# Asymptotic safety of gravity with matter

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We study the asymptotic safety conjecture for quantum gravity in the presence of matter fields. A general line of reasoning is put forward explaining why gravitons dominate the high-energy behavior, largely independently of the matter fields as long as these remain sufficiently weakly coupled. Our considerations are put to work for gravity coupled to Yang-Mills theories with the help of the functional renormalization group. In an expansion about flat backgrounds, explicit results for beta functions, fixed points, universal exponents, and scaling solutions are given in systematic approximations exploiting running propagators, vertices, and background couplings. Invariably, we find that the gauge coupling becomes asymptotically free while the gravitational sector becomes asymptotically safe. The dependence on matter field multiplicities is weak. We also explain how the scheme dependence, which is more pronounced, can be handled without changing the physics. Our findings offer a new interpretation of many earlier results, which is explained in detail. The results generalize to theories with minimally coupled scalar and fermionic matter. Some implications for the ultraviolet closure of the Standard Model or its extensions are given.

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## I. INTRODUCTION

The Standard Model of particle physics combines three of the four fundamentally known forces of nature. It remains an open challenge to understand whether a quantum theory for gravity can be established under the same set of basic principles. Steven Weinberg's seminal asymptotic safety conjecture stipulates that it can, provided the high energy behavior of gravity is controlled by an interacting fixed point [1,2]. By now, the scenario has become a viable contender with many applications ranging from particle physics to cosmology [3–8].

Fixed points for quantum gravity have been obtained from the renormalization group in increasingly sophisticated approximations ranging from the Einstein-Hilbert theory [9–39] to higher derivative and higher curvature extensions and variants thereof [40–63]. Strong quantum effects invariably modify the high-energy limit. Interestingly, however, canonical mass dimension continues to be a good ordering principle [54]: classically relevant couplings remain relevant while classically irrelevant couplings remain irrelevant [59], including the notorious Goroff-Sagnotti term [60]. Further aspects such as diffeomorphism invariance in the presence of a cutoff and the role of background fields have also been clarified.

It then becomes natural to include matter fields and to clarify the impact of matter on asymptotic safety for gravity [64–92]. In general it is found that matter fields constrain asymptotic safety for gravity, although not all specifics for this are fully settled yet. In expansions about flat backgrounds, it was noticed that the graviton dominates over free matter field fluctuations, either via an enhancement of the graviton propagator or the growth of the graviton coupling [79]. This pattern should play a role for asymptotic safety of the fully coupled theory and for weak gravity bounds [82,85,86]. In a similar vein, the impact of quantized gravity on gauge theories has been investigated within perturbation theory [93–98] by treating gravity as an effective field theory [99] and within the asymptotic safety scenario [67–69]. Modulo gauge and scheme dependences, all studies find the same negative sign for the Yang-Mills beta function ($\beta < 0$) in support of asymptotic freedom. The reason for this was uncovered in [68,69]: due to an important kinematical identity (Fig. 2), related to diffeomorphism and gauge invariance, $\beta < 0$ follows automatically, and irrespective of the gauge or regularization.

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In this paper, we want to understand the prospect for asymptotic safety of quantum gravity coupled to matter. To that end, we combine general, formal considerations with detailed and explicit studies using functional renormalization. A main new addition is a formal line of reasoning, which explains why and how gravitons dominate the high-energy behavior, largely independently of the matter fields as long as these remain sufficiently weakly coupled. Using functional renormalization, this is then put to work for $SU(N_c)$ Yang-Mills theory coupled to gravity. In an expansion about flat backgrounds, explicit results for beta functions, fixed points, universal exponents, and scaling solutions are given. Systematic approximations exploiting running propagators, the three-graviton and the graviton-gauge vertices are performed up to including independent couplings for gauge-gravity and pure gravity interactions and for the background couplings. Care is taken to distinguish fluctuating and background fields. Invariably, we find that the gauge coupling becomes asymptotically free while the gravitational sector becomes asymptotically safe. The dependence on matter field multiplicities is weak. We also investigate the scheme dependence, which is found to be more pronounced, and explain how it can be handled without changing the physics. This allows us to offer a new interpretation of many earlier results and to lift some of the tensions amongst previous findings.

This paper is organized as follows. In Sec. II, we present a formal argument for asymptotic safety of Yang-Mills gravity and extensions to general matter–gravity systems. In Sec. III, we introduce the renormalization group for Yang-Mills gravity, and some notation and conventions. In Sec. IV, we analyze whether asymptotic freedom in Yang-Mills theories is maintained when coupled to a dynamical graviton. Conversely, in Sec. V, the influence of gluon fluctuations on UV-complete theories for gravity are studied. In Sec. VI, asymptotic safety of the fully coupled Yang-Mills gravity system is investigated in the standard uniform approximation with a unique Newton's coupling. We further discuss the stable large-$N_c$ limit of this system. In Sec. VII, we lift the uniform approximation and discuss the system with separate Newton's couplings for gauge-gravity and pure gravity interactions. We also discuss the renormalization group (RG) scheme dependence and relate our findings with earlier ones in the literature. In Sec. VIII, we briefly summarize our findings. The Appendixes comprise the technical details.

## II. FROM ASYMPTOTIC FREEDOM TO ASYMPTOTIC SAFETY

In this section, we provide our main line of reasoning for why matter fields, which are free or sufficiently weakly coupled in the UV—such as in asymptotic freedom—entail asymptotic safety in the full theory including gravity. Throughout, Yang-Mills theory serves as the principle example.

### A. Yang-Mills coupled to gravity: The setup

Any correlation function approach to gravity works within an expansion of the theory about some generic metric. The necessity of gauge fixing in such an approach introduces a background metric into the approach. Hence, we use a background field approach in the gauge sector, giving us a setting with a combined background $\bar{g}_{\mu\nu}$, $\bar{A}^a_\mu$. Background independence is then ensured with the help of Nielsen or split Ward-Takahashi identities and the accompanying Slavnov-Taylor identities (STIs) for both the metric fluctuations and the gauge field fluctuations. The superfield $\phi$ comprises all fluctuations or quantum fields with

$$
\begin{aligned}
&A_\mu = \bar{A}_\mu + a_\mu, \qquad g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{G}h_{\mu\nu},\\
&\phi = (h_{\mu\nu}, c_\mu, \bar{c}_\mu, a_\mu, c, \bar{c}), \qquad (1)
\end{aligned}
$$

with the dynamical fluctuation graviton $h_{\mu\nu}$ and gauge field $a_\mu$. In (1), $c_\mu$ and $c$ are the gravity and Yang-Mills ghosts, respectively. The classical Euclidean action of the Yang-Mills–gravity system is given by the sum of the gauge-fixed Yang-Mills and Einstein-Hilbert actions,

$$
S_{\mathrm{cl}}[\bar{g}, \bar{A}; \phi] = S_{\mathrm{gauge}}[\bar{g}, \bar{A}; \phi] + S_{\mathrm{gravity}}[\bar{g}, \bar{A}; \phi], \qquad (2)
$$

where the two terms $S_{\mathrm{gauge}} = S_A + S_{A,\mathrm{gf}} + S_{A,\mathrm{gh}}$ and $S_{\mathrm{gravity}} = S_{\mathrm{EH}} + S_{g,\mathrm{gf}} + S_{g,\mathrm{gh}}$ are the fully gauge fixed actions of Yang-Mills theory and gravity, respectively. The Yang-Mills action reads

$$
S_A[g, A] = \frac{1}{2}\int \mathrm{d}^4x \sqrt{\det g}\, g^{\mu\mu'} g^{\nu\nu'} \mathrm{tr} F_{\mu'\nu'} F_{\mu\nu}, \qquad (3)
$$

where the trace in (3) is taken in the fundamental representation, and

$$
F_{\mu\nu} = \frac{i}{g_s}[D_\mu, D_\nu], \quad D_\mu = \partial_\mu - i g_s A_\mu, \quad \mathrm{tr}\, t^a t^b = \frac{1}{2}. \qquad (4)
$$

