New light mediators for the R_K and R_{K^*} puzzles

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The measurements of R_K and R_{K^*} provide hints for the violation of lepton universality. However, it is generally difficult to explain the R_{K^*} measurement in the low q^2 range, $0.045 \le q^2 \le 1.1\ \mathrm{GeV}^2$. Light mediators offer a solution by making the Wilson coefficients q^2 dependent. We check if new lepton nonuniversal interactions mediated by a scalar (S) or vector particle (Z') of mass between 10–200 MeV can reproduce the data. We find that a 25 MeV Z' with a q^2-dependent $b-s$ coupling and that couples to the electron but not the muon can explain all three anomalies in conjunction with other measurements. A similar 25 MeV S provides a good fit to all relevant data except R_{K^*} in the low q^2 bin. A 25 MeV Z' with a q^2-dependent $b-s$ coupling and that couples to the muon but not the electron provides a good fit to the combination of the R_K and R_{K^*} data, but does not fit R_{K^*} in the low q^2 bin well.

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I. INTRODUCTION

The search for new physics in B decays is an ongoing endeavor. Recently, anomalies in semileptonic B decays have received a lot of attention. These anomalies are found in the charged current $b \to c\tau^-\bar{\nu}_\tau$ and neutral current $b \to s\ell^+\ell^-$ transitions. Here we focus on the neutral current anomalies though the anomalies might be related [1]. Other anomalies appear in $B \to K^*\mu^+\mu^-$, where the LHCb [2,3] and Belle [4] Collaborations find deviations from the standard model (SM) predictions, particularly in the angular observable P_5' [5]. The ATLAS [6] and CMS [7] Collaborations have also made measurements of the $B \to K^*\mu^+\mu^-$ angular distribution with results consistent with LHCb. Further, the LHCb has made measurements of the branching ratios and angular distributions in $B_s^0 \to \phi\mu^+\mu^-$ [8,9] which are at variance with SM predictions based on lattice QCD [10,11] and QCD sum rules [12].

The measurements discussed above are subject to unknown hadronic uncertainties [13] making it necessary to construct clean observables to test for new physics (NP). One such observable is $R_K \equiv \mathcal{B}(B^+ \to K^+\mu^+\mu^-)/\mathcal{B}(B^+ \to K^+e^+e^-)$ [14,15], which has been measured by LHCb [16]:

$$R_K^{\mathrm{expt}} = 0.745^{+0.090}_{-0.074}(\mathrm{stat}) \pm 0.036(\mathrm{syst}), \quad 1 \le q^2 \le 6.0\ \mathrm{GeV}^2. \tag{1}$$

This differs from the SM prediction, $R_K^{\mathrm{SM}} = 1 \pm 0.01$ [17] by 2.6σ. Note, the observable R_K is a measure of lepton flavor universality and requires different new physics for the muons versus the electrons, while it is possible to explain the anomalies in the angular observables in $b \to s\mu^+\mu^-$ in terms of lepton flavor universal new physics [18].

Recently, the LHCb Collaboration reported the measurement of the ratio $R_{K^*} \equiv \mathcal{B}(B^0 \to K^{*0}\mu^+\mu^-)/\mathcal{B}(B^0 \to K^{*0}e^+e^-)$ in two different ranges of the dilepton invariant mass-squared q^2 [19]:

$$R_{K^*}^{\rm expt} = \begin{cases} 0.660^{+0.110}_{-0.070}({\rm stat}) \pm 0.024({\rm syst}), & 0.045 \le q^2 \le 1.1\ {\rm GeV}^2, \quad ({\rm low}\ q^2) \\ 0.685^{+0.113}_{-0.069}({\rm stat}) \pm 0.047({\rm syst}), & 1.1 \le q^2 \le 6.0\ {\rm GeV}^2, \quad ({\rm central}\ q^2). \end{cases} \tag{2}$$

These differ from the SM predictions by 2.2-2.4σ (low q^2) and 2.4-2.5σ (central q^2), which further strengthens the hint of lepton nonuniversality observed in R_K.

Lepton universality violating new physics may occur in $b \to s\mu^+\mu^-$ and/or $b \to se^+e^-$ transitions. The fact that the measurement of $\mathcal{B}(B^+ \to K^+e^+e^-)$ is found to be consistent with the prediction of the SM may lead one to conclude that NP is more likely to be in $b \to s\mu^+\mu^-$. However, the branching ratios suffer from hadronic uncertainties [20] unlike the ratios R_K and R_{K^*} and so new physics in $b \to s\mu^+\mu^-$ and/or in $b \to se^+e^-$ is still allowed.

Since the announcement of the R_{K^*} result, a number of papers have analyzed the new measurements, mostly in terms of new physics with heavy mediators [21–35]. The general conclusion is that there is a significant disagreement with the SM, possibly as large as $\sim 6\sigma$, and that theoretical hadronic uncertainties [36–38] are insufficient to understand the data. However, with heavy new physics it is difficult to understand the R_{K^*} measurement in the very low q^2 bin $0.045 \le q^2 \le 1.1$ GeV^2, although the predictions are consistent with measurements within 1.5σ. A resolution to this problem may be possible if the new physics is light.

