Double Collins effect in $e^+e^- \to \Lambda\bar{\Lambda}X$ and $e^+e^- \to \Lambda\pi X$ processes in a diquark spectator model

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We study the Collins function $H_1^\perp$ of the Λ hyperon, which describes the fragmentation of a transversely polarized quark into an unpolarized Λ hyperon. We calculate $H_1^\perp$ for light quarks of the Λ hyperon, in the diquark spectator model with a Gaussian form factor for the hyperon-quark-diquark vertex. The model calculation includes contributions from both the scalar diquark and vector diquark spectators. Using the model result, we estimate the azimuthal asymmetry A_{12}, which appears in the ratio of unlike-sign events to like-sign events contributed by double Collins effects, in the processes $e^+e^- \to \Lambda\bar{\Lambda}X$ and $e^+e^- \to \Lambda\pi X$. The QCD evolution effects for the half k_T moment of the Collins function and the unpolarized fragmentation function $D_1(z)$ are also included. The results show that the asymmetries are sizable and measurable at the kinematical configurations of Belle and *BABAR* experiments. We also find that the evolution effects play an important role in the phenomenological analysis.

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I. INTRODUCTION

The Collins function [1] $H_1^\perp(z, k_T^2)$ is a novel transverse momentum dependent (TMD) fragmentation function that encodes the correlation between the transverse spin of the fragmenting quark and the transverse momentum of the unpolarized final-state hadron. As a time-reversal-odd (T-odd) function, the Collins function can be served as a quark spin analyzer and can also be used to explore the nonperturbative fragmentation mechanism of hadrons. The experimental measurements of the pion Collins function came from several single transverse spin asymmetries in semi-inclusive deep inelastic scattering (SIDIS) [2–8] from the HERMES and the COMPASS Collaborations, and the azimuthal asymmetry in the e^+e^- annihilating process [9–13] from the *BABAR* and Belle Collaborations. Combining the experimental data from SIDIS and e^+e^- annihilating processes, one can extract the Collins function as well as the transversity distribution function [14–17], which makes the Collins function a useful tool to investigate the internal structure for hadrons. Recently, the azimuthal asymmetry of charged kaon pair production in e^+e^- annihilation was measured by the *BABAR* Collaboration [18], making the extraction [19] of the kaon Collins function possible. In addition, several model calculations of the Collins functions of the pion and kaon have been presented in Refs. [20–25] and used to make predictions on the physical observables [22,26,27].

Although in the past a lot of experimental data and theoretical analyses have provided information about the Collins functions for pion and kaon mesons, knowledge about the Collins function of the Λ hyperon is much more limited. Meanwhile there is increasing interest in the novel fragmentation mechanism of the Λ hyperon, as it is partly responsible to the observed spin polarization or spin transfer of the spin-$1/2$ Λ hyperon produced in the high-energy inclusive process [28–36]. A T-odd spin-dependent TMD fragmentation $D_{1T}^\perp(z, k_T^2)$, which describes the number density of a transversely polarized Λ hyperon fragmented from an unpolarized quark, is found to play an important role in this aspect and has been studied intensively [1,37–42]. As the chiral-odd partner of the fragmentation function $D_{1T}^\perp(z, k_T^2)$, the Collins function of the Λ hyperon also contains complementary information of the Λ fragmentation and can give rise to the azimuthal asymmetries in the high energy process. To understand the underlying mechanism of the transversely polarized quark fragmenting to the unpolarized Λ, we resort to a model calculation to acquire the knowledge of the corresponding nonperturbative quantity, which is the main goal of this work. For this purpose, for the first time to our knowledge, we calculate the Λ Collins function for the up, down, and strange

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quarks, using a spectator model [43,44]. The spectator model has been applied to calculate the Collins functions of the pion and kaon mesons [20], as well as the twist-3 collinear fragmentation function of the pion [45,46], with a pseudoscalar pion-quark coupling and Gaussian form factors at the pion-quark/antiquark vertex. In these cases the quark or antiquark is taken as the spectator system. The calculation presented in Ref. [20] showed that the model resulting pion Collins function is in reasonable agreement with the available parametrization [47]. Recently, the spectator model has also been extended to calculate the fragmentation function $D_{1T}^{\perp}$ of the Λ hyperon in Ref. [48]. In this case the spectator system is a diquark, and the calculation includes contributions from both the scalar diquark and vector diquark.

The Collins function can enter the description in SIDIS, e^+e^- annihilation, and inclusive hadron production in the hadron collision. To test the feasibility of measuring the Λ Collins function in experiments, we will study the unpolarized $e^+e^- \to \Lambda\bar{\Lambda}X$ and $e^+e^- \to \Lambda\pi^+X$ processes, in which only fragmentation functions are involved. In this process, the convolution of two Collins functions can generate at leading order (in the expansion of $1/Q$) an azimuthal asymmetry with a $\cos(\phi_1+\phi_2)$ or $\cos 2\phi_0$ modulation [49,50], depending on the chosen reference frame. However, hard gluon radiation $e^+e^- \to q\bar{q}g$ also gives rise to a Collins-like asymmetry [50,51], which is the dominant background contribution. Thus, to access the Λ hyperon Collins function in e^+e^- annihilation, one has to separate the false asymmetry from the true double Collins effects. To do this, we exploit the fact that QCD radiative corrections can be canceled by making ratios of the asymmetries in unlike-sign events over that in like-sign events [10,14,16]. Using this methodology, we will calculate the $\cos(\phi_1+\phi_2)$ angular dependent asymmetric ratio (denoted by A_{12}) in the processes $e^+e^- \to \Lambda\bar{\Lambda}X$ and $e^+e^- \to \Lambda\pi X$, which can be measured by the Belle and *BABAR* experiments. The asymmetry A_{12} can be expressed as the product of the half k_T moments of two Collins $H_1^{\perp(1/2)}$. We also take into account the QCD evolution effect of $H_1^{\perp(1/2)}$ as the energy scale at those experiments is much larger than the model scale.

