Tree-level disk amplitude of three closed strings

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(Received 17 December 2017; published 21 May 2018)

It has been shown that the disk-level S-matrix elements of one Ramond-Ramond (RR) and two Neveu-Schwarz-Neveu-Schwarz (NSNS) states could be found by applying the Ward identity associated with the string duality and the gauge symmetry on a given component of the S matrix. These amplitudes have appeared as the components of six different T-dual multiplets. It is predicted in the literature that there are some nonzero disk-level scattering amplitudes, such as one RR $(p-1)$ form with zero transverse index and two *NSNS* states, could not be captured by the T-dual Ward identity. We explicitly find this amplitude in terms of a minimal context of the integral functions by the insertion of one closed string RR vertex operator and two NSNS vertex operators. From the amplitude invariance under the Ward identity associated with the NSNS gauge transformations and T-duality, we also find some integral identities.

DOI: 10.1103/PhysRevD.97.106014

I. INTRODUCTION

D-branes are of specific interest in vacuum construction. They are localized objects on which strings can end. We will consider the stable D-branes that carry the RR charges [1]. The dynamics of the D-branes of type II superstring theories at the lowest order in α' is given by the world-volume theory which is the sum of Dirac-Born-Infeld (DBI) and Chern-Simons (CS) actions [2–5].

It has been shown that the higher-order corrections to the D-brane play a pivotal role in determining the consistency of string compactifications. Without taking these terms properly into account, one would not reach correct conclusions about the space of valid constructions of string vacua [6]. Such conclusions could be learned about these higher-order terms by taking the known terms and applying T-duality to them [7,8] or computing the terms directly by evaluating scattering amplitudes [6,9,10]. In this paper, we will concentrate on the latter approach. Thus, we must evaluate scattering amplitudes in which various string fields interact with a D-brane. We will confine ourselves to tree-level calculations, so the corresponding amplitudes are given by incorporation of multiple string vertex operators on a disc world sheet. We are following [10–12], where the formalism was developed and some simple calculations of amplitudes were done. Similar to the computations in these references, the final goal is to find the corresponding Dp-brane effective actions. In [13], we will use string dualities to present all the gauge invariant completion amplitudes of three closed strings (one RR and two NSNS) in the form of T-dual multiplets that are needed to find the four derivative corrections to the D-brane action.

The complex integrals appearing in the amplitude could have closed expressions only in an expansion in α'. So we will obtain the string amplitudes in closed form only in this limit. The disk-level S-matrix element of one RR $(p-3)$-form with two, one and zero transverse indices and two NSNS states has been studied in [6,11,12,14]. In [15] we found that these three parts of the amplitude appear as the first components of three different T-dual multiplets. Three other T-dual multiplets have been found in [15] in which the first components of them are not the corresponding amplitudes of RR $(p-3)$-form. The amplitudes have one, five and fourteen integrals for the first, the second and the third parts, respectively. The integrals in the second part satisfy two constraint equations, and the integrals in the third part satisfy eight constraint equations. The T-dual Ward identity connects these three parts to the amplitudes of the RR $(p-1)$-form, $(p+1)$-form, $(p+3)$-form, and the RR $(p+5)$-form. The sum of all multiplets does not satisfy the Ward identity corresponding to the RR gauge transformation. This indicates that there should be another T-dual multiplet. T-duality predicts that the first component of this multiplet must be the corresponding amplitude of the RR $(p-1)$-form which carries zero transverse index. We showed that this amplitude could not be captured by T-duality and Ward identities corresponding to the NSNS gauge transformations [15].

From the invariance of string amplitudes under the parity transformation, it is easy to see that string amplitudes involving RR $(p-1)$-form are only nonvanishing if one NSNS field is graviton (or dilaton) and another one is BNS field [14]. We are going to calculate this disk world-sheet amplitude by the insertion of one closed string RR vertex operator and two NSNS vertex operators.

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The paper is arranged as follows: In Sec. II, we start by reviewing the approach for three closed string amplitude including one RR and two NSNS vertex operators in the RNS world-sheet formalism. In Sec. III, by considering this approach, we calculate the corresponding amplitude of scalar RR $(p-1)$-form. In addition to all the integral functions that emerged in the amplitude of other cases of three closed string amplitudes, some new integral functions appear in this case for the first time. We introduce these new integral functions in Sec. IV and, from the amplitude gauge invariance under the NSNS gauge transformations and amplitude T-dual covariance, we find some identities between these integral functions.

II. THREE-POINT AMPLITUDES

Scattering amplitudes, which explain the interaction of strings with a D-brane, are speculated as the string vertex operator insertions on a disk world sheet with Dirichlet and Neumann boundary conditions. These boundary conditions have a major role in finding the world-sheet propagators in the conformal field theory frame. In the following, we are going to study a general case of three closed amplitude of one RR and two NSNS states and introduce an approach to find relevant amplitudes. Then, using this approach we will calculate our interesting amplitude.

In such cases, one can find the tree-level string scattering amplitudes by calculating the correlation functions of the relevant vertex operators on disk. Considering the background charge of the disk world sheet, we have to pick out the vertex operators in one of the following pictures:

$$V_{\mathrm{RR}}^{(-1/2,-1/2)}V_{\mathrm{NSNS}}^{(-1,0)}V_{\mathrm{NSNS}}^{(0,0)},\qquad V_{\mathrm{RR}}^{(-1/2,-3/2)}V_{\mathrm{NSNS}}^{(0,0)}V_{\mathrm{NSNS}}^{(0,0)}.$$

We work in the latter picture where the symmetry of the NSNS states is manifest. We perform the computation in the above two different pictures. By considering the identities found in Sec. IV, we confirm that in each case the results agree with the other case. Using a conformal transformation, the amplitude can be demonstrated as a correlation function in the upper half-complex plane with the real axis as the world-sheet boundary. The amplitude can be written as

$$\begin{aligned}\mathcal{A}\sim&(P_{-}H_{1(n)}M_p)^{AB}\\&\times\int d^2z_1 :V_A^{-1/2}(p_1,z_1):V_B^{-3/2}(p_1\cdot D,\bar{z}_1):\\&\times(\varepsilon_2\cdot D)_{\alpha_1\alpha_2}\int d^2z_2 :V_0^{\alpha_1}(p_2,z_2):V_0^{\alpha_2}(p_2\cdot D,\bar{z}_2):\\&\times(\varepsilon_3\cdot D)_{\alpha_3\alpha_4}\int d^3z_3 :V_0^{\alpha_3}(p_3,z_3):V_0^{\alpha_4}(p_3\cdot D,\bar{z}_3): \qquad (1)\end{aligned}$$

The above holomorphic components are given by

$$V_A^{-1/2}(p_1,z_1)=e^{-\phi(z_1)/2}S_A(z_1)e^{ip_1\cdot X},$$
$$V_B^{-3/2}(p_1\cdot D,\bar{z}_1)=e^{-3\phi(\bar{z}_1)/2}S_B(\bar{z}_1)e^{ip_1\cdot D\cdot X},$$

and

$$V_0^{\alpha}(p_i,z_i)=(\partial X^{\alpha}+ip_i\cdot\psi\psi^{\alpha})e^{ip_i\cdot X},$$
$$V_0^{\alpha}(p_i\cdot D,\bar{z}_i)=(\partial X^{\alpha}+ip_i\cdot D\cdot\psi\psi^{\alpha})e^{ip_i\cdot D\cdot X},\ i=2,3$$

where $X^{\alpha}(z)$ (and $X^{\alpha}(\bar{z})$), $\psi^{\alpha}(z)$ (and $\psi^{\alpha}(\bar{z})$) and $\phi(z)$ (and $\phi(\bar{z})$) are bosons, fermions and picture ghosts, respectively.

The indices $A,B,\cdots$ are the Dirac spinor indices and $P_{-}=\frac{1}{2}(1-\gamma_{11})$ is the chiral projection operator, and

$$H_{1(n)}=\frac{1}{n!}\varepsilon_{1\mu_1\cdots\mu_n}\gamma^{\mu_1}\cdots\gamma^{\mu_n}$$
$$M_p=\frac{\pm1}{(p+1)!}\epsilon_{a_0\cdots a_p}\gamma^{a_0}\cdots\gamma^{a_p} \qquad (2)$$

where ϵ is the volume $(p+1)$-form of the D_p-brane. Here the matrix $D_{\mu\nu}$ is a diagonal matrix that agrees with $\eta_{\mu\nu}$ in directions along the brane (Neumann boundary conditions) and with $-\eta_{\mu\nu}$ in directions normal to the brane (Dirichlet boundary conditions). In this notation, $D^{\mu i}=-\delta^{\mu i}$, $D^{\mu a}=\delta^{\mu a}$ and thus $D_{\mu\nu}=V_{\mu\nu}-N_{\mu\nu}$.

The correlators in (1) could be calculated by using the standard world-sheet propagators and the Wick-like rule [11]. By calculating the X-correlators, one finds that there is conservation of the momenta along the brane. $SL(2,R)$ group is the conformal symmetry of the upper half-complex plane. The volume of this group, where the amplitude is divided by it, could be removed after carrying out the correlators [11]. Considering all possible combinations of Gamma matrices that appear in the amplitude, one can find that the amplitude (1) was constructed from the following term,

$$(H_{1(n)}M_p)^{AB}(\gamma^{\alpha_1\cdots\alpha_m}C^{-1})_{AB}A_{[\alpha_1\cdots\alpha_m]}, \qquad (3)$$

where $A_{[\alpha_1\cdots\alpha_m]}$ is an antisymmetric combination of the NSNS momenta p_2, p_3 and/or the polarizations ε_2, ε_3.

Considering (2), one can find that the above sentence would be zero except for $n=p\pm3$, $\mathrm{n}=\mathrm{p}\pm1$ and $n=p+5$. Using the linear T-dual Ward identity associated with the NSNS gauge transformations, we found the amplitudes corresponding to these cases in some special cases and presented them in terms of six T-dual multiplets in [15]. Our findings in [15], were in agreement with the explicit scattering calculations performed in [6,10–12,14].