In models with light mediators [30–32,39,40], the new physics cannot be integrated out, resulting in a q^2 dependence of the Wilson coefficients (WCs). If the light mediator mass is between m_B and twice the lepton mass, and the mediator width is narrow, then it is observable as a resonance in the dilepton invariant mass. To avoid constraints from the search for such states, one generally takes the mediator mass to be m_B or less than $2m_\ell$. In this paper, we study a light scalar mediator denoted by S and a light vector mediator denoted by Z'.

II. LIGHT SCALAR

We start our discussion with a light scalar S with mass in the 10–200 MeV range. For this scenario, we assume the following flavor-changing bsS vertex,

$$F(q^2)\bar{s}[g_{bs}^S P_L + g_{bs}^{S'} P_R]bS, \tag{3}$$

where $F(q^2)$ is a form factor.[1] The matrix elements for the processes $b \to s\ell^+\ell^-$ and the mass difference in B_s mixing are

$$\begin{aligned} M_{b\to s\ell^+\ell^-} &= \frac{F(q^2)}{q^2 - M_S^2}[\bar{s}(g_{bs}^S P_L + g_{bs}^{S'} P_R)b] \\ &\quad \times [\bar{\ell}(g_L^{\ell\ell} P_L + g_R^{\ell\ell} P_R)\ell], \\ \Delta M_{B_s}^{NP} &= \frac{(F(q^2))^2}{2q^2 - 2M_S^2} f_B^2 m_{B_s} \\ &\quad \times \left[-\frac{5}{12}((g_{bs}^S)^2 + (g_{bs}^{S'})^2) + 2g_{bs}^S g_{bs}^{S'} \frac{7}{12}\right], \end{aligned} \tag{5}$$

[1]In our effective theory approach, the structure in Eq. (3) is of the general form consistent with the assumed symmetries. As an illustration of how a flavor changing vertex with a q^2-dependent form factor may occur, consider the following Lagrangian at the b-quark mass scale in the gauge basis,

$$\mathcal{L} = \frac{g}{\Lambda^2}\bar{b}b\bar{\chi}\chi + g_\chi \bar{\chi}\chi S, \tag{4}$$

where χ is a hidden sector fermion (which may serve as a dark matter candidate) of mass $m_\chi \lesssim m_b$, and we have suppressed all Lorentz structures in the Lagrangian. (In the context of Sec. III, for a light vector mediator Z', one may consider a similar Lagrangian of the form, $g_\chi \bar{\chi}\gamma^\mu \chi Z'_\mu$.) The first term in the Lagrangian represents an effective coupling between the b and χ fields that might arise via the exchange of a heavy mediator of mass $\Lambda \gg m_b$, which has been integrated out of the theory at the m_b scale. Although there is no direct coupling between b and S (or Z'), a $\bar{b}bS$ (or $\bar{b}bZ'$) vertex with a q^2-dependent coupling will be generated by a χ loop. Transforming the b quark from the gauge to the mass basis then generates a $\bar{s}bS$ (or $\bar{s}bZ'$) coupling. As an estimate of the form factor we evaluate the one-loop diagram [18] and argue that for $q^2 \ll m_B^2$, the form factor $\sim q^2$. In the case of the scalar mediator, after calculating the one loop contribution, the form factor contains terms of the form, $\frac{m_\chi^2}{\Lambda^2}$ and $\frac{q^2}{\Lambda^2}$, multiplied by $A(q^2) \sim \int_0^1 dx\, x(1-x) \ln \frac{x\Lambda_c^2}{m_\chi^2 - q^2 x(1-x)}$, where Λ_c is a cut-off of the Feynman integral. For the $\frac{q^2}{\Lambda^2}$ term to dominate, $q^2 \gg m_\chi^2$, which implies that $m_\chi \lesssim 30$ MeV for the q^2 values of interest. Then, since the q^2-dependence of $A(q^2)$ is logarithmic, the form factor $\sim q^2$. For the Z' case, the leading term in the form factor goes as q^2 due to the conserved vector current [41]. Again, in the one-loop approximation the form factor $\sim q^2 A(q^2)$ and if we take $m_\chi \sim m_B$, then for $q^2 \ll m_B^2$ the form factor goes as $\sim q^2 \ln \frac{\Lambda_c^2}{m_\chi^2}$. We note that the situation is similar to the SM case where χ is replaced by the charm quark and S (or Z') by the photon. In this case the first term in the Lagrangian, of the form $\frac{g}{M_W^2}\bar{b}s\bar{c}c$, is just one of the terms in the SM effective Lagrangian after integrating out the W boson. The charm loop then induces an effective $\bar{b}s\gamma^*$ vertex which yields $\bar{b}s\ell^+\ell^-$ via $\gamma^* \to \ell^+\ell^-$.

TABLE I. The fit results and the predictions for R_K and R_{K^*} at the best fit point for three scenarios of a light mediator with a mass of 25 MeV.

