Open-flavor strong decays of open-charm and open-bottom mesons in the $^3P_0$ model

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We provide results for the open-flavor strong decays of open-charm (D and D_s) and open-bottom (B, B_s, and B_c) mesons. The decays are calculated in a modified version of the $^3P_0$ pair-creation model, assuming harmonic oscillator wave functions. The spectra of open-charm and open-bottom mesons used in the calculations are computed within Godfrey and Isgur's relativized quark model. Quantum number assignments are also provided. Our results are compared with the existing experimental data.

I. INTRODUCTION

Since the discovery of the J/Ψ and Υ resonances in the 1970s, heavy meson physics (including the physics of charmonia [1,2], bottomonia [3,4], open-charm [5,6], and open-bottom [7–9] mesons) has been extensively studied and still is the subject of intensive theoretical and experimental research [10,11]. Recently, both the charmonium and bottomonium spectra have been enriched by the discovery of new particles [10,11]; also, the knowledge of open-charm and open-bottom mesons has improved substantially with the experimental observation of new resonances, including the $D_0^*(2400)$ [12,13], $D_1(2430)^0$ [13], and $B_1(5721)$ [14,15]. See Table I. The properties and quantum numbers of a large part of the newly observed open-charm and open-bottom mesons are still not well established. Some examples are $D_J^*(2600)$ [10,16], $D(2740)^0$ [10,17], and $B_J(5970)^0$ [10,18]. This has led to remarkable theoretical efforts to provide the experimentalists predictions for the spectrum, decay modes, and so on and also attempts to make quark model assignments for new observed states.

Important information on mesons can be extracted from their possible decay modes, including electromagnetic, weak, and strong decays. The possibility of providing a theoretical description of (open- and hidden-flavor) strong decays relies mainly on phenomenological models, because the operators that describe the strong transitions between hadrons, arising from nonperturbative QCD, are essentially unknown. In the open-flavor case, they include, among other things, "hadrodynamic" models, pair-creation models, elementary meson emission models, heavy meson effective theories, and Lagrangian approaches with heavy-quark and chiral symmetries [19–27]. For a review, see Ref. [28].

TABLE I. Experimental total decay widths and masses of D, D_s, B, and B_s mesons, extracted from the particle data group (PDG) [10]. There are no data available for B_c states above the BD threshold. States labeled by \$ are omitted from the PDG summary table [10].

State	J^P	Experimental mass (MeV)	$\Gamma_{\text{tot}}^{\text{exp}}$ (MeV)
$D^*(2007)^0$	1^-	2006.85 ± 0.05	<2.1
$D_0^*(2400)^0$	0^+	2318 ± 29	267 ± 40
$D_1(2420)^0$	1^+	2420.8 ± 0.5	31.7 ± 2.5
$D_1(2430)^0$ \$	1^+	$2427 \pm 26 \pm 25$	$384^{+107}_{-75} \pm 74$
$D_{s1}(2460)^\pm$	1^+	2459.5 ± 0.6	<3.5
$D_{s1}(2536)^\pm$	1^+	2535.10 ± 0.06	$0.92 \pm 0.03 \pm 0.04$
$D_2^*(2460)^0$	2^+	2460.7 ± 0.4	47.5 ± 1.1
$D_0(2550)^0$ \$	0^-	2564 ± 20	135 ± 17
$D_{s2}^*(2573)^\pm$	2^+	2569.1 ± 0.8	16.9 ± 0.8
$D_{s1}^*(2700)^\pm$	1^-	$2708.3^{+4.0}_{-3.4}$	120 ± 11
$D_{s1}^*(2860)^\pm$ \$	1^-	$2859 \pm 12 \pm 24$	$159 \pm 23 \pm 77$
$D_3(2750)$ \$	3^-	2763.5 ± 3.4	66 ± 5
$D_{s3}^*(2860)$ \$	3^-	$2860.5 \pm 2.6 \pm 6.5$	$53 \pm 7 \pm 7$
$B_1(5721)^0$	2^+	5726.0 ± 1.3	27.5 ± 3.4
$B_2^*(5747)^0$	2^+	5739.5 ± 0.7	24.2 ± 1.7
$B_{s1}(5830)^0$	1^+	5828.63 ± 0.27	$0.5 \pm 0.3 \pm 0.3$
$B_{s2}^*(5840)^0$	2^+	5839.85 ± 0.17	1.47 ± 0.33

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TABLE II. Pair-creation model parameters for $SU_f(4)$ and $SU_f(5)$ sectors, from Refs. [43–45].

Parameter	Value in $SU_f(4)$	Value in $SU_f(5)$
γ_0	0.510	0.732
α	0.500 GeV	0.500 GeV
r_q	0.335 fm	0.335 fm
m_n	0.330 GeV	0.330 GeV
m_s	0.550 GeV	0.550 GeV
m_c	1.50 GeV	1.50 GeV
m_b	$\cdots$	4.70 GeV

In this paper, we focus on the $^3P_0$ pair-creation model, in which the decays proceed via the production of $q\bar{q}$ pairs with vacuum quantum numbers, i.e., $J^{PC} = 0^{++}$, somewhere in the hadronic medium [29]. An important feature of the $^3P_0$ model, apart from its simplicity, is that it provides the gross features of several transitions with only one free parameter, the pair-creation strength γ_0, which is a free constant to be fitted to the experimental data. More recent studies have also discussed the possibility of substituting the constant pair-creation vertex of the model with a more refined one [30–35]. Extensively applied to the study of

TABLE III. Open-flavor strong decay widths (in MeV) for D states. Column 3 gives the values of the masses of the decaying mesons: when available, we use the experimental values from the PDG, denoted by the symbol[†] [10]; otherwise, we consider the predictions of the relativized quark model (QM) for mesons [50]. Columns 4–15 show the decay width contributions (in MeV) from various channels, such as $D\pi$, $D^*\pi$, and so on. The values of the $^3P_0$ model parameters are given in Table II. The symbol—in the table means that a certain decay is forbidden by selection rules or that the decay cannot take place because it is below threshold. The calculated mixing angles are $\theta_{1P} = 25.7°$, $\theta_{2P} = 29.4°$, $\theta_{3P} = 28.1°$, $\theta_{1D} = 38.2°$, $\theta_{2D} = 37.4°$, and $\theta_{3D} = 36.9°$.

