Common origin of nonzero θ_{13} and baryon asymmetry of the Universe in a TeV scale seesaw model with A_4 flavor symmetry

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We study the possibility of generating nonzero reactor mixing angle θ_{13} and baryon asymmetry of the Universe within the framework of an A_4 flavor symmetric model. Using the conventional type I seesaw mechanism we construct the Dirac and Majorana mass matrices that give rise to the correct light neutrino mass matrix. Keeping the right-handed neutrino mass matrix structure trivial so that it gives rise to a (quasi) degenerate spectrum of heavy neutrinos suitable for resonant leptogenesis at TeV scale, we generate the nontrivial structure of Dirac neutrino mass matrix that can lead to the light neutrino mixing through the type I seesaw formula. Interestingly, such a setup naturally leads to nonzero θ_{13} due to the existence of antisymmetric contraction of the product of two triplet representations of A_4. Such an antisymmetric part of the triplet products usually vanishes for right-handed neutrino Majorana mass terms, leading to $\mu - \tau$ symmetric scenarios in the most economical setups. We constrain the model parameters from the requirement of producing the correct neutrino data as well as baryon asymmetry of the Universe for right-handed neutrino mass scale around TeV. The A_4 symmetry is augmented by additional $Z_3 \times Z_2$ symmetry to make sure that the splitting between right-handed neutrinos required for resonant leptogenesis is generated only by next to leading order terms, making it naturally small. We find that the inverted hierarchical light neutrino masses give more allowed parameter space consistent with neutrino and baryon asymmetry data.

I. INTRODUCTION

Observations of tiny but nonzero neutrino mass and large leptonic mixing [1–7] have been one of the most compelling evidences suggesting the presence of beyond standard model (BSM) physics. The present status of different neutrino parameters can be found in the latest global fit analysis [8,9], summarized in Table I. It can be seen that out of the three leptonic mixing angles, the solar and atmospheric angles are reasonably large while the reactor mixing angle is relatively small. On the other hand, only two mass squared differences are measured experimentally, keeping the lightest neutrino mass still an unknown parameter. Also the mass ordering is not settled yet, allowing both normal hierarchy (NH) as well as inverted hierarchy (IH). Cosmology experiments can however, put an upper bound

on the lightest neutrino mass from the measurement of the sum of absolute neutrino masses $\sum_i |m_i| < 0.17$ eV [10]. Although the solar and atmospheric mixing angles $(\theta_{12}, \theta_{23})$ were known to have large values, the discovery of nonzero θ_{13} is somewhat recent [3–7]. The leptonic Dirac CP phase δ is not yet measured experimentally, though a recent measurement hinted at $\delta \approx -\pi/2$ [11]. If neutrinos are Majorana fermions, then two other CP phases appear, which do not affect neutrino oscillation probabilities and hence remain undetermined in such experiments. They can however be probed at experiments looking for lepton number (L) violating processes like neutrinoless double beta decay $(0\nu\beta\beta)$.

The standard model (SM) of particle physics, in spite of its astonishing success as a low energy theory of fundamental particles and their interactions (except gravity), cannot explain the origin of neutrino mass at renormalizable level. Because of the absence of right-handed neutrinos, there is no coupling of the Higgs field responsible for the origin of mass, with neutrinos. Even if right-handed neutrinos are introduced, one requires a Yukawa coupling with the Higgs boson of the order of 10^{-12} in order to generate sub-eV neutrino masses. It also introduces a new scale, equal to the bare mass term of the right-handed neutrinos that can neither be explained nor prevented

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TABLE I. Global fit 3σ values of neutrino oscillation parameters [8,9].

Parameters	NH [8]	IH [8]	NH [9]	IH [9]
$\frac{\Delta m_{21}^2}{10^{-5}\ \text{eV}^2}$	7.03–8.09	7.02–8.09	7.05–8.14	7.05–8.14
$\frac{\|\Delta m_{31}^2\|}{10^{-3}\ \text{eV}^2}$	2.407–2.643	2.399–2.635	2.43–2.67	2.37–2.61
$\sin^2\theta_{12}$	0.271–0.345	0.271–0.345	0.273–0.379	0.273–0.379
$\sin^2\theta_{23}$	0.385–0.635	0.393–0.640	0.384–0.635	0.388–0.638
$\sin^2\theta_{13}$	0.0193–0.0239	0.0195–0.0240	0.0189–0.0239	0.0189–0.0239
δ	0°–360°	145°–391°	0°–360°	0°–31°, 142°–360°

within the SM. In an effective field theory setup, one can generate light neutrino masses through the dimension five effective operator [12] so that neutrino masses are naturally light due to the suppression by a cutoff scale Λ. Such an operator can be realized within several BSM frameworks after integrating out the heavy fields. Such renormalizable BSM frameworks are popularly known as seesaw models [13]. Apart from the tiny mass of neutrinos, another puzzling observation is their large mixing angles, in sharp contrast with small mixing angles in the quark sector. This may also be a hint that the CP violation in the leptonic sector is large compared to the quark sector. If this is true, then it can have nontrivial implications for cosmology as the quark sector CP violation is found to be too small to generate the observed matter antimatter asymmetry of the Universe, to be discussed in detail below. The observed large mixing in the leptonic sector has motivated the study of different flavor symmetry models that can predict such mixing patterns. One of the very popular flavor symmetric scenarios is the one that predicts a $\mu - \tau$ symmetric light neutrino mass matrix that predicts $\theta_{13} = 0, \theta_{23} = \frac{\pi}{4}$ whereas the value of θ_{12} depends upon the particular realization of this symmetry [14]. Among different possible realizations, the tribimaximal (TBM) [15] mixing pattern which predicts $\theta_{12} = 35.3°$ has probably been the most studied one. In fact, this mixing pattern was consistent with light neutrino data, prior to the discovery of nonzero θ_{13}. Such mixing patterns can naturally be realized within several non-Abelian discrete flavor symmetry models [16]. Among them, the discrete group A_4 which is the group of even permutations of four objects, can reproduce the TBM mixing in the most economical way [17,18]. Since the latest neutrino oscillation data are not consistent with $\theta_{13} = 0$ and hence TBM mixing, one has to go beyond the minimal $\mu - \tau$ symmetric framework. Since the measured value of θ_{13} is small compared to the other two, one can still consider the validity of $\mu - \tau$ symmetry at the leading order and generate nonzero θ_{13} by adding small $\mu - \tau$ symmetry breaking perturbations. Such corrections can originate from the charged lepton sector or the neutrino sector itself like for example, in the form of a new contribution to the neutrino mass matrix. This has led to several works including [19–26] within different BSM frameworks.

Apart from the issue of tiny neutrino mass and large leptonic mixing, another serious drawback of the SM is its inability to explain the observed baryon asymmetry of the Universe. The observed baryon asymmetry is often quoted as the baryon to photon ratio [10],

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.04 \pm 0.08 \times 10^{-10}. \qquad (1)$$

If the Universe started in a baryon symmetric manner then one has to satisfy the Sakharov's conditions [27]: baryon number (B) violation, C and CP violation, and departure from thermal equilibrium. One popular BSM scenario that can generate a net baryon asymmetry is leptogenesis. For a review, one may refer to [28]. As outlined in the original proposal by Fukugita and Yanagida thirty years back [29], this mechanism can satisfy all Sakharov's conditions [27] required to be fulfilled in order to produce a net baryon asymmetry. Here, a net leptonic asymmetry is generated first that gets converted into baryon asymmetry through $B + L$ violating electroweak sphaleron transitions [30]. The interesting feature of this scenario is that the required lepton asymmetry can be generated through out of equilibrium decay of the same heavy fields that take part in the seesaw mechanism. Although the BSM framework explaining the baryon asymmetry could be completely decoupled from the one explaining leptonic mass and mixing, it is more economical and predictive if the same model can account for both the observed phenomena. In the conventional type I seesaw mechanism for example, the heavy right-handed neutrino decay generates the required lepton asymmetry that not only depends upon the scale of right-handed neutrino mass, but also on the leptonic CP violation, which can be probed at ongoing oscillation experiments. For a hierarchical spectrum of right-handed neutrinos, there exists a lower bound on the right-handed neutrino mass $M_R > 10^9$ GeV, popularly known as the Davidson-Ibarra bound [31], from the requirement of successful leptogenesis. One can however bring down the scale of right-handed neutrino mass within the framework of resonant leptogenesis [32–35].

Motivated by this, we study an A_4 flavor symmetric model that can simultaneously explain the correct neutrino data as well as the baryon asymmetry through TeV scale

TABLE II.   Fields and their transformation properties under $SU(2)_L$ gauge symmetry as well as the A_4 symmetry.

