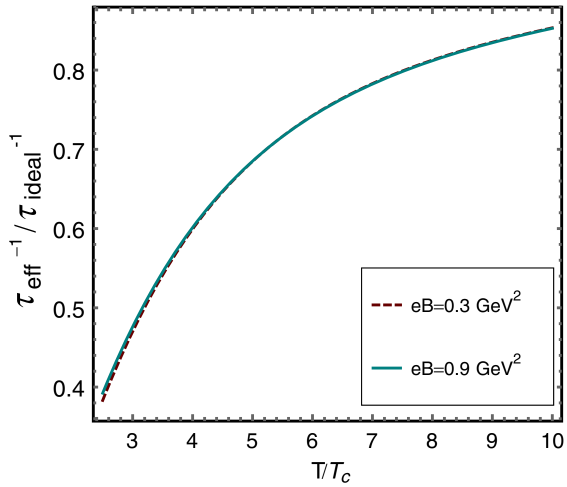

FIG. 3. Dependence of EoS on the thermal relaxation time in the strong magnetic field for the LLL quarks with $\langle p_z \rangle = T$.

HLL corrections are entering through Landau dispersion relation in the quark distribution function. The effect of higher levels in the effective coupling is understood from Eq. (12). The effective thermal relaxation time controls the behavior of bulk viscosity critically.

C. Bulk viscosity from the relaxation-time approximation

We investigated the bulk viscosity of perturbative QCD in the strong magnetic field $\vec{B} = B\hat{z}$ by adopting the EQPM for the dominant $1 \to 2$ processes. Dynamics of the system is described by the Boltzmann equation for the quasiquark distribution function,

$$(\partial_t + v_z \partial_z) f_q^l(p_z, t, z) = C(f_q^l) = -\frac{\delta f_q^l}{\tau_{\rm eff}}, \tag{22}$$

where $C(f_q^l)$ is the collision integral Eq. (17) and the longitudinal velocity $v_z \equiv \frac{\partial \omega_p^l}{\partial p_z} = \frac{p_z}{E_p}$. The equilibrium distribution function is defined as

$$\bar{f}_q^l = \frac{1}{(z_q^{-1}\exp(-\beta(E_p^l - p_z v_z)) + 1)}, \tag{23}$$

in the presence of the flow u_z. For $u_z = 0$, $\bar{f}_q^l$ reduces to Eq. (1). We consider the linear response regime of the Boltzmann equation in which u_z and δf_q^l are assumed to be small, with the appropriate collision integral to solve δf_q^l. The system in equilibrium is disturbed by an expansion in the direction of magnetic field, which gives the change in pressure (δP_L). Bulk viscosity is defined as [24]

$$\delta P_L = -3\zeta_{\rm eff}\Theta, \tag{24}$$

with $\Theta(z) \equiv \partial_z u_z$, which defines the magnitude of expansion. We investigated the QCD thermodynamic quantities such as pressure, energy density, entropy density, and the speed of sound in the strong magnetic field using the extended EQPM [38]. With LLL approximation, longitudinal pressure (in the direction of $\vec{B}$) is obtained from the fundamental thermodynamic definition,

$$P_L = \sum_f \frac{|eq_fB|}{2\pi}\frac{1}{2\pi}2N_c \times \int_{-\infty}^{\infty} dp_z \ln\left(1+z_q\exp\left(-\beta\sqrt{p_z^2+m^2}\right)\right). \tag{25}$$

Longitudinal pressure ends up as

$$P_L = \sum_f \frac{|eq_fB|}{\pi^2}N_c\int_0^\infty dp_z \frac{p_z^2}{E_p^0}\bar{f}_q^0, \tag{26}$$

where $\bar{f}_q^0$ is the momentum distribution of lowest Landau quarks ($l=0$). Similarly, the energy density of the quarks is defined as

$$\varepsilon_L = \sum_f \frac{|eq_fB|}{\pi^2}N_c\int_0^\infty dp_z \frac{(\omega_p^0)^2}{\omega_p^0}\bar{f}_q^0, \tag{27}$$

in which ω_p^0 is the single particle energy for LLL quarks. The integral can be expressed in terms of *PolyLog* functions. Change in longitudinal pressure leads to the bulk viscosity in the direction of magnetic field as given in Eq. (24) and hence

$$\zeta_{\rm eff} = \sum_f -\frac{1}{3\Theta}\frac{|eq_fB|}{\pi^2}N_c\int_0^\infty dp_z\frac{p_z^2}{E_p^0}\delta f_q^0. \tag{28}$$

