Decay width of hadronic molecule structure for quarks

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Based on the general form of the Bethe-Salpeter wave functions for the bound states consisting of two vector fields, we obtain the general formulas for the decay widths of molecular states composed of two heavy vector mesons with arbitrary spin and parity into a heavy meson plus a light meson. In this approach, our attention is still focused on the internal structure of heavy vector mesons in the molecular state. According to the molecule state model of exotic meson, we give the generalized Bethe-Salpeter wave function of molecular state as a four-quark state. Then the observed $Y(3940)$ state is considered as a molecular state consisting of two heavy vector mesons $D^{*0}\bar{D}^{*0}$ and the strong $Y(3940) \to J/\psi\omega$ decay width is calculated. The numerical result is consistent with the experimental values.

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I. INTRODUCTION

In present particle physics, the investigation of the structure of exotic meson is of great significance and the possible alternative interpretations beyond quark-antiquark state have been proposed [1–6]. Using the most general form of the Bethe-Salpeter (BS) wave functions for the bound states composed of two vector fields of arbitrary spin and definite parity [7], we have investigated the molecular states composed of two heavy vector mesons and obtained their masses and BS wave functions in Refs. [7,8]. It is obvious that our previous works are incomplete since the decay widths of the molecular states have still not been calculated. In this paper, we shall give an appropriate and accurate approach to investigate the strong decay of the molecular state of two heavy vector mesons into a heavy meson and a light meson.

Quantum chromodynamics (QCD) is the fundamental theory of strong interaction, and it is the most reasonable and fascinating to study the exotic meson from QCD. In this work, we will investigate the exotic meson as far as possible from QCD. Then the heavy meson is a bound state consisting of a quark and an antiquark and the meson-meson molecular state is actually composed of four quarks. However, the previous works [1–3] about the hadronic molecules seldom considered the internal structure of the mesons in the molecular state. In quantum field theory, Mandelstam's approach is a technique for evaluating the matrix element of a product of field operators and their derivatives between two bound states [9]. Considering the molecular state as a four-quark state and applying Mandelstam's approach, we obtain the general formulas for the matrix elements between this four-quark state and another bound state and evaluate the decay width of molecular state composed of two heavy vector mesons with arbitrary spin and parity into a heavy meson plus a light meson. When the decay width of exotic meson is much less than its mass, one can apply the perturbation theory to calculate the decay width for this physical process. The Hamiltonian describing the decay interaction is not considered at first, the BS approach can be used to investigate the molecular state. After obtaining the BS wave function of the molecular state, we consider the decay interaction and calculate the decay width. For this present purpose, we have to obtain the generalized Bethe-Salpeter (GBS) wave function of molecular state as a four-quark state. From the hadronic molecule structure, the GBS wave function for four-quark state should be the product of the BS wave function for the bound state of two heavy mesons and the BS wave functions of these two heavy mesons. To quantitatively calculate the bound state matrix element, we require the exact normalization of BS wave function for the molecular state composed of two heavy mesons.

Then this approach is applied to investigate an interesting process: the decay of $Y(3940)$ into $J/\psi\omega$ [10,11]. The structure of $Y(3940)$ does not fit the $c\bar{c}$ charmonium interpretation [12] and we assume that the $Y(3940)$ state is a molecular state consisting of two heavy vector mesons $D^{*0}\bar{D}^{*0}$. As in the effective theory at low energy QCD, we consider that the light vector meson ω is a pointlike particle

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and investigate the light meson interaction with the light quarks in these two heavy mesons, which is consistent with our previous works. Finally, the matrix element between two bound states and the strong decay width can be calculated without an extra parameter. In the framework of QCD, we systemically investigate the decay of molecule state composed of two heavy vector mesons into a heavy meson plus a light meson, and the calculated decay width $\Gamma(Y(3940)\to J/\psi\omega)$ is consistent with the experimental values. Therefore, this work provides powerful theoretical support for the molecule state model of $Y(3940)$ from QCD. Since the BS approach is derived in relativistic quantum field theory, this nonperturbative method can be used to investigate the other exotic states XYZ [13].

The paper is organized as follows. In Sec. II we give the general matrix element between four-quark state and two-quark bound state. Section III gives the GBS wave function of molecular state as a four-quark state. In Sec. IV the approach is applied to investigate the decay mode $Y(3940)\to J/\psi\omega$. After obtaining the BS wave function of heavy meson J/ψ and the normalizations of BS wave functions, we calculate this decay width. Our numerical result is reported in Secs. V and VI presents the conclusion.

II. GENERAL BOUND STATE MATRIX ELEMENT

Consider that a bound state with spin j and parity η_P is composed of two heavy vector mesons and this molecular state decays to a heavy meson and a light meson. As in effective theory at low energy QCD, we investigate the light meson interaction with the light quarks in heavy mesons and the interaction Lagrangian is

$$\mathcal{L}_I^{\mathrm{eff}} = ig_0'\begin{pmatrix}\bar{u} & \bar{d} & \bar{s}\end{pmatrix}\gamma_5\begin{pmatrix}\pi^0+\frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+\\ \sqrt{2}\pi^- & -\pi^0+\frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0\\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta\end{pmatrix}\begin{pmatrix}u\\ d\\ s\end{pmatrix} + ig_0\begin{pmatrix}\bar{u} & \bar{d} & \bar{s}\end{pmatrix}\gamma_\mu\begin{pmatrix}\rho^0+\omega & \sqrt{2}\rho^+ & \sqrt{2}K^{*+}\\ \sqrt{2}\rho^- & -\rho^0+\omega & \sqrt{2}K^{*0}\\ \sqrt{2}K^{*-} & \sqrt{2}\bar{K}^{*0} & \sqrt{2}\phi\end{pmatrix}\begin{pmatrix}u\\ d\\ s\end{pmatrix} + ig_\sigma\begin{pmatrix}\bar{u} & \bar{d}\end{pmatrix}\begin{pmatrix}u\\ d\end{pmatrix}\sigma. \quad (1)$$

The effective quark current J corresponding to the light meson can be obtained. Then the heavy vector mesons in molecular state should be bound states of a light quark and a heavy quark and the molecular state should be considered as a four-quark state; the heavy meson in the final state should be a bound state of two heavy quarks. Applying Mandelstam's approach, we can obtain the general formulas for the matrix elements of arbitrary quark current J between four-quark state and two-quark bound state

$$\langle Q|J(x_i)|P\rangle = \int d^4y_1 d^4y_2 d^4x_1 d^4x_3 d^4x_4 d^4x_2 \times\bar{\chi}_Q(y_1,y_2)T(y_1,y_2;x_i;x_1,x_3,x_4,x_2) \times\chi_P(x_1,x_3,x_4,x_2), \quad (2)$$

where $|P\rangle$ and $|Q\rangle$ are four-quark state and two-quark bound state, respectively; P and Q are their total momenta, $\chi_P(x_1,x_3,x_4,x_2)$ and $\chi_Q(y_1,y_2)$ are GBS and BS wave functions, $T(y_1,y_2;x_i;x_1,x_3,x_4,x_2)$ is the irreducible part of Green's function, shown as Fig. 1. In Fig. 1, V represents the heavy vector meson with mass M_1 and $\bar{V}'$ represents the antiparticle of heavy vector meson V' with mass M_2, MS represents the vector-vector molecular state, H and L represent the heavy and light mesons in the final state, respectively.