	$\bar{L}$	e_R	μ_R	τ_R	N	H	ϕ_E	ϕ_ν	ξ	ζ
$SU(2)_L$	2	1	1	1	1	2	1	1	1	1
A_4	3	1	$1'$	$1''$	3	1	3	3	1	$1''$
Z_3	ω	ω^2	ω^2	ω^2	ω	1	1	ω	ω	1
Z_2	1	1	1	1	-1	1	1	-1	1	-1

resonant leptogenesis. Keeping the right-handed neutrino mass matrix trivial, giving rise to a degenerate spectrum, we first try to obtain the nontrivial Dirac neutrino mass matrix responsible for nontrivial structure of the light neutrino mass matrix, to be obtained using the type I seesaw formula. We generate this nontrivial structure of Dirac neutrino mass matrix using a flavon field that, along with the lepton doublets and right-handed neutrinos transform as A_4 triplets. We find that this choice automatically gives rise to nonzero θ_{13} as the resulting light neutrino mass matrix does not possess any $\mu - \tau$ symmetry. This is due to the antisymmetric term arising out of the products of two A_4 triplets. If we generate the nontrivial leptonic mixing from a nontrivial right-handed neutrino mixing, like in the Altarelli-Feruglio-type models [18], such an antisymmetric term vanishes due to Majorana nature of this mass term. This is however not true in the case of the Dirac mass term, resulting in a nontrivial $\mu - \tau$ symmetry breaking structure in the most general case. We compare the light neutrino mass matrix derived from the model with the one from data and evaluate the model parameters for a particular choice of right-handed neutrino mass scale. The minimal such scenario is found to be rather constrained with only a handful of allowed points that satisfy all the criteria from the neutrino data point of view. We then feed these allowed points into the calculation of resonant leptogenesis and find agreement with the observed baryon asymmetry of the Universe. In the end we also briefly comment on the $\mu - \tau$ symmetric limit of these scenarios where the antisymmetric coupling term is turned off by hand.

This paper is organized as follows. In Sec. II, we discuss our A_4 flavor symmetric model with the details of different mass matrices in the lepton sector. In Sec. III, we briefly outline the mechanism of resonant leptogenesis followed by the details of numerical analysis in Sec. IV. We discuss our numerical results in Sec. V and then briefly outline the $\mu - \tau$ symmetric limit of the model in Sec. VI. We finally conclude in Sec. VII.

II. THE MODEL

The discrete group A_4 is the group of even permutations of four objects or the symmetry group of a tetrahedron. It has twelve elements and four irreducible representations with dimensions n_i such that $\sum_i n_i^2 = 12$. These four representations are denoted by **1**, **1′**, **1″** and **3**, respectively. The product rules for these representations are given

in Appendix. We consider a flavor symmetric model based on the discrete non-Abelian group A_4 augmented by $Z_3 \times Z_2$ that predicts the specific structures of different 3×3 matrices involved in the type I seesaw in a natural and minimal way. The particle content of the model is shown in Table II. The Yukawa Lagrangian for the leptons can be written as

$$\mathcal{L}_Y \supset Y_e \bar{L} H \frac{\phi_E}{\Lambda} e_R + Y_\mu \bar{L} H \frac{\phi_E}{\Lambda} \mu_R + Y_\tau \bar{L} H_d \frac{\phi_E}{\Lambda} \tau_R$$
$$+ \frac{Y_s}{\Lambda} (\phi_\nu \bar{L})_{3_s} \tilde{H} N + \frac{Y_a}{\Lambda} (\phi_\nu \bar{L})_{3_a} \tilde{H} N$$
$$+ Y_N (NN)_1 \xi + Y'_N (NN)_{1''} \xi \frac{\zeta \zeta}{\Lambda^2} + \text{H.c.} \qquad (2)$$

Using the A_4 product rules given in Appendix, we can write down the relevant leptonic mass matrices from the above Lagrangian. We denote the vacuum expectation value (vev) of the Higgs boson to be v_H and choose a specific flavon vev alignment $\langle \phi_E \rangle = (v_E, 0, 0)$, $\langle \phi_\nu \rangle = (v_\nu, v_\nu, v_\nu)$. The resulting charged lepton mass matrix is

$$M_l = \frac{v_H v_E}{\Lambda} \begin{pmatrix} Y_e & 0 & 0 \\ 0 & Y_\mu & 0 \\ 0 & 0 & Y_\tau \end{pmatrix}. \qquad (3)$$

The Dirac neutrino mass matrix is given by

$$M_D = \frac{v_H v_\nu}{\Lambda} \begin{pmatrix} \frac{2}{3} Y_s & -(\frac{Y_s}{3} + \frac{Y_a}{2}) & -(\frac{Y_s}{3} - \frac{Y_a}{2}) \\ -(\frac{Y_s}{3} - \frac{Y_a}{2}) & \frac{2}{3} Y_s & -(\frac{Y_s}{3} + \frac{Y_a}{2}) \\ -(\frac{Y_s}{3} + \frac{Y_a}{2}) & -(\frac{Y_s}{3} - \frac{Y_a}{2}) & \frac{2}{3} Y_s \end{pmatrix}. \qquad (4)$$

Considering only up to dimension five terms, the right-handed neutrino mass matrix can be written as

$$M_R = 2 Y_N v_\xi \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad (5)$$

where v_ξ is the vev of the flavon ξ. The light neutrino mass matrix can be generated using type I seesaw

$$-M_\nu = M_D M_R^{-1} M_D^T = \frac{1}{c} \begin{pmatrix} -2(a^2 - 3b^2) & (a^2 + 6ab - 3b^2) & (a^2 - 6ab - 3b^2) \\ (a^2 + 6ab - 3b^2) & (a^2 - 6ab - 3b^2) & -2(a^2 - 3b^2) \\ (a^2 - 6ab - 3b^2) & -2(a^2 - 3b^2) & (a^2 + 6ab - 3b^2) \end{pmatrix}, \tag{6}$$

where $a = \frac{1}{\Lambda} Y_a v_H v_\nu$, $b = \frac{2}{3\Lambda} Y_s v_H v_\nu$, $c = 2Y_N v_\xi$. Diagonalization of this mass matrix gives the eigenvalues as

$$m_1 = 0, \quad m_2 = -\frac{3}{c}(a^2 + 3b^2), \quad m_3 = \frac{3}{c}(a^2 + 3b^2), \tag{7}$$

which clearly disagrees with the neutrino mass data that give $\Delta m_{21}^2 \neq 0$. Even if we lift the degeneracy of the right-handed neutrino mass matrix as

$$M_R = \begin{pmatrix} c & 0 & 0 \\ 0 & 0 & c \\ 0 & c & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & d \\ 0 & d & 0 \\ d & 0 & 0 \end{pmatrix} \tag{8}$$

we still have degenerate light neutrino mass eigenvalues

$$m_1 = 0, \quad m_2 = -\frac{3(a^2 + 3b^2)}{\sqrt{c^2 - cd + d^2}},$$

$$m_3 = \frac{3(a^2 + 3b^2)}{\sqrt{c^2 - cd + d^2}}, \tag{9}$$

which is disallowed by neutrino data.

Choosing a more general vacuum alignment $\langle \phi_\nu \rangle = (v_{\nu 1}, v_{\nu 2}, v_{\nu 3})$, the Dirac neutrino mass matrix can be written as

$$M_D = \frac{v_H}{\Lambda} \begin{pmatrix} \frac{2}{3} Y_s v_{\nu 1} & -(\frac{Y_s}{3} + \frac{Y_a}{2}) v_{\nu 3} & -(\frac{Y_s}{3} - \frac{Y_a}{2}) v_{\nu 2} \\ -(\frac{Y_s}{3} - \frac{Y_a}{2}) v_{\nu 3} & \frac{2}{3} Y_s v_{\nu 2} & -(\frac{Y_s}{3} + \frac{Y_a}{2}) v_{\nu 1} \\ -(\frac{Y_s}{3} + \frac{Y_a}{2}) v_{\nu 2} & -(\frac{Y_s}{3} - \frac{Y_a}{2}) v_{\nu 1} & \frac{2}{3} Y_s v_{\nu 3} \end{pmatrix}. \tag{10}$$

Denoting $a = \frac{v_H}{\Lambda} \frac{1}{3} Y_s v_{\nu 1}$, $b = \frac{v_H}{\Lambda} \frac{1}{3} Y_a v_{\nu 1}$, $c = \frac{v_H}{\Lambda} \frac{1}{3} Y_s v_{\nu 2}$, $d = \frac{v_H}{\Lambda} \frac{1}{3} Y_s v_{\nu 3}$ we can write the Dirac neutrino mass matrix as

$$M_D = \begin{pmatrix} 2a & -d - \frac{bd}{a} & -c + \frac{bc}{a} \\ -d + \frac{bd}{a} & 2c & -a - b \\ -c - \frac{bc}{a} & -a + b & 2d \end{pmatrix}. \tag{11}$$