The classical Yang-Mills action (3) only depends on the full fields $g$, $A$ and induces gauge-field-graviton interactions via the determinant of the metric as well as the Lorentz contractions and derivatives. The gauge fixing is done in the background Lorentz gauge $\bar{D}_\mu a_\mu = 0$ with $\bar{D} = D_\mu(\bar{A})$. The gauge fixing and ghost terms read

$$
\begin{aligned}
S_{A,\mathrm{gf}} &= \frac{1}{2\xi}\int \mathrm{d}^4x \sqrt{\det \bar{g}} (\bar{g}^{\mu\nu} \bar{D}_\mu a_\nu)^2,\\
S_{A,\mathrm{gh}} &= \int \mathrm{d}^4x \sqrt{\det \bar{g}}\, \bar{g}^{\mu\nu} \bar{c} \bar{D}_\mu D_\nu c, \qquad (5)
\end{aligned}
$$

where we take the limit $\xi \to 0$. The gauge fixing and ghost terms only depend on the background metric and hence, do not couple to the dynamical graviton $h_{\mu\nu}$. The Einstein-Hilbert action is given by

$$S_{\rm EH} = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{\det g}(2\Lambda - R(g)), \tag{6}$$

with a linear gauge fixing $F_\mu$ and the corresponding ghost term,

$$\begin{aligned} S_{g,\rm gf} &= \frac{1}{2\alpha}\int \mathrm{d}^4x\sqrt{\det\bar g}\bar g^{\mu\nu}F_\mu F_\nu, \\ S_{g,\rm gh} &= \int \mathrm{d}^4x\sqrt{\det\bar g}\bar g^{\mu\mu'}\bar g^{\nu\nu'}\bar c_{\mu'}\mathcal{M}_{\mu\nu}c_{\nu'}, \end{aligned} \tag{7}$$

with the Faddeev-Popov operator $\mathcal{M}_{\mu\nu}(\bar g, h)$ of the gauge fixing $F_\mu(\bar g, h)$. We employ a linear, de-Donder type gauge fixing,

$$\begin{aligned} F_\mu &= \bar\nabla^\nu h_{\mu\nu} - \frac{1+\beta}{4}\bar\nabla_\mu h^\nu{}_\nu, \\ \mathcal{M}_{\mu\nu} &= \bar\nabla^\rho(g_{\mu\nu}\nabla_\rho + g_{\rho\nu}\nabla_\mu) - \bar\nabla_\mu\nabla_\nu, \end{aligned} \tag{8}$$

with $\beta = 1$ and the limit $\alpha\to 0$, which is a fixed point of the RG flow [100].

### B. Asymptotic freedom in Yang-Mills with gravity

Gauge theories with gauge group $U(N)$ or $SU(N)$ describe the electroweak and the strong interactions, and form the basis of the Standard Model of particle physics. A striking feature of non-Abelian gauge theories is asymptotic freedom, meaning that the theory is governed by a Gaussian fixed point in the ultraviolet, which implies that gluon interactions weaken for high energies and that perturbation theory is applicable. In fact, the great success of the Standard Model is possible only due to the presence of such a Gaussian fixed point, which allows us to neglect higher order operators in the high energy limit. The weakening of interactions is encoded in the energy dependence of the Yang-Mills coupling, which in turn is signaled by a strictly negative sign of the beta function. However, it is well-known that fermions contribute with a positive sign to the running of the Yang-Mills coupling,

$$\frac{\beta^{1-\rm loop}_{\alpha_s}}{\alpha_s^2} \equiv \mu\frac{\partial\alpha_s}{\partial\mu}\frac{1}{\alpha_s^2} = -\frac{1}{4\pi}\left(\frac{22}{3}N_c - \frac{4}{3}N_f\right), \tag{9}$$

where we have displayed only the one-loop contributions with $N_c$ and $N_f$ denoting the number of colors and fermion flavors, and $\alpha_s = g_s^2/(4\pi)$. One can see that there is a critical number of fermion flavors $N_f^{\rm crit} = \frac{11}{2}N_c$ above which the one-loop beta function changes sign. This implies that asymptotic freedom is lost. It has been noted recently that gauge theories with matter and without gravity may very well become asymptotically safe in their own right [101–106].

Returning to gravity, it has been shown in [67–69,93–98] that graviton fluctuations lead to an additional negative term $\beta_{\alpha_s,h}$ in $\beta_{\alpha_s}\to\beta_{\alpha_s,a}+\beta_{\alpha_s,h}$ where $\beta_{\alpha_s,a}$ is the pure gauge theory contribution (9). The graviton contribution has a negative sign,

$$\beta_{\alpha_s,h} \le 0. \tag{10}$$

Because of the lack of perturbative renormalizbility, this term is gauge and regularization dependent. However, it has been shown that it is always negative semidefinite [68,69], based on a kinematic identity related to diffeomorphism invariance. Hence, asymptotic freedom in Yang-Mills theories is assisted by graviton fluctuations. In the case of $U(1)$, they even trigger it. This result allows us to already get some insight into the coupled Yang-Mills–gravity system within a semianalytic consideration in an effective theory spirit: in the present work, we consider coupled Yang-Mills–gravity systems within an expansion of the pure gravity part in powers of the curvature scalar as well as taking into account the momentum dependence of correlation functions. In the Yang-Mills subsector, we consider an expansion in $\mathrm{tr}F^n$ and $(\mathrm{tr}F^2)^n$, the lowest nonclassical terms being

$$w_2(\mathrm{tr}F^2)^2, \qquad v_4\mathrm{tr}F^4. \tag{11}$$

Asymptotic freedom allows us to first integrate out the gauge field. This subsystem is well-described by integrating out the gauge field in a saddle point expansion within a one-loop approximation. Higher loop orders are suppressed by higher powers in the asymptotically free gauge coupling. This leads us to the effective action

$$\begin{aligned} \Gamma[\bar g, \bar A, \phi] &= S_{\rm gravity}[\bar g;\phi] + S_{\rm gauge}[\bar g, \bar A;\phi] \\ &\quad - \frac{1}{2}\mathrm{Tr}\ln\left[\Delta_1\delta_{\mu\nu} + \left(1-\frac{1}{\xi}\right)\nabla_\mu\nabla_\nu\right]^{k_a^{\rm UV}}_{k_a^{\rm IR}}, \end{aligned} \tag{12}$$

where $\Delta_1$ represents the spin-one Laplacian and $k_a^{\rm IR}$, $k_a^{\rm UV}$ indicate diffeomorphism-preserving infrared and ultraviolet regularizations of the one-loop determinant. Most conveniently, this is achieved by a proper-time regularization, for a comprehensive analysis within the FRG framework, see [107,108]. In any case, both regularizations depend on the metric $g_{\mu\nu}$ and the respective scales $k_a^{\rm IR}$, $k_a^{\rm UV}$. The computation can be performed with standard heat-kernel methods.

The infrared sector of the theory is not relevant for the present discussion of the fate of asymptotic safety in the ultraviolet. Note also that Yang-Mills theory exhibits an infrared mass gap with the scale $\Lambda_{\rm QCD}$ due to its confining dynamics. In covariant gauges, as used in the present work, this mass gap results in a mass gap in the gluon propagator, for a treatment within the current functional renormalization group (FRG) approach, see [109,110] and references therein. This dynamical gaping may be simulated here by simply identifying the infrared cutoff scale with $\Lambda_{\rm QCD}$.

Moreover, even though integrating out the gauge field generates higher order terms such as (11) in the UV, they are suppressed by both powers of the UV cutoff scale as well as the asymptotically free coupling. Accordingly, we drop the higher terms in the expansion of the Yang-Mills part of the effective action (12). Note that they are present in the full system as they are also generated by integrating out the graviton. This is discussed below.

It is left to discuss the pure gravity terms that are generated by ultraviolet gluon fluctuations in (12). They can be expanded in powers and inverse powers of the UV-cutoff scale $k_a = k_a^{\rm UV}$. This gives an expansion in powers of the Ricci scalar $R$ and higher order invariants. From the second line of (12), we are led to

$$(N_c^2-1)\left[c_{g,a}k_a^2\int \mathrm{d}^4x\sqrt{\det g}(2c_{\lambda,a}k_a^2-R)\right.\\ \left.+c_{R^2,a}\int \mathrm{d}^4x\sqrt{\det g}(R^2+z_aR_{\mu\nu}^2)\ln\frac{R+k_a^{\rm IR\,2}}{k_a^2}\right]\\ +\mathcal{O}\left(\frac{R^3}{k_a^2}\right), \tag{13}$$

where we suppressed potential dependences on $\Delta_g$ and $\nabla_\mu$, in particular, in the logarithmic terms. The logarithm also could contain further curvature invariants such as $R_{\mu\nu}^2$. In the spirit of the discussion of the confining infrared physics, we may substitute $k_a^{\rm IR}\to\Lambda_{\rm QCD}$ in a full nonperturbative analysis. In (13), the coefficients $c_{g,a}$, $c_{\lambda,a}$, $c_{R^2,a}$ and $z_a$, are regularization dependent and lead to contributions to Newton's coupling, the cosmological constant, as well as generating an $R^2$ term and potentially an $R_{\mu\nu}^2$ term. In the present Yang-Mills case, $c_{g,a}$ is positive for all regulators. For fermions and scalars, the respective coefficients $c_{g,\psi}$, $c_{g,\phi}$ are negative. In summary, this leaves us with an asymptotically free Yang-Mills action coupled to gravity with redefined couplings

$$G_{\rm eff}=\frac{G}{1+(N_c^2-1)c_{g,a}k_a^2G},\\ \frac{\Lambda_{\rm eff}}{G_{\rm eff}}=\frac{\Lambda}{G}+(N_c^2-1)c_{g,a}c_{\lambda,a}k_a^4. \tag{14}$$

The coupling parameters $G$, $\Lambda$ should be seen as bare couplings of the Yang-Mills–gravity system and chosen such that the (renormalized) couplings $G_{\rm eff}$, $\Lambda_{\rm eff}$ are $k_a$ independent. This corresponds to a standard renormalization procedure (introducing the standard RG scale $\mu_{\rm RG}$) and leads to $G(N_c,k_a)$, $\Lambda(N_c,k_a)$. Note that demanding $k_a$ independence of the effective couplings also eliminates their $N_c$ running. For example, for the effective Newton's coupling

$$(N_c^2-1)\partial_{(N_c^2-1)}\ln G_{\rm eff}=k_a^2\partial_{k_a^2}\ln G_{\rm eff}=0, \tag{15}$$

holds in a minimal subtraction scheme where the renormalization scale $\mu_{\rm RG}$ does not introduce further $N_c$-dependencies, most simply done with $\mu_{\rm RG}$-independent couplings $G$, $\Lambda$.

