Improved perturbative QCD formalism for B_c meson decays

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We derive the k_T resummation for doubly heavy-flavored B_c meson decays by including the charm quark mass effect into the known formula for a heavy-light system. The resultant Sudakov factor is employed in the perutrbative QCD study of the "golden channel" $B_c^+ \to J/\psi\pi^+$. With a reasonable model for the B_c meson distribution amplitude, which maintains approximate on-shell conditions of both the partonic bottom and charm quarks, it is observed that the imaginary piece of the $B_c \to J/\psi$ transition form factor appears to be power suppressed, and the $B_c^+ \to J/\psi\pi^+$ branching ratio is not lower than 10^{-3}. The above improved perturbative QCD formalism is applicable to B_c meson decays to other charmonia and charmed mesons.

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I. INTRODUCTION

A B_c meson is the ground state of the doubly heavy-flavored $\bar{b}c$ system in the Standard Model [1], different from the heavy-light one represented by a B meson and from the heavy-heavy one represented by quarkonia J/ψ and Υ in many aspects. Its weak transition can occur through the bottom quark decay with the spectator charm quark as displayed in Fig. 1(a), the charm quark decay with the spectator bottom quark in Fig. 1(b), and the pure weak annihilation channel in Fig. 1(c). Hence, B_c meson decays contain rich heavy quark dynamics in both the perturbative and nonperturbative regimes, which is worth a thorough exploration with high precision. It is certainly a challenge to develop an appropriate theoretical framework for analyzing B_c meson decays. A framework available in the literature is the perturbative QCD (PQCD) approach, which basically follows the conventional one for B meson decays, with the finite charm quark mass being included in hard decay kernels but neglected in the k_T resummation for meson distribution amplitudes. A rigorous resummation formalism for B_c meson decays, which involve multiple scales, is expected to be more complicated than for B meson decays.

In this paper, we will investigate how the charm quark mass affects the infrared structures of the B_c meson and of its decay products and derive the corresponding k_T resummation in the PQCD approach. The derivation depends on the power counting for the ratio m_c/m_b, m_b (m_c) being the bottom (charm) quark mass. Taking the limit $m_b \to \infty$ but keeping m_c finite, we treat a B_c meson as a heavy-light system, the decays of which can be analyzed in the conventional PQCD approach to B meson decays mentioned above. Taking the limit $m_b, m_c \to \infty$ but fixing the ratio m_c/m_b, we treat a B_c meson as a heavy-heavy system, the decays of which may be studied in a formalism for heavy quarkonium decays. Here, we will adopt the power counting rules proposed in Ref. [2] and regard a B_c meson as a multiscale system, which respects the hierarchy $m_b \gg m_c \gg \Lambda_{\rm QCD}$, $\Lambda_{\rm QCD}$ being the QCD scale. An intermediate impact of this power counting is that the large infrared logarithms $\ln(m_b/m_c)$, in addition to the ordinary ones $\ln(m_b/\Lambda_{\rm QCD})$, appear in the perturbative evaluation of the B_c meson distribution amplitude and need to be resummed.

The Sudakov factor from the k_T resummation with the charm quark mass effect is then employed in the PQCD study of the "golden channel" $B_c^+ \to J/\psi\pi^+$. We focus on the bottom quark decay of a B_c meson because the charm quark decay is believed to suffer from significant

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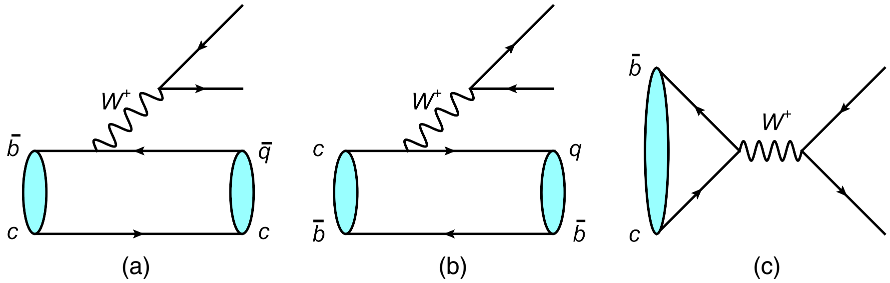

FIG. 1. Diagrams for B_c meson decays.

long-distance contributions, i.e., final-state interactions, though perturbative results for the $B_c \to B_{(s)}X$ modes have been presented in the literature [3,4]. Besides, the available models for the B_c meson distribution amplitude vary dramatically from a simple δ function [5,6] to a complicated Gaussian type [3]. We will propose a kinematic constraint on the B_c meson distribution amplitude, which allows both the partonic bottom and charm quarks to be off shell only at a power-suppressed level. It is then shown, with a reasonable model for the B_c meson distribution amplitude, that the imaginary piece of the $B_c \to J/\psi$ transition form factor, supposed to be a real object [7], is indeed power suppressed. It is also found that the $B_c^+ \to J/\psi\pi^+$ branching ratio is not lower than 10^{-3}, in agreement with those obtained in other approaches.

In Sec. II, we discuss the kinematic constraint on the charm quark momentum distribution in a B_c meson. The one-loop correction to the B_c meson distribution amplitude, which generates the double logarithm $\alpha_s \ln^2(m_b/m_c)$, α_s being the strong coupling, is calculated in Sec. III. The result hints at how the k_T resummation for B_c meson decays is modified from the known formula for B meson decays. In Sec. IV, we predict the $B_c^+ \to J/\psi\pi^+$ branching ratio in the improved PQCD framework, including the contributions from both factorizable and nonfactorizable emission diagrams. It is then stressed in the Conclusion that the formalism developed here is ready for the extension to B_c meson decays to other charmonia like η_c, χ_{cJ} $(J = 0, 1, 2), \ldots$, and charmed mesons.

