Perturbative QCD analysis of exclusive processes $e^+e^- \to VP$ and $e^+e^- \to TP$

Cai-Dian Lü,[1,2,*] Wei Wang,[3,†] Ye Xing,[3,‡] and Qi-An Zhang[1,2,§]

[1]*Institute of High Energy Physics, Chinese Academy of Science, Beijing 100049, China*
[2]*School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China*
[3]*INPAC, Shanghai Key Laboratory for Particle Physics and Cosmology, MOE Key Laboratory for Particle Physics, Astrophysics and Cosmology School of Physics and Astronomy, Shanghai JiaoTong University, Shanghai 200240, China*

(Received 3 March 2018; published 12 June 2018)

We study the $e^+e^- \to VP$ and $e^+e^- \to TP$ processes in the perturbative QCD approach based on k_T factorization, where the P, V and T denotes a light pseudoscalar, vector, and tensor meson, respectively. We point out in the case of $e^+e^- \to TP$ transition due to charge conjugation invariance, only three channels are allowed: $e^+e^- \to a_2^{\pm}\pi^{\mp}$, $e^+e^- \to K_2^{*\pm}K^{\mp}$ and the V-spin suppressed $e^+e^- \to K_2^{*0}\bar{K}^0 + \bar{K}_2^{*0}K^0$. Cross sections of $e^+e^- \to VP$ and $e^+e^- \to TP$ at $\sqrt{s} = 3.67$ GeV and $\sqrt{s} = 10.58$ GeV are calculated and the invariant mass dependence is found to favor the $1/s^4$ power law. Most of our theoretical results are consistent with the available experimental data and other predictions can be tested at the ongoing BESIII and forthcoming Belle-II experiments.

DOI: 10.1103/PhysRevD.97.114016

I. INTRODUCTION

The exclusive processes of e^+e^- annihilating into two mesons provide an opportunity to investigate various time-like meson form factors. The form factor dependence on the collision energy $\sqrt{s}$ sheds light on the structure of partonic constituents in the hadron [1,2]. It means that these processes can be used to extract the relevant information on the structure of hadrons in terms of fundamental quark and gluon degrees of freedom. Another reason to study the e^+e^- process is its similarity with annihilation contributions in charmless B decays. In two-body charmless B decays, annihilation diagrams are power-suppressed. However it has been observed that in quite a few decay modes annihilations are rather important [3–5]. Large annihilation diagrams will very presumably give considerable strong phases and as a consequence sizable CP asymmetries are induced [6,7]. This fact has an important impact in the CP violation studies of B meson decays. The $e^+e^- \to VP$, TP processes, where the P, V, and T denotes a light pseudoscalar, vector, and tensor meson, respectively, have the topology with annihilation diagrams in B decays, and thus they can provide an ideal laboratory to isolate power correction effects.

It is anticipated that hard exclusive processes with hadrons involve both perturbative and nonperturbative strong interactions. Factorization, if it exists, allows one to handle the perturbative and nonperturbative contributions separately. The short-distance hard kernels can be calculated perturbatively. With the nonperturbative inputs determined from other sources, hard exclusive processes provide an effective way to explore the factorization scheme. The factorization theorem ensures that a physical amplitude can be expressed as a convolution of hard scattering kernels and hadron distribution amplitudes. However if one directly applies the collinear factorization to the $e^+e^- \to VP$, TP, the amplitude diverges in the end point region $x \to 0$. Here x is the momentum fraction of the involved quark.

A modified perturbative QCD approach based on k_T factorization, called PQCD approach for brevity, is proposed [6–11] and has been successfully applied to many reactions [5,12–27]. In this approach, the transverse momentum of partons in the meson is kept to kill endpoint divergences. Then the physical amplitude is written as a convolution of the universal nonperturbative hadronic wave functions and hard kernels in both longitudinal and transverse directions. Double logarithms, arising from the overlap of the soft and collinear divergence, can be resumed into Sudakov factor, while single logarithms from ultraviolet divergences can be

*lucd@ihep.ac.cn
†wei.wang@sjtu.edu.cn
‡xingye_guang@sjtu.edu.cn
§zhangqa@ihep.ac.cn

handled by renormalization group equation (RGE). With Sudakov factor taken into account, the applicability of perturbative QCD can be brought down to a few GeV. In this work, we will study the $e^+e^- \to VP$ and $e^+e^- \to TP$ in the perturbative QCD (PQCD) approach [6–11] based on k_T factorization.

The rest of this paper is organized as follows. In Sec. II, we first collect the input parameters including decay constants and light-cone wave functions. Then we present the PQCD framework and give factorization formulas for the timelike form factors. Numerical results and detailed discussions are presented in Sec. III. The last section contains the conclusion.

II. PERTURBATIVE QCD CALCULATION

A. Notations

We consider the $e^+e^- \to V(T)P$, in which $V(T)$ is a vector (tensor) meson with momentum P_1 and polarization vector ϵ_μ (polarization tensor $\epsilon_{\mu\nu}$), and P denotes a pseudoscalar meson with momentum P_2 in the center of mass frame. The collision energy is denoted as $Q = \sqrt{s}$. In the standard model, such processes proceed through a virtual photon or a Z^0 boson. At low energy with $\sqrt{s} \sim$ a few GeV, the amplitude is dominated by a photon. In this case the hadron amplitude is parameterized in terms of a form factor:

$$\langle V(P_1,\epsilon_T)P(P_2)|j^{\mathrm{em}}_\mu|0\rangle = F_{\mathrm{VP}}(s)\epsilon_{\mu\nu\alpha\beta}\epsilon^\nu_T P^\alpha_1 P^\beta_2. \quad (1)$$

Notice that in Eq. (1) the vector meson is transversely polarized. We have adopted the convention $\epsilon^{0123} = 1$ for the Levi-Civita tensor.

For a tensor meson, its polarization tensor $\epsilon_{\mu\nu}$ can be constructed via the polarization vector

$$\epsilon_\mu(0) = \frac{1}{m_T}(|\vec{P}_T|, 0, 0, E_T),$$
$$\epsilon_\mu(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0). \quad (2)$$

Using the Clebsch-Gordan coefficients [28], one has

$$\epsilon_{\mu\nu}(\pm 2) = \epsilon_\mu(\pm)\epsilon_\nu(\pm),$$
$$\epsilon_{\mu\nu}(\pm 1) = \sqrt{\frac{1}{2}}[\epsilon_\mu(\pm)\epsilon_\nu(0) + \epsilon_\mu(0)\epsilon_\nu(\pm)],$$
$$\epsilon_{\mu\nu}(\pm 0) = \sqrt{\frac{1}{6}}[\epsilon_\mu(+)\epsilon_\nu(-) + \epsilon_\mu(-)\epsilon_\nu(+)] + \sqrt{\frac{2}{3}}\epsilon_\mu(0)\epsilon_\nu(0). \quad (3)$$

In the calculation it is convenient to introduce a new polarization vector ξ:

$$\xi_\mu(\lambda) = \frac{\epsilon_{\mu\nu}(\lambda)q^\nu}{P_1 \cdot q} m_T, \quad (4)$$

where $q = P_1 + P_2$ is the four momentum of the virtual photon and $q^2 = s$. Then Eq. (3) becomes

$$\xi_\mu(\pm 2) = 0, \qquad \xi_\mu(\pm 1) = \frac{1}{\sqrt{2}}\frac{Q^2\eta}{2m_T^2 + Q^2\eta}\epsilon_\mu(\pm),$$
$$\xi_\mu(0) = \sqrt{\frac{2}{3}}\frac{Q^2\eta}{2m_T^2 + Q^2\eta}\epsilon_\mu(0), \quad (5)$$

where $\eta = 1 - m_T^2/Q^2$, with m_T as the mass of the tensor meson. Here the mass of the pseudoscalar meson has been neglected. The new vector ξ plays a similar role with the ordinary polarization vector ϵ, regardless of some dimensionless constants.

Then like Eq. (1), one can define the TP form factor as

$$\langle T(P_1,\lambda)P(P_2)|j^{\mathrm{em}}_\mu|0\rangle = F_{\mathrm{TP}}\epsilon_{\mu\nu\alpha\beta}\xi^\nu(\lambda)P^\alpha_1 P^\beta_2, \quad (6)$$

in which the final state tensor meson is also transversely polarized.