We also have to include $g_{R^2}R^2$ and $g_{R_{\mu\nu}^2}R_{\mu\nu}^2$ terms in the classical gravity action in order to renormalize also these couplings,

$$g_{R^2,\rm eff}=g_{R^2}+(N_c^2-1)c_{R^2,a}\ln\frac{k_a^{\rm IR\,2}}{k_a^2},\\ g_{R_{\mu\nu}^2,\rm eff}=g_{R_{\mu\nu}^2}+(N_c^2-1)c_{R^2,a}z_a\ln\frac{k_a^{\rm IR\,2}}{k_a^2}. \tag{16}$$

Here, the minimal subtraction discussed above requires $g_{R^2}(N_c,\ln k_a/k_a^{\rm IR})$ and $g_{R_{\mu\nu}^2}(N_c,\ln k_a/k_a^{\rm IR})$. This leaves us with a theory, which includes all ultraviolet quantum effects of the Yang-Mills theory. Accordingly, in the ultraviolet, its effective action (12) resembles the Einstein-Hilbert action coupled to the classical Yang-Mills action with appropriately redefined couplings. It also has $R^2$ and $R_{\mu\nu}^2$ terms. However, the latter terms are generated in any case by graviton fluctuations so there is no structural difference to standard gravity with the Einstein-Hilbert action coupled to the classical Yang-Mills.

The only relevant $N_c$ dependence originates in the logarithmic curvature dependence of the marginal operators $R^2$ and $R_{\mu\nu}^2$ leading, e.g., to

$$(N_c^2-1)c_{R^2,a}\int \mathrm{d}^4x\sqrt{\det g}R^2\ln\left(1+\frac{R}{k_a^{\rm IR\,2}}\right). \tag{17}$$

These terms are typically generated by flows towards the infrared, for a respective computation in Yang-Mills theory, see [111]. Such a running cannot be absorbed in the pure gravity part without introducing a nonlocal classical action. From its structure, the logarithmic running in (16) resembles the one of the strong coupling in many flavor QCD: the role of the gravity part here is taken by the gluon part in many flavor QCD and that of the Yang-Mills part here is taken by the many flavors. Accordingly, a fully conclusive analysis has to take into account these induced interactions. This is left to future work, here we concentrate on the Einstein-Hilbert part. The respective truncation to matter-gravity systems have been studied at length in the literature, and the arguments presented here fully apply. Note also that the current setup (and the results in the literature) can be understood as a matter-gravity theory, where the respective terms are removed by an appropriate classical gravity action that includes, e.g., $R^2 \ln R$ terms. The discussion of these theories is also linked to the question of unitary in asymptotically safe gravity.

If we do not readjust the effective couplings within the minimal subtraction discussed above, they show already the fixed point scaling to be expected in an asymptotically safe theory of quantum gravity, see (14) and (16). This merely reflects the fact that Yang-Mills theory has no explicit scales. If we only absorb the $k_a$ running of the couplings while leaving open a general $\mu_{\mathrm{RG}}$ dependence, the effective Newton's coupling $G_{\mathrm{eff}}$ scales with $1/N_c^2$, while the effective cosmological constant scales with $N_c^0$.

In any case, we have to use $G_{\mathrm{eff}}$ for the gravity scale in the Yang-Mills–gravity system instead of $G$. For example, the expansion of the full metric $g_{\mu\nu}$ in a background and a fluctuation then reads

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{G_{\mathrm{eff}}} h_{\mu\nu}, \tag{18}$$

with the dimension-one field $h_{\mu\nu}$ in the $d = 4$ dimensional Yang-Mills–gravity system.

### C. Asymptotic safety in gravity with Yang-Mills

It is left to integrate out graviton fluctuations on the basis of the combined effective action, where the pure gravity part is of the Einstein-Hilbert type. The couplings of the pure gravity sector, in particular, Newton's coupling and the cosmological constant only receive quantum contributions from pure gravity diagrams, while pure gauge and gauge-graviton couplings only receive contributions from diagrams that contain at least one graviton line. This system is asymptotically safe in the pure gravity sector and assists asymptotic freedom for the minimal gauge coupling, see (9) and (10), and leads to graviton-induced higher-order coupling such as (11). In summary, we conclude that Yang-Mills–gravity systems are asymptotically safe. The flow of this system and its completeness is discussed in Sec. VII.

The present analysis is also important for the evaluation of general matter-gravity systems: we have argued that asymptotic freedom of the Yang-Mills theory allows us to successively integrate out the degrees of freedom, starting first with the Yang-Mills sector. Evidently, this is also true for matter-gravity systems with free matter such as treated comprehensively, e.g., in [75,79]. In the former, fermions and scalars were found to be unstable for a large flavor numbers while in the latter fermions were shown to be stable. For scalars, the situation was inconclusive as the anomalous dimension of the graviton was exceeding an upper bound, $\eta_h < 2$, beyond which a regulator of the form $R_{h,k}(p^2) \propto Z_h R_{h,k}^{(0)}(p^2)$ with $R_{h,k}^{(0)}(0) = k^2$ is no longer a regulator with the cutoff scale $k$,

$$\lim_{k\to\infty} R_{h,k}(0) \propto (k^2)^{1-\eta_h/2} \to 0, \quad \text{for } \eta_h > 2. \tag{19}$$

This bound can be pushed to $\eta_h < 4$ but also this bound was exceeded, see [79]. While the differences in the stability analysis can be partially attributed to the different approximations in [75,79] (the former does not resolve the difference between background gravitons and fluctuation gravitons in the pure gravity sector), we come to conclude here, that both (and all similar ones) analyses lack the structure discussed above. This calls for a careful reassessment of the UV flows of matter-gravity systems also in the view of relative cutoff scales. The latter is since long a well-known problem in quantum field theoretical applications of the FRG, in particular, in boson-fermion systems. For example, in condensed matter systems, it has been observed that exact results for the three-body scattering (STM), see [112], can only be obtained within a consecutive integrating out of degrees of freedom in local approximations. If identical cutoff scales are chosen, the three-body scattering only is described approximately. For a recent analysis of relative cutoff scales in multiple boson and boson-fermion systems, see [113].

In summary, the gravitationally coupled free-matter–gravity systems, Yang-Mills–gravity systems, or more generally asymptotically free gauge-matter–gravity systems are asymptotically safe, independent of the number of matter degrees of freedom if this holds for one degree of freedom or more generally if this holds for the minimal number of degrees of freedom that already has the most general interaction structure of the coupled theory. Phrased differently: simple large $N$ scaling cannot destroy asymptotic safety, with $N$ being the number of gauge-matter degrees of freedom.

We emphasize that the analysis of such a minimal system as defined above is necessary. It is not sufficient to rely on the fact that the matter or gauge part can be integrated out first as gravity necessarily induces nontrivial matter and gauge self-interactions at an asymptotically safe gravity fixed point [71,72,81,85,86]. If these self-interactions do not destroy asymptotic safety, the systems achieve asymptotic safety for a general number of matter or gauge fields by guaranteeing the ultraviolet dominance of graviton fluctuations.