Case			$R_{K^*[0.045–1.1]}$	$R_{K^*[1.1–6.0]}$	$R_{K[1.0–6.0]}$	pull
	Experimental results		0.66±0.09	0.69±0.10	0.75±0.09	
	Standard model predictions		0.93	0.99	1.0	
(i) Light scalar with electron coupling						
$F(q^2)\equiv 1,\ g^S_{ee}=2.0\times10^{-4}$	$g^S_{bs}g^S_{ee}=(12.6\pm2.2)\times10^{-9}$	$g^{S'}_{bs}g^S_{ee}=(4.0\pm1.6)\times10^{-9}$	0.70	0.91	0.69	4.3
$a_{bs}\neq0$	$g^S_{bs}g^S_{ee}=(-1.3\pm2.1)\times10^{-9}$	$g^{S'}_{bs}g^S_{ee}=(-13.1\pm2.1)\times10^{-9}$	0.58	0.85	0.75	4.7
$a_{bs}=0$	$g^S_{bs}g^S_{ee}=(2.7\pm2.6)\times10^{-8}$	$g^{S'}_{bs}g^S_{ee}=(-15.5\pm2.6)\times10^{-8}$	0.89	0.65	0.75	4.4
(ii) Light vector with muon coupling						
$F(q^2)\equiv1,\ g^{\mu\mu}_L=g^{\mu\mu}_R=8.0\times10^{-4}$	$g_{bs}g_{\mu\mu}=(2.3\pm2.0)\times10^{-10}$	$g'_{bs}g_{\mu\mu}=(1.3\pm2.2)\times10^{-10}$	0.93	0.99	0.96	1.4
$a_{bs}\neq0,\ g^{\mu\mu}_L=g^{\mu\mu}_R$	$g_{bs}g_{\mu\mu}=(6.5\pm3.5)\times10^{-10}$	$g'_{bs}g_{\mu\mu}=(1.6\pm3.6)\times10^{-10}$	0.93	0.96	0.92	2.4
$a_{bs}\neq0,\ g'_{bs}=0,\ g^{\mu\mu}_L\neq g^{\mu\mu}_R$	$g_{bs}g_{\mu\mu}=(5.7\pm2.3)\times10^{-10}$	$g_{bs}g'_{\mu\mu}=(0.2\pm0.1)\times10^{-11}$	0.89	0.95	0.93	2.9
$a_{bs}\neq0,\ g_{bs}=0,\ g^{\mu\mu}_L\neq g^{\mu\mu}_R$	$g'_{bs}g_{\mu\mu}=(-3.2\pm2.5)\times10^{-10}$	$g'_{bs}g'_{\mu\mu}=(-0.1\pm0.1)\times10^{-11}$	0.85	0.97	1.05	1.6
$a_{bs}=0,\ g^{\mu\mu}_L=g^{\mu\mu}_R$	$g_{bs}g_{\mu\mu}=(4.4\pm1.4)\times10^{-8}$	$g'_{bs}g_{\mu\mu}=(-1.9\pm1.4)\times10^{-8}$	0.86	0.72	0.76	4.6
$a_{bs}=0,\ g'_{bs}=0,\ g^{\mu\mu}_L\neq g^{\mu\mu}_R$	$g_{bs}g_{\mu\mu}=(3.9\pm1.8)\times10^{-8}$	$g_{bs}g'_{\mu\mu}=(4.4\pm4.2)\times10^{-11}$	0.87	0.80	0.69	4.4
$a_{bs}=0,\ g_{bs}=0,\ g^{\mu\mu}_L\neq g^{\mu\mu}_R$	$g'_{bs}g_{\mu\mu}=(-0.5\pm5.6)\times10^{-9}$	$g'_{bs}g'_{\mu\mu}=(0.0\pm1.5)\times10^{-11}$	0.92	0.99	1.01	0.1
(iii) Light vector with electron coupling						
$F(q^2)\equiv1,\ g^{ee}_L=g^{ee}_R=2.5\times10^{-4}$	$g_{bs}g_{ee}=(-0.6\pm1.0)\times10^{-10}$	$g'_{bs}g_{ee}=(-0.4\pm1.1)\times10^{-10}$	0.93	0.99	0.99	0.7
$a_{bs}\neq0,\ g^{ee}_L=g^{ee}_R$	$g_{bs}g_{ee}=(-1.9\pm0.6)\times10^{-9}$	$g'_{bs}g_{ee}=(-0.8\pm0.5)\times10^{-9}$	0.62	0.92	0.74	4.5
$a_{bs}\neq0,\ g'_{bs}=0,\ g^{ee}_L\neq g^{ee}_R$	$g_{bs}g_{ee}=(-4.4\pm5.9)\times10^{-10}$	$g_{bs}g'_{ee}=(7.5\pm3.3)\times10^{-10}$	0.55	0.86	0.84	4.5
$a_{bs}\neq0,\ g_{bs}=0,\ g^{ee}_L\neq g^{ee}_R$	$g'_{bs}g_{ee}=(3.9\pm4.2)\times10^{-10}$	$g'_{bs}g'_{ee}=(12.4\pm2.6)\times10^{-10}$	0.58	0.98	0.81	4.0
$a_{bs}=0,\ g^{ee}_L=g^{ee}_R$	$g_{bs}g_{ee}=(-3.9\pm1.0)\times10^{-8}$	$g'_{bs}g_{ee}=(1.4\pm1.0)\times10^{-8}$	0.78	0.60	0.75	4.8
$a_{bs}=0,\ g'_{bs}=0,\ g^{ee}_L\neq g^{ee}_R$	$g_{bs}g_{ee}=(-3.2\pm2.3)\times10^{-8}$	$g_{bs}g'_{ee}=(0.4\pm1.4)\times10^{-8}$	0.83	0.70	0.67	4.6
$a_{bs}=0,\ g_{bs}=0,\ g^{ee}_L\neq g^{ee}_R$	$g'_{bs}g_{ee}=(4.6\pm1.5)\times10^{-8}$	$g'_{bs}g'_{ee}=(2.0\pm0.3)\times10^{-8}$	0.80	0.58	0.77	4.7

where we have used Ref. [42] for B^0_s-$\bar{B}^0_s$ mixing. The mass difference in the SM for the B_s system is [43]

$$\Delta M^{\rm SM}_{B_s} = (17.4 \pm 2.6)\ {\rm ps}^{-1}, \tag{6}$$

which is consistent with experimental measurement [44],

$$\Delta M_{B_s} = (17.757 \pm 0.021)\ {\rm ps}^{-1}. \tag{7}$$

We will choose the new physics contribution, $\Delta M^{NP}_{B_s}$, to be as large as the uncertainty in the SM prediction.