One can find that the sum of the six multiplets does not satisfy the Ward identity corresponding to the RR gauge transformation. This indicates that there should be another T-dual multiplet whose first component is the amplitude corresponding to $C^{(p-1)}$ with zero transverse index. This scalar RR amplitude could not be captured by the T-dual Ward identity. So, we have to find this amplitude by explicit scattering calculation. The inability of the T-dual Ward identity to construct the S matrix indicates that the appearance of new integral functions in the scalar RR $C^{(p-1)}$ amplitude could be expected.

III. S-MATRIX ELEMENT OF ONE SCALAR RR ($p-1$) FORM AND TWO NSNS STATES

In this case, $n=0$, $p=1$ and $m=2$. There is not any contraction between the RR potential and the volume form. From these considerations, one can find that the nonzero form of the sentence (3) would be as follows:

$$32\epsilon^{a_0a_1}A_{[a_0a_1]}.$$

The correlation functions that contribute in the amplitude (1) then appear as the following,

$$(\mathcal{P})^{\beta_1\cdots\beta_q}\langle :S_A:S_B:\psi_{\beta_1}\psi^{[\alpha_1}:\cdots\rangle\mathcal{X}^{\alpha_{q+1}\cdots\alpha_4]}, \tag{4}$$

where $\mathcal{P}$'s are multiplication of NSNS momenta and $\mathcal{X}$'s are the correlators of X's. The value of m in the amplitude of our interesting case, indicates that $q=1,\ldots,4$ must be contributed. We must evaluate the above correlation functions for all allowed values of q and then find the subamplitudes corresponding to q's. The amplitude of interest would be found by adding these subamplitudes:

$$\mathcal{A}=\mathcal{A}_1+\mathcal{A}_2+\mathcal{A}_3+\mathcal{A}_4. \tag{5}$$

Let us begin with $q=4$.

A. $q=4$

By carefully using the Wick-like rule in (4) for $q=4$, one can find that the multiplication of NSNS momenta includes four momenta as

$$\mathcal{P}_4=(p_2)^{[\beta_1}(p_2\cdot D)^{\beta_2}(p_3)^{\beta_3}(p_3\cdot D)^{\beta_4]}+(2\leftrightarrow 3). \tag{6}$$

In this case, the NSNS polarizations contracte with the ψ correlators and relevant momenta multiplication. Considering the standard world-sheet propagators, one can find a nonzero correlator of X. It is easy to check that this result satisfies the $SL(2,R)$ transformation. So we can map the results to disk with unit radius. After fixing the $SL(2,R)$ symmetry as [16] in which $z_1=0$, the result for the correlator of X takes the form[1]

$$X_4=|z_2|^{2p_1\cdot p_2}|z_3|^{2p_1\cdot p_3}(1-|z_2|^2)^{p_2\cdot D\cdot p_2}(1-|z_3|^2)^{p_3\cdot D\cdot p_3}|z_2-z_3|^{2p_2\cdot p_3}|1-z_2\bar{z}_3|^{2p_2\cdot D\cdot p_3}\equiv\mathcal{X}.$$

We can choose the coordinate $z_2=r_2$ and polar coordinate $z_3=r_3e^{i\theta}$ [11]. Under this fixing the measure in (1) changes as

$$d^2z_1d^2z_2d^2z_3\to r_2dr_2r_3dr_3d\theta,\quad 0<r_2,r_3<1, 0<\theta<2\pi \tag{7}$$

and X_4 changes as

$$\mathcal{K}\equiv {r_2}^{2p_1\cdot p_2}{r_3}^{2p_1\cdot p_3}(1-{r_2}^2)^{p_2\cdot D\cdot p_2}(1-{r_3}^2)^{p_3\cdot D\cdot p_3}\times|r_2-r_3e^{i\theta}|^{2p_2\cdot p_3}|1-r_2r_3e^{i\theta}|^{2p_2\cdot D\cdot p_3}. \tag{8}$$

This function is symmetric under exchanging the momentum labels 2, 3. Replacing in (4) the correlator (8) and the momentum multiplication (6) and then using Wick-like rule, one can find that the amplitude (1) gets the following subamplitude,[2]

$$\mathcal{A}_4\sim 4\epsilon_{a_0a_1}p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_1[-2p_2^{a_1}(p_1\cdot N\cdot\varepsilon_2^A)^{a_0}+p_1\cdot N\cdot p_2(\varepsilon_2^A)^{a_0a_1}]\mathcal{I}_1, \tag{9}$$

where we use the conservation of momentum along the brane, $p_1^a+p_2^a+p_3^a=0$. ε^A is the polarization of the B-field, ε^S is the polarization of the graviton and $\mathcal{I}_1$ is an integral function as [11]

$$\mathcal{I}_1=\int_0^1dr_2\int_0^1dr_3\int_0^{2\pi}d\theta\frac{\mathcal{K}}{r_2r_3}. \tag{10}$$

where we fix the $SL(2,R)$ symmetry by using (7).

It is clear that all terms in the above amplitude and in all other amplitudes in this paper, have a scalar part including integral function(s) and/or Mandelstam variable(s). The other part we called the independent structure, including the momenta, B-field polarization, and graviton polarization that carry two indices a_0, a_1. The amplitude (9) has two independent structures in the form of $[(p\varepsilon^Sp)(p\varepsilon^A)^{a_0}(p^{a_1})]$ and $[(pp)(p\varepsilon^Sp)(\varepsilon^A)^{a_0a_1}]$.

The above integral function is the most simple integral function that appears in the three closed string scattering amplitude. It is also the only integral function that appears in the scattering amplitudes corresponding to RR ($p-3$)-form, ($p-1$)-form, ($p+1$)-form, ($p+3$)-form and ($p+5$)-form with two, three, four, five, and six transverse

[1] From this correlator, it is clear that the six independent Mandelstam variables appear in the amplitude of three closed strings. On the other hand, there are seven physical string channels, three open string channels $p_i\cdot D\cdot p_i$ and four closed string channels $(p_1+p_2+p_3)^2$ and $p_i\cdot p_j$, $i,j=1,2,3$. One can easily solve this ambiguity by using the following string channel identities:

$$(p_1+p_2+p_3)^2=2p_1\cdot p_2+2p_1\cdot p_3+2p_2\cdot p_3$$
$$p_1\cdot D\cdot p_1=p_2\cdot D\cdot p_2+p_3\cdot D\cdot p_3+2p_2\cdot p_3+2p_2\cdot D\cdot p_3.$$

[2] Our conventions set $\alpha'=2$ in the string theory amplitudes. Our index convention is that the greek letters $(\mu,\nu,\cdots)$ are the indices of the spacetime coordinates, the latin letters $(a,d,c,\cdots)$ are the world-volume indices and the letters $(i,j,k,\cdots)$ are the normal bundle indices.

indices, respectively [11,17]. This integral is invariant under the interchange of $(2 \leftrightarrow 3)$. It is shown in [17] that the T-dual Ward identity connects these amplitudes which furnish a T-dual multiplet. In fact, the scattering amplitudes that appear as the components of a T-dual multiplet carry the same integral functions.

B. $q = 3$

In this case, the multiplication of NSNS momenta has three momenta

$$\mathcal{P}_3 = (p_2)^{[\beta_i}(p_2\cdot D)^{\beta_j}((p_3)^{\beta_k} + (p_3\cdot D)^{\beta_k]}) + (2 \leftrightarrow 3). \tag{11}$$

where i, j, $k = 1$, 2, 3, 4 and $i \neq j \neq k$. After $SL(2,R)$ fixing $z_1 = 0$, the part of the X-correlator corresponding to this case that involves holomorphic coordinate and momentum is as follows,

$$\begin{aligned}\Bigg[&(p_1^{\alpha_i} + (p_1\cdot D)^{\alpha_i})\frac{1}{z_{3\bar{3}}}\left(\frac{\bar{z}_3}{z_3} - \frac{z_3}{\bar{z}_3}\right)\\ &+ (p_2\cdot D)^{\alpha_i}\left(\frac{z_{3\bar{2}}}{z_{\bar{2}\bar{3}}} - \frac{z_{\bar{3}\bar{2}}}{z_{2\bar{3}}}\right) + p_2^{\alpha_i}\left(\frac{z_{32}}{z_{2\bar{3}}} - \frac{z_{\bar{3}2}}{z_{23}}\right)\\ &+ (2 \leftrightarrow 3)\Bigg]\mathcal{X},\end{aligned} \tag{12}$$

where $z_{ij} = z_i - z_j$.

Investigating the transformation of the ψ-correlator in the Wick-like rule and the above X-correlator, one can verify that the corresponding subamplitude is invariant under the $SL(2,R)$ transformation.

In the structures containing the world-volume contraction of p_1 with NSNS polarizations and momenta, we can use the conservation of momentum along the brane to write them in terms of the world-volume contraction of p_2 and p_3. Also, using this consideration and on-shell condition, it could be found that the contraction of momentum with corresponding NSNS polarization in the transverse direction is not an independent structure, i.e.,

$$p_i\cdot N\cdot\varepsilon_i = -p_i\cdot V\cdot\varepsilon_i, \qquad i = 2, 3. \tag{13}$$

So the structures containing the world-volume contraction of the NSNS momenta with the NSNS polarizations, the transverse contraction of the NSNS momenta with the noncorresponding NSNS polarizations, and the transverse contraction of the RR momentum with the NSNS polarizations would appear as independent structures.

So the corresponding subamplitude $\mathcal{A}_3$ could be found by replacing in (4) the momentum multiplication $\mathcal{P}_3$ and correlator $\mathcal{X}^{\alpha_i}$ and then calculating the correlator of ψ's. We see that, in addition to the structure forms in $\mathcal{A}_4$, the structures in $\mathcal{A}_3$ appear as the following forms: $[(p\varepsilon^S\varepsilon^A p)\times(p^{a_0})(p^{a_1})]$, $[(pp)(p\varepsilon^S)^{a_0}(p\varepsilon^A)^{a_1}]$, $[(pp)(p^{a_0})(p\varepsilon^S\varepsilon^A)^{a_1}]$, $[(pp)(p^{a_0})(p\varepsilon^A\varepsilon^S)^{a_1}]$, $[(tr\varepsilon^S)(pp)(p^{a_0})(p\varepsilon^A)^{a_1}]$, and $[(tr\varepsilon^S)(pp)(pp)(\varepsilon^A)^{a_0a_1}]$. The integral functions in this subamplitude that represent the relevant open and closed string channels are $\mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4$, and $\mathcal{I}_7$. The explicit form of these integrals (as in (10) for the integral $\mathcal{I}_1$) are given in [11,17]. The symmetries of the integrals under the interchange of $(2 \leftrightarrow 3)$ are such that $\mathcal{I}_2 \leftrightarrow \mathcal{I}_3$ and $\mathcal{I}_4 \leftrightarrow \mathcal{I}_7$.