II. KINEMATIC CONSTRAINT ON B_c MESON DISTRIBUTION AMPLITUDE

Consider the $B_c(P_1) \to J/\psi(P_2)$ transition at the maximal recoil, where

$$P_1 = \frac{m_{B_c}}{\sqrt{2}}(1, 1, \mathbf{0}_T), \qquad P_2 = \frac{m_{B_c}}{\sqrt{2}}(1, r_{J/\psi}^2, \mathbf{0}_T) \quad (1)$$

in the light-cone coordinates label the B_c and J/ψ meson momenta, respectively, with $r_{J/\psi} = m_{J/\psi}/m_{B_c}$ and m_{B_c} ($m_{J/\psi}$) being the B_c (J/ψ) meson mass. This transition involves multiple scales the same as in the $B \to D^*$ transition, which has been studied in Ref. [2]: m_b from the initial-state B meson, m_c from the final-state D^* meson, and both the B and D^* bound states contain the nonperturbative dynamics characterized by a low hadronic scale Λ. Following the argument in Ref. [2], the scaling of the energetic J/ψ momentum $P_2 \sim (m_b, m_c^2/m_b, \mathbf{0}_T) \sim m_c(m_b/m_c, m_c/m_b, \mathbf{0}_T)$ hints that the components of a collinear gluon momentum in such a multiscale system also obeys the power counting

$$l^\mu \sim \left(\frac{m_b}{m_c}\Lambda, \frac{m_c}{m_b}\Lambda, \Lambda\right), \quad (2)$$

with a tiny invariant mass squared $l^2 \sim \mathcal{O}(\Lambda^2)$. A valence charm quark in the J/ψ meson, after emitting such a collinear gluon, can acquire the virtuality of order $P_2 \cdot l \sim m_c\Lambda$. The momentum parametrizations for the two valence charm quarks participating in the hard subprocess should be symmetric under their exchange. Denote the spectator charm quark momentum as $k_2 = x_2 P_2$ and another as $P_2 - k_2 = (1 - x_2)P_2$ with the momentum fraction x_2, and assume both of them to be off-shell at most by $\mathcal{O}(m_c\Lambda)$: $k_2^2 - m_c^2 = \mathcal{O}(m_c\Lambda)$ and $(P_2 - k_2)^2 - m_c^2 = \mathcal{O}(m_c\Lambda)$. To satisfy these two conditions simultaneously, we choose a charm quark mass $m_c \approx m_{J/\psi}/2 \sim 1.5$ GeV for $m_{J/\psi} = 3.097$ GeV, and the momentum fraction $x_2 = 1/2 \pm \delta$ can deviate from its central value by $\delta \sim \mathcal{O}(\Lambda/m_c)$. That is, the J/ψ distribution amplitude takes a substantial value in the above range of x_2 with $\delta \sim 0.3$ for $\Lambda \sim 0.5$ GeV, due to the effect of collinear gluon emissions. The model for the J/ψ meson distribution amplitude, proposed in Ref. [8] and widely employed in the PQCD analyses, does exhibit these features.

Next, we discuss the kinematic constraint on the shape of the B_c meson distribution amplitude. Label the momentum of the spectator charm quark in the B_c meson by k_1 and that of the bottom quark by $P_1 - k_1$. The approximate on-shell-ness of the partons, $k_1^2 \sim m_c^2$ and $(P_1 - k_1)^2 \sim m_b^2$, implies that the zeroth component of k_1 is of order $k_1^0 \sim m_c$. A B_c meson at rest is dominated by soft dynamics, for which the momentum of a soft gluon is characterized by the power counting [2]

$$l^\mu \sim (\Lambda, \Lambda, \Lambda), \quad (3)$$

with a tiny invariant mass squared $l^2 \sim \mathcal{O}(\Lambda^2)$. The spectator charm quark, after emitting such a soft gluon, then reaches the virtuality of order $k_1 \cdot l \sim m_c\Lambda$. Parametrize the charm quark momentum by $k_1 = x_1 P_1$, x_1 being a momentum fraction, and require the virtuality $k_1^2 - m_c^2 = \mathcal{O}(m_c\Lambda)$. Given the bottom quark mass $m_b \approx m_{B_c} - m_c \sim 4.8$ GeV for $m_{B_c} = 6.276$ GeV, we find that the B_c meson distribution amplitude takes a substantial value around the momentum fraction $x_1 \sim m_c/m_b \sim 0.3$ within the width of about $\Lambda/m_b \sim 0.1$. It can be verified, following the above discussion, that the bottom quark in the B_c meson acquires the virtuality of $(P_1 - k_1)^2 - m_b^2 \sim \mathcal{O}(m_b\Lambda)$, consistent with the soft gluon emission effect.

We then investigate the virtuality of the hard particles in the kinematic regions specified for the partonic bottom and charm quarks. First, the invariant mass of the hard gluon emitted by the spectator quark is written as

$$(k_1 - k_2)^2 \approx -\frac{m_b m_c}{2} + \mathcal{O}(m_b\Lambda), \tag{4}$$

with the insertion of $m_{B_c} \approx m_b + m_c$ and $m_{J/\psi} \approx 2m_c$ up to the first powers in m_c and in Λ. The first term on the right-hand side of Eq. (4), being $\mathcal{O}(m_b m_c)$, indicates that the hard gluon tends to be spacelike for the chosen mass scales m_b, m_c, and Λ. The hard bottom quark, to which the hard gluon attaches, remains spacelike with the virtuality

$$(P_1 - k_2)^2 - m_b^2 \approx -\frac{m_b^2}{2}. \tag{5}$$

The hard charm quark, to which the hard gluon attaches, is also spacelike with the virtuality

$$(P_2 - k_1)^2 - m_c^2 \approx -m_b m_c + \mathcal{O}(m_b\Lambda). \tag{6}$$

We conclude that, as both the partonic bottom and charm quarks are only off-shell a bit, the imaginary piece in the $B_c \to J/\psi$ transition form factor appears to be power suppressed. This observation is easily understood: the J/ψ meson mass is below the $D\bar{D}$ threshold, so the $B_c \to J/\psi$ transition hardly occurs through an intermediate state.