With these results at hand, we can now ask the question whether a "relative scaling" of gravity vs matter cutoffs maintains the observed graviton dominance. A natural "scaling hierarchy" for the cutoff scales $k_h$ in the gravity and $k_a$ in the Yang-Mills sector is motivated by the following heuristic consideration: while gravity feels the effective Newton's coupling $G_{\mathrm{eff}}$, and hence, graviton fluctuations and gravity scales should be measured in $G_{\mathrm{eff}}$, the Yang-Mills field generates contributions to the (bare) Newton's coupling $G$. Assuming that both are of a similar strength, this leads to

$$G_{\mathrm{eff}} k_h^2 \simeq G k_a^2 \tag{20}$$

for the respective cutoff scales. Interestingly though, under this hierarchy of scales, the $N_c$ dependence of the coupled system disappears, and, within an appropriate fine-tuning of the relation (20), the fixed point values of Newton's coupling and the cosmological constant show no $N_c$ dependence at all. Stated differently, a rescaling such as in (20) guarantees the dominance of graviton fluctuations over gauge or matter fluctuations as long as the gauge-matter system is asymptotically free. The phenomenon of graviton dominance as observed with identical cutoffs continues to be observed under a weighted rescaling (20).

We close this chapter with some remarks.

(1) The naturalness of the rescaling (20) is finally decided by taking into account momentum or spectral dependencies of the correlation functions. This is at the root of the question of stability and instability of matter-gravity systems. It is here where the marginal, logarithmically running, terms such as (17) come into play. They are not affected by this rescaling, which also shows their direct physics relevance.

(2) Within the above rescaling, the fixed point of the gravity-induced gauge couplings such as $w_2$ and $v_4$, see (11), are of order $g^{*4}$ of the pure gravity fixed point coupling $g^*$. Note however, that this value can be changed by readjusting the rescaling (20).

(3) Note that within the dynamical readjustment of the scales the fixed point Newton's coupling gets weak, $g^* \propto 1/N_c^2$. In other words, gravity dominates by getting weak. This is in line with the weak-gravity scenario advocated recently [82,85,86]. However, its physical foundation is different.

(4) For a sufficiently large truncation, the theory should be insensitive to a relative rescaling of the cutoff scales $k_{\mathrm{gravity}}$ and $k_{\mathrm{matter}}$ and to other changes of the regularization scheme. This is partially investigated in Sec. VII. Moreover, in all of the following renormalization group computations, we do not resort to the rescaling (20) but use identical cutoff scales $k_{\mathrm{gravity}} = k_{\mathrm{matter}}$.

In the following analysis, we will refer to the present chapter for an evaluation of our results.

## III. RENORMALIZATION GROUP

In the present work, we quantize the Yang-Mills–gravity system within the functional renormalization group approach. The general idea is to integrate-out quantum fluctuations of a given theory successively, typically in terms of momentum or energy shells, $p^2 \sim k^2$. This procedure introduces a scale dependence of the correlation functions, which is most conveniently formulated in terms of the scale-dependent effective action $\Gamma_k$, the free energy of the theory. Its scale dependence is governed by the flow equation for the effective action, the Wetterich equation [114], see also [115,116],

$$\partial_t \Gamma_k[\bar{g};\phi] = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(0,2)}[\bar{g};\phi] + R_k}\partial_t R_k\right], \tag{21}$$

where the trace sums over species of fields, space-time, Lorentz, spinor, and gauge group indices, and includes a minus sign for Grassmann valued fields. For the explicit computation, we employ the flat regulator [117,118], see Appendix A. From here on, we drop the index $k$ for notational convenience. The scale dependence of couplings, wave function renormalizations, or the effective action is implicitly understood.

The computation utilizes the systematic vertex expansion scheme as presented in [24,26,31,36,61,79] for pure gravity as well as matter-gravity systems: the scale dependent effective action that contains the graviton-gluon interactions is expanded in powers of the fluctuation super field $\phi$ defined in (1),

$$\Gamma[\bar{g},\bar{A};\phi] = \sum_n \frac{1}{n!}\Gamma^{(\phi_1\ldots\phi_n)}_{\mathbf{a_1}\ldots\mathbf{a_n}}[\bar{g},\bar{A},0]\phi_{\mathbf{a_1}}\ldots\phi_{\mathbf{a_n}}. \tag{22}$$

In (22), we resort to de-Witt's condensed notation. The bold indices sum over species of fields, space-time, Lorentz, spinor, and gauge group indices. The auxiliary background field is general. Here, we choose it as $\bar{\phi} = (\bar{A} = 0, \bar{g} = \mathbb{1})$ for computational simplicity. In this work, we truncate such that we obtain a closed system of flow equations for the gluon two- and the graviton two- and three-point functions, $\partial_t\Gamma^{(aa)}$, $\partial_t\Gamma^{(hh)}$, and $\partial_t\Gamma^{(hhh)}$. The corresponding flow equations are derived from (21) by functional differentiation.

The pure gravity part of the effective action $\Gamma_{\mathrm{grav}}$ in (22) is constructed exactly as presented in [24,26,31,36,61,79]. This construction is extended to the Yang-Mills part. Moreover, for the flow equations under consideration here, only terms with at most two gluons contribute. In summary, our approximation is based solely on the classical tensor structures $S_{\mathrm{cl}}$ that are derived from (2). The correlation functions follow as,

$$\Gamma^{(\phi_1\ldots\phi_n)}_{\mathbf{a_1}\ldots\mathbf{a_n}} = \left(\prod_{i=1}^n Z_{\phi_i}^{\frac{1}{2}}\right) S^{(\phi_1\ldots\phi_n)}_{\mathrm{cl},\mathbf{a_1}\ldots\mathbf{a_n}}(\mathbf{p};g_{\phi_1\ldots\phi_n},\lambda_{\phi_1\ldots\phi_n}), \tag{23}$$

where the $Z_{\phi_i}$ are the wave function renormalizations of the corresponding fields and $\mathbf{p} = (p_1,\ldots,p_n)$. The $g_{\phi_1\cdots\phi_n}$, $\lambda_{\phi_1\ldots\phi_n}$ are the couplings in the classical tensor structures that may differ for each vertex. In the present approximation, these couplings are extracted from the momentum dependence at the symmetric point and hence, carry part of the nontrivial momentum dependence of the vertices. The projection procedure is detailed later. We further exemplify the couplings at the example of the pure graviton and the gauge-graviton vertices. Each graviton $n$-point function, $\Gamma^{(h_1\cdots h_n)}$, depends on the dimensionless parameters

$$g_n \equiv g_{h^n} = G_n k^2, \qquad \lambda_n \equiv \lambda_{h^n} = \Lambda_n/k^2, \tag{24a}$$

and a mixed gauge-graviton $(n+2)$-point function on

$$g_{a^2h^n} = G_{a^2h^n}k^2, \qquad g_{A^2h^n} = G_{A^2h^n}k^2. \tag{24b}$$

In particular, the parameters $\lambda_n$ should not be confused with the cosmological constant, for more details see, e.g., [36]. In the present approximation, we identify all gravity couplings

$$g_{A^m h^n} = g_3 =: g, \qquad \lambda_{n>2} = \lambda_3, \qquad \lambda_2 = -\frac{1}{2}\mu, \tag{25}$$

the general case without this identification is discussed in Sec. VII. Note that the identification in (25) introduces (maximal) diffeomorphism invariance to the effective action: in order to elucidate this statement, we discuss the full effective action for constant vertices. With $g = \bar{g} + \sqrt{G}Z_h^{1/2}h$ and $A = \bar{A} + Z_a^{1/2}a$ and (25), the current approximation can schematically be written as a sum of the classical action and a mass-type term for the fluctuation graviton,

$$\Gamma[\bar{g},\bar{A};\phi] = S_{\mathrm{cl}}[g,A]|_{G=G_3,\Lambda=\Lambda_3} + \Delta\Gamma[\bar{g}] + \frac{k^4}{2}Z_h(\mu+2\lambda_3)h_{\mathbf{a}}\mathcal{T}_{\mathbf{ab}}h_{\mathbf{b}}, \tag{26}$$

where $\mathcal{T}_{\mathbf{ab}} = S^{(hh)}_{\mathrm{EH}\,\mathbf{ab}}(p^2=0; g=1,\lambda=1)$ is the tensor structure of the second derivative of the cosmological constant term. The $\lambda_3$ term cancels with the corresponding contribution in the first line, and thus, $\mu$ is the coupling of this tensor structure. This is the minimal approximation that is susceptible to the nontrivial symmetry identities, both the modified STIs and the Nielsen identities present in gauge-fixed quantum gravity. This information requires the nontrivial running of wave function renormalizations $Z_{\bar{g}}$, $Z_{\bar{A}}$, $Z_h$, $Z_c$, $Z_a$, that of the graviton mass parameter $\mu$, as well as the dynamical gravity interactions $g$ and $\lambda_3$. Note that at a (UV) fixed point the flows of the couplings $\mu$, $g$, and $\lambda_3$ vanish while the anomalous dimensions do not vanish.