The BS wave function of the heavy meson in the final state is

$$\chi_Q(y_1,y_2) = \langle 0|T\mathcal{H}^C(y_1)\bar{\mathcal{H}}^D(y_2)|Q\rangle = \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2E(Q)}}e^{iQ\cdot Y}\chi_Q(y), \quad (3)$$

where $\mathcal{H}$ is the heavy quark operator and its superscript is a flavor label, $E(p)=\sqrt{\mathbf{p}^2+m^2}$, $Y=\eta_1'y_1+\eta_2'y_2$,

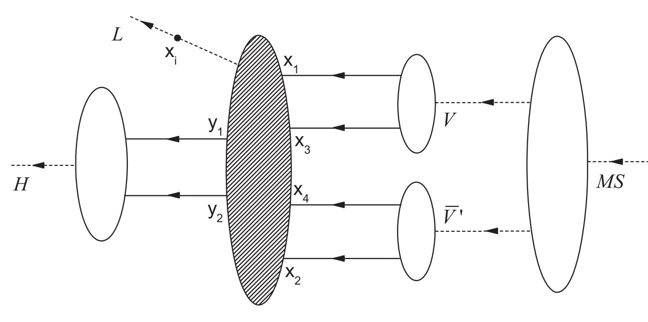

FIG. 1. General matrix element of quark current J between four-quark state and two-quark bound state. The solid lines denote quark propagators; the unfilled ellipses represent Bethe-Salpeter amplitudes; and the filled ellipse represents the irreducible part of Green's function.

$y = y_1 - y_2$, $\eta'_{1,2}$ are two positive quantities such that $\eta'_{1,2} = m_{C,D}/(m_C + m_D)$ and $m_{C,D}$ are the heavy quark masses. Extended to the four-quark bound state, the GBS wave function can be defined as

$$\chi_P(x_1, x_3, x_4, x_2) = \langle 0|T\mathcal{H}^C(x_1)\bar{\mathcal{Q}}^A(x_3)\mathcal{Q}^B(x_4)\bar{\mathcal{H}}^D(x_2)|P\rangle, \tag{4}$$

which also can be written as

$$\chi_P(x_1,x_3,x_4,x_2) = \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2E(P)}}e^{iP\cdot X}\chi_P(X',x,x'), \tag{5}$$

where $\mathcal{Q}$ is the light quark operator and its superscript is a flavor label, $X = \eta_1(\eta''_1 x_1 + \eta''_3 x_3) + \eta_2(\eta''_4 x_4 + \eta''_2 x_2)$, $X' = (\eta''_1 x_1 + \eta''_3 x_3) - (\eta''_4 x_4 + \eta''_2 x_2)$, $x = x_1 - x_3$, $x' = x_2 - x_4$, $\eta_1 + \eta_2 = 1$, $\eta''_{1,3} = m_{C,A}/(m_C + m_A)$, $\eta''_{2,4} = m_{D,B}/(m_D + m_B)$ and $m_{A,B}$ are the light quark masses.

For the sake of convenience, we assume that the final light meson is a vector meson with momentum Q', whose field operator is A_μ, and the Lagrangian for the interaction of light vector meson with quarks should be

$$\mathcal{L}_I(x_i) = ig\bar{\mathcal{Q}}^A(x_i)\gamma_\mu^{AB}\mathcal{Q}^B(x_i)A_\mu(x_i). \tag{6}$$

Then the effective quark current in Eq. (2) is $J_\mu(x_i) = \bar{\mathcal{Q}}^A(x_i)\gamma_\mu^{AB}\mathcal{Q}^B(x_i)$ and the two-particle irreducible Green's function is

$$\begin{aligned} T(y_1, y_2; x_i; x_1, x_3, x_4, x_2) &= \langle 0|T\mathcal{H}^C(y_1)\bar{\mathcal{H}}^D(y_2)\bar{\mathcal{Q}}^A(x_i)\gamma_\mu^{AB}\mathcal{Q}^B(x_i)\bar{\mathcal{H}}^C(x_1) \\ &\quad \times \mathcal{Q}^A(x_3)\bar{\mathcal{Q}}^B(x_4)\mathcal{H}^D(x_2)|0\rangle_T, \end{aligned} \tag{7}$$

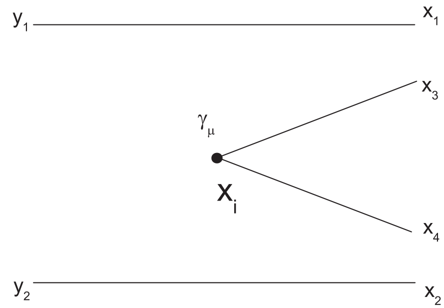

FIG. 2. The lowest order term of the two-particle irreducible Green's function.

which can be evaluated by means of perturbation theory. In present work, we retain only the lowest order term in the expansion of the right-hand side of Eq. (7), shown as in Fig. 2. It is straightforward from Eq. (7) and Fig. 2 to identify the lowest order value of $T(y_1, y_2; x_i; x_1, x_3, x_4, x_2)$ as

$$\begin{aligned} &T_0(y_1, y_2; x_i; x_1, x_3, x_4, x_2) \\ &\quad = S^C(y_1 - x_1)^{-1}\delta^{(4)}(x_3 - x_i)\gamma_\mu^{AB} \\ &\qquad \times \delta^{(4)}(x_i - x_4)S^D(x_2 - y_2)^{-1}, \end{aligned} \tag{8}$$

where the inverse propagator $S(y - x)^{-1}$ satisfies

$$S(y - x)^{-1} = \delta^{(4)}(y - x)(\gamma\cdot\partial_x + m). \tag{9}$$

Thus the bound state matrix elements of $J_\mu(x_i)$ are given in lowest order by

$$\begin{aligned} \langle Q|J_\mu(x_i)|P\rangle &= \int d^4y_1 d^4y_2 d^4x_1 d^4x_3 d^4x_4 d^4x_2 \bar{\chi}_Q(y_1, y_2)T_0(y_1, y_2; x_i; x_1, x_3, x_4, x_2)\chi_P(x_1, x_3, x_4, x_2) \\ &= \int d^4y_1 d^4y_2 \bar{\chi}_Q(y_1, y_2)(\gamma^C\cdot\partial_{y_1} + m_C)\gamma_\mu^{AB}(\gamma^D\cdot\partial_{y_2} + m_D)\chi_P(y_1, x_i, x_i, y_2). \end{aligned} \tag{10}$$

Making the Fourier transformation, we obtain the BS wave function of the heavy meson in the momentum representation

$$\chi_Q(q_1, q_2) = \int d^4y_1 d^4y_2 \chi_Q(y_1, y_2)e^{-iq_1\cdot y_1}e^{-iq_2\cdot y_2} = \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2E(Q)}}(2\pi)^4\delta^{(4)}(Q - q_1 - q_2)\chi(Q, q), \tag{11}$$

where q_1 and q_2 are the momenta of the quark and antiquark, respectively; $q = \eta'_2 q_1 - \eta'_1 q_2$ is the relative momentum between quark and antiquark. Furthermore, the GBS wave function of four-quark bound state in the momentum representation is

$$\begin{aligned} \chi_P(p_1, p_3, p_4, p_2) &= \int d^4x_1 d^4x_3 d^4x_4 d^4x_2 \chi_P(x_1, x_3, x_4, x_2)e^{-ip_1\cdot x_1}e^{-ip_3\cdot x_3}e^{-ip_4\cdot x_4}e^{-ip_2\cdot x_2} \\ &= \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2E(P)}}(2\pi)^4\delta^{(4)}(P - p_1 - p_3 - p_4 - p_2)\chi(P, p, k, k'), \end{aligned} \tag{12}$$

where p_1, p_3, p_4, p_2 are the momenta of four quarks; p, k, k' are the conjugate variables to X', x, x', respectively; and $p = \eta_2(p_1 + p_3) - \eta_1(p_4 + p_2)$, $k = \eta_3'' p_1 - \eta_1'' p_3$, $k' = \eta_4'' p_2 - \eta_2'' p_4$. In the hadronic molecule structure, p is the relative momentum between two heavy vector mesons in molecular state, k and k' are the relative momenta between quark and antiquark in two heavy vector mesons, respectively, shown as in Fig. 3.