The amplitudes corresponding to the RR $(p-3)$-form, $(p-1)$-form, $(p+1)$-form, $(p+3)$-form and $(p+5)$-form with one, two, three, four and five transverse indices, respectively, carry the same collection of the above integral functions. These amplitudes can be expressed in terms of two T-dual multiplets. The related gauge symmetries connect the corresponding components of these multiplets [17].

C. $q = 2$

The multiplication of NSNS momenta has two momenta here

$$\mathcal{P}_2 = (p_2)^{[\beta_i}((p_3)^{\beta_j} + (p_3\cdot D)^{\beta_j]}) + (2 \leftrightarrow 3). \tag{14}$$

Using the standard world-sheet propagator of X's, the part of the X-correlator corresponding to this case that contains holomorphic coordinate and momentum becomes as follows:

$$\begin{aligned}\Bigg[&\frac{1}{z_{3\bar{3}}z_{2\bar{2}}}\Bigg(\frac{z_3z_2}{\bar{z}_3\,\bar{z}_2}[(p_1\cdot D)^{\alpha_i}(p_1\cdot D)^{\alpha_j} + p_1^{\alpha_i}p_1^{\alpha_j} + (p_1\cdot D)^{\alpha_i}(p_1)^{\alpha_j}] + \frac{z_{3\bar{2}}z_{2\bar{3}}}{z_{\bar{2}\bar{3}}z_{\bar{3}\bar{2}}}[(p_2\cdot D)^{\alpha_i}(p_2\cdot D)^{\alpha_j} + (p_2\cdot D)^{\alpha_i}(p_3\cdot D)^{\alpha_j}]\\ &+ 2\left(\frac{z_2z_{3\bar{2}}}{\bar{z}_2z_{\bar{2}\bar{3}}}\right)[(p_1\cdot D)^{\alpha_i}(p_2\cdot D)^{\alpha_j} + p_1^{\alpha_i}(p_2\cdot D)^{\alpha_j}] + 2\left(\frac{z_2z_{32}}{\bar{z}_2z_{2\bar{3}}}\right)[(p_1\cdot D)^{\alpha_i}p_2^{\alpha_j} + p_1^{\alpha_i}p_2^{\alpha_j}] + \frac{z_{32}z_{23}}{z_{2\bar{3}}z_{3\bar{2}}}[p_2^{\alpha_i}p_2^{\alpha_j} + p_2^{\alpha_i}p_3^{\alpha_j}]\\ &+ 2\frac{z_{32}z_{2\bar{3}}}{z_{2\bar{3}}z_{\bar{3}\bar{2}}}p_2^{\alpha_i}(p_3\cdot D)^{\alpha_j}\Bigg) + \frac{\eta^{\alpha_1\alpha_3}}{z_{23}z_{32}} + \frac{\eta^{\alpha_2\alpha_3}}{z_{\bar{2}3}z_{3\bar{2}}} + \frac{\eta^{\alpha_2\alpha_4}}{z_{\bar{2}\bar{3}}z_{\bar{3}\bar{2}}} + (2 \leftrightarrow 3)\Bigg]\mathcal{X},\end{aligned} \tag{15}$$

where i, $j = 1$, 2, 3, 4 and $i \neq j$.

Using the fact that the above correlator contracts with the NSNS polarization tensors and considering all permutations of α_i in (4), one observes that the contribution of the above correlator in the corresponding subamplitude is as eight different terms, but two pairs of these terms are equal.

Considering the conservation of momentum along the brane and the on-shell conditions and also using the Wick-like rule for the correlation function involving four ψ's and two S's, the subamplitude corresponding to $q = 2$, namely $\mathcal{A}_2$, appears in terms of the structure forms $[(tr\varepsilon^S)(p^{a_0})\times(p^{a_1})(p\varepsilon^A p)], [(p^{a_0})(p\varepsilon^S)^{a_1}(p\varepsilon^A p)]$, and $[(pp)(pp)\times(\varepsilon^S\varepsilon^A)^{a_0a_1}]$, in addition to the structure forms in preview subamplitudes. The subamplitude $\mathcal{A}_2$ has eight independent integral functions $\mathcal{J}, \mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3, \mathcal{J}_4, \mathcal{J}_5, \mathcal{J}_{12}, \mathcal{J}_{13}$, and $\mathcal{J}_{14}$ (in which $\mathcal{J}_{15} = \mathcal{J}_{13} - \mathcal{J}_{14}$ and $\mathcal{J}_{16} = \mathcal{J}_{13} + \mathcal{J}_{14}$) whose explicit forms are given in [12]. The symmetries of the integrals under the interchange of $(2 \leftrightarrow 3)$ are such that $\mathcal{J}, \mathcal{J}_3, \mathcal{J}_{13}, \mathcal{J}_{14}$ are invariant, $\mathcal{J}_1 \leftrightarrow \mathcal{J}_4$, $\mathcal{J}_2 \leftrightarrow \mathcal{J}_{12}$, and $\mathcal{J}_5 \leftrightarrow -\mathcal{J}_5$.

It is shown in [15] that there are three T-dual multiplets in which all 11 components contain the above 14 integral functions. The components of these multiplets are the amplitudes corresponding to the RR $(p-3)$-form, $(p-1)$-form, $(p+1)$-form, $(p+3)$-form, and $(p+5)$-form with zero, one, two, three, and four transverse indices, respectively.

D. $q = 1$

In this case, the multiplication of NSNS momenta has one momentum as

$$\mathcal{P}_1 = (p_2)^{\beta_i} + (p_2 \cdot D)^{\beta_i} + (2 \leftrightarrow 3), \qquad (16)$$

and the part of the X-correlator corresponding to this case that involves the holomorphic coordinate and momentum can be found in Appendix A.

To find the corresponding subamplitude $\mathcal{A}_1$, we encounter a new contraction of some terms of the above correlator with NSNS polarizations. This indicates that we would have some new integral functions in $\mathcal{A}_1$. However, we find that no new structure appears in the subamplitude $\mathcal{A}_1$. In fact, this subamplitude includes the familiar structures. By considering the amplitude invariance under the Ward symmetry and T-duality covariance of the amplitude, which is explained in the next section, the number of new independent integrals appearing in $\mathcal{A}_2$ is then reduced to twelve integrals. We give these integrals the names: $\mathcal{K}_1, \mathcal{K}_2, \ldots, \mathcal{K}_{11}$. The symmetries of the integrals under the interchange of $(2 \leftrightarrow 3)$ are such that $\mathcal{K}_1 \leftrightarrow \mathcal{K}_2$, $\mathcal{K}_3 \leftrightarrow -\mathcal{K}_3$, $\mathcal{K}_4 \leftrightarrow -\mathcal{K}_5$, $\mathcal{K}_6 \leftrightarrow \mathcal{K}_9$, $\mathcal{K}_7 \leftrightarrow \mathcal{K}_{10}$, and $\mathcal{K}_8 \leftrightarrow \mathcal{K}_{11}$. The explicit form of these new integral functions appears in Appendix B.

To apply the Ward identity on the subamplitudes, it is convenient to write the amplitude (5) in terms of four different subamplitudes $\mathcal{A}^I, \mathcal{A}^{II}, \mathcal{A}^{IM}$, and $\mathcal{A}^{IIM}$ where the terms of these subamplitudes contain four different kinds of scalar parts. The scalar parts in the first subamplitude have one integral function; in the second subamplitude, they have a combination of integral functions; in the third subamplitude, they have one integral function and one Mandelstam variable; and in the fourth subamplitude, they have a combination of integral functions and Mandelstam variables. Hence, it could be shown that $\mathcal{A}_4$ and $\mathcal{A}_3$ have no contribution to $\mathcal{A}^{II}$ and also that $\mathcal{A}_4$ has no contribution to $\mathcal{A}^{IIM}$. Since the results are long and elaborate, we list them below without further commentary:

$$\begin{aligned}
\mathcal{A}^I = {} & 4p_2^{a_0}p_3^{a_1}tr(\varepsilon_3^S\cdot V)(p_1\cdot N\cdot\varepsilon_2^A\cdot V\cdot p_3\mathcal{J}_{12} - p_1\cdot N\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{J}_4 - 2p_2\cdot V\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{K}_5 - 2p_2\cdot V\cdot\varepsilon_2^A\cdot V\cdot p_3\mathcal{K}_4 \\
& - 2p_3\cdot V\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{K}_8) + 2p_2^{a_0}p_3^{a_1}(2p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A\cdot N\cdot p_1\mathcal{I}_2 - 2p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A\cdot N\cdot p_1\mathcal{I}_3 \\
& - 4p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A\cdot V\cdot p_2\mathcal{J}_2 + 4p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A\cdot V\cdot p_2\mathcal{J}_1 - p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{J}_{15} \\
& - p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A\cdot V\cdot p_3\mathcal{J}_{15} - 2p_2\cdot V\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A\cdot V\cdot\mathcal{K}_9 - 2p_2\cdot V\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A\cdot V\cdot p_2\mathcal{K}_{10} \\
& - 2p_2\cdot N\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A\cdot V\cdot p_2\mathcal{K}_{10} - 2p_2\cdot N\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A\cdot V\cdot p_2\mathcal{K}_9 + 2p_2\cdot V\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{K}_1 \\
& + 2p_2\cdot V\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{K}_2 - 2p_2\cdot N\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A\cdot V\cdot p_3\mathcal{K}_2 - 2p_2\cdot N\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A\cdot V\cdot p_3\mathcal{K}_1 \\
& + 2p_3\cdot V\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A\cdot V\cdot p_3\mathcal{K}_7 + 2p_3\cdot V\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{K}_7 - p_1\cdot N\cdot\varepsilon_2^A\cdot V\cdot\varepsilon_3^S\cdot N\cdot p_2\mathcal{J}_{15} \\
& - p_1\cdot N\cdot\varepsilon_2^A\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_2\mathcal{J}_{15} - 4p_1\cdot N\cdot\varepsilon_2^A\cdot V\cdot\varepsilon_3^S\cdot V\cdot p_3\mathcal{J}_{12} + 4p_1\cdot N\cdot\varepsilon_2^A\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_3\mathcal{J}_4 \\
& + 8p_2\cdot V\cdot\varepsilon_2^A\cdot V\cdot\varepsilon_3^S\cdot V\cdot p_3\mathcal{K}_4 + 8p_2\cdot V\cdot\varepsilon_2^A\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_3\mathcal{K}_4) + 4(p_1\cdot N\cdot\varepsilon_2^A)^{a_0}(2p_1^{a_1}p_2\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_3\mathcal{J}_4 \\
& - p_1^{a_1}p_2\cdot V\cdot\varepsilon_3^S\cdot N\cdot p_2\mathcal{J}_{15} + 2p_1^{a_1}p_2\cdot V\cdot\varepsilon_3^S\cdot V\cdot p_3\mathcal{J}_{12} + (2p_2^{a_1} + p_3^{a_1})p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_2\mathcal{I}_2 \\
& - (2p_2^{a_1} + p_3^{a_1})p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_2\mathcal{I}_3) + 4(p_2\cdot V\cdot\varepsilon_2^A)^{a_0}(4p_2^{a_1}(p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_1)\mathcal{I}_7 - 4p_1^{a_1}p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_2\mathcal{J}_1 \\
& + 2p_1^{a_1}p_2\cdot V\cdot\varepsilon_3^S\cdot N\cdot p_2\mathcal{K}_{10} + p_1^{a_1}p_2\cdot V\cdot\varepsilon_3^S\cdot V\cdot p_2\mathcal{K}_9 + p_1^{a_1}p_2\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_2\mathcal{K}_9 \\
& + 4p_1^{a_1}p_2\cdot V\cdot\varepsilon_3^S\cdot V\cdot p_3\mathcal{K}_4 + 4p_1^{a_1}p_2\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_3\mathcal{K}_5 - 2(2p_2^{a_1} + p_3^{a_1})p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_2\mathcal{J}_2) \\
& - 2(p_3\cdot V\cdot\varepsilon_2^A)^{a_0}(2p_2^{a_1}p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_1\mathcal{I}_2 + p_3^{a_1}p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_2\mathcal{J}_{15} - 4p_2^{a_1}p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_3\mathcal{J}_4 \\
& - 2p_1^{a_1}p_2\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_2\mathcal{K}_1 + 2p_1^{a_1}p_2\cdot V\cdot\varepsilon_3^S\cdot V\cdot p_3\mathcal{K}_7 - 2p_1^{a_1}p_2\cdot V\cdot\varepsilon_3^S\cdot N\cdot p_2\mathcal{K}_2)
\end{aligned}$$

$$
\begin{aligned}
&+2(p_3\cdot N\cdot\varepsilon_2^A)^{a_0}(2p_2^{a_1}p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_1\mathcal{I}_3-p_3^{a_1}p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_2\mathcal{J}_{15}-4p_2^{a_1}p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_3\mathcal{J}_{12}\\
&-2p_1^{a_1}p_2\cdot V\cdot\varepsilon_3^S\cdot V\cdot p_2\mathcal{K}_2-2p_1^{a_1}p_2\cdot V\cdot\varepsilon_3^S\cdot N\cdot p_2\mathcal{K}_1-2p_1^{a_1}p_2\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_3\mathcal{K}_7)\\
&+4(p_1\cdot N\cdot\varepsilon_3^S)^{a_0}p_2^{a_1}(-p_1\cdot N\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{I}_2+p_1\cdot N\cdot\varepsilon_2^A\cdot V\cdot p_3\mathcal{I}_3-2p_2\cdot V\cdot\varepsilon_2^A\cdot V\cdot p_3\mathcal{J}_1\\
&+2p_2\cdot V\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{J}_2)+2(p_2\cdot V\cdot\varepsilon_3^S)^{a_0}p_1^{a_1}(p_1\cdot N\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{J}_{15}-2p_2\cdot V\cdot\varepsilon_2^A\cdot V\cdot p_3\mathcal{K}_9\\
&-2p_2\cdot V\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{K}_{10}-2p_3\cdot V\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{K}_2)+2(p_2\cdot N\cdot\varepsilon_3^S)^{a_0}p_1^{a_1}(p_1\cdot N\cdot\varepsilon_2^A\cdot V\cdot p_3\mathcal{J}_{15}\\
&-2p_2\cdot V\cdot\varepsilon_2^A\cdot V\cdot p_3\mathcal{K}_{10}-2p_2\cdot V\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{K}_9-2p_3\cdot V\cdot\varepsilon_2^A\cdot N\cdot p_3\mathcal{K}_1),
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
\mathcal{A}^{II}={}&2p_2^{a_0}p_3^{a_1}(p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A\cdot V\cdot p_3(4\mathcal{J}-\mathcal{J}_{16}+2\mathcal{J}_5)-p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A\cdot N\cdot p_3(\mathcal{J}_{16}+2\mathcal{J}_5)\\
&+2p_3\cdot V\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A\cdot N\cdot p_3(2\mathcal{K}_8+\mathcal{K}_6)+2p_3\cdot V\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A\cdot V\cdot p_3(-2\mathcal{K}_8+\mathcal{K}_6)\\
&+p_1\cdot N\cdot\varepsilon_2^A\cdot V\cdot\varepsilon_3^S\cdot V\cdot p_2(4\mathcal{J}-\mathcal{J}_{16}-2\mathcal{J}_5)-p_1\cdot N\cdot\varepsilon_2^A\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_2(\mathcal{J}_{16}-2\mathcal{J}_5))\\
&-2(p_1\cdot N\cdot\varepsilon_2^A)^{a_0}p_1^{a_1}(p_2\cdot V\cdot\varepsilon_3^S\cdot V\cdot p_2(-4\mathcal{J}+\mathcal{J}_{16}+2\mathcal{J}_5)+p_2\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_2(\mathcal{J}_{16}-2\mathcal{J}_5))\\
&+2(p_3\cdot V\cdot\varepsilon_2^A)^{a_0}(p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_2[p_2^{a_1}(2\mathcal{J}-\mathcal{J}_{16})+p_3^{a_1}(4\mathcal{J}-\mathcal{J}_{16}+2\mathcal{J}_5)]\\
&-2p_1^{a_1}p_2\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_3(-2\mathcal{K}_8+\mathcal{K}_6)-p_2^{a_1}p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_2(8\mathcal{J}+\mathcal{J}_{15}))\\
&+2(p_3\cdot N\cdot\varepsilon_2^A)^{a_0}(p_2^{a_1}p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_2(2\mathcal{J}-\mathcal{J}_{16})-p_3^{a_1}p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_2(\mathcal{J}_{16}+2\mathcal{J}_5)\\
&+p_2^{a_1}p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_2(8\mathcal{J}-\mathcal{J}_{15})-2p_1^{a_1}p_2\cdot V\cdot\varepsilon_3^S\cdot V\cdot p_3(2\mathcal{K}_8+\mathcal{K}_6))\\
&+(p_2\cdot V\cdot\varepsilon_3^S)^{a_0}(p_1\cdot N\cdot\varepsilon_2^A\cdot V\cdot p_3[p_2^{a_1}(3\mathcal{J}-\mathcal{J}_{16}-\mathcal{J}_5)+2p_3^{a_1}(8\mathcal{J}-\mathcal{J}_{16}-2\mathcal{J}_5)])\\
&-2(p_2\cdot N\cdot\varepsilon_3^S)^{a_0}(p_1\cdot N\cdot\varepsilon_2^A\cdot N\cdot p_3[p_2^{a_1}(\mathcal{J}_{16}-2\mathcal{J}_5)+p_3^{a_1}(4\mathcal{J}+\mathcal{J}_{16}-2\mathcal{J}_5)])\\
&-8(p_1\cdot N\cdot\varepsilon_3^S)^{a_0}p_2^{a_1}p_3\cdot V\cdot\varepsilon_2^A\cdot N\cdot p_3(\mathcal{J}+\mathcal{J}_5)
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
\mathcal{A}^{IM}={}&4(p_2\cdot N\cdot\varepsilon_3^S)^{a_0}(-t\mathcal{I}_2(p_1\cdot N\cdot\varepsilon_2^A)^{a_1}+2t\mathcal{J}_2(p_2\cdot V\cdot\varepsilon_2^A)^{a_1})+4(p_2\cdot V\cdot\varepsilon_3^S)^{a_0}(t\mathcal{I}_3(p_1\cdot N\cdot\varepsilon_2^A)^{a_1}\\
&-2t\mathcal{J}_1(p_2\cdot V\cdot\varepsilon_2^A)^{a_1})+4(s\mathcal{J}_4tr(\varepsilon_3^S\cdot V)(p_3\cdot N\cdot\varepsilon_2^A)^{a_0}p_3^{a_1}-s\mathcal{J}_{12}p_2\cdot V\cdot\varepsilon_3^S\cdot V\cdot p_3(\varepsilon_2^A)^{a_0a_1})
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
\mathcal{A}^{IIM}={}&-2(p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A)^{a_0}(-v(\mathcal{J}_{16}+2\mathcal{J}_5)p_1^{a_1}-u\mathcal{J}_{15}p_1^{a_1}+4v\mathcal{J}p_2^{a_1}+2s\mathcal{I}_3p_3^{a_1})\\
&+(p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A)^{a_0}(2u\mathcal{J}_{16}p_1^{a_1}-4u\mathcal{J}_5p_1^{a_1}+2v\mathcal{J}_{15}p_1^{a_1}+4s\mathcal{I}_2p_3^{a_1}+8u\mathcal{J}(2p_2^{a_1}+p_3^{a_1}))\\
&+2(p_1\cdot N\cdot\varepsilon_2^A\cdot V\cdot\varepsilon_3^S)^{a_0}(u\mathcal{J}_{16}p_2^{a_1}-2u\mathcal{J}_5p_1^{a_1}+v\mathcal{J}_{15}p_2^{a_1}-2t\mathcal{I}_3p_2^{a_1}-2u\mathcal{J}(2p_2^{a_1}+3p_3^{a_1}))\\
&+2(p_1\cdot N\cdot\varepsilon_2^A\cdot N\cdot\varepsilon_3^S)^{a_0}(2t\mathcal{I}_2p_2^{a_1}+2v\mathcal{J}_5p_1^{a_1}+(3p_2^{a_1}+p_3^{a_1})(u\mathcal{J}_{15}+v\mathcal{J}_{16})+4v\mathcal{J}(p_2^{a_1}-p_3^{a_1}))\\
&+(p_2\cdot V\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A)^{a_0}(-4p(\mathcal{K}_9-2\mathcal{K}_{11})p_1^{a_1}+4v\mathcal{K}_2p_1^{a_1}-4v\mathcal{K}_3p_2^{a_1}-2t\mathcal{J}_{15}p_2^{a_1}+4s\mathcal{J}_5(p_2^{a_1}-p_3^{a_1})\\
&+2s(2p_2^{a_1}+p_3^{a_1})(4\mathcal{J}+\mathcal{J}_{16}))-2(p_2\cdot V\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A)^{a_0}(s\mathcal{J}_{15}(2p_2^{a_1}+p_3^{a_1})-2v\mathcal{K}_1p_1^{a_1}+v\mathcal{K}_3p_2^{a_1}\\
&+2p\mathcal{K}_{10}p_2^{a_1}+t(\mathcal{J}_{16}-2\mathcal{J}_5)p_2^{a_1})-2(p_2\cdot N\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A)^{a_0}(s\mathcal{J}_{15}(2p_2^{a_1}+p_3^{a_1})+t(-4\mathcal{J}+\mathcal{J}_{16}+2\mathcal{J}_5)p_2^{a_1}\\
&+v\mathcal{K}_3p_2^{a_1}+2p\mathcal{K}_{10}p_1^{a_1}+2u\mathcal{K}_2p_1^{a_1})+(p_2\cdot N\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A)^{a_0}(-2s\mathcal{J}_{16}(2p_2^{a_1}+p_3^{a_1})+4s\mathcal{J}_5(p_2^{a_1}-3p_3^{a_1})\\
&-2t\mathcal{J}_{15}p_2^{a_1}+8v\mathcal{K}_2p_1^{a_1}+4u\mathcal{K}_1p_1^{a_1}-2v\mathcal{K}_3p_2^{a_1}-4u(\mathcal{K}_9-2\mathcal{K}_{11})p_1^{a_1}-16s\mathcal{J}p_1^{a_1})\\
&+(p_3\cdot V\cdot\varepsilon_3^S\cdot V\cdot\varepsilon_2^A)^{a_0}(-4v(2\mathcal{K}_8+\mathcal{K}_6)p_1^{a_1}-4u\mathcal{K}_7p_1^{a_1}+8s\mathcal{J}_{12}p_3^{a_1})+(p_3\cdot V\cdot\varepsilon_3^S\cdot N\cdot\varepsilon_2^A)^{a_0}(-4v\mathcal{K}_7p_1^{a_1}\\
&-4u(-2\mathcal{K}_8+\mathcal{K}_6)p_1^{a_1}-8s\mathcal{J}_4p_3^{a_1})+(p_2\cdot V\cdot\varepsilon_2^A\cdot V\cdot\varepsilon_3^S)^{a_0}(8t\mathcal{J}_1p_2^{a_1}+4(v\mathcal{K}_{10}+u\mathcal{K}_9)p_1^{a_1})\\
&+(p_2\cdot V\cdot\varepsilon_2^A\cdot N\cdot\varepsilon_3^S)^{a_0}(-8t\mathcal{J}_2p_2^{a_1}+4(v\mathcal{K}_9+u\mathcal{K}_{10})p_1^{a_1})+(p_3\cdot V\cdot\varepsilon_2^A\cdot V\cdot\varepsilon_3^S)^{a_0}(16s\mathcal{J}p_1^{a_1}-2t\mathcal{J}_{15}p_2^{a_1}\\
&+4v\mathcal{K}_2p_1^{a_1}-4p(\mathcal{K}_9-2\mathcal{K}_{11})p_1^{a_1}-2(2p_2^{a_1}+p_3^{a_1})(s\mathcal{J}_{16}+2s\mathcal{J}_5))+(p_3\cdot V\cdot\varepsilon_2^A\cdot N\cdot\varepsilon_3^S)^{a_0}(-4s\mathcal{J}_{15}p_2^{a_1}\\
&-2t(-4\mathcal{J}+\mathcal{J}_{16}-2\mathcal{J}_5)p_2^{a_1}+4(v\mathcal{K}_1-p\mathcal{K}_{10})p_1^{a_1})+(p_3\cdot N\cdot\varepsilon_2^A\cdot V\cdot\varepsilon_3^S)^{a_0}(-2s\mathcal{J}_{15}(2p_2^{a_1}+p_3^{a_1})\\
&-2t(\mathcal{J}_{16}+2\mathcal{J}_5)p_2^{a_1}-4(p\mathcal{K}_{10}+u\mathcal{K}_2)p_1^{a_1})+(p_3\cdot N\cdot\varepsilon_2^A\cdot N\cdot\varepsilon_3^S)^{a_0}(2s(2p_2^{a_1}+p_3^{a_1})(2\mathcal{J}-\mathcal{J}_{16})\\
&-4s\mathcal{J}_5(2p_2^{a_1}+3p_3^{a_1})-2t\mathcal{J}_{15}p_2^{a_1}+4(2v\mathcal{K}_2-p(\mathcal{K}_9-2\mathcal{K}_{11})+u\mathcal{K}_1)p_1^{a_1})\\
&+(\varepsilon_2^A)^{a_0a_1}(-2p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_1(v\mathcal{I}_3+u\mathcal{I}_2)+2p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_2(2s\mathcal{I}_3+u\mathcal{J}_{15}+v(-2\mathcal{J}+\mathcal{J}_{16}))
\end{aligned}
$$