The remaining content of this paper is organized as follows. In Sec. II, we calculate the T-odd Collins function $H_1^{\perp}$ in the diquark spectator model by including both the scalar diquark and the vector diquark spectators. The QCD evolution effect of the half k_T moment of Collins function $H_1^{\perp(1/2)}(z)$ is also studied. In Sec. III, we numerically estimate the azimuthal asymmetry A_{12} for the processes $e^+e^- \to \Lambda\bar{\Lambda}X$ and $e^+e^- \to \Lambda\pi X$ at the energy scale around the Belle and *BABAR* kinematics including the QCD evolution effects of both $H_1^{\perp(1/2)}(z)$ and $D_1(z)$. We summarize this work in Sec. IV.

II. MODEL CALCULATION OF THE COLLINS FUNCTION FOR Λ HYPERON

In this section, we calculate the Collins function $H_1^{\perp}(x,\boldsymbol{k}_T^2)$, which describes the number density of an unpolarized Λ hyperon fragmented from a transversely polarized quark [52],

$$D_{\Lambda/q^\uparrow}(z,\boldsymbol{P}_{\Lambda T}) - D_{\Lambda/q^\uparrow}(z,-\boldsymbol{P}_{\Lambda T}) = \Delta D_{\Lambda/q^\uparrow}(z,\boldsymbol{P}_{\Lambda T}^2)\frac{(\hat{\boldsymbol{k}}\times\boldsymbol{P}_{\Lambda T})\cdot\boldsymbol{S}_q}{zM_\Lambda}, \tag{1}$$

where $\boldsymbol{P}_{\Lambda T}$ is the transverse momentum of the Λ hyperon with respect to the quark momentum $\boldsymbol{k}$, $\boldsymbol{S}_q$ is the spin vector of the fragmenting quark, and z and M_Λ are the light-cone momentum fraction and the mass of the produced Λ hyperon, respectively. Either $H_1^{\perp}$ or $\Delta D_{\Lambda/q^\uparrow}$ may be referred to as the Collins function defined in Refs. [52–54]. The relation between them is

$$\Delta D_{\Lambda/q^\uparrow}(z,\boldsymbol{k}_T^2) = \frac{2|\boldsymbol{P}_{\Lambda T}|}{zM_\Lambda}H_1^{\perp q}(z,\boldsymbol{k}_T^2) = \frac{2|\boldsymbol{k}_T|}{M_\Lambda}H_1^{\perp q}(z,\boldsymbol{k}_T^2), \tag{2}$$

where $\boldsymbol{k}_T$ is related to $\boldsymbol{P}_{\Lambda T}$ by $\boldsymbol{k}_T = -\boldsymbol{P}_{\Lambda T}/z$.

The Collins function can be calculated from the following trace:

$$\frac{\epsilon_T^{\alpha\rho}k_{T\rho}}{M_\Lambda}H_1^{\perp} = \frac{1}{4}\mathrm{Tr}[(\Delta(z,k_T;S_\Lambda)+\Delta(z,k_T;-S_\Lambda))i\sigma^{\alpha-}\gamma_5]. \tag{3}$$

Here, the quark-quark fragmentation correlation function $\Delta(z,k_T;S_\Lambda)$ is defined as [55,56]

$$\begin{aligned}\Delta(z,k_T;S_\Lambda) &= \frac{1}{2z}\int dk^+\Delta(k,P_\Lambda;S_\Lambda)\\ &\equiv \sum_X\int\frac{d\xi^+d^2\boldsymbol{\xi}_T}{2z(2\pi)^3}e^{ik\cdot\xi}\\ &\quad\times\langle 0|\mathcal{U}^{n^+}_{(+\infty,\xi)}\psi(\xi)|P_\Lambda,S_\Lambda;X\rangle\\ &\quad\times\langle P_\Lambda,S_\Lambda;X|\bar{\psi}(0)\mathcal{U}^{n^+}_{(0,+\infty)}|0\rangle\Big|_{\xi^-=0},\end{aligned} \tag{4}$$

with $k^- = \frac{P_\Lambda^-}{z}$. The Wilson line $\mathcal{U}$ is used to ensure gauge invariance of the operator [57,58]. The final state $|P_\Lambda,S_\Lambda;X\rangle$ describes the outgoing Λ hyperon with momentum P_Λ and spin S_Λ together with the intermediate unobserved states. In this paper we perform the calculation in a diquark spectator model [43,44], which includes both the spin-0 (scalar diquark) and the spin-1 (vector diquark) spectator systems [55,59]. The quark fragmentation process (taking up quark as an example) can be modeled as

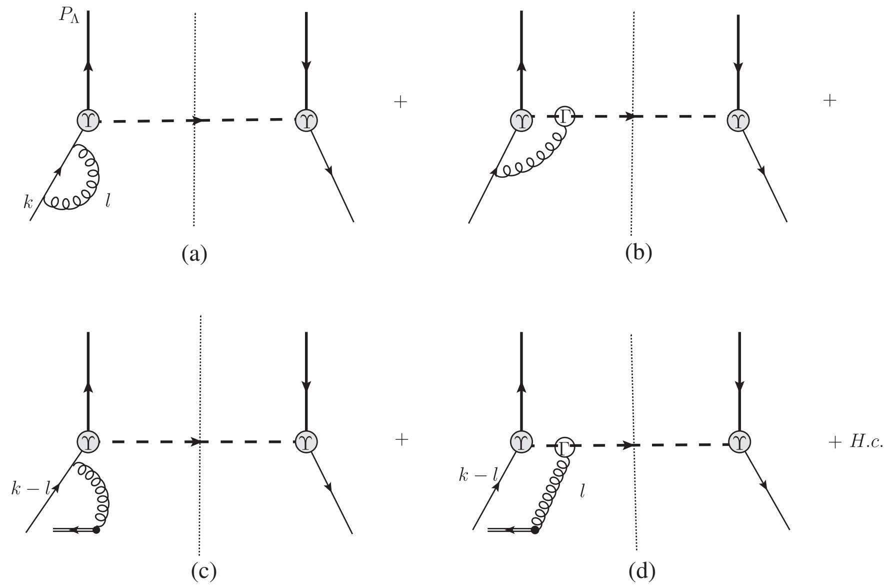

FIG. 1. One loop corrections to the fragmentation of a quark to a Λ hyperon in the spectator model. The double lines in (c) and (d) represent the eikonal lines. Here "H.c." stands for the Hermitian conjugations of these diagrams.