Then the bound state matrix elements of $J_\mu(x_i)$ in Eq. (10) become

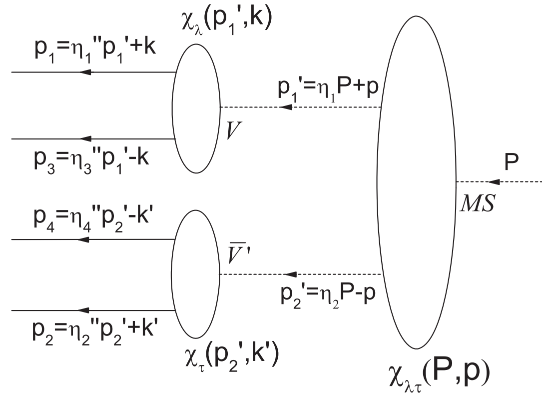

FIG. 3. Generalized Bethe-Salpeter wave function for four-quark state in the momentum representation.

$$\begin{aligned}\langle Q|J_\mu(x_i)|P\rangle &= \int d^4y_1 d^4y_2 \frac{1}{(2\pi)^8}\int d^4q_1 d^4q_2 \bar{\chi}_Q(q_1,q_2)e^{-iq_1\cdot y_1}e^{-iq_2\cdot y_2}(\gamma^C\cdot\partial_{y_1}+m_C)\gamma_\mu^{AB}(\gamma^D\cdot\partial_{y_2}+m_D)\\ &\quad\times\frac{1}{(2\pi)^{16}}\int d^4p_1 d^4p_3 d^4p_4 d^4p_2 \chi_P(p_1,p_3,p_4,p_2)e^{ip_1\cdot y_1}e^{ip_3\cdot x_i}e^{ip_4\cdot x_i}e^{ip_2\cdot y_2}\\ &= \frac{1}{(2\pi)^{16}}\int d^4p_1 d^4p_3 d^4p_4 d^4p_2 \bar{\chi}_Q(p_1,p_2)(i\gamma^C\cdot p_1+m_C)\gamma_\mu^{AB}(i\gamma^D\cdot p_2+m_D)\\ &\quad\times\chi_P(p_1,p_3,p_4,p_2)e^{ip_3\cdot x_i}e^{ip_4\cdot x_i}, \end{aligned} \tag{13}$$

where $q_1 = p_1$ and $q_2 = p_2$. Substituting Eqs. (11) and (12) into (13), we obtain the bound state matrix elements of $J_\mu(x_i)$ in the momentum representation

$$\langle Q|J_\mu(x_i)|P\rangle = \frac{1}{(2\pi)^{11}}\frac{1}{\sqrt{2E(Q)}}\frac{1}{\sqrt{2E(P)}}\int d^4k d^4p \bar{\chi}(Q,q)S^C(p_1)^{-1}\gamma_\mu^{AB}S^D(p_2)^{-1}\chi(P,p,k,k')e^{i(P-Q)\cdot x_i}, \tag{14}$$

where $S(p)$ is the quark propagator, $S(p) = 1/(i\gamma\cdot p+m)$, $q = \eta_1''(\eta_1 P + p) + k - \eta_1' Q$, $p_1 = \eta_1''(\eta_1 P + p) + k$, $p_2 = (\eta_2 P - p) - [P - Q - \eta_3''(\eta_1 P + p) + k]$, $p_3 = \eta_3''(\eta_1 P + p) - k$, $p_4 = P - Q - \eta_3''(\eta_1 P + p) + k$ and $k' = \eta_4''(\eta_2 P - p) - [P - Q - \eta_3''(\eta_1 P + p) + k]$. This is shown in Fig. 4.

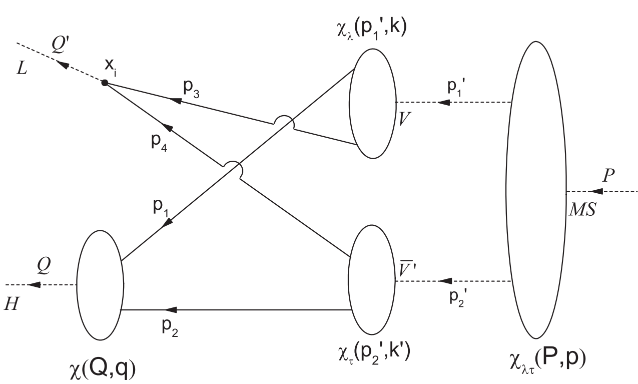

FIG. 4. The lowest order matrix element between bound states in the momentum representation.

III. GBS WAVE FUNCTION FOR FOUR-QUARK STATE

From Fig. 3, one can find three two-body systems in molecular state: a meson-meson bound state and two quark-antiquark bound states. The BS wave functions of these two-body systems can be defined as $\chi_P(p_1', p_2')$, $\chi_{p_1'}(p_1, p_3)$, $\chi_{p_2'}(p_4, p_2)$, respectively. Similar to Eq. (11), the BS wave function for the bound state of two heavy vector mesons has the form

$$\chi_P(p_1', p_2') = \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2E(P)}} \times (2\pi)^4\delta^{(4)}(P - p_1' - p_2')\chi(P,p), \tag{15}$$

and the BS wave functions of two heavy vector mesons are

$$\chi_{p_1'}(p_1, p_3) = \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2E(p_1')}} \times (2\pi)^4\delta^{(4)}(p_1' - p_1 - p_3)\chi(p_1', k), \tag{16}$$

$$\chi_{p_2'}(p_4,p_2)=\frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2E(p_2')}}\times(2\pi)^4\delta^{(4)}(p_2'-p_4-p_2)\chi(p_2',k'), \quad (17)$$

where p_1' and p_2' are the momenta of two vector mesons, respectively, $p_1'=\eta_1P+p$, $p_2'=\eta_2P-p$ and $\eta_{1,2}=M_{1,2}/(M_1+M_2)$.

In Ref. [7], we have given the general form of the BS wave functions for the bound states composed of two massive vector fields with spin j and parity η_P, for $\eta_P=(-1)^j$,

$$\chi_{\lambda\tau}^{j=0}(P,p)=\frac{1}{\mathcal{N}^j}(T^1_{\lambda\tau}\phi_1+T^2_{\lambda\tau}\phi_2), \quad (18)$$

$$\chi_{\lambda\tau}^{j\neq0}(P,p)=\frac{1}{\mathcal{N}^j}\eta_{\mu_1\ldots\mu_j}[p_{\mu_1}\ldots p_{\mu_j}(T^1_{\lambda\tau}\phi_1+T^2_{\lambda\tau}\phi_2)+T^3_{\lambda\tau}\phi_3+T^4_{\lambda\tau}\phi_4], \quad (19)$$

and, for $\eta_P=(-1)^{j+1}$,

$$\chi_{\lambda\tau}^{j=0}(P,p)=\frac{1}{\mathcal{N}^j}\epsilon_{\lambda\tau\xi\zeta}p_\xi P_\zeta\psi_1, \quad (20)$$

$$\chi_{\lambda\tau}^{j\neq0}(P,p)=\frac{1}{\mathcal{N}^j}\eta_{\mu_1\ldots\mu_j}(p_{\mu_1}\ldots p_{\mu_j}\epsilon_{\lambda\tau\xi\zeta}p_\xi P_\zeta\psi_1+T^5_{\lambda\tau}\psi_2+T^6_{\lambda\tau}\psi_3+T^7_{\lambda\tau}\psi_4+T^8_{\lambda\tau}\psi_5), \quad (21)$$

where $\mathcal{N}^j$ is the normalization, $\eta_{\mu_1\ldots\mu_j}$ is the polarization tensor describing the spin of the bound state, the subscripts λ and τ are derived from these two vector fields, the independent tensor structures $T^i_{\lambda\tau}$ are given in Appendix A, $\phi_i(P\cdot p,p^2)$ and $\psi_i(P\cdot p,p^2)$ are independent scalar functions. For the heavy vector mesons, the authors of Ref. [14–17] have investigated these bound states composed of a dressed-quark and -antiquark and obtained their BS amplitudes [16,17]:

$$\Gamma^V_\lambda(p_1';k)=\frac{1}{\mathcal{N}^V}\left(\gamma_\lambda+p'_{1\lambda}\frac{\gamma\cdot p_1'}{M_V^2}\right)\varphi_V(k^2),$$
$$\Gamma^{\bar{V}'}_\tau(p_2';k')=\frac{1}{\mathcal{N}^{\bar{V}'}}\left(\gamma_\tau+p'_{2\tau}\frac{\gamma\cdot p_2'}{M_{\bar{V}'}^2}\right)\varphi_{\bar{V}'}(k'^2), \quad (22)$$