The B_c meson wave function with an intrinsic k_T dependence is parametrized in a Gaussian form as

$$\phi_{B_c}(x, k_T) = \frac{f_{B_c}}{2\sqrt{2N_c}} \frac{\pi}{2\beta_{B_c}^2} N_{B_c} \exp\left[-\frac{1}{8\beta_{B_c}^2}\left(\frac{|\mathbf{k}_T|^2 + m_c^2}{x} + \frac{|-\mathbf{k}_T|^2 + m_b^2}{1-x}\right)\right], \tag{7}$$

in which $\mathbf{k}_T$ $(-\mathbf{k}_T)$ is the transverse momentum carried by the charm (bottom) quark, N_c is the number of colors, β_{B_c} is the shape parameter, and N_{B_c} is the normalization constant. The B_c meson distribution amplitude is given by

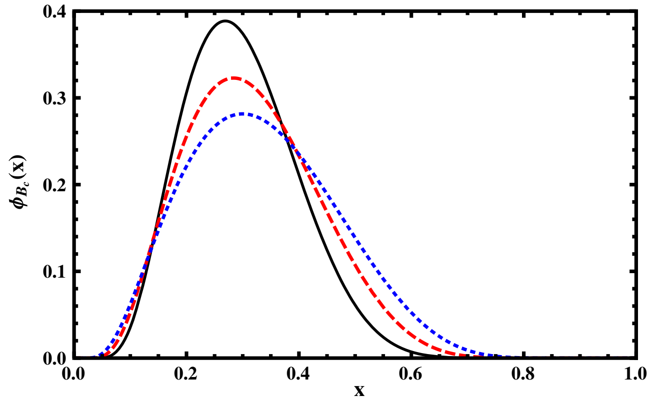

FIG. 2. Behavior of $\phi_{B_c}(x)$ for the different shape parameters $\beta_{B_c} = 0.8$ GeV (black-solid curve), 1.0 GeV (red-dashed curve), and 1.2 GeV (blue-dotted curve).

$$\phi_{B_c}(x, b) = \frac{f_{B_c}}{2\sqrt{2N_c}} N_{B_c} x(1-x) \exp\left[-\frac{(1-x)m_c^2 + x m_b^2}{8\beta_{B_c}^2 x(1-x)}\right] \times \exp[-2\beta_{B_c}^2 x(1-x) b^2], \tag{8}$$

with the impact parameter b being conjugate to k_T. The normalization constant N_{B_c} is fixed by the relation

$$\int_0^1 \phi_{B_c}(x, b=0) dx \equiv \int_0^1 \phi_{B_c}(x) dx = \frac{f_{B_c}}{2\sqrt{2N_c}}, \tag{9}$$

where the decay constant $f_{B_c} = 0.489 \pm 0.005$ GeV has been obtained in lattice QCD by the TWQCD Collaboration [9]. Figure 2, in which the behavior of $\phi_{B_c}(x)$ is plotted for the different shape parameters β_{B_c}, indicates that the peak of $\phi_{B_c}(x)$ shifts toward larger x and becomes broader with the increase of β_{B_c}. Note that data for B_c meson decay branching ratios are not yet available, so it is difficult to determine β_{B_c} unambiguously. However, the kinematic constraint derived above hints that $\beta_{B_c} =$ 1.0 GeV seems to be a reasonable choice. On the other hand, the existent models [3,10] of the B_c meson distribution amplitude roughly correspond to the range [0.6, 1.0] GeV of the parameter β_{B_c}.

III. k_T RESUMMATION FOR B_c MESON DECAYS

A theoretical challenge from the $B_c \to J/\psi$ transition is to derive the k_T resummation for energetic charm quarks with a finite mass. To proceed, we construct a transverse momentum–dependent J/ψ meson wave function in the k_T factorization theorem [11,12] and then perform the perturbative evaluation according to the wave-function definition as a hadronic matrix element of a nonlocal operator. The double logarithms attributed to the overlap of the collinear and soft radiative corrections are expected to differ from those in B meson decays into light mesons, which have been elaborated in Ref. [13]. According to the one-loop

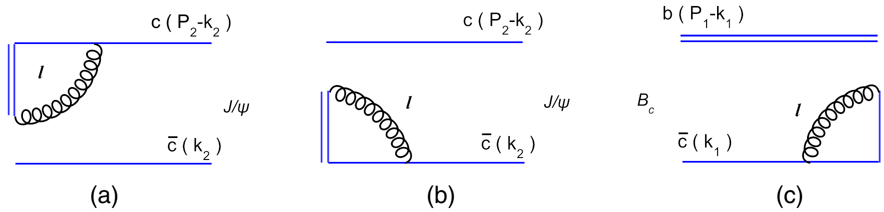

FIG. 3. $\mathcal{O}(\alpha_s)$ effective diagrams for the J/ψ and B_c mesons wave functions, which are relevant to the Sudakov factor $s_c(Q, b)$.

analysis in Ref. [14], the only source of the double logarithms is the correction to the quark-Wilson-line vertex as displayed in Fig. 3(a), in which the loop momentum does not flow into a hard subprocess. When the gluon in Fig. 3(a) attaches to the lower piece of the Wilson lines, the loop momentum flows through a hard subprocess. Since the region with small parton momenta dominates in the k_T factorization, the large collinear gluon momentum induces power suppression on the hard kernel [12], such that this one-loop diagram does not generate the double logarithm. The similar vertex diagram with the gluon being radiated by the spectator charm quark either in the J/ψ meson [Fig. 3(b)] or in the B_c meson [Fig. 3(c)] may produce the double logarithms. Nevertheless, their effects ought to be weaker, due to the lack of phase space for collinear gluons from less energetic quarks.