$u \to \Lambda(uds) + D(\bar{d}\,\bar{s})$, with D denoting a diquark. The matrix element appearing in the right-hand side of Eq. (4) has the following form:

$$\langle P_\Lambda, S_\Lambda; X|\bar{\psi}(0)|0\rangle = \begin{cases} \bar{U}(P_\Lambda, S_\Lambda)\Upsilon_s \frac{i}{\not{k}-m_q} & \text{scalar diquark}, \\ \bar{U}(P_\Lambda, S_\Lambda)\Upsilon_v^\mu \frac{i}{\not{k}-m_q}\varepsilon_\mu & \text{vector diquark}. \end{cases} \tag{5}$$

Here Υ_D ($D = s$ or v) is the hyperon-quark-diquark vertex and ε_μ is the polarization vector of the spin-1 vector diquark. In our work, the vertex structure is chosen as follows [44,48]:

$$\Upsilon_s = \mathbf{1}g_s, \qquad \Upsilon_v^\mu = \frac{g_v}{\sqrt{3}}\gamma_5\left(\gamma^\mu + \frac{P_\Lambda^\mu}{M_\Lambda}\right), \tag{6}$$

where g_D ($D = s$ or v) is the suitable coupling for the hyperon-quark-diquark vertex. In this work we assume that g_s and g_v are the same: $g_s = g_v = g_D$, and we adopt the Gaussian form for g_D:

$$g_D(k^2) = \frac{g'_D}{z} e^{\frac{-k^2}{\lambda^2 z^\alpha (1-z)^\beta}} \tag{7}$$

where g'_D, λ, α and β are the model parameters.

In the diquark model, the nonvanishing Collins function comes from the one-loop corrections that provide the necessary imaginary phases in the scattering amplitude [60,61]. At one-loop level, there are four diagrams that can generate imaginary phases, as shown in Fig. 1. In Figs. 1(b) and 1(d), the notation Γ is used to depict the gluon-diquark vertex, and we apply the following rules for the vertex between the gluon and the scalar diquark (Γ_s) or the vector diquark (Γ_v):

$$\Gamma_s^{\rho,a} = igT^a(2k - 2P_\Lambda - l)^\rho, \tag{8}$$

$$\Gamma_v^{\rho,\mu\nu,a} = -igT^a[(2k - 2P_\Lambda - l)^\rho g^{\mu\nu} - (k - P_\Lambda - l)^\nu g^{\rho\mu} - (k - P_\Lambda)^\mu g^{\rho\nu}]. \tag{9}$$

Here, T^a is the Gell-Mann matrix, and g is the coupling constant of QCD. Since the Λ hyperon is colorless, it is expected that the spectator diquark should have the same color as that of the parent quark. The Feynman rules for the eikonal line as well as the vertex between the eikonal line and the gluon can be found in Refs. [20,57,62].

Following the previous work [48] in which the fragmentation function $D_{1T}^\perp$ for the Λ hyperon has been calculated in the same model, we perform the integration over the loop momentum l with the help of the Cutkosky cutting rules. In the left-hand side of Figs. 1(b) and 1(d), in principle the momentum l enters the form factor for the hyperon-quark-diquark vertex with the form $g_D((k-l)^2)$. To simplify the integration we choose that in any case the form factor g_D depends only on the initial quark momentum k, since the main effect of the form factor is to introduce a cutoff in the high k_T region. The same choice has also been used in Refs. [20,45].

The expression for $H_1^\perp$ of the Λ hyperon, coming from the scalar diquark component, is as follows:

$$H_1^{\perp(s)}(z,k_T^2) = \frac{\alpha_s g_D^2 C_F}{(2\pi)^4} \frac{e^{\frac{-2k^2}{\lambda^2 z^\alpha (1-z)^\beta}}}{z^2(1-z)} \frac{1}{(k^2-m_q^2)} (H_{1(a)}^{\perp(s)}(z,k_T^2) + H_{1(b)}^{\perp(s)}(z,k_T^2) + H_{1(c)}^{\perp(s)}(z,k_T^2) + H_{1(d)}^{\perp(s)}(z,k_T^2)), \tag{10}$$

where

$$H_{1(a)}^{\perp(s)}(z,k_T^2) = \frac{m_q M_\Lambda}{(k^2-m_q^2)} \left(3 - \frac{m_q^2}{k^2}\right) I_1, \tag{11}$$

$$H_{1(b)}^{\perp(s)}(z,k_T^2) = M_\Lambda\{m_q(2I_2 - \mathcal{A}) - M_\Lambda(\mathcal{B} - 2I_2 + 2\mathcal{A})\}, \tag{12}$$

$$H_{1(c)}^{\perp(s)}(z,k_T^2) = 0, \tag{13}$$

$$H_{1(d)}^{\perp(s)}(z,k_T^2) = \frac{M_\Lambda}{z}\{2(1-z)(m_q \mathcal{C}P_h^- - M_\Lambda \mathcal{D}P_h^-) - z(M_\Lambda \mathcal{B} - m_q \mathcal{A})\}. \tag{14}$$

Similarly, using the gluon vertex given in Eq. (9), we can also calculate the expression for $H_1^\perp$ contributed by the vector diquark component

$$H_1^{\perp(v)}(z,k_T^2) = \frac{\alpha_s g_D^2 C_F}{(2\pi)^4} \frac{e^{\frac{-2k^2}{\lambda^2 z^\alpha (1-z)^\beta}}}{z^2(1-z)} \frac{1}{(k^2-m_q^2)} (H_{1(a)}^{\perp(v)}(z,k_T^2) + H_{1(b)}^{\perp(v)}(z,k_T^2) + H_{1(c)}^{\perp(v)}(z,k_T^2) + H_{1(d)}^{\perp(v)}(z,k_T^2)), \tag{15}$$