We now consider $b \to s\ell^+\ell^-$ transitions. For light scalars coupling to muons, R_K and R_{K^*} are generally increased from their SM values in contradiction with experiment. Moreover, the measured $B_s \to \mu^+\mu^-$ rate also puts strong constraints on new scalar couplings to muons.

We, therefore, suppose the scalar couples mainly to electrons in which case the matrix element for $b \to se^+e^-$ from Eq. (5) is

$$M^{S,S'}_{b\to se^+e^-} = \frac{g^S_{ee}}{q^2 - M^2_S} F(q^2)[g^S_{bs}(\bar{s}P_L b) + g^{S'}_{bs}(\bar{s}P_R b)](\bar{e}e) + \frac{g^{S'}_{ee}}{q^2 - M^2_S} F(q^2)[g^S_{bs}(\bar{s}P_L b) + g^{S'}_{bs}(\bar{s}P_R b)](\bar{e}\gamma_5 e), \tag{8}$$

where $g^S_{ee} \equiv (g^{ee}_L + g^{ee}_R)/2$ and $g^{S'}_{ee} \equiv (g^{ee}_R - g^{ee}_L)/2$. In the following discussion, we chose different structures for the form factor $F(q^2)$.

A. $F(q^2) \equiv 1$

First, we consider the situation in which the bsS vertex is generated either at tree level or at loop level with internal particles with masses much greater than the b quark mass. Then, the form factor $F(q^2) \equiv 1$, and to avoid a pole contribution to the measurements of $B(B^0 \to K^{*0}e^+e^-)$ in the dielectron invariant mass range, $m_{ee} = [30–1000]$ MeV [45], we choose $M_S = 25$ MeV.

Note that the *BABAR* [46] and Belle [47,48] measurements require m_{ee} to be larger than 30 [49] and 140 MeV, respectively. We fix $g^S_{ee} = 2.0 \times 10^{-4}$, which is the largest value allowed by the anomalous magnetic moment of the electron [50] for $M_S = 25$ MeV at the 2σ CL. Then we perform a χ^2-fit to the theoretically clean observables R_K and R_{K^*}, and the new physics contribution to the B_s mass difference, $\Delta M_s^{NP} = 0 \pm 2.6$ ps^{-1}. In Ref. [51], the lepton flavor dependent angular observables $Q_{4,5}$ were measured, but since the errors in the measurements are large we do not use them in our fit. We use `flavio` [52] to calculate the theoretical values of the observables $\mathcal{O}_{th}$. We then compute

$$\chi^2(g_{bs}^S, g_{bs}^{S'}) = \sum_{R_K, R_{K^*}, \Delta M_s^{NP}} (\mathcal{O}_{th}(g_{bs}^S, g_{bs}^{S'}) - \mathcal{O}_{\rm exp})^T \mathcal{C}^{-1} \times (\mathcal{O}_{th}(g_{bs}^S, g_{bs}^{S'}) - \mathcal{O}_{\rm exp}), \quad (9)$$

where $\mathcal{O}_{\rm exp}$ are the experimental measurements of the observables, and the total covariance matrix $\mathcal{C}$ is the sum of theoretical and experimental covariance matrices. The SM gives a very poor fit to the R_K and R_{K^*} measurements with

$$\chi^2_{\rm SM}/{\rm dof} = 25.5/3. \quad (10)$$

The best-fit values of the couplings g_{bs}^S and $g_{bs}^{S'}$ along with predictions at the best-fit point, for $M_S = 25$ MeV and $g_{ee}^S = 2.0 \times 10^{-4}$, are provided in Table I. As a good fit is obtained in this case, we check if these values are consistent with the various measured branching ratios in $b \to s e^+ e^-$ modes. If S can decay to e^+e^- with a branching ratio ~ 1, then the decays $B \to K^{(*)} e^+ e^-$ will be dominated by the two-body decays, $B \to K^{(*)} S$, with S decaying to e^+e^-.

For the two-body $B \to KS$ decay, the branching ratio is

$$\mathcal{B}(B \to KS) = \frac{(g_{bs}^S + g_{bs}^{S'})^2 |\vec{p}_K| (m_B^2 - m_K^2)^2 f_0^2(m_S^2/m_B^2) \tau_B}{32\pi m_b^2 m_B^2}, \quad (11)$$

where the form factor $f_0(z)$ can be found in Ref. [53].