Using the form factors in Eqs. (1), (6), one can derive the cross sections for $e^+e^- \to VP$, TP

$$\sigma(e^+e^- \to VP) = \frac{\pi\alpha^2_{\mathrm{em}}}{6}|F_{\mathrm{VP}}|^2\Phi^{3/2}(s), \quad (7)$$

$$\sigma(e^+e^- \to TP) = \frac{\pi\alpha^2_{\mathrm{em}}}{3}\left(\frac{s\eta}{2m_T^2 + s\eta}\right)^2|F_{\mathrm{TP}}|^2\Phi^{3/2}(s), \quad (8)$$

with the fine structure constant $\alpha_{\mathrm{em}} = 1/137$, and

$$\Phi(s) = \left[1 - \frac{(m_{V(T)} + m_P)^2}{s}\right]\left[1 - \frac{(m_{V(T)} - m_P)^2}{s}\right]. \quad (9)$$

B. Decay constants and light cone wave functions

Decay constants for a pseudoscalar meson and a vector meson are defined by:

$$\langle P(p)|\bar{q}_2\gamma_\mu\gamma_5 q_1|0\rangle = -if_P p_\mu, \quad (10)$$

$$\langle V(p,\epsilon)|\bar{q}_2\gamma_\mu q_1|0\rangle = f_V m_V \epsilon_\mu,$$
$$\langle V(p,\epsilon)|\bar{q}_2\sigma_{\mu\nu} q_1|0\rangle = -if^T_V(\epsilon_\mu p_\nu - \epsilon_\nu p_\mu). \quad (11)$$

Tensor meson decay constants are defined as [29]

$$\langle T(P,\lambda)|j_{\mu\nu}(0)|0\rangle = f_T m_T^2 \epsilon^{(\lambda)*}_{\mu\nu},$$
$$\langle T(P,\lambda)|j^\perp_{\mu\nu\delta}(0)|0\rangle = -if^\perp_T m_T(\epsilon^{(\lambda)*}_{\mu\delta}P_\nu - \epsilon^{(\lambda)*}_{\nu\delta}P_\mu). \quad (12)$$

The interpolating currents are chosen as

TABLE I. Decay constants of the relevant light mesons (in units of MeV).

f_π	f_K	f_ρ	f_ρ^T	f_ω	f_ω^T	f_{K^*}	$f_{K^*}^T$	f_ϕ	f_ϕ^T	f_{a_2}	$f_{a_2}^T$	$f_{K_2^*}$	$f_{K_2^*}^T$
131	160	209 ± 2	165 ± 9	195 ± 3	145 ± 10	217 ± 5	185 ± 10	231 ± 4	200 ± 10	107 ± 6	105 ± 21	118 ± 5	77 ± 14

$$j_{\mu\nu}(0)=\frac{1}{2}(\bar{q}_1(0)\gamma_\mu i\overleftrightarrow{D}_\nu q_2(0)+\bar{q}_1(0)\gamma_\nu i\overleftrightarrow{D}_\mu q_2(0)), \tag{13}$$

$$j^{\perp\dagger}_{\mu\nu\delta}(0)=\bar{q}_2(0)\sigma_{\mu\nu}i\overleftrightarrow{D}_\delta q_1(0), \tag{14}$$

with the covariant derivative $\overleftrightarrow{D}_\mu=\vec{D}_\mu-\overleftarrow{D}_\mu$ with $\vec{D}_\mu=\vec{\partial}_\mu+ig_sA^a_\alpha\lambda^a/2$ and $\overleftarrow{D}_\mu=\overleftarrow{\partial}_\mu-ig_sA^a_\alpha\lambda^a/2$.

The pseudoscalar and vector decay constants can be determined from various reactions, $\pi^-\to e^-\bar{\nu}$, $\tau^-\to(\pi^-,K^-\rho^-,K^{*-})\nu_\tau$ and $V^0\to e^+e^-$ [28]. For tensor mesons, their decay constants can be calculated in QCD sum rules [30,31] and we quote the recently updated results from Ref. [29]. Results for decay constants are collected in Table I.

The light-cone distribution amplitudes (LCDAs) are defined as matrix elements of nonlocal operators at the lightlike separations z_μ with $z^2=0$, and sandwiched between the vacuum and the meson state. The two-particle LCDAs of a pseudoscalar meson, up to twist-3 accuracy, are defined by [32]

$$\begin{aligned}&\langle P(p)|\bar{q}_{2\beta}(z)q_{1\alpha}(0)|0\rangle\\&\quad=\frac{-i}{\sqrt{2N_C}}\int_0^1 dx e^{ixp\cdot z}[\gamma_5 \not{p}\phi_P^A(x)+m_0\gamma_5\phi_P^P(x)\\&\qquad+m_0\gamma_5(\not{n}\not{v}-1)\phi_P^T(x)]_{\alpha\beta},\end{aligned} \tag{15}$$

where n, v are two lightlike vectors. The final-state P meson is moving on the n direction with v the opposite direction. x is the momentum fraction carried by the quark q_2. The chiral enhancement parameter $m_0=m_P^2/(m_{q_1}+m_{q_2})$, is used in our work as $m_0^\pi=1.4\pm0.1$ GeV, $m_0^K=1.6\pm0.1$ GeV [33,34].

We use the following form for leading twist LCDAs derived from the conformal symmetry:

$$\phi_P^A(x)=\frac{3f_P}{\sqrt{2N_C}}x(1-x)[1+a_1^PC_1^{3/2}(t)+a_2^PC_2^{3/2}(t)], \tag{16}$$

where $N_C=3$ and $t=2x-1$. $C_i^{3/2}(i=1,2)$ are Gegenbauer polynomials, with the definition

$$C_1^{3/2}(t)=3t,\qquad C_2^{3/2}(t)=\frac{3}{2}(5t^2-1). \tag{17}$$

The Gegenbauer moments at $\mu=1$ GeV are used as [33,34]:

$$a_1^\pi=0,\quad a_1^K=0.06\pm0.03,\quad a_2^{\pi,K}=0.25\pm0.15. \tag{18}$$

In this paper, we will study the collision at $\sqrt{s}=3.67$ and 10.58 GeV, and then it is plausible to adopt the asymptotic forms for twist-3 DAs for simplicity:

$$\phi_P^P(x)=\frac{f_P}{2\sqrt{2N_c}},\qquad \phi_P^T(x)=\frac{f_P}{2\sqrt{2N_C}}(1-2x). \tag{19}$$

As for the $\eta-\eta'$ mixing, we use the quark flavor basis with the mixing scheme [35,36]:

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix}=U(\phi)\begin{pmatrix}\eta_q\\ \eta_s\end{pmatrix}=\begin{pmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}\eta_q\\ \eta_s\end{pmatrix}. \tag{20}$$

The mixing angle is $\phi=39.3°\pm1.0°$ [35,36] and

$$\eta_q=\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}),\qquad \eta_s=s\bar{s}. \tag{21}$$

Their decay constants are defined as:

$$\langle 0|\bar{n}\gamma^\mu\gamma_5 n|\eta_n(P)\rangle=\frac{i}{\sqrt{2}}f_nP^\mu,\quad \langle 0|\bar{s}\gamma^\mu\gamma_5 s|\eta_s(P)\rangle=if_sP^\mu. \tag{22}$$

In the following calculation, we will assume the same wave functions for the $n\bar{n}$ and $s\bar{s}$ as the pion's wave function, except for the different decay constants [35,36] and the chiral scale parameters [37]:

$$\begin{aligned}&f_n=(1.07\pm0.02)f_\pi,\qquad f_s=(1.34\pm0.06)f_\pi,\\&m_0^n=1.07\text{ GeV},\qquad m_0^s=1.92\text{ GeV}.\end{aligned} \tag{23}$$

Similar with pseudoscalar mesons, the two-particle LCDAs for transversely polarized vector mesons up to twist-3 are parametrized as [38,39]:

$$\begin{aligned}&\langle V(p,\epsilon_T)|\bar{q}_{2\beta}(z)q_{1\alpha}(0)|0\rangle\\&\quad=\frac{1}{\sqrt{2N_C}}\int_0^1 dx e^{ixp\cdot z}[\not{\epsilon}_T\not{p}\phi_V^T(x)+m_V\not{\epsilon}_T\phi_V^v(x)\\&\qquad+m_Vi\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu\epsilon_T^\nu n^\rho v^\sigma\phi_V^p(x)]_{\alpha\beta}.\end{aligned} \tag{24}$$

The twist-2 LCDA can be expanded as:

$$\phi_V^T(x) = \frac{3f_V^T}{\sqrt{2N_C}} x(1-x)[1 + a_1^\perp C_1^{3/2}(t) + a_2^\perp C_2^{3/2}(t)], \tag{25}$$

with Gegenbauer moments at $\mu = 1$ GeV [40,41]:

$$\begin{aligned} &a_{1K^*}^\perp = 0.04 \pm 0.03, \qquad a_{1\rho}^\perp = a_{1\omega}^\perp = a_{1\phi}^\perp = 0, \\ &a_{2K^*}^\perp = 0.11 \pm 0.09, \qquad a_{2\rho}^\perp = a_{2\omega}^\perp = 0.15 \pm 0.07, \\ &a_{2\phi}^\perp = 0.06^{+0.09}_{-0.07}. \end{aligned} \tag{26}$$

As for the twist-3 LCDAs, we will also use the asymptotic forms:

$$\begin{aligned} \phi_V^v(x) &= \frac{3f_V}{8\sqrt{2N_C}}[1 + (2x-1)^2], \\ \phi_V^p(x) &= \frac{3f_V}{4\sqrt{2N_C}}(1-2x). \end{aligned} \tag{27}$$