where

$$\begin{aligned}
H_1^{\perp(v)}(a) &= \frac{m_q M_\Lambda}{(k^2-m_q^2)} \left(3 - \frac{m_q^2}{k^2}\right) I_1, \\
H_1^{\perp(v)}(b) &= \frac{1}{3}\left\{2M_\Lambda[M_\Lambda(3I_2 - 3\mathcal{A} - \mathcal{B}) + 2m_q I_2] - 2k\cdot P(I_2 - 2\mathcal{A}) + \frac{(3m_q^2 - k^2)}{4k^2} I_1 + \frac{k^2 - m_q^2}{2}(I_2 - 3\mathcal{A})\right\}, \\
H_1^{\perp(v)}(c) &= 0, \\
H_1^{\perp(v)}(d) &= \frac{M_\Lambda}{z}[2(1-z)(m_q \mathcal{C}P_h^- - M_\Lambda \mathcal{D}P_h^-) - z(M_\Lambda \mathcal{B} - m_q \mathcal{A})] \\
&\quad + \frac{1}{3M_\Lambda}\left\{4M_\Lambda(m_q M_\Lambda + k\cdot P)\mathcal{A} - \frac{2M_\Lambda}{z}[(2m_q M_\Lambda + k\cdot P)\mathcal{C}P^- - M_\Lambda^2 \mathcal{D}P^-]\right. \\
&\quad \left. + \left[M_\Lambda(k^2 - m_q^2)\mathcal{C}P^- + 2k\cdot P(m_q \mathcal{C}P^- - M_\Lambda \mathcal{D}P^-) + \frac{z m_q I_1}{2} + \frac{k^2 - m_q^2}{2}(m_q \mathcal{C}P^- - M_\Lambda \mathcal{D}P^-)\right]\right\}.
\end{aligned}$$

Here $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ are functions of k^2, m_q, m_D, and M_Λ,

$$\mathcal{A} = \frac{I_1}{\lambda(M_\Lambda, m_D)}\left(2k^2(k^2 - m_D^2 - M_\Lambda^2)\frac{I_2}{\pi} + (k^2 + M_\Lambda^2 - m_D^2)\right), \qquad \mathcal{B} = -\frac{2k^2}{\lambda(M_\Lambda, m_D)} I_1 \left(1 + \frac{k^2 + m_D^2 - M_\Lambda^2}{\pi} I_2\right),$$

$$\mathcal{C}P_\Lambda^- = \frac{I_{34}}{2k_T^2} + \frac{1}{2zk_T^2}(-zk^2 + (2-z)M_\Lambda^2 + zm_D^2)I_2, \qquad \mathcal{D}P_\Lambda^- = \frac{-I_{34}}{2zk_T^2} - \frac{1}{2zk_T^2}((1-2z)k^2 + M_\Lambda^2 - m_D^2)I_2.$$

The functions I_i in the above equations are defined as

$$I_1 = \int d^4 l \delta(l^2)\delta((k-l)^2 - m_q^2) = \frac{\pi}{2k^2}(k^2 - m_q^2), \tag{16}$$

$$I_2 = \int d^4 l \frac{\delta(l^2)\delta((k-l)^2 - m_q^2)}{(k - P_\Lambda - l)^2 - m_D^2} = \frac{\pi}{2\sqrt{\lambda(M_\Lambda, m_D)}} \ln\left(1 - \frac{2\sqrt{\lambda(M_\Lambda, m_D)}}{k^2 - M_\Lambda^2 + m_D^2 + \sqrt{\lambda(M_\Lambda, m_D)}}\right), \tag{17}$$

$$I_{34} = \pi \ln \frac{\sqrt{k^2}(1-z)}{m_D}, \tag{18}$$

with $\lambda(M_\Lambda, m_D) = (k^2 - (M_\Lambda + m_D)^2)(k^2 - (M_\Lambda - m_D)^2)$.

In the assumption of the SU(6) spin-flavor symmetry of octet baryons, the Collins function of the Λ hyperon for light quarks satisfies the following relations between different quark flavors and diquark types [63–65]:

$$H_1^{\perp u\to\Lambda} = H_1^{\perp d\to\Lambda} = \frac{1}{4} H_1^{\perp(s)} + \frac{3}{4} H_1^{\perp(v)},$$
$$H_1^{\perp s\to\Lambda} = H_1^{\perp(s)}, \tag{19}$$

where u, d, and s denote the up, down, and strange quarks, respectively. The contributions to the Collins function $H_1^\perp$ from the scalar diquark and the vector diquark are given in Eqs. (10) and (15).

It is necessary to point out that the Collins function should obey the following positivity bound [24,66], which is a useful theoretical constraint,

$$\frac{|k_T|}{M_\Lambda} |H_1^\perp(z, \boldsymbol{k}_T^2)| \le D_1(z, \boldsymbol{k}_T^2). \tag{20}$$

After performing the integration over k_T^2, we can obtain the following approximated relation:

$$2|H_1^{\perp(1/2)}(z)| \le D_1(z), \tag{21}$$

where $H_1^{\perp(1/2)}(z)$ is the half k_T moment of the Collins function defined as

$$H_1^{\perp(1/2)}(z) = z^2 \int d^2\boldsymbol{k}_T \frac{|\boldsymbol{k}_T|}{2M_\Lambda} H_1^\perp(z, z^2\boldsymbol{k}_T^2) \tag{22}$$

and $D_1(z) = z^2 \int d^2\boldsymbol{k}_T D_1(z, z^2\boldsymbol{k}_T^2)$ is the collinear unpolarized fragmentation function. In this work we would like to check whether the Collins function of the Λ hyperon in our model satisfies the positivity bound, particularly, the weaker version (21).

For the unpolarized fragmentation function $D_1(z)$ of the Λ hyperon needed in the comparison, we apply the result from the same model in Ref. [48] as

$$\begin{aligned} D_1^\Lambda(z) = {} & \frac{g_D'^2}{4(2\pi)^2} \frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4 L^2} \Big\{ z(1-z)((m_q + M_\Lambda)^2 - m_D^2) \\ & \times \exp\left(\frac{-2zL^2}{(1-z)\Lambda^2}\right) + ((1-z)\Lambda^2 \\ & - 2((m_q + M_\Lambda)^2 - m_D^2)) \frac{z^2 L^2}{\Lambda^2} \Gamma\left(0, \frac{2zL^2}{(1-z)\Lambda^2}\right) \Big\}. \end{aligned} \tag{23}$$

To obtain this result, the mass differences among the up, down, and strange quarks are neglected, and the SU(6) spin-flavor symmetry is also applied,

$$D_1^{u\to\Lambda} = D_1^{d\to\Lambda} = D_1^{s\to\Lambda} \equiv D_1^\Lambda; \tag{24}$$

that is, the light quarks fragment equally to Λ for the unpolarized fragmentation function D_1.