In this notation, the light neutrino mass matrix elements are given by

$$(-M_\nu)_{11} = \frac{4a^4 + 2a^2 cd - 2b^2 cd}{a^2 f},$$

$$(-M_\nu)_{12} = \frac{a^2(-d) + 4abd - 2ac^2 + b^2 d + 2bc^2}{af},$$

$$(-M_\nu)_{13} = -\frac{a^2 c + 4abc + 2ad^2 - b^2 c + 2bd^2}{af},$$

$$(-M_\nu)_{22} = \frac{(d - \frac{bd}{a})^2 - 4c(a + b)}{f},$$

$$(-M_\nu)_{23} = \frac{a^4 - a^2(b^2 - 5cd) - b^2 cd}{a^2 f},$$

$$(-M_\nu)_{33} = \frac{\frac{c^2(a+b)^2}{a^2} + 4d(b - a)}{f}, \tag{12}$$

where $f = 2Y_N v_\xi$ is the nonzero entry in the right-handed neutrino mass matrix given by (5). In this case, the resulting light neutrino mass matrix can give rise to the correct mass squared differences as well as mixing angles including nonzero θ_{13}. At the dimension five level however, the right-handed neutrinos remain degenerate. As we discuss below, for successful resonant leptogenesis, the right-handed neutrinos must have tiny splittings that can be generated at dimension six level in the model. This higher order contribution to the right-handed neutrino mass matrix can be written as

$$\delta M = \begin{pmatrix} 0 & 0 & r_1 \\ 0 & r_1 & 0 \\ r_1 & 0 & 0 \end{pmatrix}, \tag{13}$$

where $r_1 = Y_N' v_\xi \frac{v_\zeta^2}{\Lambda^2}$ with v_ζ being the vev of the flavon ζ. Such a small higher order term does not affect light neutrino masses and mixings considerably.

It should be noted that we have used the A_4 product rules in the T diagonal basis, as given in Appendix. This is justified in the diagonal charged lepton and Majorana light neutrino mass limit. In the S diagonal basis, the charged lepton mass matrix is nondiagonal and the light neutrino mass matrix also has a different structure due to the difference in the triple product rules. For details, one may refer to [16].

III. RESONANT LEPTOGENESIS

As pointed out by Fukugita and Yanagida [29], the out of equilibrium and *CP* violating decay of heavy Majorana neutrinos provides a natural way to create the required lepton asymmetry, as shown in Fig. 1. The asymmetry
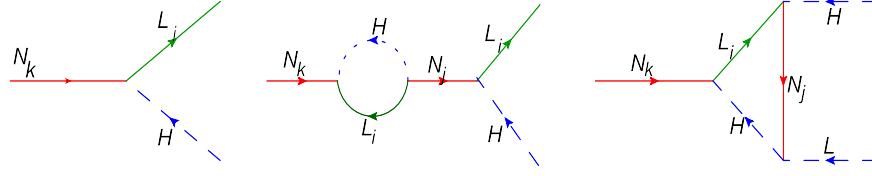
FIG. 1. Decay modes of right-handed neutrino in type I seesaw.

generated by the decay of the lightest right-handed neutrino into the lepton and Higgs boson is given by

$$\epsilon_{N_k} = -\sum \frac{\Gamma(N_k \to L_i + H^*) - \Gamma(N_k \to L_i + H)}{\Gamma(N_k \to L_i + H^*) + \Gamma(N_k \to L_i + H)}. \quad (14)$$

This lepton asymmetry is converted to the baryon asymmetry through electroweak sphaleron transitions allowing us to reproduce the observed baryon asymmetry of the Universe. As mentioned before, resonant leptogenesis is a viable alternative to high scale or vanilla leptogenesis scenarios [32–35] within the context of TeV scale minimal seesaw scenarios. Since a hierarchical spectrum of right-handed neutrinos cannot give rise to the required asymmetry at TeV scale, this mechanism gives a resonance enhancement to the lepton asymmetry by considering a very small mass splitting between the two heavy neutrinos, of the order of their average decay width.

The lepton asymmetry can be found from the following formula [36,37],

$$\epsilon_{il} = \sum_{j \neq i} \frac{\text{Im}[Y_{\nu il} Y^*_{\nu jl} (Y_\nu Y^\dagger_\nu)_{ij}] + \frac{M_i}{M_j} \text{Im}[Y_{\nu il} Y^*_{\nu jl} (Y_\nu Y^\dagger_\nu)_{ji}]}{(Y_\nu Y^\dagger_\nu)_{ii}(Y_\nu Y^\dagger_\nu)_{jj}} f_{ij}, \quad (15)$$

with the regulator f_{ij} being given as

$$f_{ij} = \frac{(M_i^2 - M_j^2) M_i \Gamma_j}{(M_i^2 - M_j^2)^2 + M_i^2 \Gamma_j^2}.$$

Here, $\Gamma_i = \frac{M_i}{8\pi}(Y_\nu Y^\dagger_\nu)_{ii}$ as the tree level heavy-neutrino decay width and Y_ν is the effective coupling between heavy and light neutrinos. Now, there is a similar contribution ϵ'_{il} to the CP asymmetry from Right Handed neutrino oscillation [36,38,39]. Its form is given by (15) with the replacement f_{ij} by f'_{ij}, where

$$f'_{ij} = \frac{(M_i^2 - M_j^2) M_i \Gamma_j}{(M_i^2 - M_j^2)^2 + (M_i \Gamma_i + M_j \Gamma_j)^2 \frac{\det[\text{Re}(Y_\nu Y^\dagger_\nu)]}{(Y_\nu Y^\dagger_\nu)_{ii}(Y_\nu Y^\dagger_\nu)_{ii}}}.$$

The total CP asymmetry is therefore the summation of these two $\epsilon^T_{il} = \epsilon_{il} + \epsilon'_{il}$. Taking into account the appropriate efficiency and dilution factors [36,40], one can write the final baryon asymmetry as

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq -\frac{28}{51} \frac{1}{27} \frac{3}{2} \sum_{l,i} \frac{\epsilon_{il}}{K^{\text{eff}}_l \min(z_c, z_l)}, \quad (16)$$

where $z_c = \frac{M_N}{T_c}$, $T_c \sim 149$ GeV being the critical temperature, $z_l \simeq 1.25 \log(25 K^{\text{eff}}_l)$ [40] and $K^{\text{eff}}_l = \kappa_l \sum_i K_i B_{il}$, with $K_i = \Gamma_i / H_N$ being the washout factor. The Hubble parameter for the radiation dominated universe is $H_N = 1.66\sqrt{g_*} M_N^2 / M_{\text{Pl}}$ at $T = M_N$ and $g^* \simeq 106.75$ is the relativistic degrees of freedom at high temperatures. B_{il}'s are the branching ratios of the N_i decay to leptons of lth flavor: $B_{il} = \frac{|Y_{\nu il}|^2}{(Y_\nu Y^\dagger_\nu)_{ii}}$. The factor κ is given by

$$\kappa_l = 2 \sum_{i,j(j \neq i)} \frac{\text{Re}[Y_{\nu il} Y^*_{\nu jl}(YY^\dagger)_{ij}] + \text{Im}[(Y_{\nu il} Y^*_{\nu jl})^2]}{\text{Re}[(Y^\dagger Y)_{ll}\{(YY^\dagger)_{ii} + (YY^\dagger)_{jj}\}]}$$
$$\times \left(1 - 2i \frac{M_i - M_j}{\Gamma_i + \Gamma_j}\right)^{-1}. \quad (17)$$

As seen from the expression (15), the lepton asymmetry is dependent on the elements of the Dirac Yukawa coupling matrix. Therefore it can be said that the same sets of model parameters that are supposed to yield correct neutrino phenomenology are also responsible for yielding an enhanced lepton asymmetry, later on generating the observed Baryon asymmetry of the universe.