$$
\begin{aligned}
&+2p_1\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_2(-2s\mathcal{I}_2+v\mathcal{J}_{15}+u(-2\mathcal{J}+\mathcal{J}_{16}))-4p_1\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_3(2s\mathcal{I}_4-v\mathcal{J}_{12}+u\mathcal{J}_4)\\
&+2p_2\cdot V\cdot\varepsilon_3^S\cdot V\cdot p_2 s(\mathcal{J}-\mathcal{J}_5)-p_2\cdot N\cdot\varepsilon_3^S\cdot N\cdot p_2(s(\mathcal{J}_{16}-2\mathcal{J}_5)+t\mathcal{J}_{15})+4p_2\cdot N\cdot\varepsilon_3^S\cdot V\cdot p_3 s\mathcal{J}_4)\\
&+2(\varepsilon_2^A\cdot V\cdot\varepsilon_3^S)^{a_0a_1}((\mathcal{J}_{16}-2\mathcal{J}_5)(tv-us)+\mathcal{J}_{15}(tu-sv)+2st\mathcal{I}_3+4us\mathcal{J})+2(\varepsilon_2^A\cdot N\cdot\varepsilon_3^S)^{a_0a_1}(2st\mathcal{I}_2\\
&-(\mathcal{J}_{16}-2\mathcal{J}_5)(sv-ut)+\mathcal{J}_{15}(tv-us)-4tu\mathcal{J})+2(p_2\cdot V\cdot\varepsilon_3^S)^{a_0}((p_3\cdot V\cdot\varepsilon_2^A)^{a_1}(t\mathcal{J}_{15}+s(-4\mathcal{J}+\mathcal{J}_{16}+2\mathcal{J}_5))\\
&+(p_3\cdot N\cdot\varepsilon_2^A)^{a_1}(s\mathcal{J}_{15}+t(\mathcal{J}_{16}+2\mathcal{J}_5)))+2(p_2\cdot N\cdot\varepsilon_3^S)^{a_0}((p_3\cdot V\cdot\varepsilon_2^A)^{a_1}(s\mathcal{J}_{15}+t(-4\mathcal{J}+\mathcal{J}_{16}-2\mathcal{J}_5))\\
&+(p_3\cdot N\cdot\varepsilon_2^A)^{a_1}(t\mathcal{J}_{15}+s(\mathcal{J}_{16}-2\mathcal{J}_5)))+4(p_1\cdot N\cdot\varepsilon_3^S)^{a_0}(2(-v\mathcal{J}_2+u\mathcal{J}_1)(p_2\cdot V\cdot\varepsilon_2^A)^{a_1}\\
&-(s\mathcal{I}_3+v(-2\mathcal{J}+\mathcal{J}_{16})+u\mathcal{J}_{15})(p_3\cdot V\cdot\varepsilon_2^A)^{a_1}+(s\mathcal{I}_2-v\mathcal{J}_{15}+u(2\mathcal{J}-\mathcal{J}_{16}))(p_3\cdot N\cdot\varepsilon_2^A)^{a_1}\\
&+(v\mathcal{I}_2+u\mathcal{I}_3)(p_1\cdot N\cdot\varepsilon_2^A)^{a_1})+4tr(\varepsilon_3^S\cdot V)((t\mathcal{J}_{12}-v\mathcal{K}_7-u\mathcal{K}_6)p_2^{a_1}(p_3\cdot N\cdot\varepsilon_2^A)^{a_0}\\
&+(p_1\cdot N\cdot\varepsilon_2^A)^{a_0}[(v\mathcal{J}_4-u\mathcal{J}_{12})p_3^{a_1}-(2t\mathcal{I}_4-v\mathcal{J}_4+u\mathcal{J}_{12})p_2^{a_1}]+2(p_2\cdot V\cdot\varepsilon_2^A)^{a_0}[(2t\mathcal{J}_3+v\mathcal{K}_5+u\mathcal{K}_4)p_2^{a_1}\\
&+(v\mathcal{K}_5+u\mathcal{K}_4)p_3^{a_1}]-(p_3\cdot V\cdot\varepsilon_2^A)^{a_0}[(t\mathcal{J}_4+v\mathcal{K}_6+u\mathcal{K}_7)p_2^{a_1}+(s\mathcal{J}_{12}+(\mathcal{K}_6+\mathcal{K}_7)(u+v))p_3^{a_1}])\\
&+2tr(\varepsilon_3^S\cdot V)(\varepsilon_2^A)^{a_0a_1}((s(2t\mathcal{I}_4-v\mathcal{J}_4+u\mathcal{J}_{12})+t(-v\mathcal{J}_{12}+u\mathcal{J}_4))), \qquad (20)
\end{aligned}
$$

where we have used the following definitions for the Mandelstam variables:

$$s=p_1\cdot N\cdot p_2,\qquad t=p_1\cdot N\cdot p_3,\qquad u=p_2\cdot V\cdot p_3\qquad p=p_2\cdot V\cdot p_2,\qquad q=p_3\cdot V\cdot p_3,\qquad v=p_2\cdot N\cdot p_3.$$

IV. CONSTRAINTS AND INTEGRAL IDENTITIES

It has been shown that scattering amplitudes are invariant under the linear T-duality transformations on the external states. Assuming that the NSNS fields are small perturbations around the flat space, e.g., $G_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$, and applying the T-duality transformation along the Killing coordinate y, the massless NSNS fields transforms linearly as

$$
\begin{aligned}
&\tilde{h}_{yy}=-h_{yy},\qquad \tilde{h}_{\mu y}=B_{\mu y},\qquad \tilde{B}_{\mu y}=h_{\mu y},\\
&\tilde{h}_{\mu\nu}=h_{\mu\nu},\qquad \tilde{B}_{\mu\nu}=B_{\mu\nu} \qquad (21)
\end{aligned}
$$

where μ, ν denote any coordinate other than y.