In Table I, we list the parameters [48] used to calculate the Λ Collins function. The values of the parameters were obtained by fitting the model result of D_1^Λ in the same model to the de Florian-Stratmann-Vogelsang (DSV) parametrization for D_1^Λ [67] at the model scale $Q_0^2 = 0.23\ \mathrm{GeV}^2$. The strong coupling constant α_s at this scale is chosen as 0.817.

In Fig. 2, we plot the numerical result of $H_1^{\perp(1/2)}(z)$ (multiplied by a factor of 2) of the Λ hyperon (solid lines), compared with the unpolarized Λ fragmentation function $D_1^\Lambda(z)$ (dashed lines) in the same model. The left panel shows the result for the up/down quarks, while the right panel depicts the result for the strange quark. The shaded areas correspond to the error bands caused by the uncertainty of the model parameters. From the curves, one can find that the size of $H_1^{\perp(1/2)}(z)$ for the up and down quarks is around several percent. Particularly, the sign of $H_{1,\Lambda/u}^{\perp(1/2)}(z)$ is negative in the small z region ($0 < z < 0.5$), while it turns to be positive in the large z region ($0.5 < z < 1$). That is, there is a node in the z dependence of the Λ Collins function for the up and down quarks. This is different from the Collins function of the pion for which no node appears. We also find that $H_1^{\perp(1/2)}(z)$ for the strange quark is consistent with zero. Finally, our model result of $H_1^\perp$ for the up and down quarks does not always satisfy the positivity bound; i.e., in the large z region ($z > 0.82$) the positivity bound is violated. We note that similar violations of the positivity bound were also observed in Refs. [48,68,69]. An explanation was given in Ref. [70], stating that the violation may arise from the fact that T-odd TMD distributions or fragmentation functions

TABLE I. Values of the parameters used in the spectator diquark model [48]. The values of the last three parameters are fixed.

m_D [GeV]	λ [GeV]	g'_D	m_q [GeV]	α	β
$0.745^{+0.03}_{-0.028}$	$5.969^{+0.274}_{-0.260}$	$1.982^{+0.119}_{-0.111}$	0.36 (fixed)	0.5 (fixed)	0 (fixed)

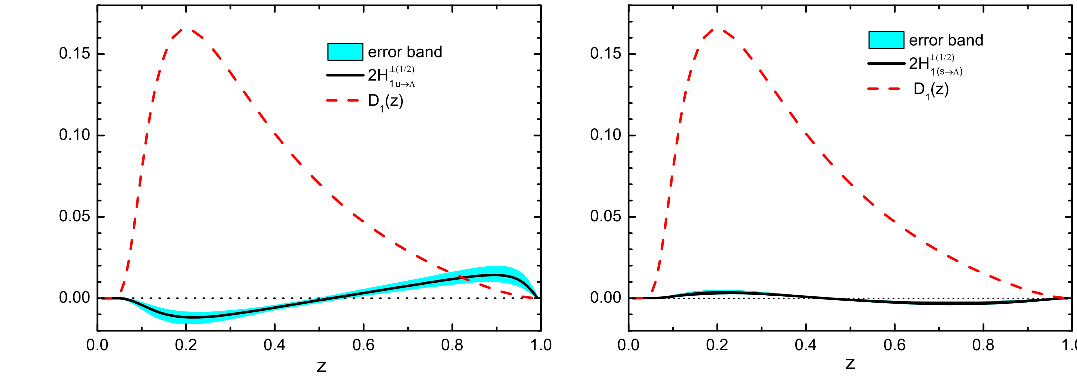

FIG. 2. Left panel: the $H_1^{\perp(1/2)}(z)$ (multiplied by 2) (solid line) for the up quark compared with $D_1(z)$ (dashed line) for the up quark at the model scale. Right panel: the $H_1^{\perp(1/2)}(z)$ (multiplied by 2) (solid line) and $D_1(z)$ (dashed line) for the strange quark at the model scale. The shaded areas correspond to the uncertainty on the model parameters.

are evaluated to $\mathcal{O}(\alpha_s)$, while in model calculations T-even TMD functions are usually truncated at the lowest order.

Since the energy scale in experiments is much higher than the model scale, it is important to include the QCD evolution of fragmentation functions to obtain reliable results for physical observables. In Refs. [71,72], the evolution equation for the twist-3 fragmentation function $\hat{H}(z)$ has been studied. This fragmentation function is proportional to the first k_T moment of Collins function via the relation

$$\hat{H}(z) = z^2 \int d^2\boldsymbol{k}_T \frac{\boldsymbol{k}_T^2}{M_\Lambda} H_1^\perp(z,\boldsymbol{k}_T^2) = 2M_\Lambda H_1^{\perp(1)}(z). \tag{25}$$

The evolution kernel for $\hat{H}(z)$ has a rather complicated form. Following Ref. [73], in this work we only consider the homogenous terms [72] in the kernel, which have the same form of the evolution kernel for the transversity distribution function h_1:

$$P_{qq}^{h_1} = C_F\left(\frac{2z}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right). \tag{26}$$

We assume that the evolution of the half k_T moment of Collins function $H_1^{\perp(1/2)}$ is the same as that of $\hat{H}$. We apply the evolution package QCDNUM [74] and customize the code to include the kernel in Eq. (26) to perform the evolution of $H_1^{\perp(1/2)}(z)$. In Fig. 3, we plot the half k_T moment of the Λ Collins function $H_1^{\perp(1/2)}(z)$. The left and right panels show the results for the up/down quarks and the strange quark at three different energy scales. The solid lines depict the model results at the initial scale $Q_0^2 = 0.23\ \mathrm{GeV}^2$, while the dashed and dotted lines show the results at $Q^2 = 1\ \mathrm{GeV}^2$ and $Q^2 = 10.52^2\ \mathrm{GeV}^2$ after applying the evolution equation for $H_1^{\perp(1/2)}(z)$. From the curves, we can see that the evolution effect for $H_1^{\perp(1/2)}(z)$ is significant; i.e., the evolution changes the shape and the size of the fragmentation functions at different Q values. It drives the peaks of $H_1^{\perp(1/2)}(z)$ to the lower z region with