For a generic tensor meson, the LCDAs up to twist-3 can be defined as [22]:

$$\begin{aligned} &\langle T(p, \pm 1)|\bar{q}_{2\beta}(z) q_{1\alpha}(0)|0\rangle \\ &\quad = \frac{1}{\sqrt{2N_C}} \int_0^1 e^{ixp\cdot z} \times [\not\xi_T \not p \phi_T^T(x) + m_T \not\xi_T \phi_T^V(x) \\ &\qquad + m_T i\epsilon_{\mu\nu\rho\sigma}\gamma_5 \gamma^\mu \xi_T^\nu n^\rho v^\sigma \phi_T^a(x)]_{\alpha\beta}. \end{aligned} \tag{28}$$

These LCDAs are related to the ones given in [29]:

$$\begin{aligned} &\phi_T^T(x) = \frac{f_T^T}{2\sqrt{2N_C}}\phi_\perp(x), \qquad \phi_T^V(x) = \frac{f_T}{2\sqrt{2N_C}} g_\perp^{(v)}(x), \\ &\phi_T^a(x) = \frac{f_T}{8\sqrt{2N_C}}\frac{d}{dx} g_\perp^{(a)}(x). \end{aligned} \tag{29}$$

The asymptotic forms will be used in the calculation:

$$\phi_{\parallel,\perp}(x) = 30x(1-x)(2x-1), \tag{30}$$

$$g_\perp^{(a)}(x) = 20x(1-x)(2x-1), \qquad g_\perp^{(v)}(x) = 5(2x-1)^3. \tag{31}$$

In the above, we have only discussed the longitudinal momentum distributions. It is reasonable that the transverse momentum also plays an important role. Thus we will include the transverse momentum dependent parton distributions (TMDs) of the final-state light mesons. Following Ref. [1], we assume no interference between the longitudinal and transverse distributions, and thus one can use the following Gaussian forms to factorize the wave functions [42,43]:

$$\psi(x, \mathbf{b}) = \phi(x) \times \exp\left(-\frac{b^2}{4\beta^2}\right), \tag{32}$$

$$\psi(x, \mathbf{b}) = \phi(x) \times \exp\left[-\frac{x(1-x)b^2}{4a^2}\right]. \tag{33}$$

In the above equation $\phi(x)$ is the longitudinal momentum distribution amplitude, and the exponential factor describes the transverse momentum distribution. The parameters β and a characterize the shape of the transverse momentum distributions. The parameter β is expected at the order of $\Lambda_{\rm QCD}$ and related with the root of the averaged transverse momentum square $\langle \mathbf{k}_T^2\rangle^{1/2}$. If we choose $\langle \mathbf{k}_T^2\rangle^{1/2} = 0.35$ GeV, $\beta^2 = 4\ \mathrm{GeV}^{-2}$. According to Ref. [43], the size parameter a follows $a^{-1} \simeq \sqrt{8}\pi f_M$, where f_M is the decay constant of the related hadron.

C. PQCD calculation

In the PQCD scheme, a form factor can be written as the convolution of a hard scattering kernel with universal hadron wave functions. In small-x region, the parton transverse momentum k_T is at the same order with the longitudinal momentum. Once k_T is introduced in the hard kernel, a transverse momentum dependent (TMD) wave function is requested. Then the form factor is factorized as:

$$\begin{aligned} F(Q^2) &= \int_0^1 dx_1 dx_2 \int d^2\mathbf{k_{T1}} d^2\mathbf{k_{T2}} \Phi_{M_1}(x_1, \mathbf{k_{T1}}, P_1, \mu) H(x_1, x_2, \mathbf{k_{T1}}, \mathbf{k_{T2}}, Q, \mu) \Phi_{M_2}(x_2, \mathbf{k_{T2}}, P_2, \mu) \\ &= \int_0^1 dx_1 dx_2 \int \frac{d^2\mathbf{b_1}}{(2\pi)^2}\frac{d^2\mathbf{b_2}}{(2\pi)^2} \mathcal{P}_{M_1}(x_1, \mathbf{b_1}, P_1, \mu) H(x_1, x_2, \mathbf{b_1}, \mathbf{b_2}, Q, \mu) \mathcal{P}_{M_2}(x_2, \mathbf{b_2}, P_2, \mu). \end{aligned} \tag{34}$$

Equation (34) is the Fourier form in the impact parameter b space. Here $\Phi_{M_i}(x_i, \mathbf{k}_{Ti}, P_i, \mu)$ and $\mathcal{P}_{M_i}(x_i, \mathbf{b}_i, P_i, \mu)$ are both the hadron wave functions, relying on k_T and b respectively.

Double logarithms arising from the overlap of soft and collinear divergences, can be resumed into Sudakov factor [44,45]:

$$\mathcal{P}_{M_i}(x_i, \mathbf{b}_i, P_i, \mu) = \exp[-s(x_i, b_i, Q) - s(1-x_i, b_i, Q)]\mathcal{P}_{M_i}(x_i, \mathbf{b}_i, \mu). \tag{35}$$

The Sudakov factor $s(\xi, b_i, Q)$, $\xi = x_i$ or $1 - x_i$, is given as [46,47]:

$$s(\xi,b,Q)=\frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right)+\frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}}-1\right)-\frac{A^{(1)}}{2\beta_1}(\hat{q}-\hat{b})-\frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(2\hat{b})+1}{\hat{b}}-\frac{\ln(2\hat{q})+1}{\hat{q}}\right]$$
$$-\left[\frac{A^{(2)}}{4\beta_1^2}-\frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right)+\frac{A^{(1)}\beta_2}{8\beta_1^3}[\ln^2(2\hat{q})-\ln^2(2\hat{b})], \quad (36)$$

where the notations have been used:

$$\hat{q}\equiv\ln\left[\frac{\xi Q}{\sqrt{2}\Lambda_{\mathrm{QCD}}}\right],\qquad \hat{b}\equiv\ln\left[\frac{1}{b\Lambda_{\mathrm{QCD}}}\right]. \quad (37)$$

The running coupling constant is given as

$$\frac{\alpha_s}{\pi}=\frac{1}{\beta_1\log(\mu^2/\Lambda_{\mathrm{QCD}}^2)}-\frac{\beta_2}{\beta_1^3}\frac{\ln\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}{\ln^2(\mu^2/\Lambda_{\mathrm{QCD}}^2)}, \quad (38)$$

and the coefficients $A^{(i)}$ and β_i are

$$\beta_1=\frac{33-2n_f}{12},\quad \beta_2=\frac{153-19n_f}{24},$$
$$A^{(1)}=\frac{4}{3},\quad A^{(2)}=\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{27}n_f+\frac{8}{3}\beta_1\ln\left(\frac{e^{\gamma_E}}{2}\right). \quad (39)$$

Here n_f is the number of the quark flavors and γ_E is the Euler constant.

Apart from the double logarithms, single logarithms from ultraviolet divergence emerge in the radiative corrections to both the hadronic wave functions and hard kernels. These are summed by the renormalization group (RG) method:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right]\mathcal{P}_{M_i}(x_i,\mathbf{b_i},P_i,\mu)=-2\gamma_q\mathcal{P}_{M_i}(x_i,\mathbf{b_i},P_i,\mu), \quad (40)$$

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right]H(x_1,x_2,\mathbf{b_1},\mathbf{b_2},Q,\mu)$$
$$=4\gamma_q H(x_1,x_2,\mathbf{b_1},\mathbf{b_2},Q,\mu). \quad (41)$$

Here the quark anomalous dimension is $\gamma_q=-\alpha_s/\pi$. In terms of the above equations, we can get the RG evolution of the hadronic wave functions and hard scattering amplitude as

$$\mathcal{P}_{M_i}(x_i,\mathbf{b_i},P_i,\mu)=\exp\left[-2\int_{1/b_i}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{u}))\right]$$
$$\times\bar{\mathcal{P}}_{M_i}(x_i,\mathbf{b_i},1/b_i), \quad (42)$$

$$H(x_1,x_2,\mathbf{b_1},\mathbf{b_2},Q,\mu)=\exp\left[-4\int_{\mu}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{u}))\right]$$
$$\times H(x_1,x_2,\mathbf{b_1},\mathbf{b_2},Q,t), \quad (43)$$

where t is the largest energy scale in the hard scattering. Then from Eqs. (35) and (42), the large-b behavior of $\mathcal{P}$ can be summarized as

$$\mathcal{P}_{M_i}(x_i,\mathbf{b}_i,P_i,\mu)=\exp[-S(x_i,b_i,Q,\mu)]\mathcal{P}_{M_i}(x_i,\mathbf{b}_i,1/b_i), \quad (44)$$

with

$$S(x_i,b_i,Q,\mu)=s(x_i,b_i,Q)+s(1-x_i,b_i,Q)$$
$$+2\int_{1/b_i}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})). \quad (45)$$

Furthermore, QCD loop corrections for the electromagnetic vertex can induce another type of double logarithms