IV. NUMERICAL ANALYSIS

As discussed before, the most general form of Dirac neutrino mass matrix (assuming a degenerate right-handed neutrino mass spectrum) can give rise to a light neutrino mass matrix from type I seesaw formula, which is consistent with $\theta_{13} \neq 0$. This is due to the presence of the antisymmetric part of the A_4 triple product that explicitly breaks $\mu - \tau$ symmetry leading to the generation of $\theta_{13} \neq 0$. Within the minimal setup, the light neutrino mass matrix is given by (24), which contains five parameters a, b, c, d, f that can in general be complex. Since this corresponds to degenerate heavy neutrino masses that cannot give rise to successful leptogenesis, we can break the degeneracy by including a higher order contribution to the right-handed neutrino mass matrix as discussed above. Taking this correction into account, we can write the right-handed neutrino mass matrix as

$$M = M_R^0 + \delta M_R = \begin{pmatrix} f & 0 & g \\ 0 & g & f \\ g & f & 0 \end{pmatrix}. \qquad (18)$$

This has eigenvalues $f + g$, $-\sqrt{f^2 - fg + g^2}$, $\sqrt{f^2 - fg + g^2}$ where f is the leading order right-handed neutrino mass and g is the parameter creating tiny mass splitting. As mentioned earlier, these parameters are related to the Lagrangian parameters as

$$a = \frac{v_H}{\Lambda} \frac{1}{3} Y_s v_{\nu 1}, \qquad b = \frac{v_H}{\Lambda} \frac{1}{3} Y_a v_{\nu 1}, \qquad c = \frac{v_H}{\Lambda} \frac{1}{3} Y_s v_{\nu 2},$$

$$d = \frac{v_H}{\Lambda} \frac{1}{3} Y_s v_{\nu 3}, \qquad f = 2 Y_N v_\xi, \qquad g = Y'_N v_\xi \frac{v_\zeta^2}{\Lambda^2}.$$

For the numerical analysis part we first fix the scale of leptogenesis by fixing the leading right-handed neutrino mass or the parameter f to be 5 TeV, say. The range of g has been chosen in such a way that we can have a tiny Majorana mass splitting required for successful leptogenesis without affecting the neutrino parameters being from their correct 3σ bound. For satisfying neutrino phenomenology and explaining leptogenesis, g has been varied randomly from 10^{-6} to 10^{-5} GeV, which gives lepton asymmetry of an

order around 10^{-7} or more. Since g is very small compared to f, its effects on light neutrino masses and mixing is not substantial. Yet, we include it while discussing the compatibility of the model with neutrino data. Thus, after making the choice of f and the range of g, we are left with four model parameters a, b, c, d that can be calculated by comparing the mass matrix predicted by the model with the one we can construct in terms of light neutrino parameters.

The leptonic mixing matrix can be written in terms of the charged lepton diagonalizing matrix (U_l) and light neutrino diagonalizing matrix U_ν as

$$U_{\text{PMNS}} = U_l^\dagger U_\nu. \qquad (19)$$

In the simple case where the charged lepton mass matrix is diagonal, which is true in our model, we can have $U_l = \mathbb{K}$. Therefore we can write $U_{\text{PMNS}} = U_\nu$. Now we can write the complete light neutrino mass matrix as

$$m_\nu = U_{\text{PMNS}} m_\nu^{\text{diag}} U_{\text{PMNS}}^T, \qquad (20)$$

where the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) leptonic mixing matrix can be parametrized as

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} U_{\text{Maj}} \qquad (21)$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and δ is the leptonic Dirac CP phase. The diagonal matrix $U_{\text{Maj}} = \text{diag}(1, e^{i\alpha}, e^{i(\zeta+\delta)})$ contains the undetermined Majorana CP phases α, ζ. The diagonal mass matrix of the light neutrinos can be written as $m_\nu^{\text{diag}} = \text{diag}(m_1, \sqrt{m_1^2 + \Delta m_{21}^2}, \sqrt{m_1^2 + \Delta m_{31}^2})$ for NH and $m_\nu^{\text{diag}} = \text{diag}(\sqrt{m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2}, \sqrt{m_3^2 + \Delta m_{23}^2}, m_3)$ for IH.

For a fixed value of right-handed neutrino mass, we can now compare the light neutrino mass matrix predicted by the model and the one calculated from the light neutrino parameters. Since there are four undetermined complex parameters of the model, we need to compare four elements. Without any loss of generality, we equate (12), (13), (22), and (33) elements of both the mass matrices. We vary the light neutrino parameters in their allowed 3σ ranges, vary the lightest neutrino mass $m_{\text{lightest}} \in (10^{-6}, 0.1)$ eV, and calculate the model parameters a, b, c, d for each set of values of neutrino parameters. However, the light neutrino mass matrix has two more independent elements as any general 3×3 complex symmetric mass matrix has six independent complex elements. On the other

hand, once a, b, c, d are calculated from the equations $(M_\nu)_{12} = (m_\nu)_{12}$, $(M_\nu)_{13} = (m_\nu)_{13}$, $(M_\nu)_{22} = (m_\nu)_{22}$, $(M_\nu)_{33} = (m_\nu)_{33}$, the other two elements $(M_\nu)_{11}$, $(M_\nu)_{23}$ are automatically determined. Since every set of values of a, b, c, d corresponds to a particular set of light neutrino parameters, we can calculate the other two light neutrino mass matrix elements $(m_\nu)_{11}$, $(m_\nu)_{23}$ for the same set of neutrino parameters. For consistency, one needs to make sure that these two elements calculated for the neutrino mass matrix predicted by the model M_ν and the ones from light neutrino parameters m_ν are equal to each other. It turns out that these two constraints tightly restrict the light neutrino parameters to a set of very specific values, resulting in a very predictive scenario. We randomly generate ten million light neutrino parameters to calculate the four model parameters a, b, c, d and restrict the parameters to only those ones that satisfy $|(m_\nu)_{11} - (M_\nu)_{11}| < 10^{-5}$, $|(m_\nu)_{23} - (M_\nu)_{23}| < 10^{-5}$. Here a tolerance of 10^{-5} is chosen to decide the equality between the two elements.

After finding the model parameters a, b, c, d as well as the light neutrino parameters satisfying the constraints relating

the two elements of the mass matrices constructed from the model and neutrino data, respectively, we calculate the lepton asymmetry for the same set of allowed parameters. The effective Dirac Yukawa coupling matrix (Y_ν) relating heavy neutrinos to the light ones appearing in the lepton asymmetry formula is considered to have the same structure as the Dirac neutrino mass matrix given in (10). Since the corrected form of the heavy neutrino mass matrix is nondiagonal [given by (18)], we first diagonalize it and find the corresponding diagonalizing matrix U_R. To keep the analysis in this basis we transform the Dirac Yukawa coupling matrices as $Y_\nu \rightarrow Y_\nu U_R$ with $U_R^* M_R U_R^\dagger = \text{diag}(M_1, M_2, M_3)$. We then calculate the baryon asymmetry for the light neutrino parameters that are consistent with neutrino data as well as the model restrictions discussed above.

V. RESULTS AND DISCUSSION

Following the procedures outlined in the previous section, we first randomly generate the light neutrino

parameters in their 3σ range [8] and for each set of values, we calculate the model parameters a, b, c, d using four equations. We then apply the constraints relating another two elements of the neutrino mass matrix and find the constrained parameter space obeying them. For normal hierarchy, we show the correlation between these model parameters in Fig. 2. Since a, b, c, d denote the strength of the Dirac neutrino mass, we can see that they lie near or below the MeV scale so that the correct light neutrino mass is generated from the type I seesaw formula where the right-handed neutrino scale is fixed at 5 TeV. We also show the variation of the same model parameters with the lightest neutrino mass m_1 for normal hierarchy in Fig. 3. It can be seen that the allowed lightest neutrino mass can have values in the range 0.01–0.1 eV, which can be sensitive to $0\nu\beta\beta$ experiments. In fact, the region of parameter space near $m_1 \sim$ 0.1 eV is ruled out by the latest bounds from $0\nu\beta\beta$ experiments as well as the cosmology upper bound on the sum of absolute neutrino masses [10]. We show similar correlations for inverted hierarchy in Figs. 4 and 5. The overfall features
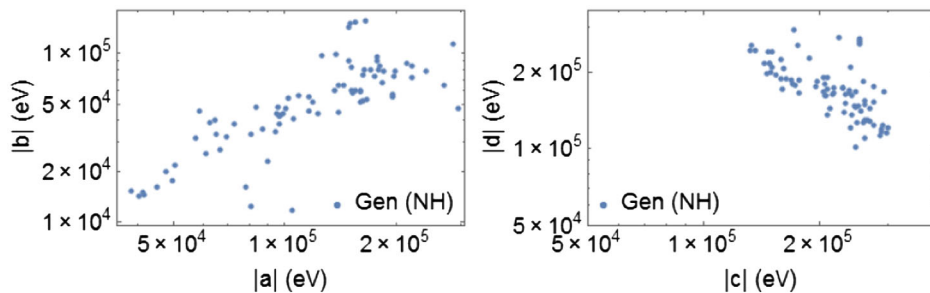