However, even when $\delta f_q^l = 0$ there is a change in pressure since the temperature ($\beta \equiv \beta(t)$) decreases in time due to the expansion. This can be directly related to the Landau-Lifshitz condition for the stress-energy tensor in the calculation of the bulk viscosity without magnetic field [67]. We subtract this effect as in Refs. [24,68], and we have

$$\delta P \to \delta\bar{P}_L \equiv \delta(P_L - \Omega\varepsilon_L), \tag{29}$$

with $\Omega \equiv \frac{\partial P_L}{\partial \varepsilon_L} = \frac{\partial P_L/\partial T}{\partial \varepsilon_L/\partial T}$. To solve this, we have used the EQPM definition of pressure and energy density in strong magnetic field as in Eqs. (26) and (27),

$$\Omega = \left\{-|eB|\frac{2T}{\pi^2}\bar{\nu}_q PolyLog[2,-z_q] + |eB|\frac{T^2}{\pi^2}\bar{\nu}_q\ln(1+z_q)(\partial_T\ln z_q)\right\}\Big/ \left\{-\frac{4|eB|T}{\pi^2}\bar{\nu}_q PolyLog[2,-z_q] + 5|eB|(T^2\partial_T\ln z_q)\frac{1}{\pi^2}\bar{\nu}_q\ln(1+z_q) + |eB|T^2(\partial_T\ln z_q)^2\frac{T}{\pi^2}\bar{\nu}_q\frac{z_q}{1+z_q} + |eB|T^2(\partial_T^2\ln z_q)\frac{T}{\pi^2}\bar{\nu}_q\ln(1+z_q)\right\}, \tag{30}$$

where $\bar{\nu}_q = \sum_f 2N_c|q_f|$ in the presence of magnetic field. Also, we need to evaluate the change in equilibrium distribution function δf_q^l for the calculation of $\delta\bar{P}_L$. Considering the linear response regime of the Boltzmann equation Eq. (22) with the distribution function as Eq. (23), we have

$$(\partial_t + v_z\partial_z)f_q^0(p_z,t,z) = -[(E_p^0 + T^2\partial_T\ln z_q)\partial_t\beta - \beta v_z p_z\Theta(z)]\bar{f}_q^0(\bar{f}_q^0-1). \tag{31}$$

Here, $z_q(\frac{T}{T_c})$ and $\beta(T)$ are functions of time since temperature changes with expansion. Detailed calculations are shown in Appendix A. In the relaxation-time approximation, we can directly connect the relaxation time $\tau_{\rm eff}$ with the collision integral $C(f_q^l)$ as shown in Eq. (22). Therefore, Eq. (31) becomes

$$\delta f_q^0 = -\tau_{\rm eff}\beta\bar{f}_q^0(\bar{f}_q^0-1)\Theta(z)(\omega_p^0\Omega - v_z p_z), \tag{32}$$

where $\partial_t\beta \equiv \beta\Omega\Theta$ as given in [24,68] and $\tau_{\rm eff}$ is the thermal relaxation time (at $l=0$ in the LLL approximation) for $1\to 2$ processes defined in Eq. (20). Now we can estimate $\zeta_{\rm eff}$ by direct substitution of Eqs. (20) and (32) and (30) to (29) and end up with

$$\zeta_{\rm eff} = \frac{1}{3}\frac{|q_feB|}{\pi^2}\frac{\beta}{m^2}\frac{(z_q+1)}{z_q}\frac{1}{\alpha_{\rm eff}^0 C_2\ln(T/m)} \times \int_0^\infty\frac{dp_z(p_z^2-\Omega\omega_p^0E_p^0)^2\bar{f}_q^0(1-\bar{f}_q^0)^2}{(\bar{f}_g+1)E_p^0}. \tag{33}$$

Here, $\bar{f}_q^0$ is the quark distribution function with $l=0$ level. Bulk viscosity $\zeta_{\rm eff}$ depends on the behavior of the term

$(p_z^2 - \Omega\omega_p^0 E_p^0)^2$ along with the momentum distribution function and the effective coupling constant.