State	J^P	Mass (MeV)	$D\pi$	$D^*\pi$	$D\rho$	$D^*\rho$	$D\eta$	$D^*\eta$	$D\omega$	$D^*\omega$	D_sK	D_s^*K	D_sK^*	$D_s^*K^*$
$D^*(2007)$ or $D_1(1^3S_1)$	1^-	2038, 2007[†]	0	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_0^*(2400)$ or $D_0(1^3P_0)$	0^+	2398, 2318[†]	66	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_1(2420)$ or $D_1(1P_1)$	1^+	2456, 2421[†]	$\cdots$	32	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_1(2430)$ or $D_1(1P_1')$	1^+	2467, 2427[†]	$\cdots$	37	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_2^*(2460)$ or $D(1^3P_2)$	2^+	2501, 2461[†]	6	2	$\cdots$	$\cdots$	0	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_0(2550)$ or $D(2^1S_0)$	0^-	2582, 2564[†]	$\cdots$	45	$\cdots$	$\cdots$	$\cdots$	0	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_1(2^3S_1)$	1^-	2645	18	36	0	$\cdots$	6	5	$\cdots$	$\cdots$	4	1	$\cdots$	$\cdots$
$D_1(1^3D_1)$	1^-	2816	20	13	13	1	10	5	4	0	6	2	$\cdots$	$\cdots$
$D_2(1D_2)$	2^-	2816	$\cdots$	25	21	6	$\cdots$	7	7	1	$\cdots$	3	$\cdots$	$\cdots$
$D_3(2750)$ or $D_3(1^3D_3)$	3^-	2833, 2764[†]	8	5	0	$\cdots$	1	0	0	$\cdots$	0	0	$\cdots$	$\cdots$
$D_2(1D_2')$	2^-	2845	$\cdots$	26	23	5	$\cdots$	8	8	1	$\cdots$	4	$\cdots$	$\cdots$
$D_1(2P_1)$	1^+	2924	$\cdots$	26	20	33	$\cdots$	7	7	11	$\cdots$	4	6	$\cdots$
$D_0(2^3P_0)$	0^+	2931	18	$\cdots$	$\cdots$	38	2	$\cdots$	$\cdots$	12	0	$\cdots$	$\cdots$	$\cdots$
$D_2(2^3P_2)$	2^+	2957	13	23	22	45	6	7	7	16	4	4	1	$\cdots$
$D_1(2P_1')$	1^+	2961	$\cdots$	21	14	29	$\cdots$	5	5	9	$\cdots$	3	8	$\cdots$
$D_0(3^1S_0)$	0^-	3067	$\cdots$	1	4	38	$\cdots$	1	1	13	$\cdots$	3	8	8
$D_1(3^3S_1)$	1^-	3111	3	2	1	31	0	0	0	11	0	1	5	15
$D_4(1^3F_4)$	4^+	3113	11	8	4	36	2	1	1	12	1	0	0	1
$D_2(1^3F_2)$	2^+	3132	10	9	11	12	5	3	4	4	2	2	1	0
$D_2(2D_2)$	2^-	3212	$\cdots$	15	15	30	$\cdots$	5	5	10	$\cdots$	3	3	5
$D_3(2^3D_3)$	3^-	3226	8	14	16	21	4	5	5	7	3	3	2	9
$D_2(2D_2')$	2^-	3248	$\cdots$	14	13	32	$\cdots$	4	4	11	$\cdots$	3	3	4
$D_1(3P_1)$	1^+	3328	$\cdots$	1	1	10	$\cdots$	0	0	3	$\cdots$	1	3	6
$D_1(2^3D_1)$	1^-	3231	7	2	0	51	1	0	0	17	0	0	1	4
$D_0(3^3P_0)$	0^+	3343	1	$\cdots$	$\cdots$	13	0	$\cdots$	$\cdots$	4	1	$\cdots$	$\cdots$	11
$D_2(3^3P_2)$	2^+	3352	2	1	0	13	0	0	0	5	0	1	1	6
$D_1(3P_1')$	1^+	3360	$\cdots$	1	1	7	$\cdots$	0	1	3	$\cdots$	1	2	8
$D_3(1^3G_3)$	3^-	3398	5	2	7	15	2	2	2	5	1	1	1	1
$D_0(4^1S_0)$	0^-	3465	$\cdots$	1	4	11	$\cdots$	1	1	4	$\cdots$	1	1	0
$D_4(2^3F_4)$	4^+	3466	5	8	10	12	2	3	3	4	2	2	2	4
$D_2(2^3F_2)$	2^+	3490	3	1	0	38	1	0	0	12	0	0	0	5
$D_2(3D_2)$	2^-	3566	$\cdots$	1	1	3	$\cdots$	0	0	1	$\cdots$	1	1	3
$D_3(3^3D_3)$	3^-	3578	2	1	0	6	0	0	0	2	0	0	1	2
$D_2(3D_2')$	2^-	3600	$\cdots$	1	1	3	$\cdots$	0	0	1	$\cdots$	1	1	3

open-flavor strong decays of light mesons [30,36,37] and baryons [38–40], the $^3P_0$ pair-creation model has also been used to compute the decays of charmonia [41–44], bottomonia [45,46], open-charm [35,47,48], and open-bottom [49] mesons.

The aim of the present paper is to provide a classification of open-charm and open-bottom mesons in terms of their masses, calculated within Godfrey and Isgur's relativized model [50,51]; open-flavor amplitudes, evaluated within a modified version of the $^3P_0$ pair-creation model [43–45,52]; and quantum number assignments, carried out by comparing our predictions to the existing data.