The loop integral corresponding to Fig. 3(a) is written as

$$\phi^{(1)} = -\frac{i}{4} g^2 C_F \mu_{\mathrm{f}}^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \times \mathrm{tr}\left[\gamma_5 \not{n}_+ \frac{\bar{\not{k}} + \not{l} + m_c}{(\bar{k}+l)^2 - m_c^2} \gamma_\nu \not{n}_- \gamma_5\right] \frac{1}{l^2} \frac{n^\nu}{n \cdot l}, \qquad (10)$$

with $\bar{k} \equiv P_2 - k_2$, the eikonal vertex n_ν, and the eikonal propagator $1/n \cdot l$. The dimensionless vector n with $n^+ > 0$ represents the direction of the Wilson lines, which is allowed to be away from the light cone [14]. The projectors $\gamma_5 \not{n}_+$ and $\not{n}_- \gamma_5$, arising from the insertion of the Fierz identity for factorizing the fermion flow, work for the selection of the logarithm $\ln(m_b/m_c)$ up to corrections in powers of m_c/m_b. A straightforward calculation leads to

$$\phi^{(1)} = \frac{\alpha_s}{4\pi} C_F \left[\frac{1}{\epsilon} + \ln \frac{4\pi \mu_f^2}{m_c^2 e^{\gamma_E}} - \ln^2 \frac{\zeta^2}{k_T^2} + \ln^2 \frac{m_c^2}{k_T^2} + \ln \frac{\zeta^2}{m_c^2} + 2 - \frac{2\pi^2}{3}\right], \qquad (11)$$

with the factorization scale μ_f, the Euler constant γ_E, and the variable $\zeta^2 \equiv 4(n \cdot \bar{k})^2/n^2$. It is found that the infrared logarithms in the above expression reproduce those in the pion case [15], as m_c is replaced by k_T. The double logarithms can be understood in the way that the soft divergence is regularized by the quark virtuality k_T, and the collinear divergence is regularized by the charm quark mass m_c, giving

$$-\ln^2 \frac{\zeta^2}{k_T^2} + \ln^2 \frac{m_c^2}{k_T^2} = -\ln \frac{\zeta^2 m_c^2}{k_T^4} \ln \frac{\zeta^2}{m_c^2}. \qquad (12)$$

The partial cancellation between the two double logarithms implies that the resummation effect in the case of energetic massive quarks is smaller than in the case of light quarks [16].

The aforementioned lack of phase space for the collinear gluons in Figs. 3(b) and 3(c) can be understood by means of the contour integration. Take Fig. 3(b), the loop integrand of which contains a denominator $(k_2 - l)^2 - m_c^2$ from the anticharm quark propagator, as an example. To get a nonvanishing contribution from the contour integration over the minus component l^- of the loop momentum, some poles of l^- have to be located in the upper half-plane, and some have to be located in the lower half-plane. This is possible only when the coefficients of l^- in the denominators of the corresponding loop integrand are not of the same sign. Hence, the plus component l^+ must take a value in the range $0 < l^+ < k_2^+$ for our gauge choice $n^+ > 0$ as stated below Eq. (10). In the dominant region with small parton momenta, i.e., with small k_2^+, the phase space for l^+ is then limited, implying a weaker double logarithmic effect.

We will not attempt a complete one-loop computation and an exact next-to-leading-logarithm resummation associated with an energetic massive quark in the present work. Instead, we will infer an approximate Sudakov exponent from the implication of Eq. (11). It has been known that the k_T resummation for an energetic light quark yields the Sudakov exponent in the b space [13,17],

$$s(Q, b) = \int_{1/b}^{Q} \frac{d\mu}{\mu} \left[\int_{1/b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} A(\alpha_s(\bar{\mu})) + B(\alpha_s(\mu))\right], \qquad (13)$$

at the next-to-leading-logarithm accuracy, where the universal anomalous dimension $A(\alpha_s)$ given to two loops is responsible for the collection of the double logarithms, the factor $B(\alpha_s)$ given to one loop is for the collection of single logarithms, and Q is related to the major light-cone component of the quark momentum through the variable ζ. The μ_f-independent logarithms in Eq. (11) can be cast into two pieces,

$$-\left(\ln^2 \frac{\zeta^2}{k_T^2} - \ln \frac{\zeta^2}{k_T^2}\right) + \left(\ln^2 \frac{m_c^2}{k_T^2} - \ln \frac{m_c^2}{k_T^2}\right), \qquad (14)$$

which are of the same form. This hints that the above infrared logarithms may be organized into the Sudakov exponents with the different upper bounds Q and m_c; namely, the Sudakov exponent $s_c(Q,b)$ for an energetic charm quark up to next-to-leading-logarithm might be expressed as the difference

$$s_c(Q,b) = s(Q,b) - s(m_c,b), \\ = \int_{m_c}^{Q} \frac{d\mu}{\mu}\left[\int_{1/b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}A(\alpha_s(\bar{\mu})) + B(\alpha_s(\mu))\right]. \quad (15)$$

This observation applies to the organization of the double logarithms in Figs. 3(b) and 3(c).