It has been speculated that the subamplitudes $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$, including the familiar integral functions $\mathcal{I}_i$'s and $\mathcal{J}_i$'s, could be predicted by T-dual Ward identity and that the subamplitude $\mathcal{A}_4$ that has new integral functions could not be predicted completely [15]. By applying the T-duality transformations on the external directions of the total amplitude (1) and considering the Ward identities corresponding to its polarizations, one can find the T-dual completion of this amplitude in which we are not interested in this paper.

Now we are going to investigate the T-duality covariance of the amplitudes that we found in this paper by applying the T-duality transformation on the internal directions. The internal directions mean the indices α, β, and γ appear in the Mandelstam variables $p^\alpha p_\alpha$, in the structures $p^\alpha\varepsilon_{\alpha\mu}$, $p^\alpha\varepsilon_{\alpha\beta}p^\beta$, $\varepsilon^{\mu\alpha}\varepsilon_\alpha{}^\nu$, $p^\alpha\varepsilon_{\alpha\beta}\varepsilon^{\beta\mu}$, and $p^\alpha\varepsilon_{\alpha\beta}\varepsilon^{\beta\gamma}p_\gamma$. From the T-duality rules, we know that the background fields are independent of the Killing coordinates along which the T-duality is applied [18]. So, apart from the last three terms, it is easy to see this symmetry because the NSNS polarization tensors contract either with the volume form or with the momentum. The last three terms require further investigation. One can find that the structures made of the contraction of two NSNS polarization tensors should appear as $(\varepsilon_2^A)^{\mu a}(\varepsilon_3^S)_a{}^\nu+(\varepsilon_3^A)^{\mu n}(\varepsilon_2^S)_n{}^\nu$ to be invariant under the T-duality transformations. As a result, the amplitude that we found in this paper would be T-dual covariance if the structures $\varepsilon^{\mu\alpha}\varepsilon_\alpha{}^\nu$, $p^\alpha\varepsilon_{\alpha\beta}\varepsilon^{\beta\mu}$, and $p^\alpha\varepsilon_{\alpha\beta}\varepsilon^{\beta\gamma}p_\gamma$ appear as the following,

$$
\begin{aligned}
&(\varepsilon_2^A\cdot V\cdot\varepsilon_3^S)^{\mu\nu}+(\varepsilon_3^A\cdot N\cdot\varepsilon_2^S)^{\mu\nu},\\
&(p\cdot\varepsilon_2^A\cdot V\cdot\varepsilon_3^S)^{\mu}+(p\cdot\varepsilon_3^A\cdot N\cdot\varepsilon_2^S)^{\mu},\\
&p\cdot\varepsilon_2^A\cdot V\cdot\varepsilon_3^S\cdot p+p\cdot\varepsilon_3^A\cdot N\cdot\varepsilon_2^S\cdot p,
\end{aligned}
$$

and the corresponding terms in the $(2\leftrightarrow 3)$ part. One can find that the amplitude (5) respects this symmetry.

A scattering amplitude should satisfy the Ward identity associated with its polarizations [11]. Therefore, the amplitude (5) should be invariant under the NSNS gauge transformations. However, it satisfies the RR gauge transformation when one includes the amplitude of the RR $(p-1)$-form with two transverse indices [17], and the amplitude of the RR $(p-1)$-form with one transverse index [15].

By imposing the consistency of the amplitude (5) with the NSNS gauge transformation, i.e., under replacing the symmetric and antisymmetric NSNS polarization with

$$(\varepsilon^S)^{\mu\nu} \to p^\mu \zeta^\nu + p^\nu \zeta^\mu, \qquad (\varepsilon^A)^{\mu\nu} \to p^\mu \zeta^\nu - p^\nu \zeta^\mu, \tag{22}$$

the gauge invariant amplitude must be zero. Using this condition, one finds the following identities,

$$\begin{aligned} -2t\mathcal{I}_1 + 2q\mathcal{I}_4 + v\mathcal{I}_2 - u\mathcal{I}_3 &= 0 \\ -2s\mathcal{I}_2 + v\mathcal{J}_{15} + 2p\mathcal{J}_2 + u(-4\mathcal{J} + \mathcal{J}_{16} - 2\mathcal{J}_5) &= 0 \\ 2s\mathcal{I}_3 - 2p\mathcal{J}_1 + u\mathcal{J}_{15} + v(\mathcal{J}_{16} + 2\mathcal{J}_5) &= 0 \\ -2s\mathcal{I}_4 + v\mathcal{J}_{12} + 2p\mathcal{J}_3 - u\mathcal{J}_4 &= 0 \\ 2s\mathcal{J}_{12} + v(\mathcal{K}_6 + 2\mathcal{K}_8) - 2p\mathcal{K}_4 + u\mathcal{K}_7 &= 0 \\ -2s\mathcal{J}_4 + v\mathcal{K}_7 - 2p\mathcal{K}_5 + u(\mathcal{K}_6 - 2\mathcal{K}_8) &= 0 \\ 2s(-\mathcal{J} + \mathcal{J}_5) + v\mathcal{K}_2 + 2p\mathcal{K}_{11} + u\mathcal{K}_1 &= 0, \end{aligned} \tag{23}$$

and similar relations under the interchange of $(2 \leftrightarrow 3)$. The first four identities that include the old integrals $\mathcal{I}_i$'s and $\mathcal{J}_i$, have appeared before in [11,12,15], and the other ones that include new integrals $\mathcal{K}_i$'s are new identities. In calculating the amplitude (5), we found two integral functions including some tachyonic poles appearing in the subamplitudes A^{IM} and A^{IIM}. On the other hand, there were two integral identities, in addition to the above identities, that could fix these two tachyonic integral functions. From these additional identities, we could fix these tachyonic integrals in terms of some familiar integrals $\mathcal{J}_i$'s and $\mathcal{K}_i$'s. The integral identities can be checked at low energy. In performing this calculation, one needs the α' expansion of the integrals that appear in the identities. The α' expansion of the integrals $\mathcal{I}_i$'s and $\mathcal{J}_i$'s have been found in [10,14]. We find the corresponding expansion for integrals $\mathcal{K}_i$'s for the special kinematic setup where $u \pm \mathrm{v} = 0$ and check the identities (23).

Considering properly the symmetries coming from T-duality and Ward identities, it could be possible to write an amplitude in terms of the minimal possible number of integral functions. From these considerations, we found that the amplitude (5) includes 24 nontachyonic independent integral functions that satisfy 14 constraint equations (23). From boundary state formalism, the disk-level amplitude of an arbitrary RR state and two NSNS vertex operators has been found in the interesting paper [19], where the T-dual gauge symmetry was not taken into account and the amplitude appeared in terms of a larger context of integral functions in which some of them were tachyonic integrals.

The T-dual Ward identity connects the amplitude (5) to the amplitudes corresponding to the RR $(p+1)$-form with one transverse index and RR $(p+3)$-form with two transverse indices which furnish the following T-dual multiplet,

$$A_0(C^{(p-1)}) \to A_1(C_i^{(p+1)}) \to A_2(C_{ij}^{(p+3)}),$$

where the number in the label of A refers to the number of transverse indices of the RR potential. The components A_1 and A_2 carry the same integral that the first component A_0 [amplitude (5)] carries. To find a gauge invariant form of all elements of the above multiplet, and more generally the amplitude of an arbitrary RR state and two NSNS, from T-dual Ward identity or explicit calculation, one needs to use the identities (23). Using the low-energy expansion of the integrals appearing in the amplitudes, one can find the α' expansion of the amplitudes. Hence, this multiplet can be analyzed at low energy to extract the appropriate couplings of one RR and two NSNS states in the field theory at order α'^2. We are not interested in finding the corresponding couplings here and leave the details of these calculations for future work.

ACKNOWLEDGMENTS

We thank M. R. Garousi and A. Jalali for useful discussions and H. Zamani for very helpful conversations. This work was supported by the University of Guilan.