As widely shown by previous quark model (QM) calculations, we expect to obtain a good overall description of the properties of these mesons, with the possible exception of states close to meson-meson decay thresholds, like $D_0^*(2400)$ and $D_{s0}^*(2317)$ [53]. Indeed, it is well known that the quenched approximation may fail for states in the

region around the opening of meson-meson decay thresholds, where it is believed that continuum-coupling effects may play an important role [43,44,54–58]. A study of these particular states in the context of coupled-channel models will be addressed in a future publication.

II. FORMALISM

A. $^3P_0$ pair-creation model

In the $^3P_0$ pair-creation model, the open-flavor strong decay of a hadron A into hadrons B and C takes place in its rest frame, via the creation of an additional $q\bar{q}$ pair characterized by $J^{PC} = 0^{++}$ quantum numbers [29,38,59]. The decay widths $A \rightarrow BC$ are calculated as [29,37,38]

$$\Gamma_{A\rightarrow BC} = \Phi_{A\rightarrow BC}(q_0)\sum_{\ell}|\langle BCq_0\ell J|T^{\dagger}|A\rangle|^2. \quad (1)$$

TABLE IV. As Table III, but for D_s mesons. The calculated mixing angles are $\theta_{1P} = 37.5°$, $\theta_{2P} = 30.4°$, $\theta_{3P} = 27.7°$, $\theta_{1D} = 38.5°$, $\theta_{2D} = 37.7°$, and $\theta_{3D} = 37.2°$. In the $1P_1 - 1P_1'$ and $D_{s3}^*(2860)$ cases, the values in parentheses are calculated by using the relativized QM predictions for the decaying meson masses.

State	J^P	Mass (MeV)	DK	D^*K	DK^*	D^*K^*	$D_s\eta'$	$D_s^*\eta'$	$D_s\phi$	$D_s^*\phi$
$D_{s1}(2460)$ or $D_{s1}(1P_1)$	1^+	2549, 2460†	$\cdots$	$\cdots$ (46)	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_{s1}(2536)$ or $D_{s1}(1P_1')$	1^+	2556, 2535†	$\cdots$	56 (54)	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_{s2}^*(2573)$ or $D_{s2}(1^3P_2)$	2^+	2591, 2569†	4	0	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_{s0}(2^1S_0)$	0^-	2675	$\cdots$	53	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_{s1}^*(2700)$ or $D_{s1}(2^3S_1)$	1^-	2735, 2708†	28	42	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_{s1}^*(2860)$ or $D_{s1}(1^3D_1)$	1^-	2898, 2859†	43	23	13	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_{s3}^*(2860)$ or $D_{s3}(1^3D_3)$	3^-	2916, 2861†	10 (14)	5 (8)	0 (1)	$\cdots$ (5)	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_{s2}(1D_2)$	2^-	2900	$\cdots$	40	31	1	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_{s2}(1D_2')$	2^-	2926	$\cdots$	43	34	1	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_{s0}(2^3P_0)$	0^+	3005	28	$\cdots$	$\cdots$	51	14	$\cdots$	$\cdots$	$\cdots$
$D_{s1}(2P_1)$	1^+	3018	$\cdots$	37	27	44	$\cdots$	$\cdots$	7	$\cdots$
$D_{s1}(2P_1')$	1^+	3038	$\cdots$	31	20	35	$\cdots$	$\cdots$	10	$\cdots$
$D_{s2}(2^3P_2)$	2^+	3049	19	33	31	64	1	$\cdots$	0	$\cdots$
$D_{s0}(3^1S_0)$	0^-	3153	$\cdots$	2	3	50	$\cdots$	6	8	3
$D_{s4}(1^3F_4)$	4^+	3190	17	11	5	60	0	0	0	0
$D_{s1}(3^3S_1)$	1^-	3194	6	3	0	39	2	5	5	11
$D_{s2}(1^3F_2)$	2^+	3208	21	17	19	18	3	1	1	4
$D_{s2}(2D_2)$	2^-	3298	$\cdots$	24	22	44	$\cdots$	3	3	5
$D_{s1}(2^3D_1)$	1^-	3306	13	3	1	74	1	2	1	4
$D_{s3}(2^3D_3)$	3^-	3311	2	20	23	29	2	1	2	10
$D_{s2}(2D_2')$	2^-	3323	$\cdots$	22	21	47	$\cdots$	3	3	5
$D_{s0}(3^3P_0)$	0^+	3412	3	$\cdots$	$\cdots$	15	2	$\cdots$	$\cdots$	12
$D_{s1}(3P_1)$	1^+	3416	$\cdots$	3	1	10	$\cdots$	3	3	7
$D_{s1}(3P_1')$	1^+	3433	$\cdots$	2	1	7	$\cdots$	3	2	8
$D_{s2}(3^3P_2)$	2^+	3439	6	3	1	14	1	2	1	6
$D_{s3}(1^3G_3)$	3^-	3469	12	12	13	27	2	1	1	1
$D_{s0}(4^1S_0)$	0^-	3544	$\cdots$	1	4	16	$\cdots$	1	1	1
$D_{s4}(2^3F_4)$	4^+	3544	7	12	15	18	2	1	2	5
$D_{s2}(2^3F_2)$	2^+	3562	8	3	2	59	0	0	0	6
$D_{s2}(3D_2)$	2^-	3650	$\cdots$	3	2	3	$\cdots$	1	1	3
$D_{s3}(3^3D_3)$	3^-	3661	5	3	1	7	0	1	0	3
$D_{s2}(3D_2')$	2^-	3672	$\cdots$	3	1	3	$\cdots$	1	1	3

The coefficient

$$\Phi_{A \to BC}(q_0) = 2\pi q_0 \frac{E_b(q_0) E_c(q_0)}{M_a}, \qquad (2)$$

depending on the relative momentum q_0 between B and C and the energies of the two decay products, $E_{b,c} = \sqrt{M_{b,c}^2 + q_0^2}$, is the phase space factor for the decay. We assume harmonic oscillator wave functions, depending on a single oscillator parameter α. The final state is characterized by the relative orbital angular momentum ℓ between B and C and a total angular momentum $\vec{J} = \vec{J}_b + \vec{J}_c + \vec{\ell}$.