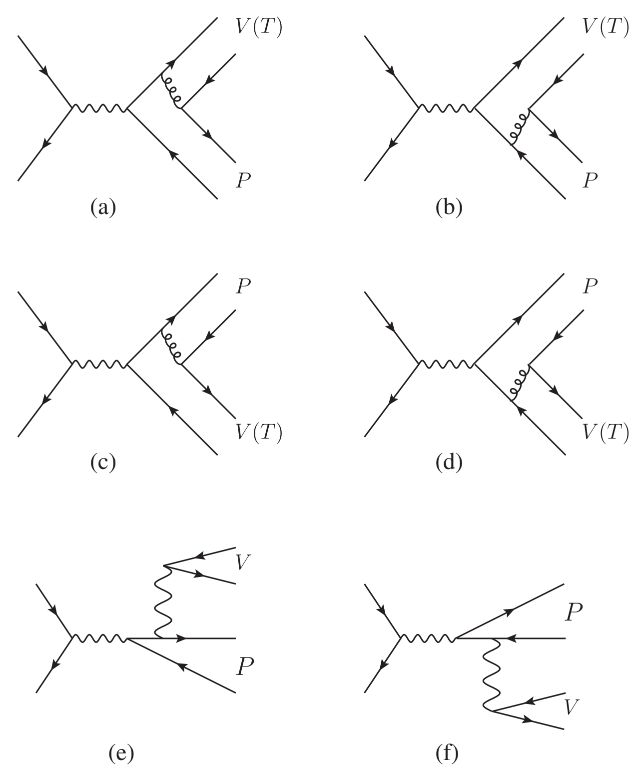

FIG. 1. Feynman diagrams for $e^+e^-\to VP$, TP. In the first four panels, a hard momentum transfer occur through the highly virtual gluon. In the last two panels, the neutral vector meson is generated by a photon.

$\alpha_s \ln^2 x_i$. They are usually factorized from the hard amplitude and resummed into the jet function $S_t(x_i)$ to further suppress the endpoint contribution. It should be pointed out that Sudakov factor from threshold resummation is universal and independent on the flavors of internal quarks, twist, and topologies of hard scattering amplitudes and the specific process [48–52]. The following approximate parametrization is proposed in [53] for the convenience of phenomenological applications

$$S_t(x, Q) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \tag{46}$$

in which the c is a parameter depending on Q. Reference [54] proposed a parabolic parametrization of the Q^2 dependence:

$$c(Q^2) = 0.04Q^2 - 0.51Q + 1.87. \tag{47}$$

The threshold resummation modifies the end point behavior of the hadron wave functions, rendering them vanish faster in this region.

Taking into account all the above ingredients, one can obtain the analytic results of the first four diagrams in Fig. 1 in k_T factorization:

$$F_a = 16\pi C_F Q \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 E(t_a) h(\bar{x}_1, x_2, b_1, b_2) S_t(x_2)\{r_1[\phi_1^{p(a)}(x_1, b_1) - \phi_1^v(x_1, b_1)]\phi_2^A(x_2, b_2)\}, \tag{48}$$

$$\begin{aligned} F_b = 16\pi C_F Q \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 E(t_b) h(x_2, \bar{x}_1, b_2, b_1) S_t(\bar{x}_1) \\ \times \{r_1 \bar{x}_1[\phi_1^{p(a)}(x_1, b_1) + \phi_1^v(x_1, b_1)]\phi_2^A(x_2, b_2) - 2r_2\phi_1^T(x_1, b_1)\phi_2^P(x_2, b_2)\}, \end{aligned} \tag{49}$$

$$\begin{aligned} F_c = -16\pi C_F Q \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 E(t_c) h(\bar{x}_2, x_1, b_2, b_1) S_t(x_1) \\ \times \{r_1 x_1[\phi_1^{p(a)}(x_1, b_1) - \phi_1^v(x_1, b_1)]\phi_2^A(x_2, b_2) + 2r_2\phi_1^T(x_1, b_1)\phi_2^P(x_2, b_2)\}, \end{aligned} \tag{50}$$

$$F_d = -16\pi C_F Q \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 E(t_d) h(x_1, \bar{x}_2, b_1, b_2) S_t(\bar{x}_2)\{r_1[\phi_1^{p(a)}(x_1, b_1) + \phi_1^v(x_1, b_1)]\phi_2^A(x_2, b_2)\}, \tag{51}$$

where $E(t_i)$ and h are given as

$$E(x_1, x_2, b_1, b_2, Q, t_i) = \alpha_s(t_i)\exp[-S_1(x_1, b_1, Q, t_i) - S_2(x_2, b_2, Q, t_i)], \tag{52}$$

$$\begin{aligned} h(x_1, x_2, b_1, b_2, Q) = \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(\sqrt{x_1 x_2}Qb_1)[\theta(b_1 - b_2)H_0^{(1)}(\sqrt{x_2}Qb_1)J_0(\sqrt{x_2}Qb_2) \\ + \theta(b_2 - b_1)H_0^{(1)}(\sqrt{x_2}Qb_2)J_0(\sqrt{x_2}Qb_1)], \end{aligned} \tag{53}$$

where J_0 and $H_0^{(1)}$ are both Bessel functions. We take $\bar{x} = 1 - x$ for short and define $r_i = m_i/Q$, with the index $i = 1$, 2 for the cases of final state meson is vector(tensor) or pseudoscalar meson. The factorization scale t is chosen as the largest mass scale involved in the hard scattering:

$$\begin{aligned} t_a &= \max(\sqrt{x_2}Q, 1/b_1, 1/b_2), \\ t_b &= \max(\sqrt{\bar{x}_1}Q, 1/b_1, 1/b_2), \\ t_c &= \max(\sqrt{x_1}Q, 1/b_1, 1/b_2), \\ t_d &= \max(\sqrt{\bar{x}_2}Q, 1/b_1, 1/b_2). \end{aligned} \tag{54}$$

If the final state meson is not a strange meson, the distribution amplitudes are completely symmetric or antisymmetric under the interchange of the quark and antiquark's momentum fraction x and $1 - x$. Then one can obtain

$$F_a(VP) = F_d(VP), \qquad F_b(VP) = F_c(VP); \tag{55}$$

$$F_a(TP) = -F_d(TP), \qquad F_b(TP) = -F_c(TP). \tag{56}$$

The contributions from a photon radiated from the interaction point into a vector meson, shown as the last two panels in Fig. 1, might be sizable. Although these diagrams are suppressed by $\alpha_{\rm em}$, they are enhanced by the almost on-shell photon propagator ($1/m_V^2$) compared with the gluon propagator in the first four diagrams ($\sim 1/s$) [55–58]. These two amplitudes can be calculated in collinear factorization due to the absence of endpoint singularities in these two diagrams. In particular, they are equal after integrating out the momentum fractions:

$$F_e = F_f = \frac{12\pi\alpha_{\rm em}^2 f_P f_V}{m_V s}(1 + a_2^P). \quad (57)$$

Finally, the form factors for the explicit channels of $e^+e^- \to VP$ process are combinations of the six amplitudes F_{a-f}:

$$F_{\rho^+\pi^-} = F_{\rho^-\pi^+} = \frac{1}{3}[F_a(\rho\pi) + F_b(\rho\pi)], \quad (58)$$

$$F_{\rho^0\pi^0} = \frac{1}{3}[F_a(\rho\pi) + F_b(\rho\pi)] + \frac{1}{6}[F_e(\rho\pi) + F_f(\rho\pi)], \quad (59)$$

$$F_{K^{*+}K^-} = \frac{2}{3}[F_a(K^*K) + F_b(K^*K)] - \frac{1}{3}[F_c(K^*K) + F_d(K^*K)], \quad (60)$$

$$F_{K^{*-}K^+} = -\frac{1}{3}[F_a(K^*K) + F_b(K^*K)] + \frac{2}{3}[F_c(K^*K) + F_d(K^*K)], \quad (61)$$

$$F_{K^{*0}\bar{K}^0} = F_{\bar{K}^{*0}K^0} = -\frac{1}{3}[F_a(K^*K) + F_b(K^*K)] - \frac{1}{3}[F_c(K^*K) + F_d(K^*K)], \quad (62)$$

$$F_{\omega\pi^0} = [F_a(\omega\pi) + F_b(\omega\pi)] + \frac{1}{18}[F_e(\omega\pi) + F_f(\omega\pi)], \quad (63)$$

$$F_{\phi\pi^0} = \frac{\sqrt{2}}{18}[F_e(\phi\pi) + F_f(\phi\pi)]. \quad (64)$$

The form factors for $e^+e^- \to V(T)\eta^{(\prime)}$ are mixtures of the η_q and η_s components:

$$F_{V(T)\eta} = \cos\theta F_{V(T)\eta_q} - \sin\theta F_{V(T)\eta_s}, \quad (65)$$

$$F_{V(T)\eta'} = \sin\theta F_{V(T)\eta_q} + \cos\theta F_{V(T)\eta_s}, \quad (66)$$

where $V = \rho^0$, ω, ϕ and

$$F_{\rho^0\eta_q} = [F_a(\rho\eta_q) + F_b(\rho\eta_q)] + \frac{5}{18}[F_e(\rho\eta_q) + F_f(\rho\eta_q)], \quad (67)$$

$$F_{\rho^0\eta_s} = -\frac{\sqrt{2}}{6}[F_e(\rho\eta_s) + F_f(\rho\eta_s)], \quad (68)$$

$$F_{\omega\eta_q} = \frac{1}{3}[F_a(\omega\eta_q) + F_b(\omega\eta_q)] + \frac{5}{54}[F_e(\omega\eta_q) + F_f(\omega\eta_q)], \quad (69)$$

$$F_{\omega\eta_s} = -\frac{\sqrt{2}}{18}[F_e(\omega\eta_s) + F_f(\omega\eta_s)], \quad (70)$$

$$F_{\phi\eta_q} = -\frac{5\sqrt{2}}{54}[F_e(\phi\eta_q) + F_f(\phi\eta_q)], \quad (71)$$

$$F_{\phi\eta_s} = -\frac{2}{3}[F_a(\phi\eta_s) + F_b(\phi\eta_s)] - \frac{1}{27}[F_e(\phi\eta_s) + F_f(\phi\eta_s)]. \quad (72)$$