The last identification in (25) reflects the fact that $-2\lambda_2$ is the dimensionless mass parameter of the graviton. Note however that $\mu$ is not a physical mass of the graviton in the sense of massive gravity: in the classical regime of gravity, it is identical to the cosmological constant, $\bar{\lambda} = -\frac{1}{2}\mu$. Higher order operators in particular $g_{a^n}$ may couple back in an indirect fashion, see, e.g., [85]. In summary, this leads us to an expansion of the mixed fluctuation terms (with both, powers of $a$ and powers of $h$) of the effective action (22)

$$\begin{aligned}\Gamma[\bar{g},\bar{A};\phi]|_{\mathrm{mixed}} &= \Gamma^{(ah)}_{\mathbf{a}_1\mathbf{a}_2}a_{\mathbf{a}_1}h_{\mathbf{a}_2} + \frac{1}{2}\Gamma^{(ahh)}_{\mathbf{a}_1\mathbf{a}_2\mathbf{a}_3}a_{\mathbf{a}_1}h_{\mathbf{a}_2}h_{\mathbf{a}_3} \\ &\quad + \frac{1}{2}\Gamma^{(aah)}_{\mathbf{a}_1\mathbf{a}_2\mathbf{a}_3}a_{\mathbf{a}_1}a_{\mathbf{a}_2}h_{\mathbf{a}_3} + \frac{1}{4}\Gamma^{(aahh)}_{\mathbf{a}_1\mathbf{a}_2\mathbf{a}_3\mathbf{a}_4}a_{\mathbf{a}_1}a_{\mathbf{a}_2}h_{\mathbf{a}_3}h_{\mathbf{a}_4} \\ &\quad + \frac{1}{12}\Gamma^{(aahhh)}_{\mathbf{a}_1\mathbf{a}_2\mathbf{a}_3\mathbf{a}_4\mathbf{a}_5}a_{\mathbf{a}_1}a_{\mathbf{a}_2}h_{\mathbf{a}_3}h_{\mathbf{a}_4}h_{\mathbf{a}_5} + \mathcal{O}(a^3h, ah^3). \end{aligned} \tag{27}$$

As we consider also correlation functions of the background gluon, we need the expansion of the fluctuation vertices in (27) in the background field, i.e.,

$$\Gamma^{(ah)}_{\mathbf{a}_1\mathbf{h}_2}[\bar{A}] = \Gamma^{(ah)}_{\mathbf{a}_1\mathbf{h}_2}[0] + \Gamma^{(\bar{A}ah)}_{\mathbf{b}_1\mathbf{a}_1\mathbf{h}_2}[0]\bar{A}_{\mathbf{b}_1} + O(\bar{A}^2), \tag{28}$$

in an expansion about vanishing background gauge field. In the following, we consider trivial metric and gluon backgrounds $\bar{g} = \mathbb{1}$ and $\bar{A} = 0$. In this background, the terms of the order $\mathcal{O}(a^3h, ah^3)$ do not enter the flow equations of the gluon and graviton propagators nor that of the graviton three-point function. This is the reason why they have not been displayed explicitly in (27). Note that with this background choice, the terms linear in $a$ in the second line in (27) vanish.

In this trivial background, we can use standard Fourier representations for our correlation functions. In momentum space, the above correlation functions are given as follows: the gluon two-point function reads

$$\Gamma^{(aa)}_{\mu\nu}(p_1,p_2) = Z_a^{\frac{1}{2}}(p_1^2)Z_a^{\frac{1}{2}}(p_2^2)\left.\frac{\delta^2 S_A}{\delta a^\mu(p_1)\delta a^\nu(p_2)}\right|_{\phi=0}. \tag{29}$$

The graviton two-point function is parameterized according to the prescription presented in [24,26,31,36,61,79],

$$\Gamma^{(hh)}_{\mu\nu\alpha\beta}(p_1,p_2) = Z_h^{\frac{1}{2}}(p_1^2)Z_h^{\frac{1}{2}}(p_2^2)\left.\frac{G_2\delta^2 S_{\mathrm{EH}}(G_2,\Lambda_2)}{\delta h^{\mu\nu}(p_1)\delta h^{\alpha\beta}(p_2)}\right|_{\phi=0}, \tag{30}$$

where $-2\Lambda_2 = \mu k^2$ as introduced in (25). Note that the right-hand side of (30) does not depend on $G_2$. The two-gluon–one-graviton vertex is given by

$$\Gamma^{(aah)}_{\mu\nu\alpha\beta}(p_1,p_2,p_3) = Z_a^{\frac{1}{2}}(p_1^2)Z_a^{\frac{1}{2}}(p_2^2)Z_h^{\frac{1}{2}}(p_3^2) \times \left.\frac{G_3^{\frac{1}{2}}\delta^3 S_A}{\delta a^\mu(p_1)\delta a^\nu(p_2)\delta h^{\alpha\beta}(p_3)}\right|_{\phi=0}, \tag{31}$$

with scale- and momentum-dependent wave function renormalizations $Z_a$ for the gluon and $Z_h$ for the graviton and a scale-dependent gravitational coupling $G_3$. The other $n$-point functions have a completely analogous construction, which is not displayed here.

In addition to the fluctuation vertices, we also need mixed vertices involving two background gluons and the fluctuation fields as in (28), $\Gamma^{A^2h^n}$ and $\Gamma^{A^2a^n}$ with $n = 1, 2$. They are parameterized as in (29)–(31) with $Z_a \to Z_A$. We also would like to emphasize two structures that facilitate the present computations:

(1) As we consider the flow equations for the gluon two-point function, and the graviton two- and three-point functions, only the terms quadratic in $a_\mu$ in (27) contribute to the graviton-gluon interactions in the flow equations. The non-Abelian parts in the $F^2$ term do not contribute since they are of order 3 and higher. Hence, modulo trivial color factors $\delta^{ab}$, the vertices defined above are identical for $SU(N)$ and $U(1)$ gauge theories.

(2) In principle, the derivatives in $F^{\mu\nu}$ are covariant derivatives with respect to the Levi-Civita connection. However, since $F^{\mu\nu}$ is asymmetric, and the Christoffel-symbols symmetric in the paired index, the latter cancel out, and the covariant derivatives can be replaced by partial derivatives.

In the end, we are interested in the gravitational corrections to the Yang-Mills beta function, and the Yang-Mills contributions to the running in the gravity sector. The beta functions of the latter have been discussed in great detail in [24,26,31,36,61]. In the Yang-Mills sector, we make use of the fact that the wave function renormalization $Z_A$ of the background gluon is related to the background (minimal) coupling by

$$Z_{\alpha_s} = Z_A^{-1}, \tag{32}$$

which is derived from background gauge invariance of the theory. The latter can be related to quantum gauge invariance with Nielsen identities, see [23,119–122] in the present framework. This also relates the background minimal coupling to the dynamical minimal coupling of the fluctuation field. Note that this relation is modified in the presence of the regulator, in particular, for momenta $p^2 < k^2$. There the interpretation of the background minimal coupling requires some care. The running of the background coupling is then determined by

$$\partial_t \alpha_s = \beta_{\alpha_s} = \eta_A \alpha_s, \tag{33}$$

with the gluon anomalous dimension

$$\eta_A := -\frac{\partial_t Z_A}{Z_A}. \tag{34}$$

Note that in general all these relations carry a momentum dependence as $Z_A(p^2)$ carries a momentum dependence. This will become important in the next section for the physics interpretation of the results.

## IV. GRAVITON CONTRIBUTIONS TO YANG-MILLS

In this section, we compute the gravitational corrections to the running of the gauge coupling. The key question is if graviton-gluon interactions destroy or preserve the property of asymptotic freedom in the Yang-Mills sector. The running of the gauge coupling can be calculated from the background gluon wave function renormalization. Its flow equation is derived from (21) with two functional derivatives with respect to $\bar{A}$. Schematically, it reads

$$\partial_t \Gamma^{(\bar{A}\bar{A})}(p) = \mathrm{Flow}_A^{(\bar{A}\bar{A})}(p) + \mathrm{Flow}_h^{(\bar{A}\bar{A})}(p), \tag{35}$$

where the first term contains only gluon fluctuations and the second term is induced by graviton-gluon interactions. The diagrammatic form of the second term is displayed in Fig. 1. This split is reflected in a corresponding split of the anomalous dimension

$$\eta_A(p^2) = \eta_{A,A}(p^2) + \eta_{A,h}(p^2). \tag{36}$$

Note that in the present approximation, we have $\eta_{A,h} = \eta_{a,h}$. This originates in the fact that the fluctuation graviton only couples to gauge invariant operators.