APPENDIX A: ON X CORRELATION

Here, we present the part of the X-correlator corresponding to $q=1$ that involves holomorphic coordinates and momenta.

$$\begin{aligned} \mathcal{X}\Big[&\frac{1}{Z_2 Z_3}(p_1^{\alpha_1} p_1^{\alpha_3} + p_1^{\alpha_3}(p_1.D)^{\alpha_1} + p_1^{\alpha_1}(p_1.D)^{\alpha_3} + (p_1.D)^{\alpha_1}(p_1.D)^{\alpha_3}) + \frac{1}{Z_3 \bar{Z}_2}(p_1^{\alpha_2} p_1^{\alpha_3} + p_1^{\alpha_2}(p_1.D)^{\alpha_3} + (p_1.D)^{\alpha_2} p_1^{\alpha_3} \\ &+ (p_1.D)^{\alpha_2}(p_1.D)^{\alpha_3}) + \frac{1}{|Z_2|^2}(p_1^{\alpha_2} p_1^{\alpha_1} + p_1^{\alpha_2}(p_1.D)^{\alpha_1} + (p_1.D)^{\alpha_2} p_1^{\alpha_1} + (p_1.D)^{\alpha_2}(p_1.D)^{\alpha_1}) \\ &- \frac{1}{Z_3 Z_{2\bar{2}}}(p_1^{\alpha_3} p_2^{\alpha_2} + (p_1.D)^{\alpha_3} p_2^{\alpha_2} - (p_1.D)^{\alpha_3}(p_2.D)^{\alpha_1} + p_1^{\alpha_3}(p_2.D)^{\alpha_1}) + \frac{1}{Z_2 Z_{\bar{2}3}}(p_1^{\alpha_1} p_3^{\alpha_2} + (p_1.D)^{\alpha_1} p_3^{\alpha_2} - p_1^{\alpha_1}(p_2.D)^{\alpha_3} \\ &- (p_1.D)^{\alpha_1}(p_2.D)^{\alpha_3}) - \frac{1}{\bar{Z}_2 Z_{3\bar{3}}}(p_1^{\alpha_2} p_3^{\alpha_4} + (p_1.D)^{\alpha_2} p_3^{\alpha_4} - p_1^{\alpha_2}(p_3.D)^{\alpha_3} - (p_1.D)^{\alpha_2}(p_3.D)^{\alpha_3}) \\ &+ \frac{1}{Z_{2\bar{2}} Z_{3\bar{3}}}(p_2^{\alpha_2} p_3^{\alpha_4} - (p_2.D)^{\alpha_1}(p_3.D)^{\alpha_3} - (p_2.D)^{\alpha_1} p_3^{\alpha_4} + p_2^{\alpha_2}(p_3.D)^{\alpha_3}) + \frac{1}{\bar{Z}_3 Z_{\bar{2}3}}(p_1^{\alpha_4} p_3^{\alpha_2} - p_1^{\alpha_4}(p_2.D)^{\alpha_3} \end{aligned}$$

$$
\begin{aligned}
&+(p_1.D)^{\alpha_4}p_3^{\alpha_2}-(p_1.D)^{\alpha_4}(p_2.D)^{\alpha_3})-\frac{1}{Z_3Z_{2\bar{3}}}(p_1^{\alpha_3}p_2^{\alpha_4}+(p_1.D)^{\alpha_3}p_2^{\alpha_4}-p_1^{\alpha_3}(p_3.D)^{\alpha_1}-(p_1.D)^{\alpha_3}(p_3.D)^{\alpha_1})\\
&-\frac{1}{\bar{Z}_2Z_{2\bar{3}}}(p_1^{\alpha_2}p_2^{\alpha_4}+(p_1.D)^{\alpha_2}p_2^{\alpha_4}-p_1^{\alpha_2}(p_3.D)^{\alpha_1}-(p_1.D)^{\alpha_2}(p_3.D)^{\alpha_1})-\frac{1}{Z_{\bar{2}\bar{3}}Z_{2\bar{3}}}((p_2.D)^{\alpha_4}(p_3.D)^{\alpha_1}-p_2^{\alpha_4}(p_3.D)^{\alpha_2}\\
&-(p_3.D)^{\alpha_1}(p_3.D)^{\alpha_2})-\frac{1}{Z_{3\bar{3}}Z_{\bar{2}\bar{3}}}((p_2.D)^{\alpha_4}(p_3.D)^{\alpha_3}-p_3^{\alpha_4}(p_3.D)^{\alpha_2}-(p_3.D)_{\alpha_2}(p_3.D)_{\alpha_3})\\
&+\frac{1}{Z_2\bar{Z}_3}(p_1^{\alpha_4}p_1^{\alpha_1}+p_1^{\alpha_4}(p_1.D)^{\alpha_1}+(p_1.D)^{\alpha_1}(p_1.D)^{\alpha_4}+p_1^{\alpha_1}(p_1.D)^{\alpha_4})+\frac{1}{\bar{Z}_2\bar{Z}_3}(p_1^{\alpha_4}p_1^{\alpha_2}+p_1^{\alpha_4}(p_1.D)^{\alpha_2}\\
&+(p_1.D)^{\alpha_4}(p_1.D)^{\alpha_2}+(p_1.D)^{\alpha_4}p_1^{\alpha_2})-\frac{1}{\bar{Z}_3Z_{2\bar{2}}}(p_1^{\alpha_4}p_2^{\alpha_2}-p_1^{\alpha_4}(p_2.D)^{\alpha_1}+(p_1.D)^{\alpha_4}p_2^{\alpha_2}-(p_1.D)^{\alpha_4}(p_2.D)^{\alpha_1})\\
&+\frac{1}{Z_{23}Z_{\bar{2}\bar{3}}}(p_2^{\alpha_3}(p_2.D)^{\alpha_4}-p_3^{\alpha_1}(p_3.D)^{\alpha_2}-(p_2.D)^{\alpha_4}p_3^{\alpha_1}+p_2^{\alpha_3}(p_3.D)^{\alpha_2})-\frac{1}{Z_{\bar{2}3}Z_{\bar{2}\bar{3}}}((p_2.D)^{\alpha_4}p_3^{\alpha_2}-(p_2.D)^{\alpha_3}(p_3.D)^{\alpha_2}\\
&-(p_2.D)^{\alpha_3}(p_2.D)^{\alpha_4})+\frac{1}{Z_{\bar{2}3}Z_{2\bar{2}}}((p_2.D)^{\alpha_1}p_3^{\alpha_2}+p_2^{\alpha_2}(p_2.D)^{\alpha_3}-(p_2.D)^{\alpha_1}(p_2.D)^{\alpha_3})\\
&-\frac{1}{Z_{3\bar{3}}Z_{2\bar{3}}}(p_3^{\alpha_4}(p_3.D)^{\alpha_1}-p_2^{\alpha_4}(p_3.D)^{\alpha_3}-(p_3.D)^{\alpha_1}(p_3.D)^{\alpha_3})+\frac{1}{\bar{Z}_2Z_{23}}(p_1^{\alpha_2}p_2^{\alpha_3}+p_1^{\alpha_2}p_3^{\alpha_1}-(p_1.D)^{\alpha_2}p_2^{\alpha_3}\\
&+(p_1.D)^{\alpha_2}p_3^{\alpha_1})+\frac{1}{Z_3Z_{23}}(p_1^{\alpha_3}p_3^{\alpha_1}+(p_1.D)^{\alpha_3}p_3^{\alpha_1}-p_1^{\alpha_4}p_2^{\alpha_3})-\frac{1}{Z_{23}Z_{\bar{2}3}}(p_2^{\alpha_3}p_3^{\alpha_2}+(p_2.D)^{\alpha_3}(p_3.D)^{\alpha_1}+p_3^{\alpha_1}p_3^{\alpha_2})\\
&-\frac{1}{Z_{2\bar{2}}Z_{2\bar{3}}}(p_2^{\alpha_4}(p_2.D)^{\alpha_1}-p_2^{\alpha_2}p_2^{\alpha_4}-p_2^{\alpha_2}(p_3.D)^{\alpha_1})+\frac{1}{Z_{\bar{2}3}Z_{2\bar{3}}}(p_2^{\alpha_4}(p_2.D)^{\alpha_3}+p_3^{\alpha_2}(p_3.D)^{\alpha_1}-p_2^{\alpha_4}p_3^{\alpha_2})\\
&+\frac{1}{Z_{23}Z_{2\bar{3}}}(p_2^{\alpha_3}p_2^{\alpha_4}-p_2^{\alpha_4}p_3^{\alpha_1}-p_2^{\alpha_3}(p_3.D)^{\alpha_1})+\frac{1}{\bar{Z}_3Z_{2\bar{3}}}(p_1^{\alpha_4}(p_3.D)^{\alpha_1}+(p_1.D)^{\alpha_4}(p_3.D)^{\alpha_1})\\
&+\frac{1}{\bar{Z}_3Z_{3\bar{3}}}(p_1^{\alpha_4}(p_3.D)^{\alpha_3}+(p_1.D)^{\alpha_4}(p_3.D)^{\alpha_3})+\frac{1}{\bar{Z}_3Z_{\bar{2}\bar{3}}}(p_1^{\alpha_4}(p_3.D)^{\alpha_2}+(p_1.D)^{\alpha_4}(p_3.D)^{\alpha_2})\\
&+\frac{1}{\bar{Z}_2Z_{2\bar{2}}}(p_1^{\alpha_2}(p_2.D)^{\alpha_1}+(p_1.D)^{\alpha_2}(p_2.D)^{\alpha_1})-\frac{1}{\bar{Z}_2Z_{\bar{2}3}}(p_1^{\alpha_2}(p_2.D)^{\alpha_3}+(p_1.D)^{\alpha_2}(p_2.D)^{\alpha_3})\\
&+\frac{1}{\bar{Z}_3Z_{23}}(p_1^{\alpha_4}p_3^{\alpha_1}-(p_1.D)^{\alpha_4}p_2^{\alpha_3}+(p_1.D)^{\alpha_4}p_3^{\alpha_1})-\frac{1}{Z_{\bar{2}3}Z_{3\bar{3}}}(p_3^{\alpha_2}p_3^{\alpha_4}-(p_2.D)^{\alpha_3}p_3^{\alpha_4}+p_3^{\alpha_2}(p_3.D)^{\alpha_3})\\
&-\frac{1}{\bar{Z}_2Z_{\bar{2}\bar{3}}}(p_1^{\alpha_2}(p_2.D)^{\alpha_4}+(p_1.D)^{\alpha_2}(p_2.D)^{\alpha_4})-\frac{1}{Z_2Z_{23}}(p_1^{\alpha_1}p_2^{\alpha_3}+(p_1.D)^{\alpha_1}p_2^{\alpha_3})-\frac{1}{Z_{23}{}^2}(p_2^{\alpha_3}p_3^{\alpha_1}-\eta^{\alpha_1\alpha_3})\\
&+\frac{1}{Z_{23}Z_{2\bar{2}}}(p_2^{\alpha_2}p_2^{\alpha_3}-2p_2^{\alpha_2}p_3^{\alpha_1})-\frac{1}{Z_2Z_{2\bar{2}}}(p_2^{\alpha_2}p_1^{\alpha_1}+p_2^{\alpha_2}(p_1.D)^{\alpha_1})+\frac{1}{Z_3Z_{\bar{2}3}}(p_3^{\alpha_2}p_1^{\alpha_3}+p_3^{\alpha_2}(p_1.D)^{\alpha_3})\\
&-\frac{1}{Z_3Z_{3\bar{3}}}(p_1^{\alpha_3}p_3^{\alpha_4}+(p_1.D)^{\alpha_3}p_3^{\alpha_4})+\frac{1}{Z_{23}Z_{3\bar{3}}}(p_2^{\alpha_3}p_3^{\alpha_4}-p_3^{\alpha_1}p_3^{\alpha_4})+\frac{1}{Z_{2\bar{3}}{}^2}(\eta^{\alpha_1\alpha_4}-p_2^{\alpha_4}(p_3.D)^{\alpha_1})\\
&-\frac{1}{Z_2Z_{2\bar{3}}}(p_1^{\alpha_1}p_2^{\alpha_4}+(p_1.D)^{\alpha_1}p_2^{\alpha_4})-\frac{1}{Z_2Z_{3\bar{3}}}(p_1^{\alpha_1}p_3^{\alpha_4}+(p_1.D)^{\alpha_1}p_3^{\alpha_4})-\frac{1}{Z_{\bar{2}\bar{3}}{}^2}((p_2.D)^{\alpha_4}(p_3.D)^{\alpha_2}-\eta^{\alpha_2\alpha_4})\\
&+\frac{1}{Z_{\bar{2}3}{}^2}(\eta^{\alpha_2\alpha_3}-(p_2.D)^{\alpha_3}p_3^{\alpha_2})+\frac{1}{Z_{2\bar{2}}Z_{\bar{2}\bar{3}}}((p_2.D)^{\alpha_1}(p_3.D)^{\alpha_2}+p_2^{\alpha_2}(p_2.D)^{\alpha_4}\\
&-(p_2.D)^{\alpha_1}(p_2.D)^{\alpha_4}+p_2^{\alpha_2}(p_2.D)^{\alpha_4}-(p_2.D)^{\alpha_1}(p_2.D)^{\alpha_4})-\frac{1}{Z_{23}{}^2}(p_2^{\alpha_3}p_3^{\alpha_1}-\eta^{\alpha_1\alpha_3})+\frac{\eta^{\alpha_1\alpha_2}}{|Z_2|^4}+\frac{\eta^{\alpha_3\alpha_4}}{|Z_3|^4}-\frac{1}{Z_{2\bar{2}}{}^2}p_2^{\alpha_2}(p_2.D)^{\alpha_1}\\
&-\frac{1}{Z_{3\bar{3}}{}^2}(p_3^{\alpha_4}(p_3.D)^{\alpha_3}-p_1^{\alpha_4}p_1^{\alpha_3}-p_1^{\alpha_4}(p_1.D)^{\alpha_3}-(p_1.D)^{\alpha_4}p_1^{\alpha_3}-(p_1.D)^{\alpha_4}(p_1.D)^{\alpha_3})\\
&-\frac{1}{Z_3Z_{\bar{2}\bar{3}}}(p_1^{\alpha_3}(p_2.D)^{\alpha_4}+(p_1.D)^{\alpha_3}(p_2.D)^{\alpha_4}-(p_1.D)^{\alpha_3}(p_3.D)^{\alpha_2}-p_1^{\alpha_3}(p_3.D)^{\alpha_2})-\frac{1}{Z_2Z_{\bar{2}\bar{3}}}(p_1^{\alpha_1}(p_2.D)^{\alpha_4}\\
&+(p_1.D)^{\alpha_1}(p_2.D)^{\alpha_4}-(p_1.D)^{\alpha_1}(p_3.D)^{\alpha_2}-p_1^{\alpha_1}(p_3.D)^{\alpha_2})\Big].
\end{aligned}
$$