D. Bulk viscosity beyond LLL approximation

The effect of HLLs on the effective coupling α_{eff}^l and thermal relaxation τ_{eff} is defined in Eqs. (12) and (19), respectively. Higher order Landau level corrections to the QCD thermodynamics (pressure, entropy density, etc.) are described in our previous work [38] and utilized in the present work wherever required. The longitudinal pressure and energy density with HLL corrections have the form

$$P_L = \sum_f \frac{|eq_f B|}{\pi^2} N_c \int_0^\infty dp_z (2 - \delta_{0l}) \frac{p_z^2}{E_p^l} \bar{f}_q^l, \quad (34)$$

and

$$\varepsilon_L = \sum_f \frac{|eq_f B|}{\pi^2} N_c \int_0^\infty dp_z (2 - \delta_{0l}) \frac{(\omega_p^l)^2}{\omega_p^l} \bar{f}_q^l, \quad (35)$$

in which $E_p^l = \sqrt{p_z^2 + m^2 + 2l|q_f eB|}$ is the Landau level of order l. The integration phase factor and quasiquark distribution function are defined in Eqs. (7) and (1), respectively. Incorporating these, we can calculate $\bar{\Omega} \equiv \frac{\partial P}{\partial \epsilon}$ with higher order corrections. Finally, the bulk viscosity with higher Landau corrections has the following form:

$$\zeta_{\mathrm{eff}} = \frac{|q_f eB|}{3\pi^2} \sum_{l=0}^\infty (2 - \delta_{0l}) \frac{\beta}{m^2} \frac{1}{\alpha_{\mathrm{eff}}^l C_2 \ln(T/m)} \times \frac{(z_q + 1)}{z_q} \int_0^\infty \frac{dp_z (p_z^2 - \bar{\Omega}\omega_p^l E_p^l)^2 \bar{f}_q^l (1 - \bar{f}_q^l)^2}{(\bar{f}_g + 1) E_p^l}. \quad (36)$$

In transport theory, the viscosity to entropy ratio ζ_{eff}/s has significant importance. The temperature behavior and the effects of HLLs on ζ_{eff}/s are discussed in the next section.

III. RESULTS AND DISCUSSIONS

We initiate our discussions with the hot QCD medium dependence on the thermal relaxation time τ_{eff} and the effective coupling α_{eff}^l. The medium dependence on α_{eff}^l and τ_{eff} is explicitly shown in Figs. 2 and 3, respectively. Thermal relaxation time defined in Eq. (20) encoded the microscopic interactions of the system, which are the dynamical inputs for the estimation of bulk viscosity. The hot QCD medium effects embedded through EoS dependence on the bulk viscosity of $1 \to 2$ processes can be inferred from Eq. (33). The EoS dependence is entering through the quasiparton momentum distribution functions along with the effective coupling. We plotted the variation of $\zeta_{\mathrm{eff}}/\zeta_{\mathrm{ideal}}$ with T/T_c for $|eB| = 0.3\ \mathrm{GeV}^2$ and $|eB| = 0.9\ \mathrm{GeV}^2$ in Fig. 4. We can see that medium effects are

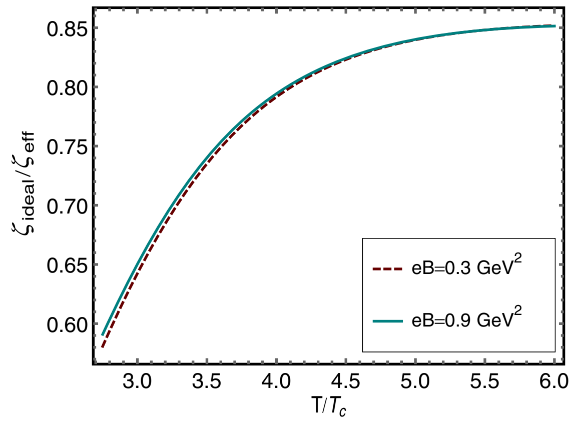

FIG. 4. Dependence of EoS on the bulk viscosity in the strong magnetic field with LLL approximation.

weakly depending on the magnitude of magnetic field. ζ_{ideal}, bulk viscosity without the medium effects, is shown in Ref. [24]. Asymptotically, the ratio approaches unity. Hence, the estimation of bulk viscosity with quasiparticle modeling agrees with the order of magnitude of the results in Ref. [24] at high temperature.