Following Refs. [43–45,52,60], we introduce a few changes in the $^3P_0$ model operator, $T^\dagger$. These modifications include the substitution of the pair-creation strength, γ_0, with an effective one [34,43–45,52,60],

$$\gamma_0^{\text{eff}} = \frac{m_n}{m_i} \gamma_0, \qquad (3)$$

with $i = n$ (i.e., u or d), s, c, and b (see Table II), to suppress heavy quark pair creation. Something similar was also done in Ref. [35], though the authors used a different form for γ_0^{eff}. We also introduce a Gaussian quark form factor, with parameter r_q, because the pair of created quarks has an effective size [43–45,52,60].

The values of the pair-creation model parameters for the $SU(4)_f$ and $SU(5)_f$ sectors, reported in Table II, are extracted from Refs. [43–45]. These are the values that we use in our calculations.

B. Godfrey and Isgur's relativized quark model

The relativized quark model [50,51] is based on an effective potential, the dynamics of which is governed by a one-gluon exchange interaction at short distances plus a long-range linear confining one.

The Hamiltonian of the model is given by [50]

$$H = \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2} + V_{\text{conf}} + V_{\text{hyp}} + V_{\text{so}}, \quad (4)$$

where m_1 and m_2 are the masses of the constituent quark and antiquark; q is their relative momentum (with conjugate coordinate r); V_{conf}, V_{hyp}, and V_{so} the confining, hyperfine, and spin-orbit potentials, respectively.

The confining potential is the sum of three terms [50],

$$V_{\text{conf}} = -\left(\frac{3}{4}c + \frac{3}{4}br - \frac{\alpha_s(r)}{r}\right)\vec{F}_1 \cdot \vec{F}_2, \qquad (5)$$

with $\langle q\bar{q}|\vec{F}_1 \cdot \vec{F}_2|q\bar{q}\rangle = -\frac{4}{3}$. The first term is a constant; the second is a spin-independent linear confining one, with parameter b; and the third is a Coulomb-like interaction. The hyperfine interaction has the standard form [50]

$$V_{\text{hyp}} = -\frac{\alpha_s(r)}{m_1 m_2}\left[\frac{8\pi}{3}\vec{S}_1 \cdot \vec{S}_2 \delta^3(\vec{r})\right.$$
$$\left. + \frac{1}{r^3}\left(\frac{3\vec{S}_1 \cdot \vec{r}\vec{S}_2 \cdot \vec{r}}{r^2} - \vec{S}_1 \cdot \vec{S}_2\right)\right]\vec{F}_i \cdot \vec{F}_j. \quad (6)$$

The spin-orbit potential [50],

TABLE V. Strong decay amplitude ratios. Our $^3P_0$ model results are compared to those of heavy meson effective theories (HMET) of Refs. [23,27], Lagrangian approaches with heavy quark and chiral symmetries (HQCS) of Refs. [24], and the experimental data [10,16,61–64].

Ratio	$^3P_0$	HMET and HQCS	Exp.
$\frac{\Gamma_{D_2^*(2460) \to D\pi}}{\Gamma_{D_2^*(2460) \to D^*\pi}}$	3.0	2.29 [27], 2.27 [24]	1.5–3.0 [16,61–63]
$\frac{\Gamma_{D(2550)}^{\text{of}}}{\Gamma_{D(2600)}^{\text{of}}}$	0.64	0.85 [23]	$\cdots$
$\frac{\Gamma_{D_{s2}^*(2573) \to D^*K}}{\Gamma_{D_{s2}^*(2573) \to DK}}$	≈ 0	0.086 [24]	<0.33 [10]
$\frac{\Gamma_{D_{s1}^*(2700) \to D^*K}}{\Gamma_{D_{s1}^*(2700) \to DK}}$	1.5	0.91 [24]	$0.91 \pm 0.13 \pm 0.12$ [10]
$\frac{\Gamma_{D_{s3}^*(2860) \to D^*K}}{\Gamma_{D_{s3}^*(2860) \to DK}}$	0.50 (0.57)[a]	0.39 [24]	$1.10 \pm 0.15 \pm 0.19$ [65]
$\frac{\Gamma(B_2^*(5747) \to B^*\pi)}{\Gamma(B_2^*(5747) \to B\pi)}$	0.75	0.87 [24]	$1.0 \pm 0.5 \pm 0.8$ [10,64]
$\frac{\Gamma(B_{s2}^*(5840) \to B^*K)}{\Gamma(B_2^*(5747) \to BK)}$	≈ 0	0.07 [24]	$0.093 \pm 0.013 \pm 0.012$ [10]
$\frac{\Gamma(B_3(1^3D_3) \to B^*\pi)}{\Gamma(B_3(2^3D_3) \to B\pi)}$	1.0	0.92 [24]	$\cdots$
$\frac{\Gamma(B_{s3}(1^3D_3) \to B^*K)}{\Gamma(B_{s3}(1^3D_3) \to BK)}$	0.94	0.815 [24]	$\cdots$

[a]See the caption of Table IV.

$$V_{so} = V_{so,cm} + V_{so,tp}, \qquad (7)$$

$$V_{so,tp} = -\frac{1}{2r}\frac{\partial V_{ij,conf}}{\partial r}\left(\frac{\vec{S}_i}{m_i^2} + \frac{\vec{S}_j}{m_j^2}\right)\cdot\vec{L} \qquad (8b)$$

is the sum of two contributions, where

the Thomas-precession one.

$$V_{so,cm} = -\frac{\alpha_s(r)}{r^3}\left(\frac{1}{m_i} + \frac{1}{m_j}\right)\left(\frac{\vec{S}_i}{m_i} + \frac{\vec{S}_j}{m_j}\right)\cdot\vec{L}\vec{F}_i\cdot\vec{F}_j \qquad (8a)$$

is the color-magnetic term and

In the case of states characterized by a quark and antiquark of unequal mass, charge conjugation is not a good quantum number. Therefore, states with different spins but the same angular momentum, $|n^1 L_J\rangle$ and $|n^3 L_J\rangle$, can mix via the spin-orbit interaction. For example, this happens in the case of $^1P_1$ and $^3P_1$ states, where we consider the linear combinations

TABLE VI. As Table III, but for B mesons. The calculated mixing angles are $\theta_{1P} = 30.3°$, $\theta_{2P} = 32.3°$, $\theta_{3P} = 31.6°$, $\theta_{1D} = 39.7°$, $\theta_{2D} = 39.0°$, and $\theta_{3D} = 38.6°$. The width denoted by the symbol \$ is calculated by using the mass of the decaying meson from the Godfrey-Isgur model. If we use the experimental value of the mass and do not introduce a mixing with $B_1(1P_1')$, we get a $B^*\pi$ width of 42 MeV. In this latter case, the $B_1(1P_1) - B_1(1P_1')$ mixing is not introduced because the masses of the two mesons differ by 60 MeV, approximately.