For the two-body $B \to K^* S$ decay, the branching ratio is

$$\mathcal{B}(B \to K^* S) = \frac{(g_{bs}^S - g_{bs}^{S'})^2 |\vec{p}_{K^*}|^3 A_0^2(m_S^2) \tau_B}{8\pi m_b^2}, \quad (12)$$

where τ_B is the lifetime of the B meson, $|\vec{p}_{K^*}| = \lambda^{1/2}(m_B^2, m_{K^*}^2, m_S^2)/2m_B$, and the form factor A_0 is taken from Ref. [54]. To bound the NP coupling constants g_{bs}^S and $g_{bs}^{S'}$, we require the $B \to K^{(*)} S$ branching ratio to be less than 1%. This choice is consistent with uncertainties in the calculation of the B meson width [55]. For M_S between 10–200 MeV, $\mathcal{B}(B^0 \to K^{*0} e^+ e^-)$ and $\mathcal{B}(B^0 \to K^0 e^+ e^-)$ impose the constraints shown in Table II. The best-fit values of the coupling given in Table I are in contradiction with these constraints. Hence, a light scalar with form factor $F(q^2) \equiv 1$ is ruled out.

B. $F(q^2) \neq 1$

Now we consider a q^2-dependent form factor $F(q^2) \neq 1$ which may be loop induced. For momentum transfer $q^2 \ll m_B^2$, $F(q^2)$ can be expanded as [39]

$$F(q^2) = a_{bs} + b_{bs} \frac{q^2}{m_B^2} + \cdots, \quad (13)$$

where m_B is the B-meson mass. We do not include the B_s mass difference and $\mathcal{B}(B_s \to e^+ e^-)$ as constraints since $F(q^2)$ is unknown for $q^2 \sim m_B^2$. We assume that S does not couple to neutrinos so that $B \to K \nu \bar{\nu}$ [56,57] does not constrain a_{bs}. Redefining $a_{bs} g_{bs}^S$ as g_{bs}^S, and $a_{bs} g_{bs}^{S'}$ as $g_{bs}^{S'}$, we perform a χ^2-fit to the theoretically clean observables R_K and R_{K^*}. The best fit values of the couplings and the predictions for R_K and R_{K^*} are shown in Table I. Taking into account the constraints on g_{bs}^S and $g_{bs}^{S'}$ from Table II along with the constraints on g_{ee} from the anomalous magnetic moment of the electron, we see that the best-fit values $\mathcal{O}(10^{-8})$ cannot be achieved in this case.

To avoid the strong constraints from the two-body decays we set $a_{bs} = 0$ in Eq. (13) (thereby also evading the $B \to K \nu \bar{\nu}$ constraint if the mediator couples to neutrinos [39]), and absorbing the factor b_{bs} to redefine g_{bs}^S and $g_{bs}^{S'}$, the matrix element for $b \to s e^+ e^-$ is given by

TABLE II. Constraints from $\mathcal{B}(B^0 \to K^0 e^+ e^-)$ and $\mathcal{B}(B^0 \to K^{*0} e^+ e^-)$. See the text for details.

	$\mathcal{B}(B^0 \to K^0 e^+ e^-)$	$\mathcal{B}(B^0 \to K^{*0} e^+ e^-)$	Combined
S, $a_{bs} \neq 0$	$\lvert g_{bs}^S + g_{bs}^{S'}\rvert \lesssim 9.9 \times 10^{-7}$	$\lvert g_{bs}^S - g_{bs}^{S'}\rvert \lesssim 9.0 \times 10^{-7}$	$\lvert g_{bs}^S\rvert, \lvert g_{bs}^{S'}\rvert \lesssim 9.5 \times 10^{-7}$
S, $a_{bs} = 0$	$\lvert g_{bs}^S + g_{bs}^{S'}\rvert \lesssim 4.4 \times 10^{-2} (\frac{25\ \rm MeV}{M_S})^2$	$\lvert g_{bs}^S - g_{bs}^{S'}\rvert \lesssim 4.0 \times 10^{-2} (\frac{25\ \rm MeV}{M_S})^2$	$\lvert g_{bs}^S\rvert, \lvert g_{bs}^{S'}\rvert \lesssim 4.2 \times 10^{-2} (\frac{25\ \rm MeV}{M_S})^2$
Z', $a_{bs} \neq 0$	$\lvert g_{bs} + g'_{bs}\rvert \lesssim 5.8 \times 10^{-9} (\frac{M_{Z'}}{25\ \rm MeV})$	$\lvert g_{bs} - g'_{bs}\rvert \lesssim 5.4 \times 10^{-9} (\frac{M_{Z'}}{25\ \rm MeV})$	$\lvert g_{bs}\rvert, \lvert g'_{bs}\rvert \lesssim 5.6 \times 10^{-9} (\frac{M_{Z'}}{25\ \rm MeV})$
Z', $a_{bs} = 0$	$\lvert g_{bs} + g'_{bs}\rvert \lesssim 2.6 \times 10^{-4} (\frac{25\ \rm MeV}{M_{Z'}})$	$\lvert g_{bs} - g'_{bs}\rvert \lesssim 2.4 \times 10^{-4} (\frac{25\ \rm MeV}{M_{Z'}})$	$\lvert g_{bs}\rvert, \lvert g'_{bs}\rvert \lesssim 2.5 \times 10^{-4} (\frac{25\ \rm MeV}{M_{Z'}})$

TABLE III. The experimental results for various $b\to se^+e^-$ observables, along with predictions for the SM and four new physics cases that fit the R_K and R_{K^*} data and satisfy the $\mathcal{B}(B\to K^{(*)}e^+e^-)$ constraints. The light mediator mass is 25 MeV, $F(q^2)\neq 1$ and $a_{bs}=0$.