Similarly, based on Eq. (56), form factors of the $e^+e^- \to TP$ channels can be written as:

$$F_{a_2^+\pi^-} = -F_{a_2^-\pi^+} = [F_a(a_2\pi) + F_b(a_2\pi)], \quad (73)$$

$$F_{K_2^{*+}K^-} = \frac{2}{3}[F_a(K_2^*K) + F_b(K_2^*K)] - \frac{1}{3}[F_c(K_2^*K) + F_d(K_2^*K)], \quad (74)$$

$$F_{K_2^{*-}K^+} = -\frac{1}{3}[F_a(K_2^*K) + F_b(K_2^*K)] + \frac{2}{3}[F_c(K_2^*K) + F_d(K_2^*K)], \quad (75)$$

$$F_{K_2^{*0}\bar{K}^0} = F_{\bar{K}_2^{*0}K^0} = -\frac{1}{3}[F_a(K_2^*K) + F_b(K_2^*K)] - \frac{1}{3}[F_c(K_2^*K) + F_d(K_2^*K)]. \quad (76)$$

The abbreviations a_2, K_2^* correspond to the tensor meson $a_2(1320)$ and $K_2^*(1430)$, respectively.

III. NUMERICAL RESULTS AND DISCUSSIONS

Using Eqs. (48)–(51), and other input parameters, we can calculate cross sections for the processes $e^+e^- \to VP$ and $e^+e^- \to TP$. In Table II, we have collected the results for cross sections at $\sqrt{s} = 3.67$ GeV, together with the experimental data from CLEO-c collaboration [59,60] (see Ref. [61] for BES measurements), and the results at $\sqrt{s} = 10.58$ GeV, together with the data measured by Belle [62] and *BABAR* [63] collaborations. As we have discussed before, three different types of transverse momentum distribution functions were used, denoted as $S1$, $S2$, and $S3$, respectively. $S1$ denotes the calculation without intrinsic transverse momentum distribution, $S2$ and $S3$ are obtained with the distributions in Eqs. (32) and (33), respectively. Theoretical errors are obtained by varying $\Lambda_{\rm QCD} = (0.25 \pm 0.05)$ GeV, and the factorization scale t from $0.75t$ to $1.25t$ (without changing $1/b_i$).

A few remarks are in order.

(i) Results at different center of mass energy $\sqrt{s}$ can be used to study the $1/s^n$ dependence of cross sections.

TABLE II. Cross sections of $e^+e^- \to VP, TP$ at $\sqrt{s} = 3.67$ GeV and $\sqrt{s} = 10.58$ GeV. $S1$ denotes the calculation without intrinsic transverse momentum distribution, $S2$ and $S3$ are obtained with the distributions as Eqs. (32) and (33). The experimental measurements from Refs. [59,60,62,63] are also shown. Theoretical errors are obtained by varying $\Lambda_{\rm QCD} = (0.25 \pm 0.05)$ GeV, and the factorization scale t from 0.75 to 1.25t (without changing $1/b_i$).

	$\sqrt{s} = 3.67$ GeV				$\sqrt{s} = 10.58$ GeV			
Channel	σ_{S1} (pb)	σ_{S2} (pb)	σ_{S3} (pb)	$\sigma_{\rm exp}$ (pb)	σ_{S1} (fb)	σ_{S2} (fb)	σ_{S3} (fb)	$\sigma_{\rm exp}$ (fb)
$\rho^\pm\pi^\mp$	6.80 ± 1.18	3.38 ± 0.53	3.95 ± 0.63	$4.8^{+1.5+0.5}_{-1.2-0.5}$	0.66 ± 0.10	0.53 ± 0.08	0.60 ± 0.09	
$\rho^0\pi^0$	3.38 ± 0.60	1.69 ± 0.27	1.99 ± 0.32	$3.1^{+1.0+0.4}_{-1.2-0.4}$	0.25 ± 0.05	0.20 ± 0.04	0.23 ± 0.04	
$K^{*\pm}K^\mp$	10.13 ± 0.91	5.27 ± 0.50	5.39 ± 0.35	$1.0^{+1.1+0.5}_{-0.7-0.5}$	1.15 ± 0.10	0.94 ± 0.08	1.02 ± 0.08	$0.18^{+0.14}_{-0.12} \pm 0.02$
$K^{*0}\bar{K}^0 + \bar{K}^{*0}K^0$	61.94 ± 13.76	31.34 ± 6.15	31.85 ± 6.25	$23.5^{+4.6+3.1}_{-3.9-3.1}$	6.65 ± 1.20	5.39 ± 0.93	5.88 ± 1.02	$7.48 \pm 0.67 \pm 0.51$
$\omega\pi^0$	24.94 ± 4.59	12.41 ± 2.08	15.18 ± 2.59	$15.2^{+2.8+1.5}_{-2.4-1.5}$	2.38 ± 0.40	1.90 ± 0.31	2.16 ± 0.35	$6.01^{+1.29}_{-1.18} \pm 0.57$
$\phi\pi^0$	1.2×10^{-4}	1.2×10^{-4}	1.2×10^{-4}	<2.2	2.2×10^{-3}	2.2×10^{-3}	2.2×10^{-3}	
$\rho^0\eta$	14.37 ± 2.10	7.21 ± 0.96	8.10 ± 1.06	$10.0^{+2.2+1.0}_{-1.9-1.0}$	1.10 ± 0.13	0.89 ± 0.11	1.03 ± 0.12	
$\rho^0\eta'$	8.22 ± 1.19	4.10 ± 0.54	4.57 ± 0.59	$2.1^{+4.7+0.2}_{-1.6-0.2}$	1.03 ± 0.11	0.83 ± 0.09	0.93 ± 0.10	
$\omega\eta$	1.31 ± 0.20	0.65 ± 0.09	0.77 ± 0.11	$2.3^{+1.8+0.5}_{-1.0-0.5}$	0.10 ± 0.01	0.081 ± 0.011	0.094 ± 0.012	
$\omega\eta'$	0.75 ± 0.11	0.37 ± 0.05	0.43 ± 0.06	<17.1	0.094 ± 0.011	0.076 ± 0.009	0.086 ± 0.010	
$\phi\eta$	17.82 ± 3.34	9.21 ± 1.51	8.23 ± 1.32	$2.1^{+1.9+0.2}_{-1.2-0.2}$	2.11 ± 0.30	1.75 ± 0.23	1.84 ± 0.25	$2.9 \pm 0.5 \pm 0.1$
$\phi\eta'$	21.97 ± 4.13	11.36 ± 1.87	10.20 ± 1.65	<12.6	2.81 ± 0.42	2.31 ± 0.33	2.47 ± 0.35	
$a_2^\pm\pi^\mp$	43.88 ± 13.98	20.34 ± 6.59	28.96 ± 8.62		6.66 ± 1.73	4.96 ± 1.30	6.06 ± 1.58	
$K_2^{*\pm}K^\mp$	60.57 ± 15.89	27.81 ± 7.45	33.81 ± 8.98		11.48 ± 2.45	8.48 ± 1.79	9.98 ± 2.15	$8.36 \pm 0.95 \pm 0.62$
$K_2^{*0}\bar{K}^0 + \bar{K}_2^{*0}K^0$	3.2×10^{-2}	1.1×10^{-2}	1.3×10^{-2}		8.8×10^{-3}	6.0×10^{-3}	7.3×10^{-3}	$1.65^{+0.86}_{-0.78} \pm 0.27$

From our results at $\sqrt{s} = 3.67$ and 10.58 GeV, the averaged value is about $n = 4.1$ for $e^+e^- \to VP$ and $n = 3.9$ for $e^+e^- \to TP$.[1] This favors the $1/s^4$ scaling, which is consistent with the constituent scaling rule [64,65]. The fitted result from experimental data is $n = 3.83 \pm 0.07$ and 3.75 ± 0.12 for $e^+e^- \to K^*(892)^0\bar{K}^0$ and $\omega\pi^0$, respectively [62].