State	J^P	Mass (MeV)	$B\pi$	$B^*\pi$	$B\rho$	$B^*\rho$	$B\eta$	$B^*\eta$	$B\omega$	$B^*\omega$	B_sK	B_s^*K	B_sK^*	$B_s^*K^*$
$B_1(5721)$ or $B_1(1P_1)$	1^+	5777, 5726†	...	55$	...	...	...	...	...	...	...	...	...	...
$B_2^*(5747)$ or $B_2(1^3P_2)$	2^+	5796, 5740†	4	3	...	...	...	...	...	...	...	...	...	...
$B_0(1^3P_0)$	0^+	5756	117	...	...	...	...	...	...	...	...	...	...	...
$B_1(1P_1')$	1^+	5784	...	73	...	...	...	...	...	...	...	...	...	...
$B_0(2^1S_0)$	0^-	5905	...	87	...	...	...	4	...	...	...	...	...	...
$B_1(2^3S_1)$	1^-	5934	30	60	...	...	7	6	...	...	3	1	...	...
$B_2(1D_2)$	2^-	6095	...	49	16	1	...	15	4	0	...	7	...	...
$B_3(1^3D_3)$	3^-	6105	16	16	0	1	1	1	0	...	0	0	...	...
$B_1(1^3D_1)$	1^-	6110	42	23	11	1	21	10	3	0	11	5	...	...
$B_2(1D_2')$	2^-	6124	...	50	17	2	...	16	5	0	...	8	...	...
$B_1(2P_1)$	1^+	6197	...	46	31	58	...	12	10	19	...	7	...	...
$B_2(2^3P_2)$	2^+	6213	22	35	21	99	9	13	7	35	6	7	...	...
$B_0(2^3P_0)$	0^+	6214	35	...	...	70	3	...	...	22	0	...	...	...
$B_1(2P_1')$	1^+	6228	...	42	29	48	...	10	10	15	...	5	...	...
$B_0(3^1S_0)$	0^-	6334	...	11	16	59	...	0	6	21	...	2	11	6
$B_1(3^3S_1)$	1^-	6355	6	9	7	53	0	0	3	19	0	1	9	16
$B_4(1^3F_4)$	4^+	6364	19	20	4	80	3	3	1	26	1	1	0	1
$B_2(1^3F_2)$	2^+	6387	20	15	23	27	10	7	8	9	5	4	1	0
$B_2(2D_2)$	2^-	6450	...	25	24	56	...	9	8	19	...	5	5	9
$B_3(2^3D_3)$	3^-	6459	12	18	26	37	6	9	8	12	4	5	2	17
$B_1(2^3D_1)$	1^-	6475	15	6	0	100	3	1	0	33	0	0	2	7
$B_2(2D_2')$	2^-	6486	...	25	22	60	...	8	7	20	...	4	6	8
$B_1(3P_1)$	1^+	6557	...	7	3	9	...	0	1	3	...	1	5	11
$B_2(3^3P_2)$	2^+	6570	6	7	0	16	1	0	0	6	0	0	3	10
$B_1(3P_1')$	1^+	6585	...	7	3	5	...	0	1	2	...	1	4	12
$B_0(3^3P_0)$	0^+	6590	6	...	...	5	0	...	...	2	0	...	...	18
$B_3(1^3G_3)$	3^-	6622	10	9	15	38	5	4	5	12	2	2	2	2
$B_4(2^3F_4)$	4^+	6679	5	9	17	22	3	5	6	7	2	3	2	7
$B_0(4^1S_0)$	0^-	6687	...	0	6	16	...	1	2	5	...	1	2	2
$B_2(2^3F_2)$	2^+	6704	9	5	12	69	2	1	0	23	0	0	0	9
$B_2(3D_2)$	2^-	6767	...	6	1	2	–	1	0	1	–	0	1	4
$B_3(3^3D_3)$	3^-	6775	5	5	1	6	1	1	0	2	0	0	1	4
$B_2(3D_2')$	2^-	6800	...	5	1	2	...	1	0	1	...	0	1	4
$B_3(2^3G_3)$	3^-	6909	5	4	2	38	1	1	0	13	0	0	0	7
$B_0(4^3P_0)$	0^+	6954	1	...	...	3	0	...	...	1	0	...	...	3
$B_4(3^3F_4)$	4^+	6966	3	4	1	3	1	1	0	1	0	0	0	2
$B_4(4^3F_4)$	4^+	7230	0	0	0	2	0	0	0	1	0	0	0	0

$$|nP\rangle = \cos\theta_{nP}|n^1P_1\rangle + \sin\theta_{nP}|n^3P_1\rangle \quad (9a)$$

and

$$|nP'\rangle = -\sin\theta_{nP}|n^1P_1\rangle + \cos\theta_{nP}|n^3P_1\rangle. \quad (9b)$$

For more details, see Refs. [50,51].

The spectrum of open-charm and open-bottom states, obtained by solving the eigenvalue problem of Eq. (4) with the values of the model paramaters of Ref. [50], is reported in Tables III–VIII, third columns.