where $\Gamma^V_\lambda(p_1';k)$ and $\Gamma^{\bar{V}'}_\tau(p_2';k')$ are transverse ($p'_{1\lambda}\Gamma^V_\lambda(p_1';k)=p'_{2\tau}\Gamma^{\bar{V}'}_\tau(p_2';k')=0$), $\mathcal{N}^V$ and $\mathcal{N}^{\bar{V}'}$ are the normalizations, $\varphi_V(k^2)=\varphi_{\bar{V}}(k^2)=\exp(-k^2/\omega_V^2)$ and ω_V is a parameter fixed by providing fits to the observables. The BS wave functions of heavy vector mesons are

$$\chi_\lambda(p_1',k)=\frac{-i}{\gamma^C\cdot p_1-im_C}\frac{1}{\mathcal{N}^V}\left(\gamma_\lambda+p'_{1\lambda}\frac{\gamma\cdot p_1'}{M_V^2}\right)\times\varphi_V(k^2)\frac{-i}{\gamma^A\cdot p_3-im_A},$$
$$\chi_\tau(p_2',k')=\frac{-i}{\gamma^B\cdot p_4-im_B}\frac{1}{\mathcal{N}^{\bar{V}'}}\left(\gamma_\tau+p'_{2\tau}\frac{\gamma\cdot p_2'}{M_{\bar{V}'}^2}\right)\times\varphi_{\bar{V}'}(k'^2)\frac{-i}{\gamma^D\cdot p_2-im_D}. \quad (23)$$

It should be noted that in Refs. [16,17] the BS amplitudes of heavy mesons are obtained in Euclidean space.

Applying the Feynman rules, we can obtain the GBS wave function for four-quark state from Fig. 3

$$\chi_P(p_1,p_3,p_4,p_2)=\frac{(2\pi)^{12}}{(2\pi)^{3/2}}\frac{1}{\sqrt{2E(P)}}\times\delta^{(4)}(P-p_1-p_3-p_4-p_2)\times\chi_\lambda(p_1',k)\chi_{\lambda\tau}(P,p)\chi_\tau(p_2',k'). \quad (24)$$

Comparing this equation with Eq. (12), we obtain

$$\chi(P,p,k,k')=(2\pi)^8\chi_\lambda(p_1',k)\chi_{\lambda\tau}(P,p)\chi_\tau(p_2',k'). \quad (25)$$

Up to now, for the molecular state composed of two heavy vector mesons with arbitrary spin and definite parity, we obtain the concrete form of its GBS wave function describing the four-quark state. Obviously, Eq. (25) can be extended to arbitrary meson-meson bound state.

IV. THE DECAY MODE $Y(3940)\to J/\psi\omega$

As an illustration, we investigate the decay process $Y(3940)\to J/\psi\omega$ in this section. Assuming that the $Y(3940)$ state is a S-wave molecule state consisting of two heavy vector mesons D^{*0} and $\bar{D}^{*0}$, we have obtained the mass of this molecular state by solving the BS equation and deduced that the spin-parity quantum numbers of the $Y(3940)$ system are 0^+ in Ref. [7]. The charmed meson D^{*0} in molecular state is composed of c-quark and u-antiquark and the heavy vector meson J/ψ in the final state is a bound state of $c\bar{c}$. In Fig. 4, V and $\bar{V}'$ become D^{*0} and $\bar{D}^{*0}$, respectively; H becomes J/ψ and L becomes ω; and in Eq. (14) the flavor labels $C=D$ and $A=B$ represent c-quark and u-quark, respectively. The Lagrangian for the interaction of ω meson with u-quark should be

$$\mathcal{L}_I^\omega(x_i)=ig_\omega\bar{\mathcal{Q}}^A(x_i)\gamma_\mu^A\mathcal{Q}^A(x_i)A_\mu^\omega(x_i). \quad (26)$$

Then the bound state matrix elements of vector current become

$$\langle Q|J_\mu(x_i)|P\rangle = \frac{1}{(2\pi)^{11}}\frac{1}{\sqrt{2E(Q)}}\frac{1}{\sqrt{2E(P)}}\int d^4k d^4p \times \bar{\chi}(Q,q)S^C(p_1)^{-1}\gamma_\mu^A S^C(p_2)^{-1} \times \chi(P,p,k,k')e^{i(P-Q)\cdot x_i}. \quad (27)$$

The GBS wave function for four-quark state is

$$\chi(P,p,k,k') = (2\pi)^8\chi_\lambda(p_1',k)\chi^{0^+}_{\lambda\tau}(P,p)\chi_\tau(p_2',k') = (2\pi)^8\frac{-i}{\gamma^C\cdot p_1 - im_c}\frac{1}{\mathcal{N}^{D^{*0}}}\left(\gamma_\lambda + p'_{1\lambda}\frac{\gamma\cdot p_1'}{M^2_{D^{*0}}}\right) \times\varphi_{D^{*0}}(k^2)\frac{-i}{\gamma^A\cdot p_3 - im_u}\frac{1}{\mathcal{N}^{0^+}}(T^1_{\lambda\tau}\mathcal{F}_1 + T^2_{\lambda\tau}\mathcal{F}_2) \times\frac{-i}{\gamma^A\cdot p_4 - im_u}\frac{1}{\mathcal{N}^{\bar{D}^{*0}}}\left(\gamma_\tau + p'_{2\tau}\frac{\gamma\cdot p_2'}{M^2_{\bar{D}^{*0}}}\right) \times\varphi_{\bar{D}^{*0}}(k'^2)\frac{-i}{\gamma^C\cdot p_2 - im_c}, \quad (28)$$

where $m_{c,u}$ are the constituent quark masses, $\omega_{D^{*0}} = 1.50$ GeV [17] and the momentum of this bound state is set as $P = (0,0,0,iM)$ in the rest frame. The BS wave function of the physically observed J/ψ meson has the form

$$\chi(Q,q) = \varepsilon^\kappa_\nu(Q)\chi_\nu(Q,q) = \varepsilon^\kappa_\nu(Q)\frac{-i}{\gamma^C\cdot q_1 - im_c}\frac{1}{\mathcal{N}^{J/\psi}}\left(\gamma_\nu + Q_\nu\frac{\gamma\cdot Q}{M^2_{J/\psi}}\right) \times\varphi_{J/\psi}(q^2)\frac{-i}{\gamma^C\cdot q_2 - im_c}, \quad (29)$$

where $\varepsilon^{\kappa=1,2,3}_\nu(Q)$ is the polarization vector of heavy vector meson J/ψ and $\varepsilon^\kappa(Q)\cdot Q = 0$. Because we consider that the heaviest quark carries all the heavy-meson momentum as in heavy-quark effective theory (HQET) [18], these momenta in Eq. (14) become

$$p_1' = P/2 + p, \qquad p_2' = P/2 - p,$$
$$q_1 = Q/2 + q, \qquad q_2 = Q/2 - q,$$
$$p_1 = P/2 + p + k, \qquad p_2 = Q - P/2 - p - k,$$
$$p_3 = -k, \qquad p_4 = -k',$$
$$k' = Q - P - k, \qquad q = P/2 - Q/2 + p + k. \quad (30)$$

In this paper we consider that the BS wave function of J/ψ has the form given in Refs. [16,17], however, the parameter $\omega_{J/\psi}$ has not been determined. Fortunately, a credible wave function of J/ψ from lattice QCD has been given in a series of studies [19–21] and we can obtain the value of $\omega_{J/\psi}$ as follows. In Refs. [20,21] the wave function of J/ψ is considered as the equal-time $c\bar{c}$ BS wave function in the coordinate representation, which is equivalent to the instantaneous approximation of BS wave function in the momentum representation. From Eq. (29), we obtain

$$\chi_\nu(Q,q) = \frac{-i}{\gamma\cdot(Q/2+q) - im_c}\frac{1}{\mathcal{N}^{J/\psi}}\left(\gamma_\nu + Q_\nu\frac{\gamma\cdot Q}{M^2_{J/\psi}}\right) \times\varphi_{J/\psi}(q^2)\frac{-i}{\gamma\cdot(Q/2-q) - im_c}, \quad (31)$$

where q is the relative momentum between c-quark and c-antiquark, Q is set as the momentum of the heavy meson in the rest frame, q and Q are not the momenta presented in the decay process. The heavy vector meson wave function in instantaneous approximation can be obtained from the BS wave function $\chi_\nu(Q,q)$ in Eq. (31) [22]

$$\Psi^{J/\psi}_i(\mathbf{q}) = \int dq_4\frac{1}{4}\mathrm{tr}\{\gamma_i\chi_i(Q,q)\} \quad i = 1,2,3. \quad (32)$$