(ii) From Table II, we can see that, cross sections for many processes are large enough to be measured, such as the $e^+e^- \to \rho\pi$, $\rho\eta$, $\omega\pi$ and $a_2^\pm\pi^\mp$ at $\sqrt{s} = 10.58$ GeV, and $e^+e^- \to a_2^\pm\pi^\mp$, $K_2^{*\pm}K^\mp$ at $\sqrt{s} = 3.67$ GeV. We suggest the experimentalists to measure these channels especially at BESIII [66] and Belle-II in future.

(iii) For the channels $e^+e^- \to K^{*\pm}K^\mp$, there are very poor measurements from CLEO collaboration [60], since the charged K^* meson is reconstructed by three–body decays: $K^{*\pm} \to K^0\pi^\pm \to 3\pi$, with large systematic uncertainties. Our results are larger than the central of experimental data. We hope the future experimental measurements can clarify this difference more clearly.

(iv) If we neglect the photon-enhanced amplitudes $F_{e,f}$, and assume the flavor SU(3) symmetry, one has the relations for cross sections: $\sigma(\omega\pi^0):\sigma(\rho^\pm\pi^\mp):\sigma(\rho^0\pi^0):\sigma(K^{*\pm}K^\mp):\sigma(K^{*0}\bar{K}^0 + \bar{K}^{*0}K^0) = 1:2/9:1/9:2/9:8/9$.

(v) At $\sqrt{s} = 3.67$ GeV, we have $\sigma(e^+e^- \to \rho^\pm\pi^\mp) = 2\sigma(e^+e^- \to \rho^0\pi^0)$, while the photon-enhanced contribution becomes more important at $\sqrt{s} = 10.58$ GeV, and the ratio $\sigma(e^+e^- \to \rho^\pm\pi^\mp)/\sigma(e^+e^- \to \rho^0\pi^0)$ is approximately 2.5.

(vi) In the SU(3) limit, we expect $\sigma(\omega\pi^0)/\sigma(K^{*0}\bar{K}^0 + \bar{K}^{*0}K^0) = 9/8 > 1$, however our calculation has indicated that the cross section $\sigma(\omega\pi^0)$ is smaller than that for $e^+e^- \to K^{*0}\bar{K}^0 + \bar{K}^{*0}K^0$ by a factor of 2 to 3. One reason arises from the fact that the decay constants $f_\pi f_\omega$ is about 30% smaller than $f_K f_{K^*}$. The chiral scale parameter m_0^K will further enhance the cross sections.

(vii) On the experiment side, the ratios R_{VP} and R_{TP} are introduced to explore the SU(3) symmetry breaking effect in the $e^+e^- \to K^*K$ and $e^+e^- \to K_2^*K$ processes, with the definition

$$R_{VP} = \frac{\sigma(e^+e^- \to K^*(892)^0\bar{K}^0)}{\sigma(e^+e^- \to K^*(892)^-K^+)},$$
$$R_{TP} = \frac{\sigma(e^+e^- \to K_2^*(1430)^0\bar{K}^0)}{\sigma(e^+e^- \to K_2^*(1430)^-K^+)}. \quad (77)$$

In the PQCD framework, this ratio can be written as

$$R = \left|\frac{(F_a+F_b)+(F_c+F_d)}{2(F_a+F_b)-(F_c+F_d)}\right|^2 = \left|\frac{1+\frac{F_c+F_d}{F_a+F_b}}{2-\frac{F_c+F_d}{F_a+F_b}}\right|^2. \quad (78)$$

In SU(3) symmetry limit, the wave functions of K, K^*, and K_2^* is symmetric or antisymmetric under the exchange of the momentum fractions of quark and antiquark, and thus the relations in Eq. (55) are obtained. Then one can drive $R_{VP} = 4$. One source of the SU(3) symmetry breaking is that the s quark is heavier than $q(=u,d)$ quark and carries more momentum in the final state meson, therefore the gluon which generates $\bar{s}s$ is harder than the $\bar{q}q$ one. In this case, the coupling constant in the $\bar{s}s$ process is smaller. Consequently, the amplitude $|F_a + F_b|$ will be smaller than $|F_c + F_d|$, and thus R_{VP} is expected larger than 4.

From Table II, one can obtain theoretical results for R_{VP}:

$$R_{VP}(\sqrt{s} = 3.67\ \text{GeV}) \simeq 5.99,$$
$$R_{VP}(\sqrt{s} = 10.58\ \text{GeV}) \simeq 5.76. \quad (79)$$

(viii) At $\sqrt{s} = 3.67$ GeV, the CLEO-c collaboration [60] has measured the ratio:

$$R_{VP}^{\text{Exp}}(\sqrt{s} = 3.67\ \text{GeV}) = 23.5^{+17.1}_{-26.1} \pm 12.2, \quad (80)$$

with very large error-bar. Its central value is significantly larger, but within the errors it is consistent with our theoretical results. Belle collaboration gives the results at $\sqrt{s} = 10.52$, 10.58, and 10.876 GeV, respectively [62]

$$R_{VP}^{\text{Exp}} > 4.3, \qquad 20.0, \qquad 5.4. \quad (81)$$

Note that in the region near 10.58 GeV, Belle result is significantly larger than our expectation, which might come from the $\Upsilon(4S)$ resonance contribution. Off the $\Upsilon(4S)$ resonance, the experimental results are consistent with our theoretical calculations. Especially for $e^+e^- \to \omega\pi^0$, results for the Born cross sections in Ref. [62] indicate that the contribution of $\Upsilon(4S)$ resonance is significant in this channel.

(ix) Due to the charge conjugation invariance, we have the relations for the $e^+e^- \to TP$ transition amplitude given in Eq. (56). Thus only three channels are allowed: $e^+e^- \to a_2^\pm\pi^\mp$, $e^+e^- \to K_2^{*\pm}K^\mp$, and $e^+e^- \to K_2^{*0}\bar{K}^0 + \bar{K}_2^{*0}K^0$.

If one further assumes V-spin symmetry, the process $e^+e^- \to K_2^{*0}\bar{K}^0 + \bar{K}_2^{*0}K^0$ is highly suppressed since $F_a + F_b \sim -(F_c + F_d)$. From Table II, one can obtain theoretical results for R_{TP}:

$$R_{TP} \lesssim 10^{-4}. \quad (82)$$

[1] We correct here the improper statement in Ref. [1].

TABLE III. Gegenbauer moments for the pion/Kaon and ρ meson.

Lattice QCD	Lattice QCD ($\mu = 1$ GeV)	QCD sum rules [33,34]
$a_2^\pi(2\ \mathrm{GeV}) = 0.1364(154)(145)$ [72]	0.1896 ± 0.0294	0.25 ± 0.15
$a_2^\pi(2\ \mathrm{GeV}) = 0.233(29)(58)$ [71]	0.3239 ± 0.0901	
$a_2^\pi(4\ \mathrm{GeV}) = 0.201(114)$ [70]	0.3519 ± 0.1996	
$a_1^K(2\ \mathrm{GeV}) = 0.061(2)(4)$ [71]	0.0791 ± 0.0058	0.06 ± 0.03
$a_1^K(4\ \mathrm{GeV}) = 0.0453(9)(29)$ [70]	0.0705 ± 0.0047	
$a_2^K(4\ \mathrm{GeV}) = 0.175(18)(47)$ [70]	0.3064 ± 0.0881	0.25 ± 0.15
$a_{2\rho}^\perp(2\ \mathrm{GeV}) = 0.101(22)$ [73]	0.1404 ± 0.0306	0.15 ± 0.07

This is consistent with the Belle data [62]:

$$R_{TP}^{\mathrm{Exp}} < 1.1, \qquad 0.4, \qquad 0.6. \tag{83}$$

(x) The theoretical uncertainties in our calculation are mainly from the uncertainties of the meson wave functions. The longitudinal distribution amplitudes in exclusive B decays will give about 10%–20% uncertainties [43]. When the transverse momentum distribution functions are introduced in Eqs. (32) and (33), the contribution from the large-b region will be suppressed. This suppression makes the PQCD approach more self-consistent. Comparing the different results in Table II, one can observe severe suppressions especially at $\sqrt{s} = 3.67$ GeV: the suppression is about 50% for $S2$ and about 40% for $S3$. Since the results depend on the explicit form of transverse momentum distribution, more accurate transverse momentum dependent wave functions and more experimental results would be valuable.

(xi) In this calculation, we have limited ourselves to the leading-order accuracy. The next-to-leading order (NLO) calculation is complicated [67–69] that will be presented in a future publication. As an estimation of the size of the NLO contribution, we vary Λ_{QCD} and the factorization scale t in Eq. (54): $\Lambda_{\mathrm{QCD}} = (0.25 \pm 0.05)$ GeV, and changing the hard scale t from 0.75 to $1.25t$ (without changing $1/b_i$). We find that our results are not sensitive to these variations. It implies that the NLO contributions are presumably not very large.