At last, the μ_f-dependent logarithm $\ln(\mu_f^2/m_c^2)$ in Eq. (11) means that the J/ψ (as well as B_c) meson distribution amplitude is defined at the scale m_c and that the renormalization-group evolution for the $B_c \to J/\psi$ transition runs from $\mu_f = m_c$ to the hard scale of the process. We summarize the exponents of the total evolution factors for the B_c and J/ψ meson distribution amplitudes as

$$S_{B_c} = s_c(x_1P_1^-, b_1) + \frac{5}{3}\int_{m_c}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \\ S_{J/\psi} = s_c(x_2P_2^+, b_2) + s_c((1-x_2)P_2^+, b_2) \\ + 2\int_{m_c}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \quad (16)$$

with the hard scale t, and the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$, that governs the aforementioned renormalization-group evolution. The coefficient 5/3 in the first line of Eq. (16) differs from the coefficient 2 in the second line, since we have employed the effective heavy quark field for the bottom quark in the definition of the B_c meson distribution amplitude, as exhibited by the horizontal double line in Fig. 3(c). For the numerical analysis below, we insert the one-loop running coupling constant α_s into Eq. (16) in order to match the expected next-to-leading-logarithm accuracy of our resummation formula.

IV. $B_c^+ \to J/\psi\pi^+$ DECAY

After the pioneering paper on B_c meson decays by Bjorken in 1986 [18], numerous investigations in different formalisms have been devoted to this subject, but the predictions vary in a wide range. For example, the $B_c^+ \to J/\psi\pi^+$ branching ratio was predicted to be between orders of 10^{-4} and 10^{-2} [19–23]. In particular, it takes the values 1.2×10^{-3} in the QCD factorization approach [20] and $(1.4\sim2.5)\times10^{-3}$ [21], $2.33^{+0.63+0.16+0.48}_{-0.58-0.16-0.12}\times10^{-3}$ [22], and $2.6^{+0.6+0.2+0.8}_{-0.4-0.2-0.2}\times10^{-3}$ [23] in the conventional PQCD approach. These results manifest the sensitivity to the hadronic inputs in the theoretical frameworks for B_c meson decays. However, the current data, appearing only as the ratios of the decay rates because of experimentally complicated background, such as

$$R_{K/\pi}^{J/\psi} \equiv \frac{\mathrm{Br}(B_c \to J/\psi K^+)}{\mathrm{Br}(B_c \to J/\psi\pi^+)}, \quad (17)$$

cannot be used to discriminate the branching-ratio predictions. The factorizable emission diagrams in Figs. 4(a) and 4(b) dominate the $B_c^+ \to J/\psi K^+$ and $B_c^+ \to J/\psi\pi^+$ modes, so the associated uncertain $B_c \to J/\psi$ transition form factor cancels in the ratio. This explains why the various formalisms lead to similar $R_{K/\pi}^{J/\psi}$ in agreement with the latest measurement [24], although they give quite distinct values for the individual branching ratios.

In this section, we calculate the $B_c \to J/\psi$ transition form factor and the $B_c^+ \to J/\psi\pi^+$ branching ratio in the improved PQCD approach developed in Sec. III. The relevant weak effective Hamiltonian $H_{\rm eff}$ is written as [25]

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}V_{cb}^*V_{ud}[C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] + \mathrm{H.c.}, \quad (18)$$

where $C_{1,2}(\mu)$ are the Wilson coefficients evaluated at the renormalization scale μ and the local four-quark operators are

$$O_1 = \bar{d}_\alpha\gamma_\mu(1-\gamma_5)u_\beta \quad \bar{c}_\beta\gamma_\mu(1-\gamma_5)b_\alpha, \\ O_2 = \bar{d}_\alpha\gamma_\mu(1-\gamma_5)u_\alpha \quad \bar{c}_\beta\gamma_\mu(1-\gamma_5)b_\beta, \quad (19)$$

with the color indices α and β and the Fermi constant $G_F = 1.16639\times10^{-5}\ \mathrm{GeV}^{-2}$. For the Cabibbo-Kobayashi-Maskawa matrix elements V_{cb} and V_{ud}, we employ the Wolfenstein parametrization at leading order with the

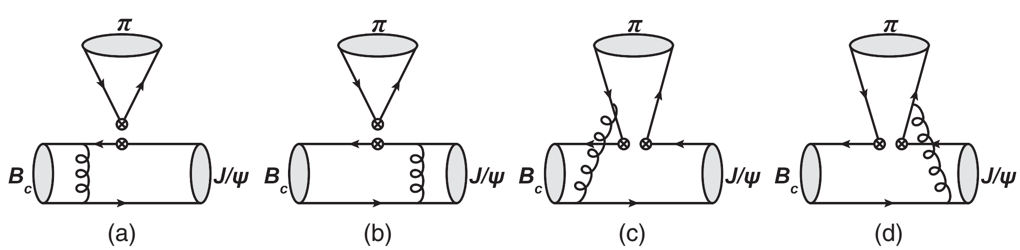

FIG. 4. Leading-order diagrams for the $B_c^+ \to J/\psi\pi^+$ decay in the PQCD approach.

parameters $A = 0.811$ and $\lambda = 0.22506$ [26]. The momenta of the B_c and J/ψ mesons have been chosen in Eq. (1), from which the pion momentum is given by $P_3 = m_{B_c}/\sqrt{2}(0, 1 - r^2_{J/\psi}, \mathbf{0}_T)$, for the vanishing pion mass. The momenta of the spectator quarks in the involved hadrons are parametrized as

$$\begin{aligned} k_1 &= (x_1 P_1^+, x_1 P_1^-, \mathbf{k}_{1T}), \\ k_2 &= (x_2 P_2^+, x_2 P_2^-, \mathbf{k}_{2T}), \\ k_3 &= (x_3 P_3^+, x_3 P_3^-, \mathbf{k}_{3T}). \end{aligned} \tag{20}$$