Next, we present the temperature behavior of bulk viscosity to entropy ratio for the $1 \to 2$ process in strong magnetic field. Explicit dependence of temperature on ζ_{eff}/s is shown in Eqs. (33) and (36). Equation (20) shows that the coupling constant α entering through the relaxation time (and hence bulk viscosity) of $1 \to 2$ processes as $1/\alpha$ whereas for $2 \to 2$ processes as $1/\alpha^2$. In Fig. 5, we have depicted ζ_{eff}/s in the presence of the magnetic field as a function of T/T_c for both the EoS in LLL approximation. The behavior of bulk viscosity depends on the Ω. The temperature behavior of $(\epsilon - \frac{P}{\Omega})/T^4$ is shown in Fig. 5. This term is significantly important in Eq. (33) of ζ_{eff}/s. The higher value of ζ_{eff}/s near the transition temperature T_c is due to the term $(\epsilon - \frac{P}{\Omega})/T^4$. At very high temperature ζ_{eff}/s approaches 0.

We compared the bulk viscosity to entropy ratio of $1 \to 2$ processes with the results from sum rule analysis [69] and lattice data results [70] as in Fig. 6. In [69], the universal properties of bulk viscosity in the absence of magnetic field are studied from the sum rule analysis. We observe that the magnetic field enhances the ζ/s. HLL corrections are significant for the higher temperature ranges. We plotted the HLL corrections to the bulk viscosity in the strong magnetic field background in the chosen temperature range in Fig. 7. Corrections up to $l = 3$ Landau level are shown in the figure. Higher order corrections beyond third Landau level seem to be negligible in the chosen temperature range. Since the HLL thermal occupation depends on $\exp(-\sqrt{eB}/T)$, higher order corrections are significant at very high temperature. The dominant contributions of the higher order corrections are entering through the effective coupling and the

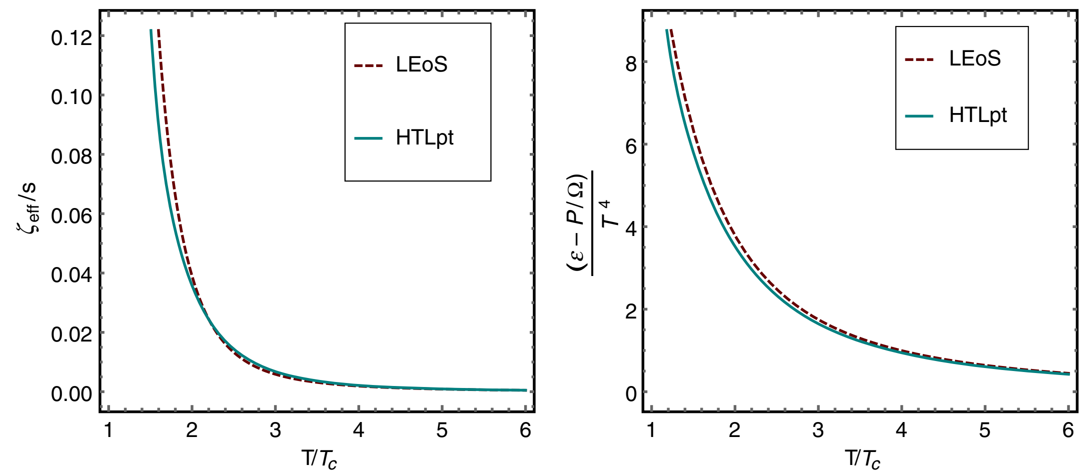

FIG. 5. Temperature behavior of the ratio of bulk viscosity to entropy (left panel) and $\frac{(\varepsilon - P/\Omega)}{T^4}$ (right panel) at $|eB| = 0.3\ \mathrm{GeV}^2$ with LLL approximation.

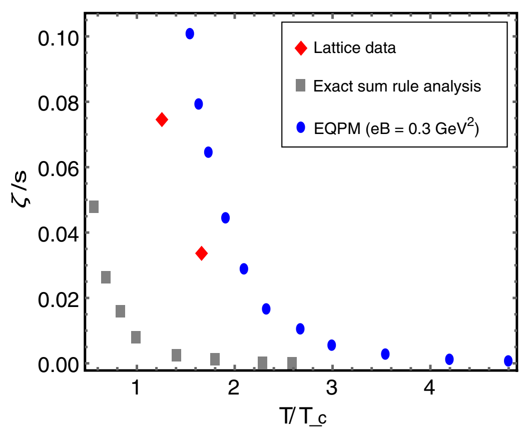

FIG. 6. Comparison of the temperature behavior of ζ/s for the $1 \to 2$ processes at $|eB| = 0.3\ \mathrm{GeV}^2$ with Lattice data [70,71] and sum rule analysis [69] in the absence of magnetic field.