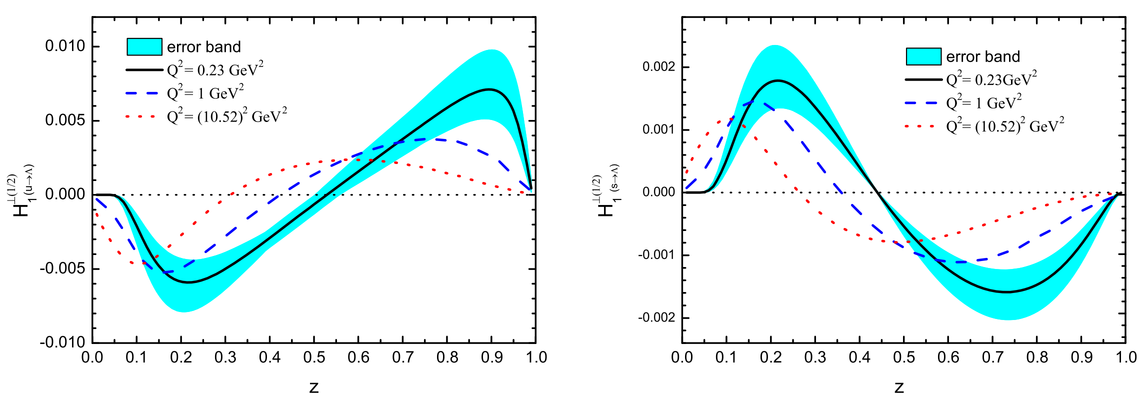

FIG. 3. The Collins function of the Λ hyperon at three different energy scales: $Q_0^2 = 0.23\ \mathrm{GeV}^2$ (solid lines), $Q^2 = 1\ \mathrm{GeV}^2$ (dashed lines), and $Q^2 = (10.52)^2\ \mathrm{GeV}^2$ (dotted lines). Left panel: $zH_1^{\perp(1/2)}(z)$ for the up and down quarks; right panel: $zH_1^{\perp(1/2)}(z)$ of the strange quark. The bands show the uncertainties from the errors of the parameters.

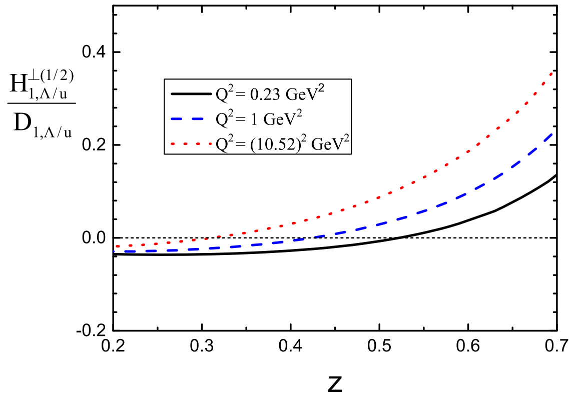

FIG. 4. The ratio $H^{\perp(1/2)}_{1,\Lambda/u}(z,Q^2)/D_1^\Lambda(z,Q^2)$ at three different energy scales: $Q_0^2 = 0.23$ GeV^2 (solid lines), $Q^2 = 1$ GeV^2 (dashed lines), and $Q^2 = (10.52)^2$ GeV^2 (dotted lines).

increasing Q. At a higher scale, the node of $H_1^{\perp(1/2)}(z)$ for the up or down quark also moves to the lower z region. The similar tendency also appeared in the transversity distribution function of the nucleon for the up quark in Ref. [55].

To demonstrate the evolution effects of fragmentation functions in the azimuthal asymmetries, in Fig. 4 we also plot the ratio $H_1^{\perp(1/2)}(z,Q^2)/D_1^\Lambda(z,Q^2)$ for the up quark at three scales. We find that, in the region $0.2 < z < 0.7$, the ratio $H^{\perp(1/2)}_{1,\Lambda/u}(z,Q^2)/D_1^\Lambda(z,Q^2)$ increases with the increasing z at any energy scale.

III. ASYMMETRIES IN THE e^+e^- ANNIHILATION PROCESS

Using the Λ Collins function calculated in Sec. II, in this section, we numerically estimate the azimuthal asymmetries in the processes

$$e^+ + e^- \to h_1 + h_2 + X, \tag{27}$$

in the case the final state hadrons h_1 and h_2 are $\Lambda\bar{\Lambda}$ or $\Lambda\pi$ at the energy scale of Belle and *BABAR* experiments. In these processes, the two leptons e^+ (with momentum l) and e^- (with momentum l') annihilate into a photon with momentum $q = (l + l')$; the photon then produces a quark-antiquark pair, which fragments into the final state hadron pair and other unobserved states. In the unpolarized process, the double Collins effect shows up at the leading order in the differential cross section (in the $1/Q$ expansion). There are two different reference frames adopted in experimental analysis (for further details and definitions, see Refs. [10,15,17,50]). The first one is the second-hadron momentum frame, in which the z axis is along the momentum of h_2 [49] and ϕ_0 is defined as the azimuthal angle of the Λ hyperon in the centre-of-mass (c.m.) frame of the incoming e^+e^- pair. In this frame a $\cos 2\phi_0$ azimuthal asymmetry appears from the convolution of the two Collins functions. The second one is a thrust reference frame, in which the jet thrust axis is used as the $\hat{z}$ and the $e^+e^- \to q\bar{q}$ scattering defines the $\hat{x}\hat{z}$ plane. In this frame a $\cos(\phi_1 + \phi_2)$ asymmetry arises, where ϕ_1 and ϕ_2 stand for the azimuthal angles of the two hadrons around the thrust axis in the Collins-Soper frame.