Asymptotic freedom is signaled by a negative sign of the gluon anomalous dimension as the beta function for the coupling is proportional to $\eta_A$. We know that the pure gluon contributions $\eta_{A,A}$ are negative. Hence, the question whether asymptotic freedom is preserved in the Yang-Mills–gravity system boils down to the sign of the gravity contributions $\eta_{A,h}$, and we arrive at

$$\eta_{A,h} \leq 0 \quad \Leftrightarrow \quad \text{asymptotic freedom}. \tag{37}$$

The anomalous dimension in (37) depends on both cutoff and momentum scales. For small momentum scales $p^2/k^2 \to 0$, the regulator induces a breaking of quantum-gauge and quantum-diffeomorphism invariance: the respective STIs of the fluctuation field correlation functions are modified. This necessitates also a careful investigation of the background observables, which only carry physics due to the relation of background gauge- and diffeomorphism invariance.

Note that asymptotic freedom as defined in (37) only applies to the minimal coupling. Higher order fluctuation couplings are not necessarily vanishing. Indeed, it has been shown that the asymptotically safe fixed points of general

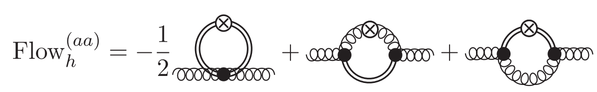


FIG. 1. Diagrammatic depiction of graviton contributions to the flow of the gluon propagator. Wiggly and double lines represent gluon and graviton propagators, respectively.

matter and gauge fields coupled to gravity can not be fully asymptotically free in the matter and gauge field sector, see [72,81,82,85,86]. In the present work, this leads to $a^4$ vertices from higher order invariants such as $(\mathrm{tr}F^2)^2$ and $\mathrm{tr}F^4$ with fixed point values proportional to $g_a^2/(1+\mu)^3$ with $g_a = g$ in our approximation. Moreover, these vertices generate a tadpole diagram that contribute to the gluon propagator. Apart from shifting the Gaussian fixed point of higher order operators in the Yang-Mills sector to an interacting one, see [85] for the $U(1)$ case, it also deforms the gluon contribution to the Yang-Mills beta function. Its qualitative properties will be discussed later, as it is important for the large $N_c$ behavior of the fixed point. However, a full inclusion is deferred to future work.

### A. Background observables

The discussion of physics content of background observables and its relation to gauge- and diffeomorphism invariance has been initiated for the Yang-Mills–gravity system in [68,69]. There it has been shown that $\eta_{a,h} = 0$ vanishes for

$$\frac{r_a}{1+r_a}\frac{1}{1+r_h} = 0 \tag{38}$$

due to a nontrivial kinematic identity. This identity relates angular averages of one- and two-graviton–two-gluon scattering vertices in the absence of a gluon regulator $r_a$, see Fig. 2. In other words, for a combination of regulators that satisfy (38) the quantum-gauge and quantum-diffeomorphism symmetry violating effects of the regulators do not effect the kinematic identity that holds in the absence of the regulator.

This structure requires some care in the interpretation of the running of background observables for $k \to \infty$: while the physics properties of the dynamical fluctuation fields should not depend on the choice of the regulators, background observables do not necessarily display physics in this limit. By now, we know of many examples for the latter deficiency ranging from the beta function of Yang-Mills theory, see [120], to the behavior of the background couplings in pure gravity, [24,26,31,36,61] and matter-gravity systems [79,82]. Moreover, we have already argued that the relation between the dynamical and the background minimal coupling only holds without modifications for sufficiently large momenta.

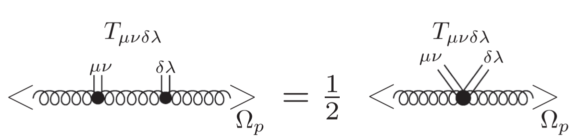


FIG. 2. Kinematic identity for the one- and two-graviton–two-gluon scattering vertices for $r_a = 0$ and $\Gamma_A^{(2)} \simeq S_A^{(2)}$, taken from [68,69].

In summary, this implies the following for the interpretation of background observables: we either choose pairs of regulators that satisfy (38) or we evaluate background observables for momentum configurations that are not dominantly affected by the breaking of quantum-gauge and quantum-diffeomorphism invariance. Here, we will pursue the latter option that gives us more freedom in the choice of regulators. For the computation of the graviton contribution to the running of the Yang-Mills background coupling, this implies that we have to evaluate the flow of the two-point function for sufficiently large external momenta,

$$p^2 \gtrsim k^2. \tag{39}$$

For these momenta, the three-point function diagrams effectively satisfy (38), and the anomalous dimension $\eta_{a,h}(p^2)$ carries the information about the graviton contribution of the beta function of the background coupling.

### B. Gravity supports asymptotic freedom

The results of the discussion on background observables in the previous Sec. IV A allow us to access the question of asymptotic freedom of the minimal Yang-Mills coupling. With the construction of the effective action (27), we obtain a flow equation for $\partial_t \Gamma^{(aa)}$, which is projected with the transverse projection operator

$$P_{\mathrm{T}}^{\mu\nu}(p) = \delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}. \tag{40}$$

The graviton-induced contributions to the resulting flow equation take the form

$$\begin{aligned} &P_{\mathrm{T}}^{\mu\nu}(p)\partial_t \Gamma_{\mu\nu}^{(aa)}(p) \\ &\quad = \mathrm{Flow}_h^{(aa)}(p^2) \\ &\quad = Z_a(p^2) g \int_q ((\dot{r}(q^2) - \eta_a(q^2) r(q^2)) f_a(q,p,\mu) \\ &\qquad + (\dot{r}(q^2) - \eta_h(q^2) r(q^2)) f_h(q,p,\mu)), \end{aligned} \tag{41}$$

where the terms on the right-hand side originate from diagrams with a regulator insertion in the gluon and graviton propagator, respectively. The left-hand side is simply given by

$$P_{\mathrm{T}}^{\mu\nu}\partial_t \Gamma_{\mu\nu}^{(aa)}(p) = p^2 \partial_t Z_a(p^2). \tag{42}$$

Dividing by $Z_a(p^2)$, one obtains an inhomogeneous Fredholm integral equation of the second kind for the gluon anomalous dimension,

$$\eta_a(p^2) = f(p^2) + g \int \frac{\mathrm{d}^4 q}{(2\pi)^4} K(p,q,\mu,\eta_h)\eta_a(q^2). \tag{43}$$

This integral equation can be solved using the resolvent formalism by means of a Liouville-Neumann series. In this work, we approximate the full momentum dependence by evaluating the anomalous dimension in the integrand in (43) at $q^2 = k^2$. This is justified since the integrand is peaked at $q \approx k$ due to the regulator. With this approximation, (43) can be evaluated numerically for all momenta. This approximation was already used in [79] and led to results in good qualitative agreement with the full momentum dependence. Details of the full solution are discussed in Appendix C. With the approximation to (43), we investigate the sign of the graviton contributions to the gluon propagator. These contributions are functions of the gravity couplings, which in turn depend on the truncation. It is therefore interesting to evaluate $\eta_{a,h}$ with a parametric dependence on the gravity couplings, in order to obtain general conditions under which asymptotic freedom is guaranteed.

The gluon anomalous dimension is of the form $\eta_a(p^2, g, \mu, \eta_h)$. In order to avoid the unphysical regulator dependence potentially induced by the violation of the kinematical identity (38), we choose the momentum $p^2 = k^2$ in order to satisfy (39). In summary, this provides us with a minimal coupling $\alpha_s$,

$$\partial_t \alpha_s = \beta_{\alpha_s} = \eta_a(k^2)\alpha_s. \tag{44}$$

As a main result in the present section, we conclude that

$$\beta_{\alpha_s} \leq 0 \quad \text{for } \mu > -1 \quad \text{and} \quad \eta_h(k^2) \leq 2. \tag{45}$$

The restriction to $\eta_h \leq 2$ is also the bound on the anomalous dimension advocated in [79]. To be more precise, $\eta_h > 2$ only changes the sign of the Yang-Mills beta function in the limit $\mu \to -1$. For other values of $\mu$, very large values of $\eta_h$ are necessary in order to destroy asymptotic freedom, e.g., for $\mu = -0.4$ the bound is $\eta_h \approx 50$. The precise bound is displayed in Fig. 4, where the red region indicates $\beta_{\alpha_s} > 0$.

Despite the necessary restriction to momenta $p^2 \gtrsim k^2$ for its relation to the physical background coupling, we have also evaluated $\eta_{a,h}$ for more general momentum configurations and a range of gravity parameters $\mu$ and $\eta_h$: in Fig. 3, the sign of the graviton-induced part of the gluon anomalous dimension $\eta_{a,h}$ is plotted in the momentum range $0 \leq p^2 \leq k^2$. For small momenta, $\eta_{a,h}$ changes sign for $\mu \to -1$. Again it can be shown that this does not happen for regulators with (38).