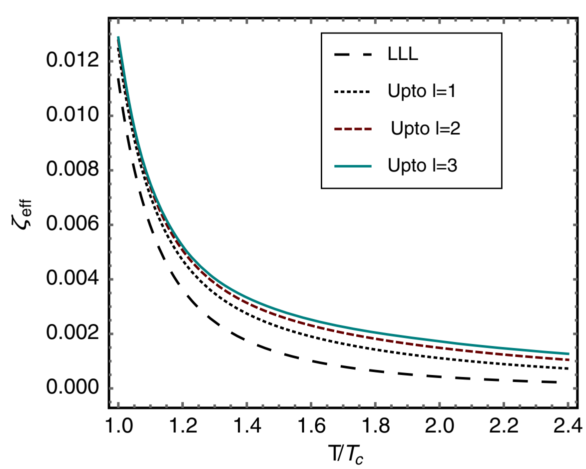

FIG. 7. Effects of HLLs on the bulk viscosity in the strong magnetic field $|eB| = 0.3\ \mathrm{GeV}^2$ in the given temperature range.

momentum distribution function. Evaluation of the higher order corrections to the matrix element of the processes is beyond the scope of this work.

IV. CONCLUSION AND OUTLOOK

In conclusion, the bulk viscosity of the hot magnetized QCD medium gets significant contributions from both the magnetic field and the EoS. The most significant contributions in the strong magnetic field limit to the bulk viscosity come from the $1 \to 2$ processes in the medium (as these are not possible in the absence of the field). The bulk viscosity has been computed from the semiclassical transport theory approach within the relaxation-time approximation. The thermal relaxation time for the quarks is obtained from their respective damping rates in the medium considering the same process. The effects of magnetic fields are encoded in the effective quark/antiquark momentum distribution functions in the form of the Landau levels and also in their energy dispersion relations. On the other hand, the gluon dynamics is affected through the effective coupling that has been obtained in our analysis, again following the transport theory approach.

The hot QCD medium effects in the thermal relaxation time of the quarks are found to be negligible at very high temperature. Furthermore, the leading order term in the bulk viscosity of hot perturbative QCD in strong field limit has been estimated from the EQPM using the relaxation-time approximation and compared against the estimations with and without the magnetic field in other approaches. The results in the present work turned out to be consistent with other recent works. All the analysis is done in LLL approximation first, and then the effects from the HLLs have been included. The HLL corrections of the bulk viscosity are found to be quite significant at the higher temperatures.

We intend to calculate other transport coefficients such as shear viscosity and charge diffusion coefficient in the strong magnetic field background with the EQPM in the near future. Looking at the nonlinear aspects of the electromagnetic response of the QGP would be another direction for work.

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APPENDIX: BOLTZMANN EQUATION IN THE LINEAR RESPONSE REGIME

We need to solve the Boltzmann equation with appropriate collision integral for $1 \to 2$ processes. We have

$$(\partial_t + v_z\partial_z) f_q^l(p_z, t, z) = -\frac{\delta f_q^l}{\tau_{\rm eff}}, \tag{A1}$$

where $\tau_{\rm eff}$ is the thermal relaxation time for $1 \to 2$ process. We consider u_z and δf_q^l to be small since the prime focus is on the linear response regime. Using extended EQPM quasiquark momentum distribution defined in Eq. (1), Eq. (A1) becomes

$$\delta f_q^0 = -\tau_{\rm eff}\bar{f}_q^0(\bar{f}_q^0 - 1) \times [E_p^0\partial_t\beta + z_q\partial_t z_q^{-1} - \beta v_z p_z \Theta(z)], \tag{A2}$$

with $\Theta(z) \equiv (\partial_z u_z)$. Since temperature is time dependent, Eq. (A2) becomes

$$\delta f_q^0 = -\tau_{\rm eff}\bar{f}_q^0(\bar{f}_q^0 - 1) \times [(E_p^0 - \partial_\beta \ln z_q)(\partial_t\beta) - \beta v_z p_z \Theta(z)]. \tag{A3}$$

Finally, we have used $\partial_t\beta = \beta\Omega\Theta(z)$ as defined in Ref. [24]. Thus we end up with

$$\delta f_q^0 = -\tau_{\rm eff}\beta\bar{f}_q^0(\bar{f}_q^0 - 1)\Theta(z) \times [(E_p^0 + T^2\partial_T \ln z_q)\Omega - v_z p_z], \tag{A4}$$

where $(E_p^0 + T^2\partial_T \ln z_q) \equiv \omega_p^0$ is the single particle energy in EQPM.

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