III. RESULTS AND DISCUSSION

Below, we discuss our results of Tables III–VIII for the open-flavor strong decays of open-charm (D and D_s) and open-bottom (B, B_s and B_c) mesons. The decays are computed in the $^3P_0$ pair-creation model formalism of Sec. II A, with the values of the model parameters of Table II and Refs. [43–45]. There, the parameters of the $SU_f(4)$ and $SU_f(5)$ sectors were fitted to the charmonium and bottomonium strong decay amplitudes, respectively. When available, we calculate the amplitudes by using the experimental values of the meson masses, extracted from the particle data group (PDG) [10]; otherwise, we use the

TABLE VII. As Table III, but for B_s mesons. The calculated mixing angles are $\theta_{1P} = 39.1°$, $\theta_{2P} = 33.1°$, $\theta_{3P} = 31.6°$, $\theta_{1D} = 40.0°$, $\theta_{2D} = 39.5°$, and $\theta_{3D} = 39.1°$. In the $B_{s1}(5830)$ case, the value in brackets is calculated by using the experimental value for the mass, without mixing with $B_{s1}(1P'_1)$.

State	J^P	Mass (MeV)	BK	B^*K	BK^*	B^*K^*	$B_s\eta'$	$B_s^*\eta'$	$B_s\phi$	$B_s^*\phi$
$B_{s1}(5830)$ or $B_{s1}(1P_1)$	1^+	5857, 5829	$\cdots$	85 (30)	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s0}(1^3P_0)$	0^+	5830	208	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s2}^*(5840)$ or $B_{s2}(1^3P_2)$	2^+	5875, 5840†	1	0	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s1}(1P'_1)$	1^+	5861	$\cdots$	98	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s0}(2^1S_0)$	0^-	5985	$\cdots$	106	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s1}(2^3S_1)$	1^-	6013	46	81	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s2}(1D_2)$	2^-	6169	$\cdots$	82	3	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s3}(1^3D_3)$	3^-	6178	17	16	0	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s1}(1^3D_1)$	1^-	6181	89	47	1	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s2}(1D'_2)$	2^-	6196	$\cdots$	86	3	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s0}(2^3P_0)$	0^+	6279	52	$\cdots$	$\cdots$	71	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s1}(2P_1)$	1^+	6279	$\cdots$	70	47	81	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s2}(2^3P_2)$	2^+	6295	35	56	24	172	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s1}(2P'_1)$	1^+	6296	$\cdots$	63	49	51	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$B_{s0}(3^1S_0)$	0^-	6409	$\cdots$	16	24	92	$\cdots$	6	4	$\cdots$
$B_{s1}(3^3S_1)$	1^-	6429	10	21	10	84	5	6	6	$\cdots$
$B_{s4}(1^3F_4)$	4^+	6431	28	27	4	125	0	0	0	$\cdots$
$B_{s2}(1^3F_2)$	2^+	6453	46	34	39	38	3	1	1	0
$B_{s2}(2D_2)$	2^-	6526	$\cdots$	44	36	83	$\cdots$	6	7	9
$B_{s3}(2^3D_3)$	3^-	6534	18	29	39	53	2	1	1	20
$B_{s1}(2^3D_1)$	1^-	6542	31	13	0	145	4	3	4	6
$B_{s2}(2D'_2)$	2^-	6553	$\cdots$	43	33	87	$\cdots$	7	7	8
$B_{s1}(3P_1)$	1^+	6635	$\cdots$	16	3	10	$\cdots$	5	5	12
$B_{s0}(3^3P_0)$	0^+	6638	13	$\cdots$	$\cdots$	10	5	$\cdots$	$\cdots$	21
$B_{s2}(3^3P_2)$	2^+	6647	12	14	0	19	2	4	4	11
$B_{s1}(3P'_1)$	1^+	6650	$\cdots$	14	4	5	$\cdots$	5	5	14
$B_{s3}(1^3G_3)$	3^-	6685	26	22	31	67	4	2	2	2
$B_{s4}(2^3F_4)$	4^+	6747	7	13	26	37	3	3	3	9
$B_{s0}(4^1S_0)$	0^-	6757	$\cdots$	2	6	22	$\cdots$	2	2	2
$B_{s2}(2^3F_2)$	2^+	6768	21	13	4	110	0	1	1	11
$B_{s2}(3D_2)$	2^-	6841	$\cdots$	15	4	5	$\cdots$	2	2	5
$B_{s3}(3^3D_3)$	3^-	6848	10	12	4	7	0	1	1	6
$B_{s2}(3D'_2)$	2^-	6864	$\cdots$	14	3	5	$\cdots$	2	2	5
$B_{s0}(4^3P_0)$	0^+	6949	2	$\cdots$	$\cdots$	10	1	$\cdots$	$\cdots$	4
$B_{s3}(2^3G_3)$	3^-	6970	15	10	6	62	0	0	0	10
$B_{s4}(3^3F_4)$	4^+	7034	8	9	5	5	0	0	0	3
$B_{s4}(4^3F_4)$	4^+	7297	3	2	0	3	0	0	0	1

relativized QM predictions reported in the third columns of Tables III and IV, and VI–VIII. See also Table I, which shows the existing experimental data for the total widths of D, D_s, B, and B_s resonances. There are no data available for higher B_c resonances [10]. Our theoretical results reproduce the global trend of the PDG data [10] with a few exceptions.

In more detail, starting from the D sector, our result for the open-flavor width of the $D^*(2007)^0$, $\Gamma_{of}^{th} = 4$ keV, is compatible with the total experimental width $\Gamma_{tot}^{exp} < 2.1$ MeV [10]. A more refined prediction would require the introduction of coupled-channel effects, the mass of $D^*(2007)^0$ being very close to the $D\pi$ threshold. The same applies to $D_0^*(2400)$, where the presence of higher Fock components in the meson wave function may lower the relativized QM prediction for the mass, 2398 MeV, down to the experimental value, 2318 ± 29 MeV, and also contribute to the open-flavor amplitude. In the $D_1(2420)$ case, which should mainly decay into $D^*\pi$ with the possible chain $D^*\pi \to D\pi\pi$, our $^3P_0$ model prediction is compatible with the data, while this is not true for $D_1(2430)$, being $\Gamma_{of}^{th} \ll \Gamma_{tot}^{exp}$. Nevertheless, it is worth noting that, in this second case, the experimental error is still very large; moreover, if $D_1(2420)$ and $D_1(2430)$ are mixed by spin-orbit forces, their open-flavor widths are likely to be of the same order of magnitude. Our results for the total open-flavor widths of $D_2^*(2460)$ and $D_0(2550)$ are compatible with the present experimental data, being $\Gamma_{of}^{th} < \Gamma_{tot}^{exp}$. In the $D_2^*(2460)$ case, it is also interesting to calculate the ratio between the kinematically allowed open-flavor decays, namely $\mathcal{R}_{D_2^*(2460)}^{of} = \Gamma_{D_2^*(2460) \to D\pi}/\Gamma_{D_2^*(2460) \to D^*\pi}$. Our estimate, $\mathcal{R}_{D_2^*(2460)}^{of} \approx 3$, is close to the heavy meson