APPENDIX B: NEW INTEGRALS

In this appendix, we present the explicit form of the integrals appearing in subamplitude $\mathcal{A}_1$. These combinations appear naturally from the contractions,

$$\begin{aligned}
\mathcal{K}_1 &= \int d^2z_2 d^2z_3 \frac{(1-|z_2|^2)((1+|z_3|^2)(\bar{z}_2 z_3 + z_2\bar{z}_3) + 2|z_3|^2(1+|z_2|^2))}{|z_2|^2|z_2-z_3|^2|z_3|^2|-1+\bar{z}_2 z_3|^2},\\
\mathcal{K}_2 &= \int d^2z_2 d^2z_3 \frac{(1-|z_3|^2)((1+|z_2|^2)(\bar{z}_2 z_3 + z_2\bar{z}_3) + 2|z_2|^2(1+|z_3|^2))}{|z_2|^2|z_2-z_3|^2|z_3|^2|-1+\bar{z}_2 z_3|^2},\\
\mathcal{K}_3 &= \int d^2z_2 d^2z_3 \frac{2|z_2+z_3|^2(|z_2|^2-|z_3|^2)}{|z_2|^2|z_3|^2|z_2-z_3|^4},\\
\mathcal{K}_4 &= \int d^2z_2 d^2z_3 \frac{(1+|z_2|^2)(1-|z_3|^4)[2|z_2|^2(1+|z_3|^2) - (1+|z_2|^2)(\bar{z}_2 z_3 + z_2\bar{z}_3)]}{|z_2|^2|z_3|^2(-1+|z_2|^2)(-1+|z_3|^2)|-1+\bar{z}_2 z_3|^2|z_2-z_3|^2},\\
\mathcal{K}_5 &= \int d^2z_2 d^2z_3 \frac{(1-|z_2|^4)(1+|z_3|^2)[(1+|z_3|^2)(\bar{z}_2 z_3 + z_2\bar{z}_3) - 2|z_3|^2(1+|z_2|^2)]}{|z_2|^2|z_3|^2(-1+|z_2|^2)(-1+|z_3|^2)|-1+\bar{z}_2 z_3|^2|z_2-z_3|^2},\\
\mathcal{K}_6 &= \int d^2z_2 d^2z_3 \frac{-2(1+|z_3|^2)(\bar{z}_2{}^2 z_3^2 + \bar{z}_3{}^2 z_2^2 - |z_3|^2 - |z_2|^2 + |z_2|^2|z_3|^2(2-|z_2|^2-|z_3|^2))}{|z_2|^2|z_3|^2(1-|z_3|^2)|z_2-z_3|^2|-1+\bar{z}_3 z_2|^2},\\
\mathcal{K}_7 &= \int d^2z_2 d^2z_3 \frac{2(1-|z_2|^2)(1-|z_3|^4)(\bar{z}_2 z_3 + \bar{z}_3 z_2)}{|z_2|^2|z_3|^2(1-|z_3|^2)|z_2-z_3|^2|-1+\bar{z}_3 z_2|^2},\\
\mathcal{K}_8 &= \int d^2z_2 d^2z_3 \frac{(1+|z_3|^2)[(|z_2|^2-|z_3|^2)(1-|z_2|^2|z_3|^2) + (\bar{z}_3 z_2 - \bar{z}_2 z_3)^2]}{|z_2|^2|z_3|^2|z_2-z_3|^2(1-|z_3|^2)|-1+\bar{z}_2 z_3|^2}\\
\mathcal{K}_9 &= \int d^2z_2 d^2z_3 \frac{-2(1+|z_2|^2)(\bar{z}_2{}^2 z_3^2 + \bar{z}_3{}^2 z_2^2 - |z_3|^2 - |z_2|^2 + |z_2|^2|z_3|^2(2-|z_2|^2-|z_3|^2))}{|z_2|^2|z_3|^2(1-|z_2|^2)|z_2-z_3|^2|-1+\bar{z}_2 z_3|^2},\\
\mathcal{K}_{10} &= \int d^2z_2 d^2z_3 \frac{2(1-|z_2|^4)(1-|z_3|^2)(\bar{z}_2 z_3 + \bar{z}_3 z_2)}{|z_2|^2|z_3|^2(1-|z_2|^2)|z_2-z_3|^2|-1+\bar{z}_2 z_3|^2},\\
\mathcal{K}_{11} &= \int d^2z_2 d^2z_3 \frac{(1+|z_2|^2)[(|z_2|^2-|z_3|^2)(-1+|z_2|^2|z_3|^2) + (\bar{z}_3 z_2 - \bar{z}_2 z_3)^2]}{|z_2|^2|z_3|^2|z_2-z_3|^2(1-|z_2|^2)|-1+\bar{z}_2 z_3|^2},
\end{aligned}$$

where we apply the $SL(2,R)$ symmetry fixing as [16] in which $z_1 = 0$.

[1] K. Becker, G. Guo, and D. Robbins, J. High Energy Phys. 09 (2010) 029.
[2] R. G. Leigh, Mod. Phys. Lett. A **04**, 2767 (1989).
[3] C. Bachas, Phys. Lett. B **374**, 37 (1996).
[4] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995).
[5] M. R. Douglas, arXiv:hep-th/9512077.
[6] K. Becker, G. Guo, and D. Robbins, J. High Energy Phys. 01 (2012) 127.
[7] M. R. Garousi, A. Ghodsi, T. Houri, and G. Jafari, J. High Energy Phys. 10 (2013) 103.
[8] M. R. Garousi, Phys. Lett. B **718**, 1481 (2013).
[9] K. B. Velni and A. Jalali, Phys. Rev. D **95**, 086010 (2017).
[10] K. B. Velni, Phys. Rev. D **93**, 066009 (2016).
[11] M. R. Garousi and M. Mir, J. High Energy Phys. 02 (2011) 008.
[12] M. R. Garousi and M. Mir, J. High Energy Phys. 05 (2011) 066.
[13] K. B. Velni and A. Jalali (to be published).
[14] K. Becker, G. Guo, and D. Robbins, J. High Energy Phys. 12 (2011) 050.
[15] K. B. Velni and M. R. Garousi, Phys. Rev. D **89**, 106002 (2014).
[16] B. Craps and F. Roose, Phys. Lett. B **445**, 150 (1998).
[17] K. B. Velni and M. R. Garousi, Nucl. Phys. **B869**, 216 (2013).
[18] T. Buscher, Phys. Lett. B **194**, 59 (1987); Phys. Lett. B **201**, 466 (1988).
[19] K. Becker, M. Becker, D. Robbins, and N. Su, Nucl. Phys. **B907**, 360 (2016).