In order to understand the patterns behind Figs. 3 and 4 it is illuminating to examine $\eta_{a,h}(p^2 = 0)$ for flat regulators (A1) with a $p^2$ derivative. It reads

$$\eta_{a,h} = -\frac{g}{8\pi}\left(\frac{8-\eta_a}{1+\mu} - \frac{4-\eta_h}{(1+\mu)^2}\right). \tag{46}$$

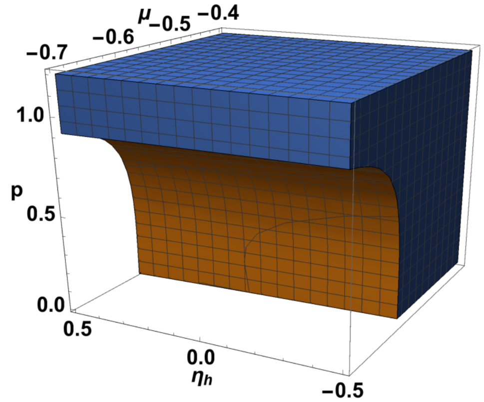


FIG. 3. Sign of the graviton contributions to the gluon anomalous dimension $\eta_{a,h}$ as a function of $\eta_h$, $\mu$, and $p$. The colored region indicates sgn$\eta_{a,h} < 0$. At $p = k$, the whole displayed region supports asymptotic freedom.

The first term on the right-hand side stems from $\partial_t R_{k,a}$ and is positive for $\eta_a < 8$. The second stems from $\partial_t R_{h,k}$. It is nonvanishing for $\eta_h = 0$ and hence, already contributes at one-loop order. Its very presence reflects the breaking of the nontrivial kinematical identity depicted in Fig. 2 as it is proportional to it. The interpretation of $\eta_{a,h}$ as the graviton-induced running of the Yang-Mills background coupling crucially hinges on physical quantum gauge invariance: it is important to realize that only with the relation between the auxiliary background gauge invariance and quantum gauge invariance the latter carries physics. In turn, in the momentum regime where the kinematical identity is violated, physical gauge invariance is not guaranteed, and background gauge invariance reduces to an auxiliary symmetry with no physical content. Accordingly, one either has to evaluate $\eta_{a,h}(p^2)$ for sufficiently large momenta $p^2 \gtrsim k^2$ or utilizes regulators that keep the kinematical identity Fig. 2 at least approximately for all momenta.

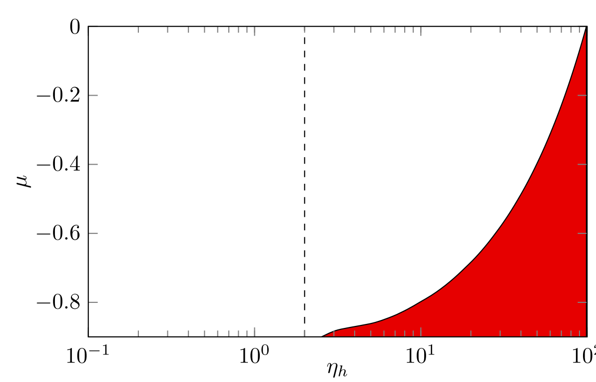


FIG. 4. Sign of the graviton contributions to the gluon anomalous dimension $\eta_{a,h}(k^2)$ as a function of $\eta_h$ and $\mu$. The red region indicates sgn$\eta_{a,h}(k^2) > 0$ and the loss of asymptotic freedom. The dashed line marks $\eta_h = 2$.

In summary, Figs. 3 and 4 entail that $\mathrm{sgn}(\eta_{a,h}) < 0$ holds for physically relevant momenta and values of the gravity couplings. Thus asymptotic freedom is preserved. We have argued that (44) provides the correct definition for the beta function of the minimal coupling of Yang-Mills theory with $\mathrm{sgn}(\beta_{\alpha_s}) \leq 0$. Hence, we conclude that an ultraviolet fixed point in the spirit of the asymptotic safety scenario is compatible with asymptotic freedom of the minimal coupling in Yang-Mills theories. In Appendix D, we utilize different approximations to the gluon anomalous dimension, and we discuss in detail the regimes, where it changes the sign in the parameter space of the gravity couplings.

## V. YANG-MILLS CONTRIBUTIONS TO GRAVITY

This section is concerned with the impact of gluon fluctuations on the gravity sector. The fully coupled system is analyzed subsequently in Sec. VI.

### A. General structure

For the question of asymptotic safety, we have to investigate the gluon contributions to the graviton propagator as well as to the graviton three-point function. This allows us to compute the corrections to the running of the gravity couplings $(\mu, g, \lambda_3)$ due to gluon fluctuations.

The gluon corrections to the graviton two- and three-point function split analogously to the graviton corrections to Yang-Mills theory in the preceding section, since for any graviton $n$-point function the structure is given by

$$\mathrm{Flow}^{(nh)} = \mathrm{Flow}_h^{(nh)} + \mathrm{Flow}_a^{(nh)}, \tag{47}$$

with graviton and gluon contributions denoted by $\mathrm{Flow}_h^{(nh)}$ and $\mathrm{Flow}_a^{(nh)}$, respectively. For example, the gluon contributions to the flow of the graviton two- and three-point function are depicted in Figs. 5 and 6. Accordingly, the beta function for Newton's coupling including gluon corrections has the structure

$$\begin{aligned}\partial_t g = {} & (2 + 3\eta_h) g + g^2 (A_h(\mu, \lambda_3) + \eta_h B_h(\mu, \lambda_3) \\ & + C_a + \eta_a D_a),\end{aligned} \tag{48}$$

where we have used the identifications (25). In (48), $A_h$ and $B_h$ originate from graviton loops, and they depend on $\mu$ and $\lambda_3$, while $C_a$ and $D_a$ are generated by gluon loops and are

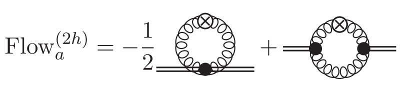


FIG. 5. Diagrammatic depiction of the gluon contributions to the flow of the graviton propagator. Wiggly and double lines represent gluon and graviton propagators, respectively.

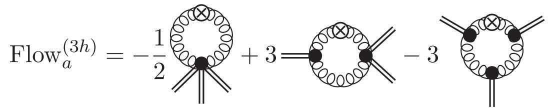


FIG. 6. Diagrammatic depiction of the gluon contributions to the flow of the graviton three-point function. Wiggly and double lines represent gluon and graviton propagators, respectively.

just numbers. Similarly, the beta function for $\lambda_3$ has the structure

$$\begin{aligned}\partial_t \lambda_3 = {} & \left(-1 + \frac{2}{3}\eta_h + \frac{\partial_t g}{2g}\right)\lambda_3 + g(E_h(\mu, \lambda_3) \\ & + \eta_h F_h(\mu, \lambda_3) + G_a + \eta_a H_a).\end{aligned} \tag{49}$$

Throughout this chapter, we display the anomalous dimensions $\eta_h, \eta_a$ as momentum independent. Note, however, that they are momentum dependent, and we approximate their momentum dependence by evaluating them at $p = k$ if they appear in an integral, see [79] for details.

Moreover, the Yang-Mills contributions to the graviton propagator enter the above beta function (48) via the graviton anomalous dimension $\eta_h$ and the graviton mass parameter $\mu$. These equations have the general form

$$\begin{aligned}\eta_h &= g(I_h(\mu, \lambda_3) + \eta_h J_h(\mu, \lambda_3) + K_a + \eta_a L_a), \\ \partial_t \mu &= (\eta_h - 2)\mu + g(M_h(\mu, \lambda_3) + N_h(\mu, \lambda_3)\eta_h \\ & \quad + O_a + \eta_a P_a),\end{aligned} \tag{50}$$

where again all pure gravity contributions are labeled with an index $h$ and the one generated by gluons with an index $a$. Note again that all the Yang-Mills contributions do not depend on $\mu$ and $\lambda_3$, as the corresponding diagrams do not involve graviton propagators and pure graviton vertices, see Figs. 5 and 6. In particular, this implies that these terms have no $1/(1+\mu)$ singularity in the limit $\mu \to -1$. Furthermore, all these diagrams contain a closed gluon loop, and hence, all the factors in the above equations with an index $a$ are proportional to $N_c^2 - 1$.

### B. Contributions to the graviton propagator

The gluon contribution to the graviton propagator has been studied in a derivative expansion around $p^2 = 0$ in [69], where it was shown that this projection is insufficient due to the nontrivial momentum dependence of the flow. The latter is characterized by a dip at $p^2 \approx k^2$. It has been shown in [24] that this structure is also present in the full flow, i.e., including the graviton contributions and that projections at momentum scales close to the cutoff are necessary, see also [31,36]. We have rederived the momentum dependence of $\mathrm{Flow}_a^{(2h)}(p^2)$, see Fig. 7.