effective theory one of Ref. [27], i.e., 2.29. Our result can also be compared to the existing experimental data: $3.0 \pm 1.1 \pm 1.5$ [61], $2.2 \pm 0.7 \pm 0.6$ [62], $2.8 \pm 0.8_{-0.6}^{+0.5}$ [63], and $1.47 \pm 0.03 \pm 0.16$ [16]. Other interesting information is the ratio between the total decay widths of $D(2550)$ and $D(2600)$, under the hypothesis that $D(2600)$ has 2^3S_1 quantum numbers. Also, in this second case, our result, $\mathcal{R}_{D(2550)/D(2600)}^{of} = \Gamma_{D(2550)}^{of}/\Gamma_{D(2600)}^{of} = 0.64$, is similar to the heavy meson effective theory one of Ref. [23], $\mathcal{R}_{D(2550)/D(2600)}^{of} = 0.85$.

Moving to the D_s sector, our predictions for $D_{s1}(2460)$ are compatible with the data [10], while those for $D_{s1}(2536)$ are not. The former meson has a narrow width and mainly decays to D_s^* via photon or π^0 emission, which are normally suppressed decay modes [6]. Because of the large mass difference between $D_{s1}(2460)$ and $D_{s1}(2536)$, which cannot be explained in terms of hyperfine or spin-orbit splittings, these mesons may have exotic nature. Our results for the total widths of $D_{s2}^*(2573)$, $D_{s1}^*(2700)$, $D_{s1}^*(2860)$, and $D_{s3}^*(2860)$ are compatible with the experimental data [10], being $\Gamma_{of}^{th} < \Gamma_{tot}^{exp}$. We cannot say much on the single channels, as the PDG only provides some preliminary results for a few branching fractions, except that, in the $D_{s2}^*(2573)$ case, our predictions are compatible with $\Gamma(D^*K)/\Gamma(DK) < 0.33$ [10]. In the $D_{s3}^*(2860)$ case, we also show predictions extracted by using the relativized QM mass for the decaying meson because I) There is a large difference between experimental and calculated masses and II) the experimental data are not very reliable as, at the moment, the state is excluded from the PDG summary table [10].

Finally, we discuss our predictions for the B and B_s sectors. Our results for the total open-flavor widths of

TABLE VIII. As Table III, but for B_c mesons.

State	J^P	Mass (MeV)	BD	B^*D	BD^*	B^*D^*	B_sD_s	$B_s^*D_s$	$B_sD_s^*$	$B_s^*D_s^*$
$B_{c2}(2^3P_2)$	2^+	7164	2	⋯	⋯	⋯	⋯	⋯	⋯	⋯
$B_{c0}(3^1S_0)$	0^-	7249	⋯	107	⋯	⋯	⋯	⋯	⋯	⋯
$B_{c2}(1^3F_2)$	2^+	7269	90	29	⋯	⋯	⋯	⋯	⋯	⋯
$B_{c4}(1^3F_4)$	4^+	7271	3	1	⋯	⋯	⋯	⋯	⋯	⋯
$B_{c1}(3^3S_1)$	1^-	7272	13	64	⋯	⋯	⋯	⋯	⋯	⋯
$B_{c1}(2^3D_1)$	1^-	7365	3	2	37	27	10	⋯	⋯	⋯
$B_{c3}(2^3D_3)$	3^-	7379	46	52	15	184	0	⋯	⋯	⋯
$B_{c0}(3^3P_0)$	0^+	7454	0	–	–	157	4	⋯	⋯	⋯
$B_{c3}(1^3G_3)$	3^-	7474	93	66	45	27	5	1	⋯	⋯
$B_{c2}(3^3P_2)$	2^+	7487	10	4	7	105	4	7	0	⋯
$B_{c2}(2^3F_2)$	2^+	7565	36	13	0	133	1	3	4	1
$B_{c0}(4^1S_0)$	0^-	7567	⋯	2	23	55	⋯	2	2	14
$B_{c4}(2^3F_4)$	4^+	7568	21	34	39	52	4	3	0	6
$B_{c3}(3^3D_3)$	3^-	7669	20	20	6	19	0	1	3	5
$B_{c0}(4^3P_0)$	0^+	7740	4	⋯	⋯	22	3	⋯	⋯	1
$B_{c3}(2^3G_3)$	3^-	7743	42	25	12	104	0	0	1	9
$B_{c4}(3^3F_4)$	4^+	7834	15	19	15	12	1	0	0	6
$B_{c4}(4^3F_4)$	4^+	8077	10	12	7	6	0	0	0	3

$B_2^*(5747)$ and $B_{s2}^*(5840)$ and for the ratio $\frac{\Gamma(B_2^*(5747)\to B^*\pi)}{\Gamma(B_2^*(5747)\to B\pi)}$ are compatible with the experimental data [10,64]. By contrast, our result for the open-flavor width of $B_{s1}(5830)$ is incompatible with the data. Our prediction is very sensitive to the value of the decaying meson mass—as $B_{s1}(5830)$ is close to the B^*K threshold—and thus a few MeV mass difference can produce large deviations in the calculated decay amplitude. It is also interesting to discuss the possible quantum number assignments for $B_J(5970)^0$, the width of which is 81 ± 12 MeV [10]. Following Ref. [26], the three possible assignments are 2^3S_1, 1^3D_3, and 1^3D_1. According to our results, the most likely is 2^3S_1.