The B_c, J/ψ, and π meson distribution amplitudes have the structures

$$\Phi_{B_c}(x,b) \equiv \frac{i}{\sqrt{2N_c}} (\not{P}_{B_c} + m_{B_c})\gamma_5 \phi_{B_c}(x,b), \tag{21}$$

$$\begin{aligned} \Phi_\pi(x) \equiv \frac{i}{\sqrt{2N_c}} \gamma_5 [\not{P}_\pi \phi^A_\pi(x) + m_0^\pi \phi^P_\pi(x) \\ + m_0^\pi (\not{n}\not{v} - 1)\phi^T_\pi(x)], \end{aligned} \tag{22}$$

$$\Phi^L_{J/\psi}(x) \equiv \frac{1}{\sqrt{2N_c}} [m_{J/\psi} \not{\epsilon}^L_{J/\psi} \phi^L_{J/\psi}(x) + \not{\epsilon}^L_{J/\psi} \not{P}_{J/\psi} \phi^t_{J/\psi}(x)], \tag{23}$$

with the dimensionless vectors $n = (0, 1, \mathbf{0}_T)$ and $v = (1, 0, \mathbf{0}_T)$ and the longitudinal polarization vector for the J/ψ meson

$$\epsilon^L_{J/\psi} = \frac{1}{\sqrt{2} r_{J/\psi}} (1, -r^2_{J/\psi}, \mathbf{0}_T). \tag{24}$$

Owing to the experimental status stated before, we adopt the shape parameter $\beta_{B_c} = 1$ GeV for the B_c meson distribution amplitude inferred from the kinematic constraint. The light-cone pion distribution amplitudes ϕ^A_π (twist 2) and ϕ^P_π and ϕ^T_π (twist 3) have been parametrized as [27–29]

$$\begin{aligned} \phi^A_\pi(x) = \frac{f_\pi}{2\sqrt{2N_c}} 6x(1-x)[1 + a_2^\pi C_2^{3/2}(2x-1) \\ + a_4^\pi C_4^{3/2}(2x-1)], \end{aligned} \tag{25}$$

$$\begin{aligned} \phi^P_\pi(x) = \frac{f_\pi}{2\sqrt{2N_c}} \bigg[1 + \left(30\eta_3 - \frac{5}{2}\rho_\pi^2\right) C_2^{1/2}(2x-1) \\ - 3\left(\eta_3\omega_3 + \frac{9}{20}\rho_\pi^2(1 + 6a_2^\pi)\right) C_4^{1/2}(2x-1) \bigg], \end{aligned} \tag{26}$$

$$\begin{aligned} \phi^T_\pi(x) = \frac{f_\pi}{2\sqrt{2N_c}} (1-2x) \bigg[1 + 6\bigg(5\eta_3 - \frac{1}{2}\eta_3\omega_3 \\ - \frac{7}{20}\rho_\pi^2 - \frac{3}{5}\rho_\pi^2 a_2^\pi\bigg)(1 - 10x + 10x^2) \bigg], \end{aligned} \tag{27}$$

with the decay constant $f_\pi = 0.130$ GeV; the Gegenbauer moments $a_2^\pi = 0.115 \pm 0.115$ and $a_4^\pi = -0.015$; the parameters $\eta_3 = 0.015$ and $\omega_3 = -3$ [27,28]; the mass ratio $\rho_\pi = m_\pi/m_0^\pi$, $m_0^\pi = 1.4$ GeV being the pion chiral mass; and the Gegenbauer polynomials $C_n^\nu(t)$,

$$\begin{aligned} C_2^{1/2}(t) &= \frac{1}{2}(3t^2 - 1), \qquad C_4^{1/2}(t) = \frac{1}{8}(3 - 30t^2 + 35t^4), \\ C_2^{3/2}(t) &= \frac{3}{2}(5t^2 - 1), \qquad C_4^{3/2}(t) = \frac{15}{8}(1 - 14t^2 + 21t^4). \end{aligned} \tag{28}$$

The J/ψ meson distribution amplitudes $\phi^L_{J/\psi}$ (twist 2) and $\phi^t_{J/\psi}$ (twist 3) have been derived as [8]

$$\phi^L_{J/\psi}(x) = 9.58 \frac{f_{J/\psi}}{2\sqrt{2N_c}} x(1-x) \left[\frac{x(1-x)}{1 - 2.8x(1-x)}\right]^{0.7}, \tag{29}$$

$$\phi^t_{J/\psi}(x) = 10.94 \frac{f_{J/\psi}}{2\sqrt{2N_c}} (1-2x)^2 \left[\frac{x(1-x)}{1 - 2.8x(1-x)}\right]^{0.7}, \tag{30}$$

with the decay constant $f_{J/\psi} = 0.405 \pm 0.014$ GeV.