However, perturbative calculations [50,51] show that the hard gluon radiation process $e^+e^- \to q\bar{q}g$ also contributes the same azimuthal angular dependence as the double Collins effect does and should be separated in order to obtain the pure Collins effect. As suggested in Refs. [10,14,16], the QCD radiative corrections can be canceled by making the ratio of the asymmetry in unlike-sign events A_U (h_1 and h_2 are unlike sign) to that in like-sign events A_L (h_1 and h_2 are like sign). For the $\cos(\phi_1 + \phi_2)$ asymmetry, the ratio in the process $e^+e^- \to \Lambda\bar{\Lambda} + X$ can be cast to

$$\begin{aligned} R_{12} \equiv \frac{A_{12}^U}{A_{12}^L} &= \frac{1 + \frac{1}{4}\cos(\phi_1+\phi_2)\frac{\langle\sin^2\theta\rangle}{\langle 1+\cos^2\theta\rangle}P_U}{1 + \frac{1}{4}\cos(\phi_1+\phi_2)\frac{\langle\sin^2\theta\rangle}{\langle 1+\cos^2\theta\rangle}P_L} \\ &\simeq 1 + \frac{1}{4}\cos(\phi_1+\phi_2)\frac{\langle\sin^2\theta\rangle}{\langle 1+\cos^2\theta\rangle}(P_U - P_L) \\ &\equiv 1 + \cos(\phi_1+\phi_2)A_{12}(z_1,z_2), \end{aligned} \tag{28}$$

with P_U and P_L having the form

$$P_U = \frac{\sum_q e_q^2[\Delta^N D_{\Lambda/q^\uparrow}(z_1)\Delta^N D_{\bar{\Lambda}/\bar{q}^\uparrow}(z_2) + \Delta^N D_{\bar{\Lambda}/q^\uparrow}(z_1)\Delta^N D_{\Lambda/\bar{q}^\uparrow}(z_2)]}{\sum_q e_q^2[D_{1,\Lambda/q}(z_1)D_{1,\bar{\Lambda}/\bar{q}}(z_2) + D_{1,\bar{\Lambda}/q}(z_1)D_{1,\Lambda/\bar{q}}(z_2)]}, \tag{29}$$

$$P_L = \frac{\sum_q e_q^2[\Delta^N D_{\Lambda/q^\uparrow}(z_1)\Delta^N D_{\Lambda/\bar{q}^\uparrow}(z_2) + \Delta^N D_{\bar{\Lambda}/q^\uparrow}(z_1)\Delta^N D_{\bar{\Lambda}/\bar{q}^\uparrow}(z_2)]}{\sum_q e_q^2[D_{1,\Lambda/q}(z_1)D_{1,\Lambda/\bar{q}}(z_2) + D_{1,\bar{\Lambda}/q}(z_1)D_{1,\bar{\Lambda}/\bar{q}}(z_2)]}, \tag{30}$$

and $A_{12}(z_1,z_2)$ the asymmetry appearing in the ratio with a $\cos(\phi_1 + \phi_2)$ azimuthal angular dependence

$$A_{12}(z_1,z_2) = \frac{1}{4}\frac{\langle\sin^2\theta\rangle}{\langle 1+\cos^2\theta\rangle}(P_U - P_L). \tag{31}$$

Here for Λ pair production we define like-sign events as $\Lambda\Lambda$ and unlike-sign events as $\bar{\Lambda}\Lambda$. The asymmetry in the $\Lambda\pi$ production can be defined similarly by using $\Lambda\pi^+$ as "like-sign" events and $\bar{\Lambda}\pi^+$ as "unlike-sign" events.

Note that in Eqs. (29) and (30),

$$\Delta^N D_{h/q^\uparrow}(z) = \int d^2\boldsymbol{k}_T \Delta^N D_{h/q^\uparrow}(z, \boldsymbol{k}_T^2) = \int d^2\boldsymbol{k}_T \frac{2|\boldsymbol{P}_{hT}|}{zM_h} H_1^{\perp q}(z, \boldsymbol{k}_T^2) = 4H_1^{\perp(1/2)q}(z). \tag{32}$$

Thus the product half k_T moments of the Collins function appear in the A_{12} asymmetry.

In principle one may also adopt the $\cos(2\phi_0)$ method to study the ratio of unlike-sign events to like-sign events. However, in this case, only in the Gaussian model [14,75] for the k_T dependence of $H_1^\perp$ can the asymmetry be expressed as the product of two $H_1^{\perp(1/2)}(z)$. Thus in the following we choose to estimate the asymmetry A_{12}.

In the following we calculate the asymmetry A_{12} instead of calculating directly the azimuthal angular dependence appearing in the cross section. Since our model in Ref. [48] does not distinguish favored fragmentation functions and disfavored fragmentation functions for D_1, in a practical calculation we rescale them with $(1+z)$ for favored fragmentation and $(1-z)$ for disfavored fragmentation functions according to the assumption in Refs. [10,76]. For the Collins functions, the Schäfer-Teryaev sum rule shows that [77]

$$\sum_h \int_0^1 dz H_{1(q\to h)}^{\perp(1)}(z) = 0 \quad \text{with}$$
$$H_1^{\perp(1)}(z) = \pi z^2 \int_0^\infty dk_T^2 \frac{\boldsymbol{k}_T^2}{2M_h^2} H_1^\perp(z, k_T^2). \tag{33}$$

We adopt the same assumption in Ref. [20] that the sum rule holds in a strong sense; i.e., for the Λ hyperon, it satisfies

$$H_{1(u\to\bar{\Lambda})}^{\perp(1/2)} = -H_{1(u\to\Lambda)}^{\perp(1/2)}. \tag{34}$$

The other disfavored Collins functions are related to the above result by isospin and charge symmetries.

Using the framework setting above, we estimate the azimuthal asymmetry A_{12} in the process $e^+e^- \to \Lambda\bar{\Lambda}X$ at $Q = 10.52$ GeV, which is the scale of the Belle measurement [10] and which is also close to the kinematics available at *BABAR*. As the energy scales in these experiments are much higher than the model scale, we need to take into account the QCD evolution effects of the fragmentation functions. To study the impact of the evolution effect, we adopt two different ways to calculate the azimuthal asymmetry A_{12} in $e^+e^- \to \Lambda\bar{\Lambda}X$. One is to assume that all the fragmentation functions do not evolve with the energy scale, which is an extreme condition. The other is to apply the evolution kernel in (26) for $H_1^{\perp(1/2)}(z)$ and the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution for $D_1(z)$. For the factor $\langle \sin^2\theta\rangle / \langle 1+\cos^2\theta\rangle$ in Eq. (31), the mean value in each (z_1, z_2) bin is given in Ref. [10], and here we take 0.7 as a rough estimate.