FIG. 2. Correlation between different model parameters for normal hierarchy. The label Gen refers to the most general structure of the mass matrix discussed in the text.
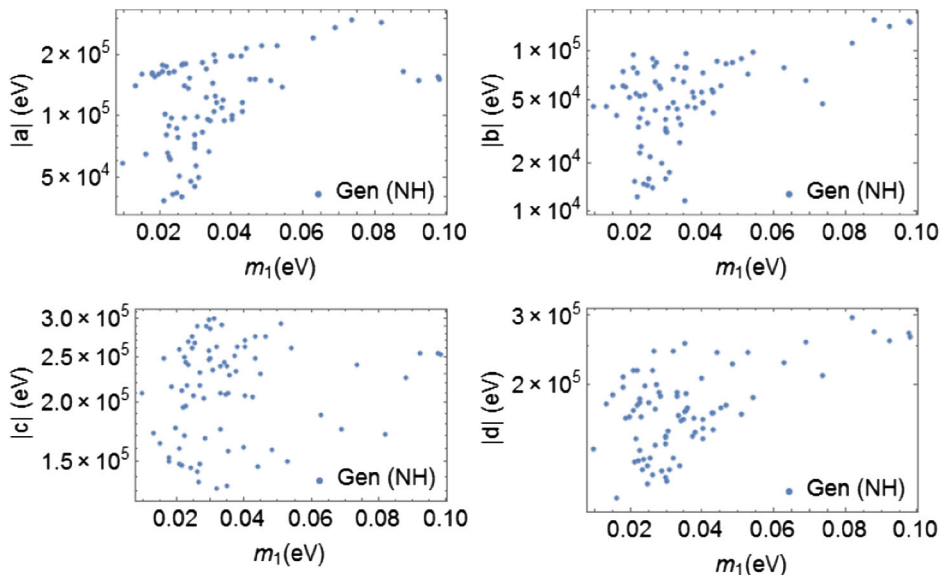

FIG. 3. Model parameters as a function of the lightest neutrino mass for normal hierarchy. The label Gen refers to the most general structure of the mass matrix discussed in the text.
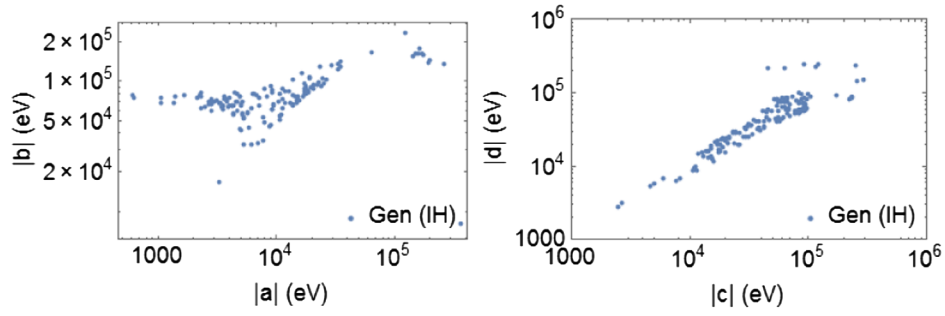
FIG. 4.   Correlation between different model parameters for inverted hierarchy. The label Gen refers to the most general structure of the mass matrix discussed in the text.
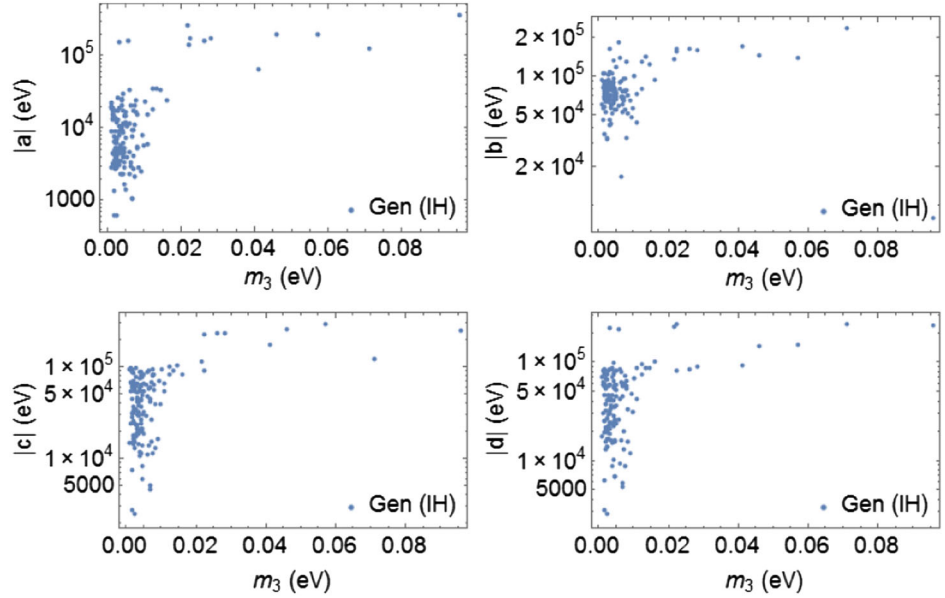

FIG. 5.   Model parameters as a function of the lightest neutrino mass for inverted hierarchy. The label Gen refers to the most general structure of the mass matrix discussed in the text.
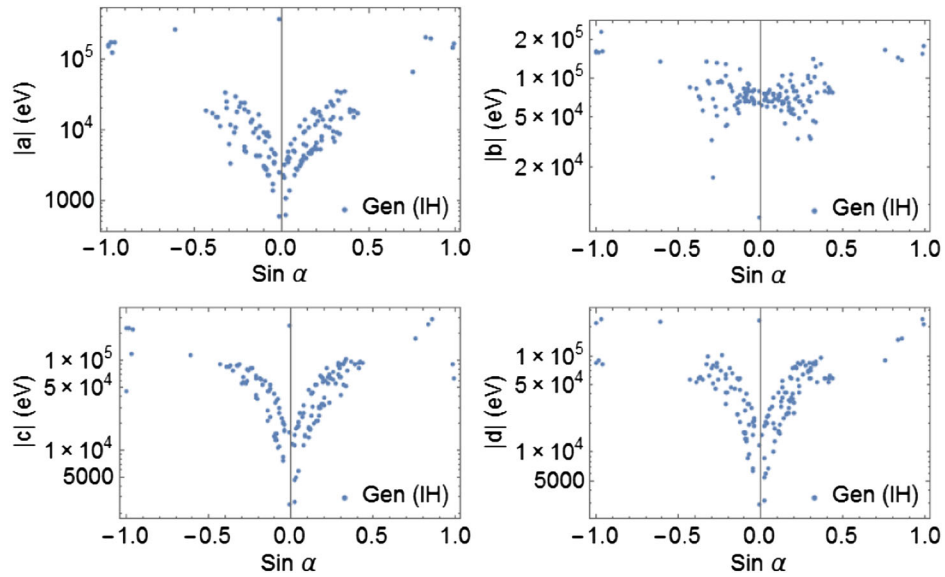