To be consistent with the lattice QCD, we obtain the three-vector wave function of J/ψ in Euclidean space

$$\Psi^{J/\psi}(\mathbf{q}) = \int dq_4\frac{1}{\mathcal{N}^{J/\psi}}\exp\left(\frac{-\mathbf{q}^2 - q_4^2}{\omega^2_{J/\psi}}\right) \times\frac{\mathbf{q}^2/3 + q_4^2 - M^2_{J/\psi}/4 + m_c^2}{(\mathbf{q}^2 + q_4^2 + m_c^2 + M^2_{J/\psi}/4)^2 - q_4^2M^2_{J/\psi}}\sqrt{3}\hat{\mathbf{q}}, \quad (33)$$

where $\hat{\mathbf{q}}$ is the unit momentum. The q_4 integral can be numerically calculated and the wave function in the coordinate representation $\Psi^{J/\psi}(\mathbf{r})$ can be obtained. Multiplying $\Psi^{J/\psi}(\mathbf{r})$ by the spatial distance r and then comparing the numerical result with the wave function of J/ψ shown as Fig. 2 in Ref. [21], we obtain the parameter $\omega_{J/\psi} = 0.826$ GeV.

Now, we consider these normalizations $\mathcal{N}^{J/\psi}$, $\mathcal{N}^{D^{*0}}$, and $\mathcal{N}^{0^+}$. In Refs. [16,17] the BS equation for quark-antiquark state is treated in the ladder approximation, the reduced normalization condition for the BS wave function of J/ψ meson expressed as Eq. (31) is

$$\frac{-i}{(2\pi)^4}\frac{1}{3}\int d^4q\bar{\chi}_\nu(Q,q) \times\frac{\partial}{\partial Q_0}[S(Q/2+q)^{-1}S(Q/2-q)^{-1}]\chi_\nu(Q,q) = 2Q_0, \quad (34)$$

where the factor $1/3$ appears for the three transverse directions are summed. Similarly, for D^{*0} meson, its BS wave function can be written as

$$\chi_\nu(Q,q) = \frac{-i}{\gamma\cdot(Q+q) - im_c}\frac{1}{\mathcal{N}^{D^{*0}}}\left(\gamma_\nu + Q_\nu\frac{\gamma\cdot Q}{M^2_{D^{*0}}}\right) \times\varphi_{D^{*0}}(q^2)\frac{-i}{\gamma\cdot(-q) - im_u}, \quad (35)$$

and the reduced normalization condition is

$$\frac{-i}{(2\pi)^4}\frac{1}{3}\int d^4q\bar{\chi}_\nu(Q,q)\frac{\partial}{\partial Q_0}[S(Q+q)^{-1}]S(-q)^{-1}\chi_\nu(Q,q)=2Q_0. \tag{36}$$

In Ref. [7] we have obtained the BS wave function of the molecular state $D^{*0}\bar{D}^{*0}$ with 0^+

$$\chi^{0^+}_{\lambda\tau}(P,p)=\frac{1}{\mathcal{N}^{0^+}}[T^1_{\lambda\tau}\mathcal{F}_1(P\cdot p,p^2)+T^2_{\lambda\tau}\mathcal{F}_2(P\cdot p,p^2)], \tag{37}$$

and also employed the ladder approximation to solve the BS equation. The reduced normalization condition for $\chi^{0^+}_{\lambda\tau}(P,p)$ is

$$\frac{-i}{(2\pi)^4}\int d^4p\bar{\chi}_{\lambda\tau}(P,p)\frac{\partial}{\partial P_0}[\Delta_{F\lambda\lambda'}(P/2+p)^{-1}\Delta_{F\tau\tau'}(P/2-p)^{-1}]\chi_{\lambda'\tau'}(P,p)=2P_0, \tag{38}$$

where $\Delta_{F\beta\alpha'}(p)^{-1}$ is the inverse propagator for the vector field with mass m. The propagator for the vector field $\Delta_{F\alpha\beta}(p)=(\delta_{\alpha\beta}+\frac{p_\alpha p_\beta}{m^2})\frac{-i}{p^2+m^2-i\epsilon}$ and $\Delta_{F\beta\alpha'}(p)^{-1}$ satisfy the relation $\Delta_{F\alpha\beta}(p)\Delta_{F\beta\alpha'}(p)^{-1}=\delta_{\alpha\alpha'}$, and we obtain

$$\Delta_{F\beta\alpha'}(p)^{-1}=i\left(\delta_{\beta\alpha'}-\frac{p_\beta p_{\alpha'}}{p^2+m^2}\right)(p^2+m^2). \tag{39}$$

For the scalar functions $\mathcal{F}_1$ and $\mathcal{F}_2$, we can obtain two individual equations [7]

$$\mathcal{F}_1(P\cdot p,p^2)=\frac{1}{(P/2+p)^2+M_1^2-i\epsilon}\frac{1}{(P/2-p)^2+M_2^2-i\epsilon}\int\frac{d^4q'}{(2\pi)^4}V_1(p,q';P)\mathcal{F}_1(P\cdot q',q'^2),$$

$$(P/2-p)^2\mathcal{F}_2(P\cdot p,p^2)=\frac{1}{(P/2+p)^2+M_1^2-i\epsilon}\frac{1}{(P/2-p)^2+M_2^2-i\epsilon}\int\frac{d^4q'}{(2\pi)^4}V_2(p,q';P)(P/2-q')^2\mathcal{F}_2(P\cdot q',q'^2),$$

where $V_1(p,q';P)$ and $V_2(p,q';P)$ are derived from the interaction kernel between $D^{*0}\bar{D}^{*0}$. Though in Ref. [7] we solved these two equations in instantaneous approximation and obtained these wave functions $\Psi^{0^+}_1(\mathbf{p})=\int dp_0\mathcal{F}_1(P\cdot p,p^2)$ and $\Psi^{0^+}_2(\mathbf{p})=\int dp_0(P/2-p)^2\mathcal{F}_2(P\cdot p,p^2)$, it is easy to obtain $\mathcal{F}_1$ and $\mathcal{F}_2$ from $\Psi^{0^+}_1$ and $\Psi^{0^+}_2$, respectively.