	$R_{K[0.045\text{–}1.0]}$	$\mathcal{B}(B\to Ke^+e^-)_{[1.0\text{–}6.0]}$	$\mathcal{B}(B\to X_s e^+e^-)_{[1.0\text{–}6.0]}$	$B(B^0\to K^{*0}e^+e^-)_{[0.03^2\text{–}1]}$
Experimental results	⋯	$(1.56\pm0.18)\times10^{-7}$ [16]	$(1.93\pm0.55)\times10^{-6}$ [62]	$(3.1\pm0.9)\times10^{-7}$ [45]
Standard model predictions	0.98	1.69×10^{-7}	1.74×10^{-6}	2.6×10^{-7}
Light scalar $g^S_{bs}g^S_{ee}=2.7\times10^{-8}$, $g^{S'}_{bs}g^S_{ee}=-15.5\times10^{-8}$	0.93	2.5×10^{-7}	2.3×10^{-6}	2.6×10^{-7}
Light vector $g_{bs}g_{ee}=-3.9\times10^{-8}$, $g'_{bs}g_{ee}=1.4\times10^{-8}$	0.73	2.4×10^{-7}	2.6×10^{-6}	2.8×10^{-7}
Light vector, $g'_{bs}=0$ $g_{bs}g_{ee}=-3.2\times10^{-8}$, $g_{bs}g'_{ee}=0.4\times10^{-8}$	0.66	2.7×10^{-7}	2.5×10^{-6}	2.7×10^{-7}
Light vector, $g_{bs}=0$ $g'_{bs}g_{ee}=4.6\times10^{-8}$, $g'_{bs}g'_{ee}=2.0\times10^{-8}$	1.04	2.4×10^{-7}	2.5×10^{-6}	2.8×10^{-7}

$$M^{S,S'}_{b\to se^+e^-}=\frac{q^2}{m_B^2}\frac{g^S_{ee}}{q^2-M_S^2}[g^S_{bs}(\bar{s}P_Lb)+g^{S'}_{bs}(\bar{s}P_Rb)](\bar{e}e)+\frac{q^2}{m_B^2}\frac{g^{S'}_{ee}}{q^2-M_S^2}[g^S_{bs}(\bar{s}P_Lb)+g^{S'}_{bs}(\bar{s}P_Rb)](\bar{e}\gamma_5 e). \tag{14}$$

With the form factor q^2/M_B^2, requiring $\mathcal{B}(B^0\to K^{*0}e^+e^-)$ and $\mathcal{B}(B^0\to K^0e^+e^-)$ to be less than 1% gives the constraints on g^S_{bs} and $g^{S'}_{bs}$ in Table II. The best-fit values can be found in Table I. A reasonable fit is obtained in this case with a pull of 4.4. We see that R_K and R_{K^*} values in the central q^2 bin can be reasonably accommodated, while the effect on R_{K^*} in the low q^2 bin is small in this case. We also evaluated the branching ratios for various $b\to se^+e^-$ observables; see Table III. Our prediction for $\mathcal{B}(B\to Ke^+e^-)_{[1.0\text{–}6.0]}$ is somewhat in tension with the experimental result. Allowing for a 10% uncertainty in the theoretical prediction [58], the discrepancy is about 2.3σ. The prediction for the inclusive mode $\mathcal{B}(B\to X_se^+e^-)_{[1.0\text{–}6.0]}$, which suffers from less hadronic uncertainties, is consistent with measurement.

Finally, we consider the case with a pseudoscalar coupling of the electron and find similar results to that of the scalar coupling.

III. LIGHT Z'

A Z' with mass less than $2m_\mu$ was recently proposed in Ref. [39] to simultaneously explain the measurements of R_K and the anomalous magnetic moment of the muon, with implications for nonstandard neutrino interactions. Such a Z' may potentially explain R_{K^*} in the low q^2 bin [31]. A Z' with a mass in the few GeV range was discussed recently [30,32] but the q^2 dependence of the WC is not strong enough to explain the R_{K^*} at low q^2 [32]. Here we focus on an MeV Z'.

We assume the flavor-changing bsZ' vertex to have the form

$$F(q^2)\bar{s}\gamma^\mu[g_{bs}P_L+g'_{bs}P_R]bZ'_\mu. \tag{15}$$

The matrix elements for $b\to s\ell^+\ell^-$ and the mass difference in B_s mixing are

$$\begin{aligned}M_{b\to s\ell^+\ell^-}&=\frac{F(q^2)}{q^2-M_{Z'}^2}[\bar{s}\gamma^\mu(g_{bs}P_L+g'_{bs}P_R)b]\\&\quad\times(\bar{\ell}\gamma^\mu(g_L^{\ell\ell}P_L+g_R^{\ell\ell}P_R)\ell)\\&\quad-\frac{F(q^2)}{q^2-M_{Z'}^2}\frac{m_bm_\ell}{M_{Z'}^2}(g_R^{\ell\ell}-g_L^{\ell\ell})\\&\quad\times[\bar{s}(g_{bs}P_R+g'_{bs}P_L)b](\bar{\ell}\gamma_5\ell),\\ \Delta M_{B_s}^{NP}&=\frac{(F(q^2))^2}{2q^2-2M_{Z'}^2}\frac{2}{3}f_B^2m_{B_s}\\&\quad\times\left[(g_{bs}^2+g'^2_{bs})\left(1-\frac{5}{8}\frac{m_b^2}{M_{Z'}^2}\right)\right.\\&\quad\left.-2g_{bs}g'_{bs}\left(\frac{5}{6}-\frac{m_b^2}{M_{Z'}^2}\frac{7}{12}\right)\right],\end{aligned} \tag{16}$$

where we have used Ref. [42] for B_s^0-$\bar{B}_s^0$ mixing. Also, we define $g_{\ell\ell}\equiv(g_L^{\ell\ell}+g_R^{\ell\ell})/2$ and $g'_{\ell\ell}\equiv(g_R^{\ell\ell}-g_L^{\ell\ell})/2$ for convenience.