FIG. 6.   Model parameters as a function of one of the Majorana phases α for inverted hierarchy. The label Gen refers to the most general structure of the mass matrix discussed in the text.
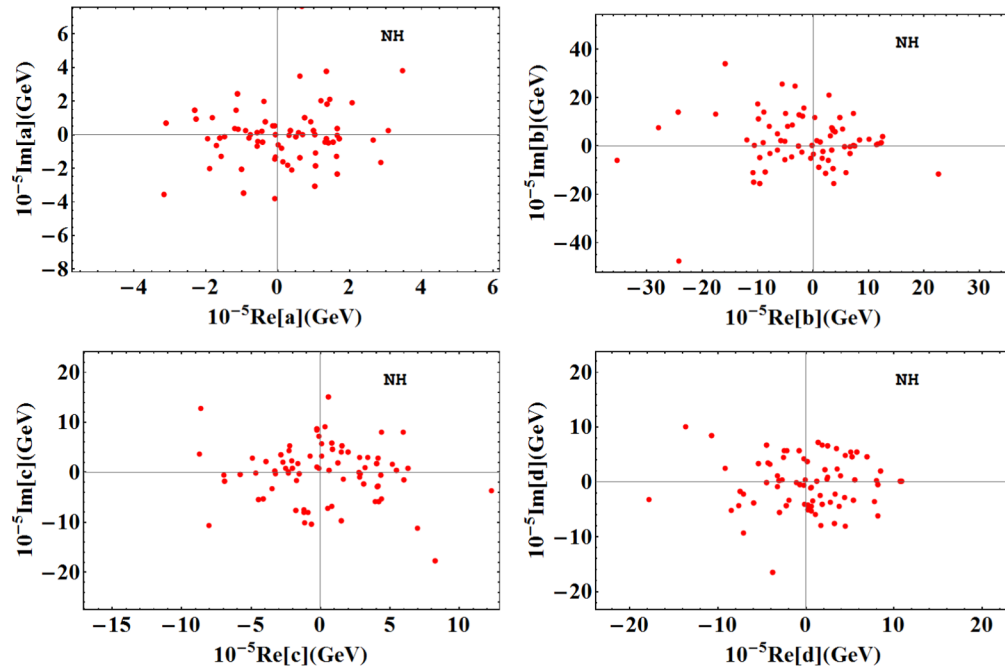
FIG. 7.   Real and imaginary parts of the model parameters for normal hierarchy with the most general structure of the mass matrix discussed in the text.
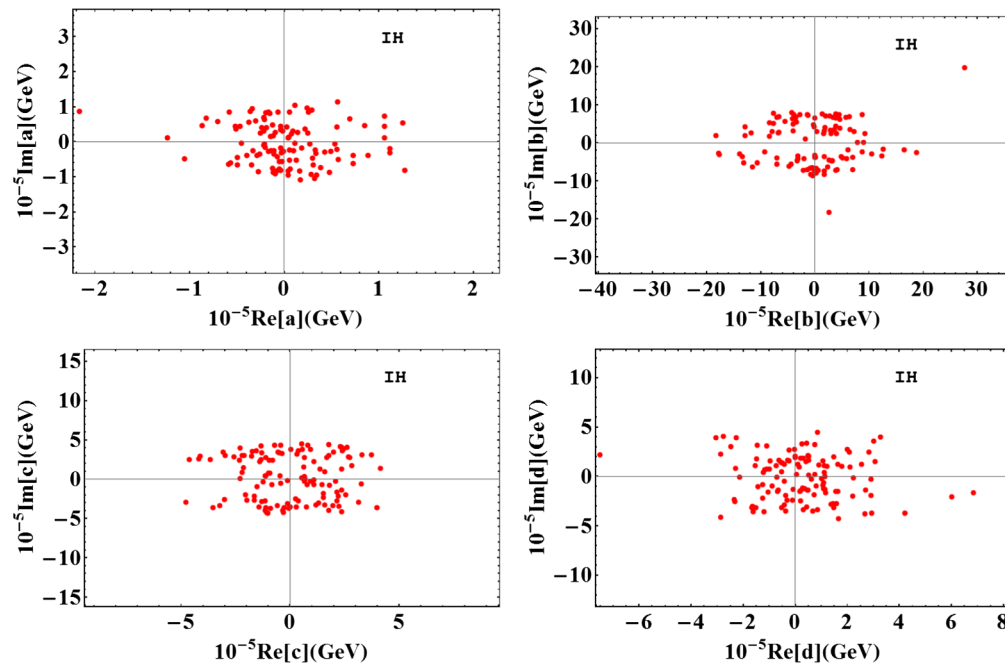

FIG. 8.   Real and imaginary parts of the model parameters for inverted hierarchy with the most general structure of the mass matrix discussed in the text.

of these correlation plots are similar the ones for normal hierarchy, shown in Figs. 2 and 3. However, for inverted hierarchy, we see a preference for smaller values of lightest neutrino mass, close to 0.01 eV, away from the upper bounds set by $0\nu\beta\beta$ and cosmology data. We then show some interesting correlations between the model parameters for

inverted hierarchy with one of the Majorana CP phases in Fig. 6. This figure also shows that the requirement of satisfying correct neutrino data constrains this CP phase to a range $|\sin\alpha| < 0.5$.

We also check if there are any correlations among the known neutrino parameters in this analysis. This could
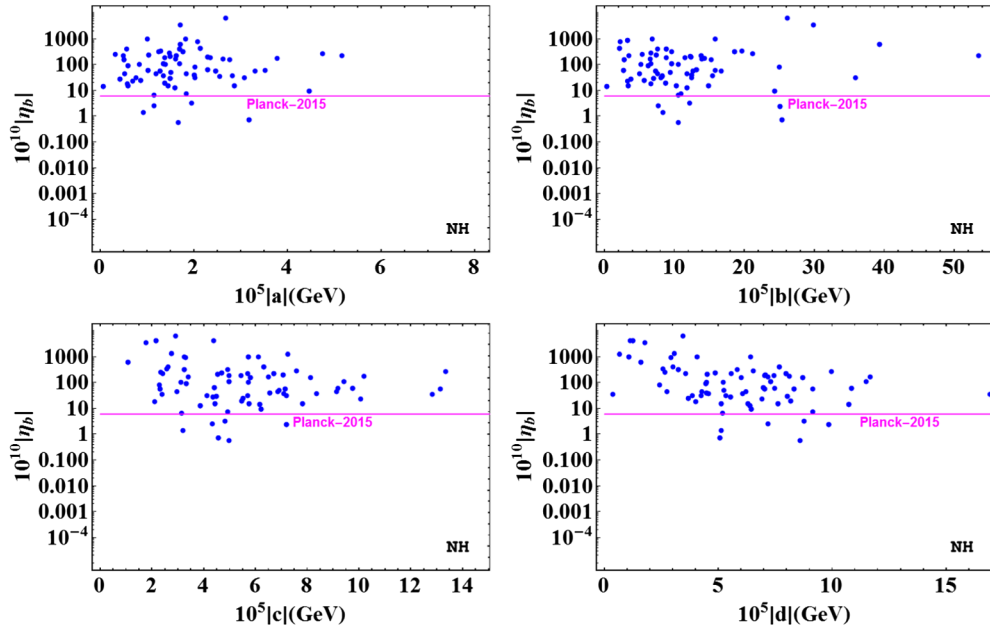
FIG. 9. Baryon asymmetry as a function of model parameters for normal hierarchy. The horizontal pink line corresponds to the Planck bound $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$ [10].
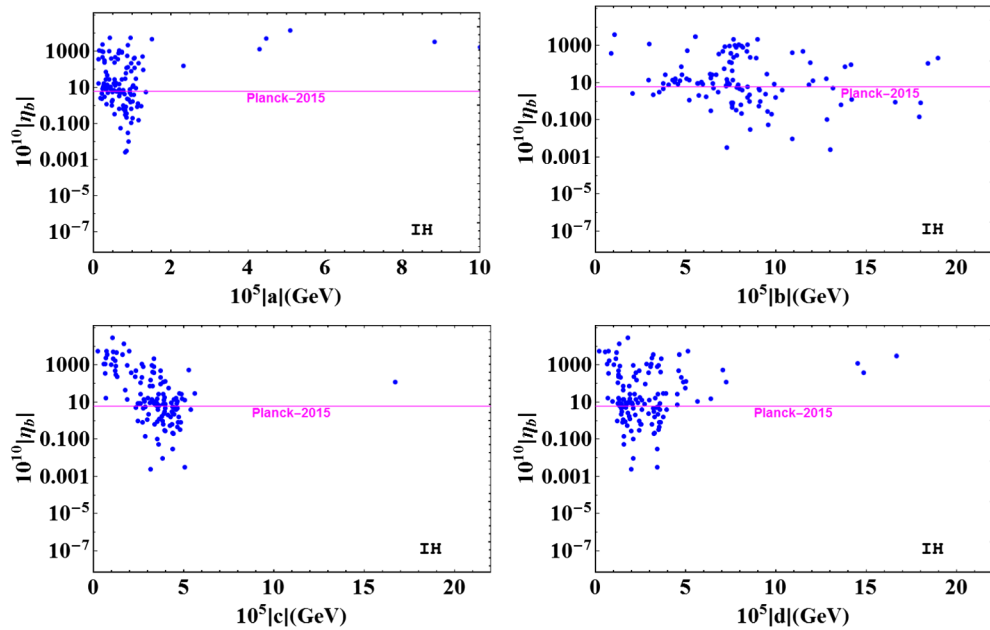

FIG. 10. Baryon asymmetry as a function of model parameters for inverted hierarchy. The horizontal pink line corresponds to the Planck bound $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$ [10].

arise due to the fact that there are only four parameters a, b, c, d that we are solving for by using more numbers of input parameters, leading to an overconstrained system. However, we did not find any such correlations between the known neutrino parameters. This is primarily due to the fact that the model parameters a, b, c, d are in general complex and hence they represent a set of eight real parameters. We show their real and imaginary parts separately in Figs. 7

and 8 for normal and inverted hierarchies, respectively. The imaginary parts of the model parameters are the source of CP phases in this model and hence play a crucial role in generating the leptonic asymmetries.