After obtaining the parameter $\omega_{J/\psi}$ and these normalizations, we can calculate the bound state matrix elements of $J_\mu(x_i)$ expressed as Eq. (27) and they become

$$\langle Q|J_\mu(x_i)|P\rangle=\frac{-1}{(2\pi)^3}\frac{1}{\sqrt{2E(Q)}}\frac{1}{\sqrt{2E(P)}}\varepsilon^{\kappa *}_\nu(Q)\mathcal{M}_{\nu\mu}e^{i(P-Q)\cdot x_i}, \tag{40}$$

where

$$\begin{aligned}\mathcal{M}_{\nu\mu}=&\int d^4kd^4p\frac{1}{\mathcal{N}^{J/\psi}}\frac{\varphi_{J/\psi}(q^2)}{p_2^2+m_c^2}\frac{1}{p_1^2+m_c^2}\frac{1}{\mathcal{N}^{D^{*0}}}\frac{\varphi_{D^{*0}}(k^2)}{k^2+m_u^2}\frac{1}{\mathcal{N}^{\bar{D}^{*0}}}\frac{\varphi_{\bar{D}^{*0}}(k'^2)}{k'^2+m_u^2}\frac{1}{\mathcal{N}^{0^+}}\\&\times\mathrm{Tr}\Big\{[\gamma\cdot p_2+im_c]\left(\gamma_\nu+Q_\nu\frac{\gamma\cdot Q}{M^2_{J/\psi}}\right)[\gamma\cdot p_1+im_c]\{[(p'_1\cdot p'_2)\gamma_\tau-(\gamma\cdot p'_2)p'_{1\tau}]\mathcal{F}_1(p)\\&+[p'^2_1p'^2_2\gamma_\tau+(p'_1\cdot p'_2)(\gamma\cdot p'_1)p'_{2\tau}-p'^2_2(\gamma\cdot p'_1)p'_{1\tau}-p'^2_1(\gamma\cdot p'_2)p'_{2\tau}]\mathcal{F}_2(p)\}\\&\times[\gamma\cdot(-k)+im_u]\gamma_\mu[\gamma\cdot(-k')+im_u]\gamma_\tau\Big\}.\end{aligned} \tag{41}$$

The trace of the product of 8 γ-matrices should contain 105 terms. Using Mathematica, we calculate this trace and the result is given in Appendix B. In this work, the p integral is also calculated in instantaneous approximation. It is some difficult to straightly compute the integral including vectors and we give a simple approach as follows. Since the tensor $\mathcal{M}_{\nu\mu}$ only depends on P and Q, it can be expressed in Minkowski space as

$$\mathcal{M}_{\nu\mu}=g_{\nu\mu}f_1(P,Q)+P_\nu Q_\mu f_2(P,Q)+P_\nu P_\mu f_3(P,Q)+Q_\nu P_\mu f_4(P,Q)+Q_\nu Q_\mu f_5(P,Q), \tag{42}$$

where $f_i(P,Q)$ are the scalar functions. Multiplying Eq. (42) by these tensor structures $g_{\nu\mu}$, $P_\nu Q_\mu$, $P_\nu P_\mu$, $Q_\nu P_\mu$, $Q_\nu Q_\mu$, respectively, we can obtain a set of equations

$$
\begin{aligned}
g_{\nu\mu}\mathcal{M}_{\nu\mu} &= h_1 = 4f_1 + (P\cdot Q)f_2 + P^2 f_3 + (P\cdot Q)f_4 + Q^2 f_5,\\
P_\nu Q_\mu \mathcal{M}_{\nu\mu} &= h_2 = (P\cdot Q)f_1 + P^2Q^2 f_2 + P^2(P\cdot Q)f_3 + (P\cdot Q)^2 f_4 + Q^2(P\cdot Q)f_5,\\
P_\nu P_\mu \mathcal{M}_{\nu\mu} &= h_3 = P^2 f_1 + P^2(P\cdot Q)f_2 + P^2P^2 f_3 + P^2(P\cdot Q)f_4 + (P\cdot Q)^2 f_5,\\
Q_\nu P_\mu \mathcal{M}_{\nu\mu} &= 0 = (P\cdot Q)f_1 + (P\cdot Q)^2 f_2 + P^2(P\cdot Q)f_3 + Q^2P^2 f_4 + Q^2(P\cdot Q)f_5,\\
Q_\nu Q_\mu \mathcal{M}_{\nu\mu} &= 0 = Q^2 f_1 + Q^2(P\cdot Q)f_2 + (P\cdot Q)^2 f_3 + Q^2(P\cdot Q)f_4 + Q^2Q^2 f_5,
\end{aligned}
\tag{43}
$$

where h_i are numbers. After numerically calculating h_i and solving this set of equations, we obtain the values of f_i.

Finally, we obtain the lowest order transition matrix element for the decay mode $Y(3940) \to J/\psi\omega$

$$
\begin{aligned}
i\int d^4x_i \langle \omega, J/\psi | \mathcal{L}_I^\omega(x_i) | Y(3940)\rangle &= i\int d^4x_i \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2E(Q')}} e^{-iQ'\cdot x_i}\varepsilon_\mu^{\kappa'*}(Q')\langle Q|ig_\omega J_\mu(x_i)|P\rangle\\
&= \frac{1}{(2\pi)^{3/2}}\frac{g_\omega}{\sqrt{2E(Q')}}\frac{1}{\sqrt{2E(Q)}}\frac{1}{\sqrt{2E(P)}}(2\pi)^4\delta^{(4)}(P-Q-Q')\frac{1}{(2\pi)^3}\varepsilon_\mu^{\kappa'}(Q')\varepsilon_\nu^{\kappa}(Q)\mathcal{M}_{\nu\mu},
\end{aligned}
\tag{44}
$$

where $\varepsilon_\mu^{\kappa'}(Q')$ is the polarization vector of ω meson with mass M_ω, $E(Q') = \sqrt{\mathbf{Q}'^2 + M_\omega^2}$, $E(Q) = \sqrt{\mathbf{Q}^2 + M_{J/\psi}^2}$ and the meson-quark coupling constant $g_\omega^2 = 2.42$ was obtained within QCD sum rules approach [23]. Then the decay width for this process becomes

$$
\begin{aligned}
\Gamma &= \int d^3Q d^3Q'(2\pi)^4\delta^{(4)}(P-Q-Q')\frac{g_\omega^2}{2E(Q')}\frac{1}{2E(Q)}\frac{1}{2E(P)}\frac{1}{(2\pi)^6}\sum_{\kappa'=1}^{3}\sum_{\kappa=1}^{3}|\varepsilon_\mu^{\kappa'}(Q')\varepsilon_\nu^{\kappa}(Q)\mathcal{M}_{\nu\mu}|^2\\
&= \frac{1}{(2\pi)}\frac{g_\omega^2\mathbf{Q}_\omega^2}{4M\sqrt{\mathbf{Q}_\omega^2 + M_\omega^2}\sqrt{\mathbf{Q}_\omega^2 + M_{J/\psi}^2}}W(\mathbf{Q}_\omega^2),
\end{aligned}
\tag{45}
$$

where $\mathbf{Q}_\omega^2 = [M^2 - (M_{J/\psi} + M_\omega)^2][M^2 - (M_{J/\psi} - M_\omega)^2]/(4M^2)$. Using the transverse condition $\varepsilon^\kappa(Q)\cdot Q = 0$ and the completeness relation, we obtain

$$
\begin{aligned}
W(\mathbf{Q}_\omega^2) &= f_1^* f_1\left[4 + \frac{Q^2}{M_{J/\psi}^2} + \frac{Q'^2}{M_\omega^2} + \frac{(Q\cdot Q')^2}{M_\omega^2 M_{J/\psi}^2}\right] + f_2^* f_2\left[P^2Q^2 + \frac{(P\cdot Q)^2Q^2}{M_{J/\psi}^2} + \frac{P^2(Q\cdot Q')^2}{M_\omega^2} + \frac{(P\cdot Q)^2(Q\cdot Q')^2}{M_\omega^2 M_{J/\psi}^2}\right]\\
&+ f_3^* f_3\left[P^4 + \frac{P^2(P\cdot Q)^2}{M_{J/\psi}^2} + \frac{P^2(P\cdot Q')^2}{M_\omega^2} + \frac{(P\cdot Q)^2(P\cdot Q')^2}{M_\omega^2 M_{J/\psi}^2}\right]\\
&+ (f_1^* f_2 + f_2^* f_1)\left[P\cdot Q + \frac{(P\cdot Q)Q^2}{M_{J/\psi}^2} + \frac{(P\cdot Q')(Q\cdot Q')}{M_\omega^2} + \frac{(P\cdot Q)(Q\cdot Q')^2}{M_\omega^2 M_{J/\psi}^2}\right]\\
&+ (f_1^* f_3 + f_3^* f_1)\left[P^2 + \frac{(P\cdot Q)^2}{M_{J/\psi}^2} + \frac{(P\cdot Q')^2}{M_\omega^2} + \frac{(P\cdot Q)(P\cdot Q')(Q\cdot Q')}{M_\omega^2 M_{J/\psi}^2}\right]\\
&+ (f_2^* f_3 + f_3^* f_2)\left[P^2(P\cdot Q) + \frac{(P\cdot Q)^3}{M_{J/\psi}^2} + \frac{P^2(P\cdot Q')(Q\cdot Q')}{M_\omega^2} + \frac{(P\cdot Q)^2(P\cdot Q')(Q\cdot Q')}{M_\omega^2 M_{J/\psi}^2}\right],
\end{aligned}
\tag{46}
$$

where $P = (0,0,0,iM)$, $Q = (\mathbf{Q}_\omega, i\sqrt{\mathbf{Q}_\omega^2 + M_{J/\psi}^2})$ and $Q' = (-\mathbf{Q}_\omega, i\sqrt{\mathbf{Q}_\omega^2 + M_\omega^2})$.