The $B_c^+ \to J/\psi\pi^+$ decay amplitude is decomposed into

$$\mathcal{A}(B_c \to J/\psi\pi) = V^*_{cb} V_{ud} (f_\pi F + M). \tag{31}$$

The factorizable emission diagrams, i.e., Figs. 4(a) and 4(b), give the factorization formula

$$\begin{aligned} F = 8\pi C_F m^2_{B_c} \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1, b_1)(r^2_{J/\psi} - 1) \\ \times \{[r_{J/\psi}(r_b + 2x_2 - 2)\phi^t_{J/\psi}(x_2) - (2r_b + x_2 - 1)\phi^L_{J/\psi}(x_2)] h_a(x_1, x_2, b_1, b_2) E_f(t_a) \\ + [r^2_{J/\psi}(x_1 - 1) - r_c]\phi^L_{J/\psi}(x_2) h_b(x_1, x_2, b_1, b_2) E_f(t_b)\}, \end{aligned} \tag{32}$$

where the ratios $r_b = m_b/m_{B_c}$ and $r_c = m_c/m_{B_c}$ and b_i are the impact parameters conjugate to the transverse momenta k_{iT}. It is known that the above formula is related to the transition form factor $A_0^{B_c \to J/\psi}(q^2 = 0)$ [30–32] with $q = P_1 - P_2$. As pointed out in the Introduction, the PQCD approach is applicable to the evaluation of the nonfactorizable emission diagrams, i.e., Figs. 4(c) and 4(d). The corresponding factorization formula is expressed as

$$M = -\frac{32}{\sqrt{6}}\pi C_F m_{B_c}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_c}(x_1, b_1)\phi_\pi^A(x_3)(r_{J/\psi}^2 - 1)$$
$$\times \{[(r_{J/\psi}^2 - 1)(x_1 + x_3 - 1)\phi_{J/\psi}^L(x_2) + r_{J/\psi}(x_2 - x_1)\phi_{J/\psi}^t(x_2)]h_c(x_1, x_2, x_3, b_1, b_3)E_f(t_c)$$
$$+ [(2x_1 - (x_2 + x_3) + r_{J/\psi}^2(x_3 - x_2))\phi_{J/\psi}^L(x_2) + r_{J/\psi}(x_2 - x_1)\phi_{J/\psi}^t(x_2)]h_d(x_1, x_2, x_3, b_1, b_3)E_f(t_d)\}. \quad (33)$$

In the above expressions, the hard functions $h_{a,b,c,d}$ are defined by

$$h_a(x_1, x_2, b_1, b_2) = [\theta(b_2 - b_1)I_0(\sqrt{\beta_a}b_1)K_0(\sqrt{\beta_a}b_2) + (b_1 \leftrightarrow b_2)]K_0(\sqrt{\alpha}b_1), \quad (34)$$

$$h_b(x_1, x_2, b_1, b_2) = [\theta(b_2 - b_1)I_0(\sqrt{\beta_b}b_1)K_0(\sqrt{\beta_b}b_2) + (b_1 \leftrightarrow b_2)]K_0(\sqrt{\alpha}b_2), \quad (35)$$

$$h_{c,d}(x_1, x_2, x_3, b_1, b_3) = [\theta(b_3 - b_1)I_0(\sqrt{\alpha}b_1)K_0(\sqrt{\alpha}b_3) + (b_1 \leftrightarrow b_3)]K_0(\sqrt{\beta_{c,d}}b_3), \quad (36)$$

with the factors α and $\beta_{a,b,c,d}$ and the hard scales $t_{a,b,c,d}$,

$$\alpha = -[(x_1 - x_2)(x_1 - x_2 r_{J/\psi}^2)]m_{B_c}^2, \quad (37)$$

$$\beta_a = -[(1 - x_2)(1 - x_2 r_{J/\psi}^2) - r_b^2]m_{B_c}^2, \qquad \beta_b = -[(1 - x_1)(r_{J/\psi}^2 - x_1) - r_c^2]m_{B_c}^2, \quad (38)$$

$$\beta_c = -[(x_2 r_{J/\psi}^2 + (1 - x_3)(1 - r_{J/\psi}^2) - x_1)(x_2 - x_1)]m_{B_c}^2, \quad (39)$$

$$\beta_d = -[(x_2 r_{J/\psi}^2 + x_3(1 - r_{J/\psi}^2) - x_1)(x_2 - x_1)]m_{B_c}^2, \quad (40)$$

$$t_a = \max(\sqrt{|\alpha|}, \sqrt{|\beta_a|}, 1/b_1, 1/b_2), \qquad t_b = \max(\sqrt{|\alpha|}, \sqrt{|\beta_b|}, 1/b_1, 1/b_2), \quad (41)$$

$$t_c = \max(\sqrt{|\alpha|}, \sqrt{|\beta_c|}, 1/b_1, 1/b_3), \qquad t_d = \max(\sqrt{|\alpha|}, \sqrt{|\beta_d|}, 1/b_1, 1/b_3). \quad (42)$$

Note that, as α and $\beta_{a,b,c,d}$ are negative, the associated Bessel functions transform as

$$K_0(\sqrt{y}) = K_0(i\sqrt{|y|}) = \frac{i\pi}{2}[J_0(\sqrt{|y|}) + iN_0(\sqrt{|y|})], \qquad I_0(\sqrt{y}) = J_0(\sqrt{|y|}) \quad (43)$$

for $y < 0$. The evolution functions $E_f(t) = \alpha_s(t)C_i(t)S_i(t)$ contain the Wilson coefficients

$$C_{ab}(t) = \frac{1}{3}C_1(t) + C_2(t), \qquad C_{cd}(t) = C_1(t) \quad (44)$$

and the Sudakov factors

$$S_{ab}(t) = s_c(x_1 P_1^-, b_1) + s_c(x_2 P_2^+, b_2) + s_c((1 - x_2)P_2^+, b_2) - \frac{1}{\beta_1}\left[\frac{11}{6}\ln\frac{\ln(t/\Lambda)}{\ln(m_c/\Lambda)}\right], \quad (45)$$

$$S_{cd}(t) = s_c(x_1 P_1^-, b_1) + s_c(x_2 P_2^+, b_1) + s_c((1 - x_2)P_2^+, b_1) + s(x_3 P_3^-, b_3) + s((1 - x_3)P_3^-, b_3)$$
$$- \frac{1}{\beta_1}\left[\frac{11}{6}\ln\frac{\ln(t/\Lambda)}{\ln(m_c/\Lambda)} + \ln\frac{\ln(t/\Lambda)}{-\ln(b_3\Lambda)}\right], \quad (46)$$

where the explicit expression of the Sudakov exponent $s(Q, b)$ for an energetic light quark is referred to Refs. [30,31].