After finding the allowed neutrino as well as model parameters from the requirement of satisfying the latest neutrino oscillation data, we feed them to the calculation of the baryon asymmetry through resonant leptogenesis.
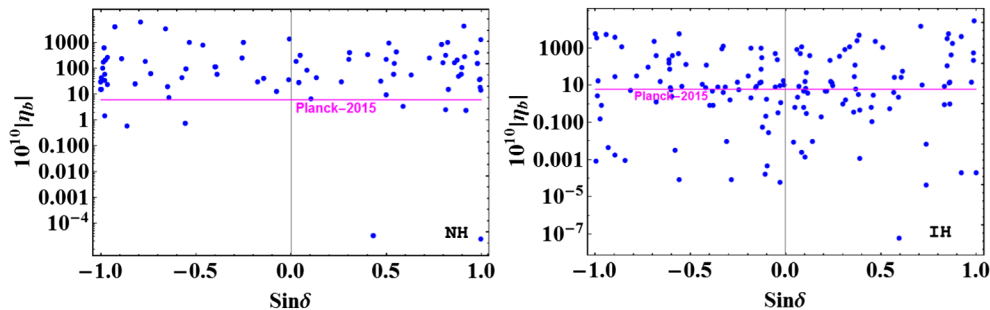
FIG. 11. Baryon asymmetry as a function of Dirac CP phase for normal and inverted hierarchy. The horizontal pink line corresponds to the Planck bound $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$ [10].
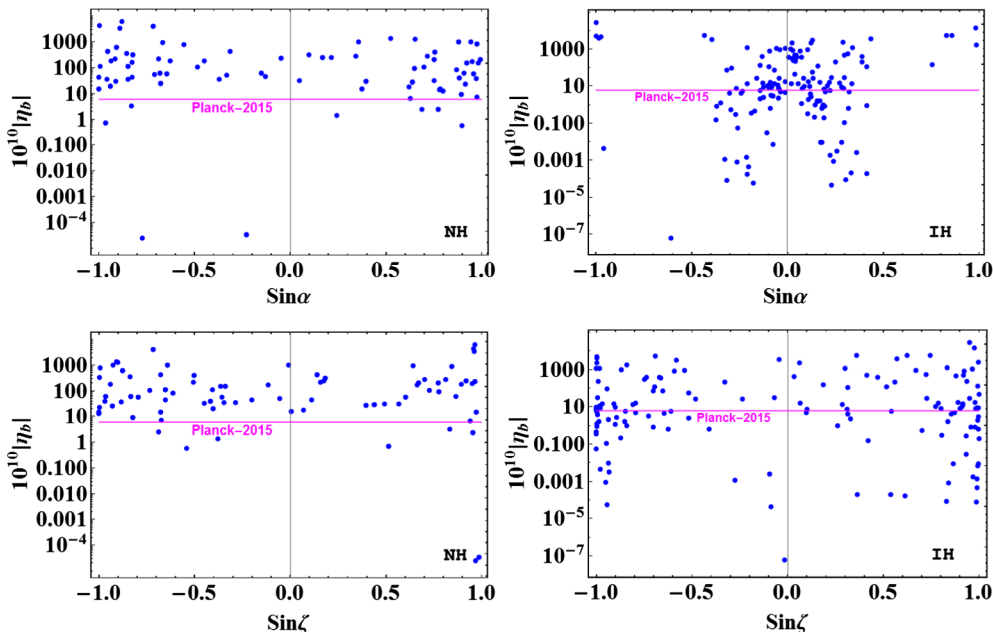

FIG. 12. Baryon asymmetry as a function of Majorana CP phases for normal and inverted hierarchy. The horizontal pink line corresponds to the Planck bound $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$ [10].

The resulting values of η_B are shown for normal hierarchy as a function of the model parameters in Fig. 9. We can see that there are several points that satisfy the Planck 2015 bound on baryon asymmetry $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$ [10]. We find more allowed parameters that satisfy the Planck bound for inverted hierarchy, as can be seen from the plots shown in Fig. 10. We also show the baryon asymmetry versus Dirac CP phase δ in Fig. 11. It can be seen from this plot that we do not see preference for any particular value of Dirac CP phase. To show the variation of η_B with Majorana CP phases, we show the plots in Fig. 12 for both normal and inverted hierarchy.

Here we note that there is a difference of around 9 orders of magnitudes between the mass splitting between the right-handed neutrinos (of keV order) and their masses (of TeV order). Although in this model we have generated such tiny mass splitting naturally, by forbidding it at leading order and

generating it only at higher orders (mass splitting term is suppressed by Λ^2 compared to the dimension four mass term without any suppression, as discussed above), we still need to make sure that these splittings are stable under quantum corrections. That is, if we generate this tiny splitting naturally at the scale of the flavor symmetry breaking $\sim\Lambda$, such splittings should not be disturbed significantly while running them down to the scale at which the lepton asymmetry is being generated, $T \sim M_R \sim \mathcal{O}(\text{TeV})$. Several earlier works discussed such radiative origin of mass splittings [41] by considering a degenerate spectrum at high energy scale [39,42]. Such splittings at the scale of leptogenesis ($T \sim M_R$) originating from renormalization group (RG) effects from a scale Λ to M_R can be estimated as

$$\Delta M_R^{\text{RG}} \approx -\frac{M_R}{8\pi^2} \ln\left(\frac{\Lambda}{M_R}\right) \text{Re}[Y_\nu^\dagger(\Lambda) Y_\nu(\Lambda)]. \quad (22)$$

TABLE III. Fields and their transformation properties under $SU(2)_L$ gauge symmetry as well as the A_4 symmetry in the $\mu - \tau$ symmetric limit.

	$\bar{L}$	e_R	μ_R	τ_R	N	H	ϕ_E	ϕ_ν	ξ	ζ	η
$SU(2)_L$	2	1	1	1	1	2	1	1	1	1	1
A_4	3	1	$1'$	$1''$	3	1	3	3	1	$1''$	1
Z_3	ω	ω^2	ω^2	ω^2	ω	1	1	ω	ω	1	ω
Z_2	1	1	1	1	-1	1	1	-1	1	-1	-1

The effective Yukawa couplings Y_ν here can be derived from the model parameters a, b, c, d by taking their ratio with the Higgs vev $v_H \sim 100$ GeV. As seen from Figs. 7 and 8, the parameters a, b, c, d can be as large as of order 10^{-4} GeV and hence the effective Yukawa couplings Y_ν are of the order of 10^{-6}. Thus, the mass splitting from RG effects can be estimated to be approximately

$$\Delta M_R^{\text{RG}} \approx (x - 3) \times 3 \times 10^{-11} \text{ GeV}$$

where $\Lambda = 10^x$ GeV, $M_R \sim \mathcal{O}(\text{TeV})$ is used. Therefore, the splitting from RG effects is usually small for TeV scale M_R and the values of Yukawa couplings we have in our model. In fact, as pointed out by [43], the pure radiative splitting scenario gives rise to vanishing lepton asymmetry at order $\mathcal{O}(Y_\nu^4)$, showing more preference to the nonminimal scenario where splitting is generated by an extra term in the Lagrangian, like the one we have in our model.

VI. $\mu - \tau$ SYMMETRIC LIMIT OF THE MODEL

In the most general case discussed above, the light neutrino mass matrix derived from the type I seesaw formula turns out to break $\mu - \tau$ symmetry resulting in nonzero θ_{13}. The antisymmetric part of the triplet multiplications $\frac{Y_a}{\Lambda}(\phi_\nu \bar{L})_{3_a} \tilde{H} N$ in the Dirac mass term is responsible for breaking the $\mu - \tau$ symmetry and in the limit of $Y_a \to 0$, the $\mu - \tau$ symmetry in the light neutrino mass matrix can be recovered. In this limit, for the simple flavon vev alignment $\langle \phi_E \rangle = (v_E, 0, 0)$, $\langle \phi_\nu \rangle = (v_\nu, v_\nu, v_\nu)$, the charged lepton mass matrix is diagonal as before whereas the Dirac neutrino mass matrix takes a simpler form given by

$$M_D = \begin{pmatrix} 2a & -a & -a \\ -a & 2a & -a \\ -a & -a & 2a \end{pmatrix}, \quad (23)$$

where $a = \frac{v_H}{\Lambda} \frac{1}{3} Y_s v_\nu$. Using the right-handed neutrino mass matrix given by (5), the light neutrino mass matrix from the type I seesaw formula can be written as

$$-M_\nu = M_D M_R^{-1} M_D^T = \frac{3a^2}{b} \begin{pmatrix} 2 & -1 & -1 \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{pmatrix}, \quad (24)$$

where $b = 2Y_N v_\xi$. This light neutrino mass matrix is clearly $\mu - \tau$ symmetric but it predicts two degenerate massive neutrinos and one massless neutrino, inconsistent with the observed mass squared differences.