(xii) In this calculation, we have used the LCDAs derived from the QCD sum rules, while there have been Lattice QCD calculations of the Gegenbauer moments of the LCDAs for the light pion/kaon and ρ mesons [70–73]. These results are collected in Table III. We evolve the lattice QCD results to $\mu = 1$ GeV, using the renormalization group equation for the Gegenbauer moments a_n:

$$a_n(\mu) = a_n(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{(\gamma_n-\gamma_0)/\beta_0}, \tag{84}$$

with $\beta_0 = 11 - \frac{2}{3}N_f$, and one-loop anomalous dimension

$$\gamma_n = C_F\left[1 - \frac{2}{(n+1)(n+2)} + 4\sum_{j=2}^{n+1}\frac{1}{j}\right]. \tag{85}$$

In the above γ_0 is the anomalous dimension of the local current. As a comparison the QCD sum rules results are also given in this table. From this table, we can see that all these results are well consistent with each other. From a phenomenological viewpoint, using the Lattice QCD results for LCDAs will not induce sizable corrections, especially compared to other uncertainties. However one should in principle use the lattice QCD results when available.

(xiii) In our calculation, we have included the twist-3 light-cone distribution amplitudes, which are proportional to the meson mass or chiral scale parameter. Higher-order meson mass corrections have been neglected. In Ref. [74], a comprehensive analysis of the exclusive process $\gamma\gamma^* \to f_2$ has been presented, in which the meson mass corrections have been taken into account. On the one hand, at the energy $\sqrt{s} = 3.67$ GeV, meson mass corrections might play an important role [74], while at high energy with $\sqrt{s} = 10.58$ GeV, we expect the mass effects are less important. On the other hand, at the energy $\sqrt{s} = 3.67$ GeV, the perturbative QCD expansion in α_s may not work well either and thus sizable corrections are anticipated. To have a precise prediction for the cross section of processes at $\sqrt{s} = 3.67$ GeV must take into account all these corrections. In addition, it is not clear to us whether the k_T factorization are valid for higher power corrections. Thus using the results in this work one should bear in mind that the power corrections might be large especially at $\sqrt{s} = 3.67$ GeV.

IV. CONCLUSION

Hard exclusive processes $e^+e^- \to VP$ and $e^+e^- \to TP$ at center of mass energy $\sqrt{s} = 3.67$ and 10.58 GeV are investigated in the perturbative QCD framework in this work. For the wave functions of the light mesons involved in the factorization amplitudes, we have employed various models of transverse momentum dependence of wave functions. At the center of mass energy $\sqrt{s} = 3.67$ GeV, two different transverse momentum distribution functions can give about 50% and 40% suppressions, respectively. The value R_{VP} and R_{TP} obtained from our results are consistent with the experimental data. We found that our theoretical results favor the $1/s^4$ scaling law for the cross sections. Most of our results are consistent with the experimental data and the others can be tested at the ongoing BESIII and forthcoming Belle-ILL experiments.

ACKNOWLEDGMENTS

The authors are grateful to Jian-Ping Dai, Hsiang-nan Li, Cheng-Ping Shen, and Yu-Ming Wang for valuable discussions. This work is supported in part by National Natural Science Foundation of China under Grants No. 11575110, No. 11521505, No. 11655002, No. 11621131001, No. 11735010, Natural Science Foundation of Shanghai under Grant No. 15DZ2272100, by Shanghai Key Laboratory for Particle Physics and Cosmology, and by Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education.

[1] C. D. Lü, W. Wang, and Y. M. Wang, Exclusive processes $e^+e^- \to VP$ in k(T) factorization, Phys. Rev. D **75**, 094020 (2007).

[2] V. V. Braguta, A. K. Likhoded, and A. V. Luchinsky, Study of exclusive processes $e^+e^- \to VP$, Phys. Rev. D **78**, 074032 (2008).

[3] C. D. Lü and K. Ukai, Branching ratios of $B \to D(s)$ K decays in perturbative QCD approach, Eur. Phys. J. C **28**, 305 (2003).

[4] Y. Li and C. D. Lü, Study of pure annihilation type decays $B \to D*(s)$ K, J. Phys. G **29**, 2115 (2003).

[5] Y. Li, C. D. Lü, Z. J. Xiao, and X. Q. Yu, Branching ratio and CP asymmetry of $B(s) \to \pi^+\pi^-$ decays in the perturbative QCD approach, Phys. Rev. D **70**, 034009 (2004).

[6] Y. Y. Keum, H.-n. Li, and A. I. Sanda, Penguin enhancement and $B \to K\pi$ decays in perturbative QCD, Phys. Rev. D **63**, 054008 (2001).

[7] C. D. Lü, K. Ukai, and M. Z. Yang, Branching ratio and CP violation of $B \to \pi\pi$ decays in perturbative QCD approach, Phys. Rev. D **63**, 074009 (2001).

[8] H.-n. Li and G. F. Sterman, The perturbative pion form-factor with Sudakov suppression, Nucl. Phys. **B381**, 129 (1992).

[9] H.-n. Li and H. L. Yu, Extraction of V(ub) from Decay $B \to \pi$ Lepton Neutrino, Phys. Rev. Lett. **74**, 4388 (1995).

[10] Y. Y. Keum, H.-n. Li, and A. I. Sanda, Fat penguins and imaginary penguins in perturbative QCD, Phys. Lett. B **504**, 6 (2001).

[11] C. D. Lü and M. Z. Yang, $B \to \pi\rho$, $\pi\omega$ decays in perturbative QCD approach, Eur. Phys. J. C **23**, 275 (2002).

[12] H.-n. Li, Perturbative QCD factorization of $\pi\gamma* \to \gamma(\pi)$ and $B \to \gamma(\pi)$ lepton anti-neutrino, Phys. Rev. D **64**, 014019 (2001).

[13] M. Nagashima and H.-n. Li, k(T) factorization of exclusive processes, Phys. Rev. D **67**, 034001 (2003).

[14] M. Nagashima and H.-n. Li, Two parton twist three factorization in perturbative QCD, Eur. Phys. J. C **40**, 395 (2005).

[15] X. Liu, H. s. Wang, Z. j. Xiao, L. Guo, and C. D. Lü, Branching ratio and CP asymmetry of $B \to \rho\eta - (prime)$ decays in the perturbative QCD approach, Phys. Rev. D **73**, 074002 (2006).

[16] H. s. Wang, X. Liu, Z. j. Xiao, L. b. Guo, and C. D. Lü, Branching ratio and CP asymmetry of $B \to \pi\eta - (prime)$ decays in the perturbative QCD approach, Nucl. Phys. **B738**, 243 (2006).

[17] Y. L. Shen, W. Wang, J. Zhu, and C. D. Lü, Study of $K*_0(1430)$ and $a_0(980)$ from $B \to K*_0(1430)\pi$ and $B \to a_0(980)K$ decays, Eur. Phys. J. C **50**, 877 (2007).

[18] A. Ali, G. Kramer, Y. Li, C. D. Lü, Y. L. Shen, W. Wang, and Y. M. Wang, Charmless non-leptonic B_s decays to PP, PV and VV final states in the pQCD approach, Phys. Rev. D **76**, 074018 (2007).

[19] Z. Q. Zhang and Z. J. Xiao, NLO contributions to $B \to KK*$ decays in the pQCD approach, Eur. Phys. J. C **59**, 49 (2009).

[20] R. H. Li, C. D. Lü, W. Wang, and X. X. Wang, $B \to S$ transition form factors in the PQCD approach, Phys. Rev. D **79**, 014013 (2009).

[21] R. H. Li, C. D. Lü, and W. Wang, Transition form factors of B decays into p-wave axial-vector mesons in the perturbative QCD approach, Phys. Rev. D **79**, 034014 (2009).

[22] W. Wang, B to tensor meson form factors in the perturbative QCD approach, Phys. Rev. D **83**, 014008 (2011).

[23] Z. J. Xiao, W. F. Wang, and Y. Y. Fan, Revisiting the pure annihilation decays $B_s \to \pi^+\pi^-$ and $B^0 \to K^+K^-$: the data and the pQCD predictions, Phys. Rev. D **85**, 094003 (2012).

[24] C. S. Kim, R. H. Li, and W. Wang, $B \to DK^*_{0,2}$ decays: PQCD analysis to determine CP violation phase angle γ, Phys. Rev. D **88**, 034003 (2013).

[25] W. Bai, M. Liu, Y. Y. Fan, W. F. Wang, S. Cheng, and Z. J. Xiao, Revisiting $K\pi$ puzzle in the pQCD factorization approach, Chin. Phys. C **38**, 033101 (2014).

[26] Y. l. Zhang, S. Cheng, J. Hua, and Z. J. Xiao, Perturbative QCD factorization of $\rho\gamma^\star \to \rho$, Phys. Rev. D **93**, 036002 (2016).