A. Z' with muon coupling

We begin with the case where the Z' couples to muons and not to the electrons.

1. $F(q^2)\equiv 1$

We first assume that $F(q^2)\equiv 1$ and consider the case $g_L^{\mu\mu}=g_R^{\mu\mu}=g_{\mu\mu}$, so the leptonic term is a purely vector current. We perform a fit to the R_K and R_{K^*} data, and the new physics contribution to the B_s mass difference. We choose $M_{Z'} = 25$ MeV and fix $g_{\mu\mu} = 8.0 \times 10^{-4}$, which is the 2σ upper bound from the anomalous magnetic moment of the muon. The fit results are shown in Table I. We see that the overall improvement over the SM is insignificant because g^{S}_{bs} and $g^{S'}_{bs}$ are suppressed by B_s mixing.

2. $F(q^2) \neq 1$

Now we consider $F(q^2) \neq 1$ and assume an expansion as in Eq. (13). Keeping only the leading a_{bs} term, we perform a fit to the observables R_K and R_{K^*} for $M_S = 25$ MeV. We do not employ the new physics contribution to the B_s mass difference as a constraint since $F(q^2)$ is unknown for $q^2 \sim m_B^2$. The fit results are shown in Table I. The overall improvement over the SM is poor, with a pull of 2.4. Clearly, a light Z' with pure vector coupling to the muon is unable to explain the $R_{K[1.0\text{–}6.0]}$, $R_{K^*[0.045\text{–}1.1]}$ and $R_{K^*[1.1\text{–}6.0]}$ anomalies simultaneously. However, on removing $R_{K^*[0.045\text{–}1.1]}$ from the fit, one can easily accommodate the measured values of $R_{K[1.0\text{–}6.0]}$ and $R_{K^*[1.1\text{–}6.0]}$, and a pull of around 4.0 is obtained.

We next consider the case with $a_{bs} \neq 0$ and the Z' also has nonzero axial vector coupling with the muons, i.e., $g_L^{\ell\ell} \neq g_R^{\ell\ell}$. To keep the number of new couplings unchanged, we take either $g'_{bs} = 0$ or $g_{bs} = 0$. This case also does not give a good fit to the data; see Table I.

As can be seen from Table I, overall two of the scenarios with $a_{bs} = 0$ provide good fits except to the R_{K^*} measurement in the low q^2 bin. Morevover, a Z' with purely vector muon coupling is easily compatible with other $b \to s\ell^+\ell^-$ observables [32].

B. Z' with electron coupling

We now consider the case where the Z' couples to electrons and not to muons.

1. $F(q^2) \equiv 1$

We first assume that $F(q^2) \equiv 1$ and we start by considering the case $g_L^{ee} = g_R^{ee} = g_{ee}$ so the leptonic term is a purely vector current. We perform a fit to the R_K and R_{K^*} data, and the new physics contribution to the B_s mass difference. We fix $g_{ee} = 2.5 \times 10^{-4}$, which is within the 90% CL upper limit from NA48/2 [59]. The fit results are shown in Table I. The fit to R_K and R_{K^*} is close to the SM predictions because of B_s mixing.

2. $F(q^2) \neq 1$

Now we consider $F(q^2) \neq 1$. We fit to the observables R_K and R_{K^*} only since $F(q^2)$ is unknown for $q^2 \sim m_B^2$. The best-fit results are shown in Table I. While a good fit to R_K and R_{K^*} is obtained, we need to check if these couplings are consistent with other measurements. As in the scalar case there is a two-body contribution to $\mathcal{B}(B \to K^{(*)}e^+e^-)$ from $B \to K^{(*)}Z'$ and Z' decaying to e^+e^- with a branching ratio ~ 1.

The branching ratio for $B \to KZ'$ is [60,61]

$$\mathcal{B}(B \to KZ') = \frac{|g_{bs} + g'_{bs}|^2}{64\pi} \frac{m_B^2 \beta^3_{BKZ'}}{M^2_{Z'}\Gamma_B} [f_+^{BK}(M^2_{Z'})]^2, \tag{17}$$

where $\beta_{XYZ} = \lambda^{1/2}(1, M_Y^2/M_X^2, M_Z^2/M_X^2)$ and f_+^{BK} is a form factor. For $B \to K^*Z'$, the branching ratio is given by

$$\mathcal{B}(B \to K^*Z') = \frac{\beta_{BK^*Z}}{16\pi m_B \Gamma_B}(|H_0|^2 + |H_+|^2 + |H_-|^2), \tag{18}$$

where the helicity amplitudes are defined as

$$H_0 = (g_{bs} - g'_{bs})\left[-\frac{1}{2}(m_B + M_{K^*})\xi A_1(M^2_{Z'}) + \frac{M_{K^*}M_{Z'}}{m_B + M_{K^*}}\sqrt{\xi^2 - 1}A_2(M^2_{Z'})\right], \tag{19}$$

and

$$H_\pm = \frac{1}{2}(g_{bs} - g'_{bs})[(m_B + M_{K^*})A_1(M^2_{Z'})] \pm (g_{bs} + g'_{bs})\frac{M_{K^*}M_{Z'}}{m_B + M_{K^*}}\sqrt{\xi^2 - 1}V(M^2_{Z'}). \tag{20}$$