We suitably modify the field content to arrive at a more realistic $\mu - \tau$ symmetric light neutrino mass matrix, as shown in Table III. In the limit of the vanishing antisymmetric part of the A_4 triplet products, the Yukawa Lagrangian for the Dirac neutrino mass terms can be written as

$$\mathcal{L}_Y \supset \frac{Y_s}{\Lambda} (\phi_\nu \bar{L})_{3_s} \tilde{H} N + \frac{Y'}{\Lambda} (\bar{L} N)_1 \tilde{H} \eta + \text{H.c.} \quad (25)$$

In this case, the Dirac neutrino mass matrix can be written as

$$M_D = \begin{pmatrix} a + 2b & -b & -b \\ -b & 2b & a - b \\ -b & a - b & 2b \end{pmatrix}, \quad (26)$$

where $b = \frac{v_H}{\Lambda} \frac{1}{3} Y_s v_\nu$, $a = \frac{v_H}{\Lambda} \frac{1}{3} Y' v_\eta$, with v_η being the vev of the flavon field η. Using the same leading order right-handed neutrino mass matrix given by (5), we can derive a $\mu - \tau$ symmetric light neutrino mass matrix using the type I seesaw formula. In fact, this gives rise to TBM-type mixing, one of the widely studied neutrino mixing frameworks that was consistent with neutrino data prior to the discovery of nonzero θ_{13}. Since the TBM can still be considered as a leading order approximation due to the smallness of θ_{13} compared to other mixing angles, such a scenario can be realistic provided a small deviation to it can be realized in order to generate nonzero θ_{13}. This can be done simply by incorporating another flavon field ψ that has the following transformation:

$$\psi(SU(2)_L:1, \qquad A_4:1', \qquad Z_3:\omega, \qquad Z_2:-1).$$

This allows one more contribution to the Dirac neutrino mass term in the form of

$$\mathcal{L}_Y \supset \frac{Y''}{\Lambda} (\bar{L} N)_{1''} \tilde{H} \psi + \text{H.c.} \quad (27)$$

After the flavon field ψ gets a vev v_ψ, this introduces a $\mu - \tau$ symmetry breaking correction to the Dirac mass term given by

$$\delta M_D = \begin{pmatrix} 0 & 0 & f \\ 0 & f & 0 \\ f & 0 & 0 \end{pmatrix}, \quad (28)$$

where $f = \frac{v_H}{\Lambda} \frac{1}{3} Y'' v_\psi$. Since this is a limiting case of the most general case based on an assumption of vanishing antisymmetric terms, we do not perform any numerical calculations for this scenario. The calculations are similar to generic A_4 models where nonzero θ_{13} is generated by considering corrections to a leading order $\mu - \tau$ symmetric light neutrino mass matrix. For example, the work [24] considered such a scenario.

VII. CONCLUSION

We have studied an extension of the standard model by discrete flavor symmetry $A_4 \times Z_3 \times Z_2$ that can simultaneously explain the correct neutrino oscillation data and the observed baryon asymmetry of the Universe. Considering a TeV scale type I seesaw we adopt the mechanism of resonant leptogenesis to generate a lepton asymmetry through out of equilibrium CP violating decay of right-handed neutrinos, which later gets converted into the required baryon asymmetry through electroweak sphalerons. The field content and its transformation under the flavor symmetry are chosen in such a way that the leading order right-handed neutrino mass matrix has a trivial structure giving a degenerate spectrum. The tiny splitting between the right-handed neutrino masses (required for resonant leptogenesis) arises through higher dimension mass terms, naturally suppressing the splitting. Because of the trivial structure of the right-handed neutrino mass matrix, the leptonic mixing arises through the nontrivial structure of the Dirac neutrino mass matrix within a type I seesaw framework. This automatically leads to a $\mu - \tau$ symmetry breaking light neutrino mass matrix due to the existence of antisymmetric terms arising from the product of two triplet representations of A_4. Although such terms vanish for the right-handed neutrino mass matrix due to the Majorana nature, they do not vanish in general for the Dirac neutrino mass matrix. Within a minimal setup, we then compare the $\mu - \tau$ symmetry breaking light neutrino mass matrix with the one constructed from light neutrino parameters and find the model parameters, while fixing the right-handed neutrino mass at 5 TeV. Since there are only four independent complex parameters of the model that can be evaluated comparing four mass matrix elements, it gives rise to two constraints due to the existence of six independent complex elements of a light neutrino mass matrix that is complex symmetric if the light neutrinos are of Majorana type. These two constraints severely restrict the allowed parameter space to a narrow range, which we evaluate numerically by doing a random scan of ten million neutrino data points in the allowed 3σ range, for both normal and inverted hierarchical patterns of light neutrino masses. Among the unknown light neutrino parameters namely, the lightest neutrino mass, one Dirac and two Majorana CP phases, we get some interesting restrictions on some of these parameters from the requirement of satisfying the correct neutrino data within the model framework.

After finding the model and neutrino parameters consistent with the basic setup, we then feed the allowed parameters to the resonant leptogenesis formulas and calculate the baryon asymmetry of the Universe. We find that both the normal and inverted hierarchical scenarios can satisfy the Planck 2015 bound on baryon asymmetry $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$ [10]. We however get more allowed points for the inverted hierarchical scenario compared to the normal one. Finally, we also briefly outline the $\mu - \tau$ symmetric limit of the model taking the approximation of vanishing antisymmetric triplet product term and a possible way to generate nonzero θ_{13} in that scenario. We however, do not perform any separate numerical calculation in this limiting scenario. We find it interesting that just trying to generate leptonic mixing through a nontrivial Dirac neutrino mass term automatically leads to broken $\mu - \tau$ symmetry, automatically generating nonzero θ_{13}. This is in fact a more economical way to generate the correct neutrino oscillation data than taking the usual route of generating the $\mu - \tau$ symmetric mass matrix at leading order followed by some next to leading order corrections responsible for generating $\theta_{13} \neq 0$, which was the usual procedure adopted after the discovery of nonzero θ_{13} in 2012. It is also interesting that the model can naturally generate the tiny mass splitting between right-handed neutrinos and generate the required baryon asymmetry through the mechanism of resonant leptogenesis. Such a TeV scale seesaw scenario can also have some other interesting implications in collider as well as rare decay experiments like lepton flavor violation, details of which can be found elsewhere. Also, such a TeV scale seesaw scenario can play a nontrivial role in restoring the electroweak vacuum stability as discussed recently by the authors of [37].

APPENDIX: DETAILS OF THE A_4 GROUP

A_4, the symmetry group of a tetrahedron, is a discrete non-Abelian group of even permutations of four objects. It has four irreducible representations: three one dimensional and one three dimensional, which are denoted by $\mathbf{1}, \mathbf{1'}, \mathbf{1''}$, and $\mathbf{3}$ respectively, being consistent with the sum of the square of the dimensions $\sum_i n_i^2 = 12$. We denote a generic permutation $(1, 2, 3, 4) \to (n_1, n_2, n_3, n_4)$ simply by $(n_1 n_2 n_3 n_4)$. The group A_4 can be generated by two basic permutations S and T given by $S = (4321), T = (2314)$. This satisfies

$$S^2 = T^3 = (ST)^3 = 1,$$

which is called a presentation of the group. Their product rules of the irreducible representations are given as

$$\mathbf{1} \otimes \mathbf{1} = \mathbf{1},$$

$$\mathbf{1'} \otimes \mathbf{1'} = \mathbf{1''},$$

$$\mathbf{1'} \otimes \mathbf{1''} = \mathbf{1},$$

$$\mathbf{1''} \otimes \mathbf{1''} = \mathbf{1'},$$

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \otimes \mathbf{1'} \otimes \mathbf{1''} \otimes \mathbf{3}_a \otimes \mathbf{3}_s,$$

where a and s in the subscript corresponds to antisymmetric and symmetric parts, respectively. Denoting two triplets as (a_1, b_1, c_1) and (a_2, b_2, c_2), respectively, their direct product can be decomposed into the direct sum mentioned above. In the S diagonal basis, the products are given as

$$\mathbf{1} \backsim a_1a_2 + b_1b_2 + c_1c_2$$

$$\mathbf{1}' \backsim a_1a_2 + \omega^2 b_1b_2 + \omega c_1c_2$$

$$\mathbf{1}'' \backsim a_1a_2 + \omega b_1b_2 + \omega^2 c_1c_2$$

$$\mathbf{3}_s \backsim (b_1c_2 + c_1b_2, c_1a_2 + a_1c_2, a_1b_2 + b_1a_2)$$

$$\mathbf{3}_a \backsim (b_1c_2 - c_1b_2, c_1a_2 - a_1c_2, a_1b_2 - b_1a_2).$$

In the T diagonal basis on the other hand, they can be written as

$$\mathbf{1} \backsim a_1a_2 + b_1c_2 + c_1b_2$$

$$\mathbf{1}' \backsim c_1c_2 + a_1b_2 + b_1a_2$$

$$\mathbf{1}'' \backsim b_1b_2 + c_1a_2 + a_1c_2$$

$$\mathbf{3}_s \backsim \frac{1}{3}(2a_1a_2 - b_1c_2 - c_1b_2, 2c_1c_2 - a_1b_2 - b_1a_2, 2b_1b_2 - a_1c_2 - c_1a_2)$$

$$\mathbf{3}_a \backsim \frac{1}{2}(b_1c_2 - c_1b_2, a_1b_2 - b_1a_2, c_1a_2 - a_1c_2).$$

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