IV. SUMMARY AND CONCLUSION

We computed the open-flavor strong decays of open-charm and open-bottom mesons within a modified version of the $^3P_0$ pair-creation model [29,38].

In the $^3P_0$ model, the open-flavor decays take place in the rest frame of the initial state, via the production of a light $q\bar{q}$ pair (i.e., $q = u$, d, or s) with $^3P_0$ quantum numbers. Heavy quark pair production is heavily suppressed, as required by the phenomenology, by substituting the pair-creation strength, γ_0, with an effective one, γ_0^{eff} [34,43–45,52,60]. Moreover, the nonpointlike nature of the pair of produced quarks is taken into account by introducing a quark form factor [43–45,52,60,66–70] into the model transition operator. The values of the $^3P_0$ model parameters in the $\text{SU}_f(4)$ and $\text{SU}_f(5)$ sectors were extracted from our previous studies on $c\bar{c}$ [43,44] and $b\bar{b}$ [44,45] meson spectroscopy and decays, where they were fitted to the existing experimental data [10].

The open-charm and open-bottom meson spectra we needed in our calculation were predicted within Godfrey and Isgur's relativized quark model [50]. This is one of the most powerful tools for the study of $q\bar{q}$ meson spectroscopy and provides a description of the meson spectrum in the light, strange, $c\bar{c}$, etc., sectors with a universal set of parameters; moreover, 30 years since its formulation, it still gives a good overall description of the experimental data.

As discussed in our previous papers [43–45,58], there may be substantial deviations between the experimental values of the masses and QM predictions [50] in the case of resonances lying close to meson-meson decay thresholds. In these cases, continuum-coupling effects may be important and determine a downward energy shift for the bare meson masses, thus improving the fit to the data; coupled-channel effects may also contribute to the open-flavor amplitudes. Such resonances may have an exotic nature, such as tetraquarks, meson-meson molecules, or $q\bar{q}$ mesons plus continuum components. For example, this may be the case of $D(1^3S_1)$, $D_0^*(2400)$, and $D_{s0}^*(2317)$ [53], where QM predictions are incompatible with the present experimental data [10]. The possible interpretations for suspected exotic open-charm and open-bottom mesons will be discussed in a future paper.

In conclusion, we think that our predictions can be useful to the experimentalists in their study of the properties of open-charm and open-bottom mesons and in the search for new resonances.

APPENDIX: FLAVOR COUPLINGS IN THE $^3P_0$ PAIR-CREATION MODEL

In the following, we show how to calculate the $\text{SU}_f(5)$ flavor couplings of the $^3P_0$ pair-creation model. The $\text{SU}_f(4)$ couplings can be computed analogously.

We consider the transition $A \to BC$, where A, B, and C are quark-antiquark mesons. The $\text{SU}_f(5)$ flavor couplings can be written as the scalar product between initial, $|A(q_1\bar{q}_2)\Phi_0(q_3\bar{q}_4)\rangle$, and final, $|B(q_1\bar{q}_4)C(q_3\bar{q}_2)\rangle$, states. Here, Φ_0 is the $\text{SU}_f(5)$ flavor singlet,

$$|\Phi_0\rangle = \frac{1}{\sqrt{n_f}} \sum_{i=1}^{n_f} q_3^i \bar{q}_4^i$$
$$= \frac{1}{\sqrt{5}} (|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle + |c\bar{c}\rangle + |b\bar{b}\rangle), \quad \text{(A1)}$$

and $n_f = 5$ is the dimension of the SU_f flavor group. In general, two different diagrams can contribute to the flavor matrix element $\langle BC|A\Phi_0\rangle$: in the first one, the quark in A ends up in B, while in the second one, it ends up in C. In the majority of cases, one of these two diagrams vanishes; however, for some matrix elements, both must be taken into account [43–45,52]. Finally, the flavor matrix elements can be calculated as

$$[\langle q_1\bar{q}_4| \otimes \langle q_3\bar{q}_2|] \left[|q_1\bar{q}_2\rangle \otimes \frac{1}{\sqrt{n_f}} \sum_{i=1}^{n_f} |q_3^i\bar{q}_4^i\rangle \right]$$
$$= \frac{1}{\sqrt{n_f}} \sum_{i=1}^{n_f} [\langle q_1\bar{q}_4 q_3\bar{q}_2|q_1\bar{q}_2 q_3^i\bar{q}_4^i\rangle$$
$$+ \langle q_3\bar{q}_2 q_1\bar{q}_4|q_1\bar{q}_2 q_3^i\bar{q}_4^i\rangle]. \quad \text{(A2)}$$

For an example, we calculate the $B^0 \to B^0\pi^0$ flavor coupling. The flavor matrix element can be written as

$$\langle B^0\pi^0|B^0\Phi_0\rangle_{\text{flavor}} = -\frac{1}{\sqrt{10}} [\langle d\bar{b}d\bar{d}|d\bar{b}d\bar{d}\rangle + \langle d\bar{d}d\bar{b}|d\bar{b}d\bar{d}\rangle$$
$$- \langle d\bar{b}u\bar{u}|d\bar{b}u\bar{u}\rangle + \cdots]$$
$$= -\frac{1}{\sqrt{10}}. \quad \text{(A3)}$$

The only surviving contribution in Eq. (A3) is $\langle d\bar{d}d\bar{b}|d\bar{b}d\bar{d}\rangle$; the others, like $\langle d\bar{b}d\bar{d}|d\bar{b}d\bar{d}\rangle$ or $\langle d\bar{b}u\bar{u}|d\bar{b}u\bar{u}\rangle$, are null [see Eq. (A2)]. In conclusion, after dividing Eq. (A3) by the corresponding SU(2) (isospin) Clebsch-Gordan coefficient, we get

$$\langle B\pi|B\Phi_0\rangle_{\text{flavor}} = -\sqrt{\frac{3}{10}}. \quad \text{(A4)}$$

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