V. NUMERICAL RESULT

The constituent quark masses $m_c = 1.55$ GeV, $m_u = 0.33$ GeV, the meson masses $M_\omega = 0.782$ GeV, $M_{D^{*0}} = 2.007$ GeV, $M_{J/\psi} = 3.097$ GeV [24]. In this work, we employ the value of $\omega_{D^{*0}}$ given in Ref. [17] rather than the one in Ref. [16]. Using the approach introduced in our previous work [7], we recalculate the mass of the molecular state

$D^{*0}\bar{D}^{*0}$ with 0^+ and obtain $M = 3.942$ GeV. Then by doing the numerical calculation, we obtain that the decay width of the $Y(3940)$ is $\Gamma(Y(3940) \to J/\psi\omega) = 66$ MeV.

Our approach involves the meson-quark coupling constant g_ω and the parameters $\omega_{D^{*0}}$, $\omega_{J/\psi}$ in the BS amplitudes of heavy vector mesons which have been fixed by providing fits to observables, so there is not an adjustable parameter. The calculated mass and decay width of the $Y(3940)$ state are consistent with the experimental data, while the values of its mass and width in experiments are $M = 3.943$ GeV, $\Gamma = 87$ MeV [10] and $M = 3.914$ GeV, $\Gamma = 34$ MeV [11]. The deduced quantum numbers of the $Y(3940)$ system are also consistent with the results given in Ref. [25]. Therefore, we can reasonably conclude that the $Y(3940)$ state is very probably a molecular state composed of $D^{*0}\bar{D}^{*0}$.

VI. CONCLUSION

The general formulas for the decay widths of molecular states composed of two heavy vector mesons with arbitrary spin and parity into a heavy meson plus a light meson is given. Taking into account the internal structure of heavy vector mesons in the molecular state and using the general form of the BS wave functions for the bound states consisting of two vector fields, we obtain the general form of the GBS wave functions for four-quark states describing this molecular structure. Then assuming that the exotic state $Y(3940)$ is a $D^{*0}\bar{D}^{*0}$ molecular state, we numerically calculate the decay width $\Gamma(Y(3940) \to J/\psi\omega)$, which is in good agreement with experiments. So far, we have established a complete and accurate theoretical approach from QCD to investigate the molecular state composed of two heavy vector mesons. In the future, one can use the general form of the GBS wave functions for four-quark states to investigate arbitrary decay mode of the molecular state including the strong and radiative decays. More importantly, the nonperturbative contribution from the vacuum condensates can be introduced into the BS wave functions and the irreducible part of Green's function, and then the calculated bound state matrix element contains more inspiration of QCD.

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APPENDIX A: THE TENSOR STRUCTURES IN THE GENERAL FORM OF THE BS WAVE FUNCTIONS

The tensor structures in Eqs. (18)–(21) are given below [7]

$$T^1_{\lambda\tau} = (\eta_1\eta_2 P^2 - \eta_1 P\cdot p + \eta_2 P\cdot p - p^2)g_{\lambda\tau} - (\eta_1\eta_2 P_\lambda P_\tau + \eta_2 P_\lambda p_\tau - \eta_1 p_\lambda P_\tau - p_\lambda p_\tau),$$

$$\begin{aligned}T^2_{\lambda\tau} = {} & (\eta_1^2 P^2 + 2\eta_1 P\cdot p + p^2)(\eta_2^2 P^2 - 2\eta_2 P\cdot p + p^2)g_{\lambda\tau} \\ & + (\eta_1\eta_2 P^2 - \eta_1 P\cdot p + \eta_2 P\cdot p - p^2)(\eta_1\eta_2 P_\lambda P_\tau - \eta_1 P_\lambda p_\tau + \eta_2 p_\lambda P_\tau - p_\lambda p_\tau) \\ & - (\eta_2^2 P^2 - 2\eta_2 P\cdot p + p^2)(\eta_1^2 P_\lambda P_\tau + \eta_1 P_\lambda p_\tau + \eta_1 p_\lambda P_\tau + p_\lambda p_\tau) \\ & - (\eta_1^2 P^2 + 2\eta_1 P\cdot p + p^2)(\eta_2^2 P_\lambda P_\tau - \eta_2 P_\lambda p_\tau - \eta_2 p_\lambda P_\tau + p_\lambda p_\tau),\end{aligned}$$

$$\begin{aligned}T^3_{\lambda\tau} = {} & \frac{1}{j!} p_{\{\mu_2}\cdots p_{\mu_j} g_{\mu_1\}\lambda}(\eta_1^2 P^2 + 2\eta_1 P\cdot p + p^2)[(\eta_2^2 P^2 - 2\eta_2 P\cdot p + p^2)(\eta_1 P + p)_\tau \\ & - (\eta_1\eta_2 P^2 - \eta_1 P\cdot p + \eta_2 P\cdot p - p^2)(\eta_2 P - p)_\tau] \\ & - p_{\mu_1}\cdots p_{\mu_j}[(\eta_2^2 P^2 - 2\eta_2 P\cdot p + p^2)(\eta_1^2 P_\lambda P_\tau + \eta_1 P_\lambda p_\tau + \eta_1 p_\lambda P_\tau + p_\lambda p_\tau) \\ & - (\eta_1\eta_2 P^2 - \eta_1 P\cdot p + \eta_2 P\cdot p - p^2)(\eta_1\eta_2 P_\lambda P_\tau - \eta_1 P_\lambda p_\tau + \eta_2 p_\lambda P_\tau - p_\lambda p_\tau)],\end{aligned}$$

$$\begin{aligned}T^4_{\lambda\tau} = {} & \frac{1}{j!} p_{\{\mu_2}\cdots p_{\mu_j} g_{\mu_1\}\tau}(\eta_2^2 P^2 - 2\eta_2 P\cdot p + p^2)[(\eta_1\eta_2 P^2 - \eta_1 P\cdot p \\ & + \eta_2 P\cdot p - p^2)(\eta_1 P + p)_\lambda - (\eta_1^2 P^2 + 2\eta_1 P\cdot p + p^2)(\eta_2 P - p)_\lambda] \\ & - p_{\mu_1}\cdots p_{\mu_j}[(\eta_1^2 P^2 + 2\eta_1 P\cdot p + p^2)(\eta_2^2 P_\lambda P_\tau - \eta_2 P_\lambda p_\tau - \eta_2 p_\lambda P_\tau + p_\lambda p_\tau) \\ & - (\eta_1\eta_2 P^2 - \eta_1 P\cdot p + \eta_2 P\cdot p - p^2)(\eta_1\eta_2 P_\lambda P_\tau - \eta_1 P_\lambda p_\tau + \eta_2 p_\lambda P_\tau - p_\lambda p_\tau)],\end{aligned}$$

$$\begin{aligned} T^5_{\lambda\tau} &= (\eta_2 P\cdot p - \eta_1 P\cdot p - 2p^2)\, p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi}p_\xi + (2\eta_1\eta_2 P\cdot p + \eta_2 p^2 - \eta_1 p^2)\, p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi}P_\xi \\ &\quad + p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\lambda\xi\zeta}p_\xi P_\zeta p_\tau + p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta}p_\xi P_\zeta p_\lambda, \end{aligned}$$

$$T^6_{\lambda\tau} = (P\cdot p)\, p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi}p_\xi - p^2 p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi}P_\xi + p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\lambda\xi\zeta}p_\xi P_\zeta p_\tau - p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta}p_\xi P_\zeta p_\lambda,$$

$$\begin{aligned} T^7_{\lambda\tau} &= (\eta_2 P^2 - \eta_1 P^2 - 2P\cdot p)\, p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi}p_\xi + (2\eta_1\eta_2 P^2 + \eta_2 P\cdot p - \eta_1 P\cdot p)\, p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi}P_\xi \\ &\quad + p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\lambda\xi\zeta}p_\xi P_\zeta P_\tau + p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta}p_\xi P_\zeta P_\lambda, \end{aligned}$$

$$T^8_{\lambda\tau} = P^2 p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi}p_\xi - (P\cdot p)\, p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi}P_\xi + p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\lambda\xi\zeta}p_\xi P_\zeta P_\tau - p_{\{\mu_2}\ldots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta}p_\xi P_\zeta P_\lambda.$$