[27] Z. T. Zou, A. Ali, C. D. Lü, X. Liu, and Y. Li, Improved estimates of the $B_{(s)} \to VV$ decays in perturbative QCD approach, Phys. Rev. D **91**, 054033 (2015).

[28] C. Patrignani *et al.* (Particle Data Group), Review of particle physics, Chin. Phys. C **40**, 100001 (2016).

[29] H. Y. Cheng, Y. Koike, and K. C. Yang, Two-parton light-cone distribution amplitudes of tensor mesons, Phys. Rev. D **82**, 054019 (2010).
[30] T. M. Aliev and M. A. Shifman, Old tensor mesons in QCD sum rules, Phys. Lett. **112B**, 401 (1982).
[31] T. M. Aliev, K. Azizi, and V. Bashiry, On the mass and decay constant of $K*_2(1430)$ tensor meson, J. Phys. G **37**, 025001 (2010).
[32] P. Ball, Theoretical update of pseudoscalar meson distribution amplitudes of higher twist: The nonsinglet case, J. High Energy Phys. 01 (1999) 010.
[33] P. Ball and R. Zwicky, New results on $B \to \pi, K, \eta$ decay formfactors from light-cone sum rules, Phys. Rev. D **71**, 014015 (2005).
[34] P. Ball, V. M. Braun, and A. Lenz, Higher-twist distribution amplitudes of the K meson in QCD, J. High Energy Phys. 05 (2006) 004.
[35] T. Feldmann, P. Kroll, and B. Stech, Mixing and decay constants of pseudoscalar mesons, Phys. Rev. D **58**, 114006 (1998).
[36] T. Feldmann, P. Kroll, and B. Stech, Mixing and decay constants of pseudoscalar mesons: The sequel, Phys. Lett. B **449**, 339 (1999).
[37] Y. Y. Charng, T. Kurimoto, and H.-n. Li, Gluonic contribution to $B \to \eta-(prime)$ form factors, Phys. Rev. D **74**, 074024 (2006); Erratum, Phys. Rev. D **78**, 059901 (2008).
[38] P. Ball, V. M. Braun, Y. Koike, and K. Tanaka, Higher twist distribution amplitudes of vector mesons in QCD: Formalism and twist—three distributions, Nucl. Phys. **B529**, 323 (1998).
[39] P. Ball and V. M. Braun, Higher twist distribution amplitudes of vector mesons in QCD: Twist—4 distributions and meson mass corrections, Nucl. Phys. **B543**, 201 (1999).
[40] P. Ball and R. Zwicky, $B_{d,s} \to \rho, \omega, K^*, \phi$ decay formfactors from light-cone sum rules revisited, Phys. Rev. D **71**, 014029 (2005).
[41] P. Ball and R. Zwicky, $|V_{td}/V_{ts}|$ from $B \to V\gamma$, J. High Energy Phys. 04 (2006) 046.
[42] N. G. Stefanis, W. Schroers, and H. C. Kim, Pion formfactors with improved infrared factorization, Phys. Lett. B **449**, 299 (1999).
[43] T. Kurimoto, Uncertainty in the leading order PQCD calculations of B meson decays, Phys. Rev. D **74**, 014027 (2006).
[44] J. Botts and G. F. Sterman, Hard elastic scattering in QCD: Leading behavior, Nucl. Phys. **B325**, 62 (1989).
[45] N. G. Stefanis, W. Schroers, and H. C. Kim, Analytic coupling and Sudakov effects in exclusive processes: Pion and $\gamma*\gamma \to \pi^0$ form-factors, Eur. Phys. J. C **18**, 137 (2000).
[46] F. g. Cao, T. Huang, and C. w. Luo, Reexamination of the perturbative pion form-factor with Sudakov suppression, Phys. Rev. D **52**, 5358 (1995).
[47] H.-n. Li and B. Tseng, Nonfactorizable soft gluons in nonleptonic heavy meson decays, Phys. Rev. D **57**, 443 (1998).
[48] G. F. Sterman, Soft gluon corrections to short distance hadronic cross-sections, Phys. Lett. B **179**, 281 (1986).
[49] G. F. Sterman, Summation of large corrections to short distance hadronic cross-sections, Nucl. Phys. **B281**, 310 (1987).
[50] S. Catani and L. Trentadue, Resummation of the QCD perturbative series for hard processes, Nucl. Phys. **B327**, 323 (1989).
[51] H.-n. Li, Unification of the k(T) and threshold resummations, Phys. Lett. B **454**, 328 (1999).
[52] H.-n. Li, Threshold resummation for exclusive B meson decays, Phys. Rev. D **66**, 094010 (2002).
[53] T. Kurimoto, H.-n. Li, and A. I. Sanda, Leading power contributions to $B \to \pi, \rho$ transition form-factors, Phys. Rev. D **65**, 014007 (2001).
[54] H.-n. Li and S. Mishima, Pion transition form factor in k(T) factorization, Phys. Rev. D **80**, 074024 (2009).
[55] M. Beneke, J. Rohrer, and D. Yang, Enhanced Electroweak Penguin Amplitude in $B \to VV$ Decays, Phys. Rev. Lett. **96**, 141801 (2006).
[56] C. D. Lü, Y. L. Shen, and W. Wang, Role of electromagnetic dipole operator in the electroweak penguin dominated vector meson decays of B meson, Chin. Phys. Lett. **23**, 2684 (2006).
[57] W. Wang, On the production of hidden-flavored hadronic states at high energy, Chin. Phys. C **42**, 043103 (2018).
[58] F. K. Guo, U. G. Meiner, and W. Wang, On the constituent counting rule for hard exclusive processes involving multi-quark states, Chin. Phys. C **41**, 053108 (2017).
[59] N. E. Adam *et al.* (CLEO Collaboration), Observation of 1^-0^- Final States from psi(2S) Decays and e^+e^- Annihilation, Phys. Rev. Lett. **94**, 012005 (2005).
[60] G. S. Adams *et al.* (CLEO Collaboration), Decay of the psi(3770) to light hadrons, Phys. Rev. D **73**, 012002 (2006).
[61] M. Ablikim *et al.* (BES Collaboration), Measurement of the final states omega pi0, rho eta, and rho eta prime from psi (2S) electromagnetic decays and e+e- annihilations, Phys. Rev. D **70**, 112007 (2004); Erratum, Phys. Rev. D**71**, 019901 (2005).
[62] C. P. Shen *et al.* (Belle Collaboration), Measurement of $e^+e^- \to \omega\pi^0$, $K^*(892)\bar{K}$ and $K_2^*(1430)\bar{K}$ at $\sqrt{s}$ near 10.6 GeV, Phys. Rev. D **88**, 052019 (2013).
[63] B. Aubert *et al.* (*BABAR* Collaboration), Observation of the exclusive reaction $e^+e^- \to \phi\eta$ at $\sqrt{s} = 10.58$-GeV, Phys. Rev. D **74**, 111103 (2006).
[64] G. P. Lepage and S. J. Brodsky, Exclusive processes in perturbative quantum chromodynamics, Phys. Rev. D **22**, 2157 (1980).
[65] S. J. Brodsky and G. P. Lepage, Helicity selection rules and tests of gluon spin in exclusive QCD processes, Phys. Rev. D **24**, 2848 (1981).
[66] D. M. Asner *et al.*, Physics at BES-III, Int. J. Mod. Phys. A **24**, 499 (2009).
[67] H.-n. Li, Y. L. Shen, and Y. M. Wang, Next-to-leading-order corrections to $B \to \pi$ form factors in k_T factorization, Phys. Rev. D **85**, 074004 (2012).
[68] S. Cheng, Y. Y. Fan, and Z. J. Xiao, NLO twist-3 contribution to the pion electromagnetic form factors in k_T factorization, Phys. Rev. D **89**, 054015 (2014).
[69] S. Cheng, Y. Y. Fan, X. Yu, C.-D. Lü, and Z. J. Xiao, The NLO twist-3 contributions to $B \to \pi$ form factors in k_T factorization, Phys. Rev. D **89**, 094004 (2014).

[70] V. M. Braun *et al.*, Moments of pseudoscalar meson distribution amplitudes from the lattice, Phys. Rev. D **74**, 074501 (2006).

[71] R. Arthur, P. A. Boyle, D. Brommel, M. A. Donnellan, J. M. Flynn, A. Juttner, T. D. Rae, and C. T. C. Sachrajda, Lattice results for low moments of light meson distribution amplitudes, Phys. Rev. D **83**, 074505 (2011).

[72] V. M. Braun, S. Collins, M. Gckeler, P. Prez-Rubio, A. Schfer, R. W. Schiel, and A. Sternbeck, Second moment of the pion light-cone distribution amplitude from lattice QCD, Phys. Rev. D **92**, 014504 (2015).

[73] V. M. Braun *et al.*, The ρ-meson light-cone distribution amplitudes from lattice QCD, J. High Energy Phys. 04 (2017) 082.

[74] V. M. Braun, N. Kivel, M. Strohmaier, and A. A. Vladimirov, Electroproduction of tensor mesons in QCD, J. High Energy Phys. 06 (2016) 039.