With the QCD scale $\Lambda_{\rm QCD}^{(4)} = 0.25$ GeV and the B_c meson lifetime $\tau_{B_c} = 0.507$ ps, we obtain $\mathrm{Br}(B_c^+ \to J/\psi\pi^+) = 1.60 \times 10^{-3}$. This result is consistent with 1.2×10^{-3} derived in the QCD factorization approach [20], in which the transition form factor $A_0^{B_c \to J/\psi}$ was treated as an input, a bit larger value of $A_0^{B_c \to J/\psi} = 0.6$ was employed, and the one-loop correction to the $b \to c$ decay vertex was included. Our prediction can be compared to the measured branching ratio of the corresponding mode with the replacement of the spectator charm quark by an up quark, $\mathrm{Br}(B^+ \to \bar{D}^{*0}\pi^+) = (5.18 \pm 0.26) \times 10^{-3}$ [26], which receives an additional color-suppressed tree contribution. The dependence of the quantities $A_0^{B_c \to J/\psi}(0)$ and $\mathrm{Br}(B_c^+ \to J/\psi\pi^+)$ on β_{B_c} in the range [0.8, 1.2] GeV is shown in Table I. It is clearly seen that the imaginary piece of the $B_c \to J/\psi$ transition form factor is greatly suppressed, being only 10%–20% of the real piece, and that the $B_c^+ \to J/\psi\pi^+$ branching ratio is unlikely to be lower than 10^{-3}. Roughly speaking, the preferred

TABLE I. Dependence on the shape parameter β_{B_c} of the quantities $A_0^{B_c\to J/\psi}(0)$ and $\mathrm{Br}(B_c^+ \to J/\psi\pi^+)$ in the improved PQCD formalism.

Shape parameter	$A_0^{B_c\to J/\psi}(0)$	$\mathrm{Br}(B_c^+ \to J/\psi\pi^+)$
$\beta_{B_c} = 0.8$ GeV	$0.488 - i0.095$	2.80×10^{-3}
$\beta_{B_c} = 0.9$ GeV	$0.434 - i0.070$	2.10×10^{-3}
$\beta_{B_c} = 1.0$ GeV	$0.384 - i0.053$	1.60×10^{-3}
$\beta_{B_c} = 1.1$ GeV	$0.341 - i0.039$	1.23×10^{-3}
$\beta_{B_c} = 1.2$ GeV	$0.306 - i0.029$	0.94×10^{-3}

range of $\mathrm{Br}(B_c^+ \to J/\psi\pi^+)$ from the PQCD approach can be preliminarily read as $[0.9, 2.8] \times 10^{-3}$. When the data are available for individual branching ratios, or for the ratios of decay rates that are more sensitive to the nonfactorizable emission contributions, it is possible to pin down the shape parameter β_{B_c} and to make more precise predictions in the PQCD approach. In the latter case, the emitted meson could be a scalar or tensor, such that the dominant nonfactorizable emission diagrams do not cancel in the ratios of decay rates.

V. CONCLUSION

In this paper, we have deduced the shape of the B_c meson distribution amplitude $\phi_{B_c}(x)$ resulting from the soft gluon emission effect based on the parton kinematic analysis and found that $\phi_{B_c}(x)$ exhibits a peak around the momentum fraction $x \sim m_c/m_b \sim 0.3$ of the spectator charm quark with a width of order $\Lambda/m_b \sim 0.1$. These features were then implemented into the parametrization of $\phi_{B_c}(x)$ in terms of a Gaussian form with the shape parameter $\beta_{B_c} \sim 1.0$ GeV. We have estimated the potential imaginary piece in the $B_c \to J/\psi$ transition form factor, which should be power suppressed according to the specified parton kinematics and the argument on the absence of intermediate states. It is worth emphasizing that the resummation formula adopted in the conventional PQCD approach to B_c meson decays [21–23] is not appropriate. We have modified the k_T resummation by taking into account the finite charm quark mass, the effect of which was shown to enhance the decay rates. We point out that this modification is exact only at the leading-logarithm level, and a precise next-to-leading-logarithm resummation formalism for a hadronic process involving the multiple scales m_b, m_c, and Λ_{QCD} is still urged; it demands a complete one-loop calculation for determining the factor $B(\alpha_s)$ in Eq. (13).

Given the B_c meson distribution amplitude preferred by the kinematic constraints and the newly derived Sudakov factor for the $B_c \to J/\psi$ transition, we have calculated, at leading order in the strong coupling, the transition form factor $A_0^{B_c\to J/\psi}(0)$ and the $B_c^+ \to J/\psi\pi^+$ branching ratio in the range [0.8,1.2] GeV of the shape parameter β_{B_c}. It was observed that the strong phase in $A_0^{B_c\to J/\psi}(0)$ is indeed largely suppressed and that the predicted $\mathrm{Br}(B_c^+ \to J/\psi\pi^+) \sim 1.60 \times 10^{-3}$ is comparable to the data $\mathrm{Br}(B^+ \to \bar{D}^{*0}\pi^+) = (5.18 \pm 0.26) \times 10^{-3}$. The definite value of the shape parameter demands the input data of some individual B_c decay channels from LHCb, with which it is then possible to make more precise predictions for various modes. At last, we stress that the improved PQCD formalism developed in this work is applicable to B_c meson decays to other charmonia and charmed mesons.

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