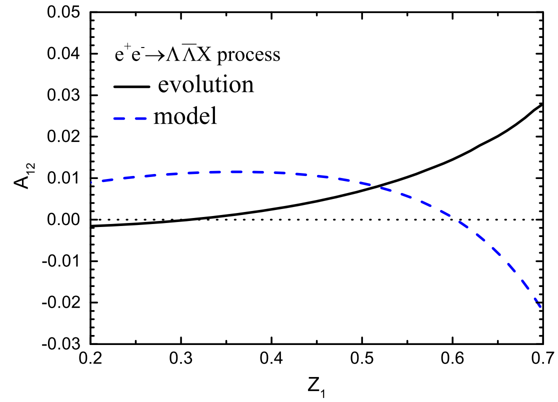

FIG. 5. The azimuthal asymmetry $A_{12}(z_1)$ for the $e^+e^- \to \Lambda\bar{\Lambda}X$ process as the function of z_1 with z_2 integrated out. The dashed line represents the asymmetry assuming the fragmentation functions do not evolve with energy scales. The solid lines denote the asymmetry considering the evolution effects of both D_1 and $H_1^{\perp(1/2)}$.

In Fig. 5, we plot the azimuthal asymmetry A_{12} of the Λ pair production in e^+e^- annihilation contributed by the double Collins effect as functions of z_1. The variable z_2 is integrated over $0.2 < z_2 < 0.7$ since the Collins function in our model violates the positivity bound at the large z region ($z > 0.8$). The dashed line in Fig. 5 shows the asymmetry under the extreme assumption in which the evolution of the fragmentation functions are ignored, while the solid lines denote the asymmetry in case the evolution effects of both $D_1(z)$ and $H_1^{\perp(1/2)}(z)$ are included. From the curves, one can find that the azimuthal asymmetry $A_{12}(z_1)$ increases with increasing z and is several percent in size. Similar results were also find in the case of pion pair production in e^+e^- annihilation [9,20]. Comparing the solid lines and the dashed line, we can also see that the evolution effects

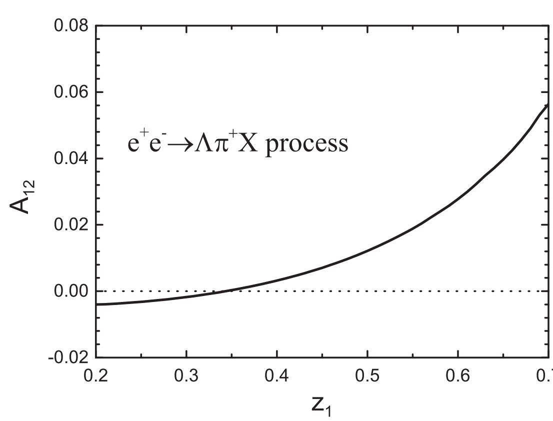

FIG. 6. The azimuthal asymmetries $A_{12}(z_1)$ for the $e^+e^- \to \Lambda\pi X$ process as the function of z_1 with z_2 integrated out.

significantly alter the asymmetry in the $e^+e^- \to \Lambda\bar{\Lambda}X$ process; thereby it should not be neglected.

We also estimate the azimuthal asymmetry A_{12} at the energy $Q = 10.52$ GeV in the process $e^+e^- \to \Lambda\pi + X$, which could be measured at Belle and *BABAR* more easily. In this case, we adopt the leading-order (LO) de Florian-Sassot-Stratmann (DSS) parametrization for the unpolarized fragmentation functions of the pion [78], and we choose the parametrization of the pion Collins function from Ref. [16] at the initial scale $Q^2 = 2.41$ GeV^2. In the calculation we consider the evolution of both the Λ and the pion Collins functions. The result is plotted in Fig. 6. We find that in $\Lambda\pi$ production, the shape of the asymmetry is similar to the case of Λ pair production, while the size of the asymmetry at large z_1 is several times larger than that of Λ pair production.

IV. CONCLUSION

In this work, we investigated the T-odd Collins function $H_1^\perp$ of the Λ hyperon for light quarks. In particular, we studied its contribution to azimuthal asymmetries A_{12} as the ratio of unlike-sign events to like-sign events in $e^+e^- \to \Lambda\bar{\Lambda}X$ and $e^+e^- \to \Lambda\pi X$ processes. We calculated the Collins function of the Λ hyperon in the diquark spectator model by considering both the scalar and the vector diquark components. In the calculation we adopted a Gaussian form factor for the hyperon-quark-diquark vertex, and we applied the values of the model parameters fitted from the DSV parametrization at the initial scale $Q_0^2 = 0.23$ GeV^2. The numerical result shows that the Collins function of the Λ hyperon for the up and down quarks dominates over that for the strange quark. We also calculated the QCD evolution of the half k_T moment of the Λ Collins function and found that the evolution effects significantly alter $H_1^{\perp(1/2)}(z)$. Applying the model results for $H_1^{\perp(1/2)}(z)$, we estimated the azimuthal asymmetry A_{12} contributed by the Collins effect in the unpolarized $e^+e^- \to \Lambda\bar{\Lambda}X$ process at $Q = 10.52$ GeV in two scenarios: one is to take into account the evolution of both $H_1^{\perp(1/2)}(z)$ and $D_1(z)$; the other is to neglect any scale dependence of fragmentation functions. The asymmetry is around several percent, and it increases with increasing z_1. We also estimated the asymmetry A_{12} in the process $e^+e^- \to \Lambda\pi X$ and found that the shape of the asymmetry is similar to the one in $e^+e^- \to \Lambda\bar{\Lambda}X$, while the size of the asymmetry is larger than that in Λ pair production. Therefore it is feasible to measure these azimuthal asymmetries through the Belle and *BABAR* experiments. We also found that the evolution effects significantly change the shape and size of the asymmetry. Our study may provide useful information on the Λ fragmentation function as well as the nonperturbative origin of the azimuthal asymmetry in e^+e^- annihilation.

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