V, A_1 and A_2 are form factors [53,54] and $\xi = (m_B^2 - M^2_{K^*} - M^2_{Z'})/(2M_{K^*}M_{Z'})$.

Assuming the decay rate of $B \to KZ'$ and $B \to K^*Z'$ to be less than 1% of the B^0 width, we obtain the constraints shown in Table II. Since g_{ee} is constrained to be less than 2.5×10^{-4} at the 90% CL for $M_{Z'} = 25$ MeV [59], the constraints in Table II exclude the best-fit values to explain the R_K and R_{K^*} measurements in this case.

We next consider the case when Z' also has nonzero axial vector coupling with the electrons, i.e., $g_L^{ee} \neq g_R^{ee}$. The best-fit results are shown in Table I. While a good fit to R_K and R_{K^*} is obtained, the best-fit values do not satisfy the two-body constraints of Table II along with the constraint on g_{ee}.

Now, to avoid the two-body constraint, like in the scalar case, we set $a_{bs} = 0$ in Eq. (13). In this case, assuming $g_L^{ee} = g_R^{ee} = g_{ee}$, i.e., pure vector coupling to the electron, and for $M_{Z'} = 25$ MeV, we fit the product $g_{ee}g_{bs}$ and $g_{ee}g'_{bs}$ to the R_K and R_{K^*} data. The results are summarized in Table I. Clearly, at the best-fit point, the predictions for R_K and R_{K^*} are within the 1σ range of the measurements. Requiring $\mathcal{B}(B^0 \to K^0e^+e^-) < 1\%$ and $\mathcal{B}(B^0 \to K^{*0}e^+e^-) < 1\%$, we get the constraints shown in Table II. The best fit satisfies all constraints on g_{bs}, g'_{bs} and g_{ee}. From Table I, we see that R_K and R_{K^*} values in all measured q^2 bins can be reasonably accommodated. We also checked that the predictions for the branching ratios to electron modes are consistent with the various observables; see Table III. Our prediction for $\mathcal{B}(B \to K e^+e^-)_{[1.0\text{–}6.0]}$ is somewhat higher than the measurement and this tension could become significant with a reduction in the theoretical and experimental uncertainties. The prediction for the inclusive mode $\mathcal{B}(B \to X_s e^+e^-)_{[1.0\text{–}6.0]}$, which suffers from less hadronic uncertainties, is consistent with measurement.

Next we consider the case when Z' also has nonzero axial vector coupling with the electrons, i.e., $g_L^{ee} \neq g_R^{ee}$. Again, we either set $g'_{bs} = 0$ or $g_{bs} = 0$. The best-fit values shown in Table I satisfy the constraints on the NP couplings, and the R_K and R_{K^*} values in all measured q^2 bins can be reasonably accommodated. The corresponding branching ratios with electron modes are provided in Table III.

IV. SUMMARY

In this work, we have addressed the recent measurement of R_{K^*} with particular attention to the low q^2 bin, $0.045 \le q^2 \le 1.1\ \mathrm{GeV}^2$. This measurement has been difficult to explain with new physics above the GeV scale. For mediators in the 10–200 MeV mass range, we find

(1) A (pseudo)scalar that only couples to muons cannot explain the R_K and R_{K^*} measurements as the predicted values are larger than in the SM, in conflict with experiment. An S coupling to only electrons can reproduce the $R_{K[1.0\text{–}6.0]}$, $R_{K*[0.045\text{–}1.1]}$ and $R_{K*[1.1\text{–}6.0]}$ data, but the desired values of the couplings are not consistent with the measurements of the branching ratios $\mathcal{B}(B \to K^{(*)}e^+e^-)$. A q^2-dependent flavor changing $b-s$ coupling to the scalar can produce compatibility with $\mathcal{B}(B \to K^{(*)}e^+e^-)$ and gives a good fit to R_K and R_{K^*} in the central q^2 bin, but the deviation of R_{K^*} from the SM in the low q^2 bin is small.

(2) A Z' with general vector and axial vector couplings to the muon and a q^2-dependent $b-s$ coupling provides a good fit to the combination of the three R_K and R_{K*} measurements, but does not fit $R_{K*[0.045\text{–}1.1]}$ well.

(3) A Z' with general vector and axial vector couplings to the electron can explain R_K and R_{K^*} data in all measured bins but the desired values of the couplings are not consistent with the measurements of $\mathcal{B}(B \to K^{(*)}e^+e^-)$. However, a q^2-dependent flavor changing $b-s$ coupling to the vector is compatible with $\mathcal{B}(B \to K^{(*)}e^+e^-)$ and gives good fits to R_K and R_{K^*}; of the cases we considered, the case with purely vector electron coupling provides the best agreement with the data with a pull of 4.8.

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