APPENDIX B: TRACE OF A PRODUCT OF 8 γ-MATRICES

If a, b, c, d, e, f, g, h are 8 arbitrary 4-vectors, we compute the trace

$$\begin{aligned} &Tr\gamma\cdot a\gamma\cdot b\gamma\cdot c\gamma\cdot d\gamma\cdot e\gamma\cdot f\gamma\cdot g\gamma\cdot h \\ &= 4(a\cdot hb\cdot gc\cdot fd\cdot e - a\cdot gb\cdot hc\cdot fd\cdot e - a\cdot hb\cdot fc\cdot gd\cdot e + a\cdot fb\cdot hc\cdot gd\cdot e + a\cdot gb\cdot fc\cdot hd\cdot e \\ &\quad - a\cdot fb\cdot gc\cdot hd\cdot e + a\cdot hb\cdot cf\cdot gd\cdot e - a\cdot cb\cdot hf\cdot gd\cdot e + a\cdot bc\cdot hf\cdot gd\cdot e - a\cdot gb\cdot cf\cdot hd\cdot e \\ &\quad + a\cdot cb\cdot gf\cdot hd\cdot e - a\cdot bc\cdot gf\cdot hd\cdot e + a\cdot fb\cdot cg\cdot hd\cdot e - a\cdot cb\cdot fg\cdot hd\cdot e + a\cdot bc\cdot fg\cdot hd\cdot e \\ &\quad - a\cdot hb\cdot gc\cdot ed\cdot f + a\cdot gb\cdot hc\cdot ed\cdot f + a\cdot hb\cdot ec\cdot gd\cdot f - a\cdot eb\cdot hc\cdot gd\cdot f - a\cdot gb\cdot ec\cdot hd\cdot f \\ &\quad + a\cdot eb\cdot gc\cdot hd\cdot f + a\cdot hb\cdot fc\cdot ed\cdot g - a\cdot fb\cdot hc\cdot ed\cdot g - a\cdot hb\cdot ec\cdot fd\cdot g + a\cdot eb\cdot hc\cdot fd\cdot g \\ &\quad + a\cdot fb\cdot ec\cdot hd\cdot g - a\cdot eb\cdot fc\cdot hd\cdot g - a\cdot gb\cdot fc\cdot ed\cdot h + a\cdot fb\cdot gc\cdot ed\cdot h + a\cdot gb\cdot ec\cdot fd\cdot h \\ &\quad - a\cdot eb\cdot gc\cdot fd\cdot h - a\cdot fb\cdot ec\cdot gd\cdot h + a\cdot eb\cdot fc\cdot gd\cdot h + a\cdot hb\cdot gc\cdot de\cdot f - a\cdot gb\cdot hc\cdot de\cdot f \\ &\quad - a\cdot hb\cdot dc\cdot ge\cdot f + a\cdot db\cdot hc\cdot ge\cdot f + a\cdot gb\cdot dc\cdot he\cdot f - a\cdot db\cdot gc\cdot he\cdot f + a\cdot hb\cdot cd\cdot ge\cdot f \\ &\quad - a\cdot cb\cdot hd\cdot ge\cdot f + a\cdot bc\cdot hd\cdot ge\cdot f - a\cdot gb\cdot cd\cdot he\cdot f + a\cdot cb\cdot gd\cdot he\cdot f - a\cdot bc\cdot gd\cdot he\cdot f \\ &\quad - a\cdot hb\cdot fc\cdot de\cdot g + a\cdot fb\cdot hc\cdot de\cdot g + a\cdot hb\cdot dc\cdot fe\cdot g - a\cdot db\cdot hc\cdot fe\cdot g - a\cdot fb\cdot dc\cdot he\cdot g \\ &\quad + a\cdot db\cdot fc\cdot he\cdot g - a\cdot hb\cdot cd\cdot fe\cdot g + a\cdot cb\cdot hd\cdot fe\cdot g - a\cdot bc\cdot hd\cdot fe\cdot g + a\cdot fb\cdot cd\cdot he\cdot g \\ &\quad - a\cdot cb\cdot fd\cdot he\cdot g + a\cdot bc\cdot fd\cdot he\cdot g + a\cdot gb\cdot fc\cdot de\cdot h - a\cdot fb\cdot gc\cdot de\cdot h - a\cdot gb\cdot dc\cdot fe\cdot h \\ &\quad + a\cdot db\cdot gc\cdot fe\cdot h + a\cdot fb\cdot dc\cdot ge\cdot h - a\cdot db\cdot fc\cdot ge\cdot h + a\cdot gb\cdot cd\cdot fe\cdot h - a\cdot cb\cdot gd\cdot fe\cdot h \\ &\quad + a\cdot bc\cdot gd\cdot fe\cdot h - a\cdot fb\cdot cd\cdot ge\cdot h + a\cdot cb\cdot fd\cdot ge\cdot h - a\cdot bc\cdot fd\cdot ge\cdot h + a\cdot hb\cdot ec\cdot df\cdot g \\ &\quad - a\cdot eb\cdot hc\cdot df\cdot g - a\cdot hb\cdot dc\cdot ef\cdot g + a\cdot db\cdot hc\cdot ef\cdot g + a\cdot eb\cdot dc\cdot hf\cdot g - a\cdot db\cdot ec\cdot hf\cdot g \\ &\quad - a\cdot eb\cdot cd\cdot hf\cdot g + a\cdot cb\cdot ed\cdot hf\cdot g - a\cdot bc\cdot ed\cdot hf\cdot g + a\cdot db\cdot ce\cdot hf\cdot g - a\cdot cb\cdot de\cdot hf\cdot g \\ &\quad + a\cdot bc\cdot de\cdot hf\cdot g - a\cdot gb\cdot ec\cdot df\cdot h + a\cdot eb\cdot gc\cdot df\cdot h + a\cdot gb\cdot dc\cdot ef\cdot h - a\cdot db\cdot gc\cdot ef\cdot h \\ &\quad - a\cdot eb\cdot dc\cdot gf\cdot h + a\cdot db\cdot ec\cdot gf\cdot h + a\cdot eb\cdot cd\cdot gf\cdot h - a\cdot cb\cdot ed\cdot gf\cdot h + a\cdot bc\cdot ed\cdot gf\cdot h \\ &\quad - a\cdot db\cdot ce\cdot gf\cdot h + a\cdot cb\cdot de\cdot gf\cdot h - a\cdot bc\cdot de\cdot gf\cdot h + a\cdot fb\cdot ec\cdot dg\cdot h - a\cdot eb\cdot fc\cdot dg\cdot h \\ &\quad - a\cdot fb\cdot dc\cdot eg\cdot h + a\cdot db\cdot fc\cdot eg\cdot h + a\cdot eb\cdot dc\cdot fg\cdot h - a\cdot db\cdot ec\cdot fg\cdot h - a\cdot eb\cdot cd\cdot fg\cdot h \\ &\quad + a\cdot cb\cdot ed\cdot fg\cdot h - a\cdot bc\cdot ed\cdot fg\cdot h + a\cdot db\cdot ce\cdot fg\cdot h - a\cdot cb\cdot de\cdot fg\cdot h + a\cdot bc\cdot de\cdot fg\cdot h). \end{aligned}$$

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