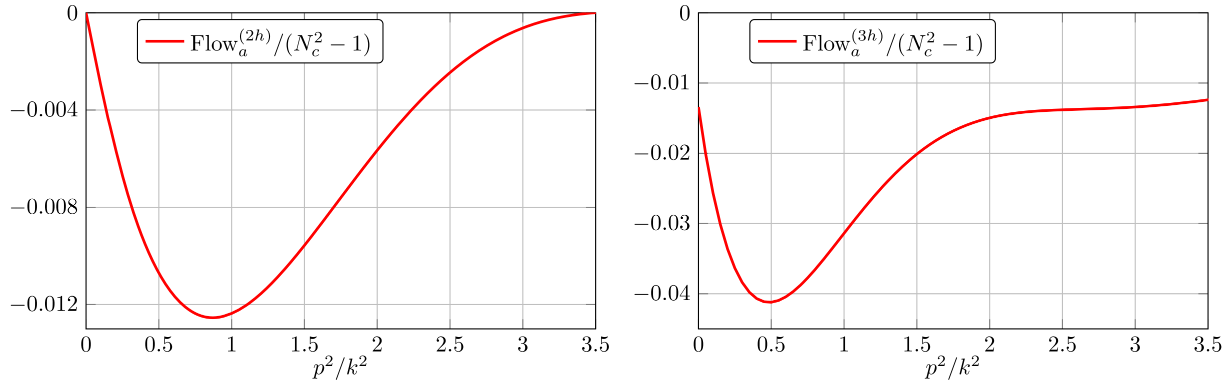


FIG. 7. The momentum dependence of $\mathrm{Flow}_a^{(2h)}/(N_c^2-1)$ (left) and $\mathrm{Flow}_a^{(3h)}/(N_c^2-1)$ (right) for $g=1$ and $\eta_a=0$ on the right-hand side of the flow.

For the projection at $p^2=0$ and flat regulators (A1), we rederive the result of [69] and obtain for the momentum-independent part

$$\mathrm{Flow}_a^{(2h)}(p^2=0) = gZ_h(N_c^2-1)\frac{1}{60\pi}\eta_a. \tag{51}$$

Surprisingly, this contribution is proportional to $\eta_a$. This happens due to a cancellation between both diagrams displayed in Fig. 5. Note that this cancellation only occurs for the flat regulator. For other regulators, the contribution can be either positive or negative. This is discussed in Appendix B and will play a crucial role in the later analysis.

For the computation of the graviton anomalous dimension, we resort to a finite difference projection, which is of the general form

$$\frac{\mathrm{Flow}_a^{(2h)}(p_1^2) - \mathrm{Flow}_a^{(2h)}(p_2^2)}{p_1^2-p_2^2} = gZ_h(N_c^2-1)(\alpha+\beta\eta_a), \tag{52}$$

where $\alpha$ and $\beta$ depend only on $p_1$ and $p_2$. This is rooted in the fact that there are only internal gluon propagators and graviton-gluon vertices, and these do not depend on $\lambda_3$ and $\mu$ as discussed in the last section. For $p_2=0$ and $p_1\to p_2$, i.e., a $p^2$ derivative at $p^2=0$, we obtain

$$\alpha=\beta=-\frac{1}{12\pi}\approx -0.027. \tag{53}$$

For a finite difference with $p_1^2=k^2$ and $p_2=0$, we obtain

$$\alpha\approx -0.012, \qquad \beta\approx -0.0033. \tag{54}$$

Equations (53) and (54) display the gluon contribution to $-\eta_h$; thus, the gluon contribution to $\eta_h$ is positive independent of the momentum projection scheme. Note however that (53) and (54) display a qualitatively different behavior, and (54) is the correct choice due to the momentum dependence of the flow. This has already been observed in the pure gravity computations in [24,26,31,36] and emphasizes the importance of the momentum dependence. In this work, we use a finite difference between $p_1^2=p^2$ and $p_2^2=-\mu k^2$ for the equation of $\eta_h(p^2)$, see [26,79] for details.

### C. Contributions to the three-point function

The contributions to the graviton three-point function enter the beta function of the Newton's coupling $g$ (48) via $C_a$ and $D_a$ and the beta function of $\lambda_3$ (49) via $G_a$ and $H_a$. The diagrammatic representation of these contributions is shown in Fig. 6. Here, the contribution to $\partial_t g$ is the momentum dependent part and the contribution to $\partial_t\lambda_3$ in the momentum independent part to the graviton three-point function. For the projection on the couplings $g$ and $\lambda_3$, we use precisely the same projection operators as in [31]. These are different projection operators for $g$ and $\lambda_3$, and we mark this with an index $G$ and $\Lambda$ in the following.

We have seen in the previous sections, that the momentum dependence of the flow plays a crucial role, and key properties may be spoiled if nontrivial momentum dependence is not taken into account properly. Therefore, we resolve the momentum dependence of the contributions $\mathrm{flow}_{G,a}^{(3h)}(p^2)$, which is shown in the right panel of Fig. 7. Interestingly, the contribution is peaked at $p^2=\frac{1}{2}k^2$ and is not well described by $p^2$ in the region $0\le p^2\le k^2$. Because of this nontrivial structure, the contribution to $\partial_t g$ depends on the momenta where it is evaluated. For general momenta $p_1^2$ and $p_2^2$, we obtain

$$\frac{\mathrm{Flow}_{G,a}^{(3h)}(p_1^2) - \mathrm{Flow}_{G,a}^{(3h)}(p_2^2)}{p_1^2-p_2^2} = g^{\frac{3}{2}}Z_h^{\frac{3}{2}}(N_c^2-1)(\gamma+\delta\eta_a), \tag{55}$$

where $\gamma$ and $\delta$ again only depend on $p_1^2$ and $p_2^2$. Evaluated as derivatives, i.e., $p_2^2=0$ and $p_1^2\to 0$, we arrive at

$$\gamma = -\frac{7}{30\pi} \approx -0.074, \qquad \delta = -\frac{1}{570\pi} \approx -0.00056. \tag{56}$$

With $p_1^2 = k^2$ and $p_2^2 = 0$, they are given by

$$\gamma \approx -0.018, \qquad \delta \approx -0.0014. \tag{57}$$

As in the case of the gluon propagator, the sign of the derivative definition agrees with the bilocal one but they differ strongly in their magnitude. In the present work, we use (57). The contribution to $\lambda_3$ is always evaluated at vanishing momentum. We obtain

$$\mathrm{Flow}^{(3h)}_{\Lambda,a}(p^2 = 0) = g^{\frac{3}{2}} Z_h^{\frac{3}{2}} (N_c^2 - 1) \frac{3 - \eta_a}{60\pi}. \tag{58}$$

### D. Mixed graviton-gluon coupling

So far, we have only considered pure gluon and pure graviton correlation functions in the coupled Yang-Mills–gravity system. Indeed, the results that will be presented in Sec. VI are based on precisely these correlation functions, and other couplings are identified according to (25). In Sec. VII, we will then discuss the stability of the results under extensions of the truncation. In particular, we will have a look at the inclusion of a flow equation for the graviton–two-gluon coupling $g_a$.

The flow equation for $g_a$ is derived analogously to the $g_3$ coupling from three-graviton vertex: we build the projection operator from the classical tensor structure $S^{(haa)}$ with a transverse traceless graviton and two transverse gluons. This projection operator is contracted with both sides of the flow equation for this specific vertex. The equation is further evaluated at the momentum symmetric point [31]. The resulting $p^2$ part gives the flow equation for $g_a$. We obtain an analytic flow equation for $g_a$ by a $p^2$ derivative at $p^2 = 0$. The resulting flow equation is given in Appendix F.

For the computations in Sec. VII, we use the preferred method of finite differences. In particular, we choose the evaluation points $p^2 = k^2$ and $p^2 = 0$. With this method, we do not obtain analytic flows but we take more nontrivial momentum dependences into account [31,36]. The computation is simplified by the fact that the present flow is actually vanishing at $p^2 = 0$. Consequently, the finite difference equals to an evaluation at $p^2 = k^2$, and the momentum derivative gives the same result as a $1/p^2$ division.

### E. Momentum locality

We close this section with a remark on the momentum locality introduced in [31] as a necessary condition for well-defined RG flows. It was shown to be related to diffeomorphism invariance of the theory. It entails that flows should not change the leading order of the large momentum behavior of correlation functions.

The asymptotics of the diagrams for the graviton two-point function, ordered as displayed in Fig. 5, are

$$\begin{aligned} \mathrm{Diag}_1^{(2h)}(p^2 \to \infty) &= -g\frac{8-\eta_a}{12\pi}, \\ \mathrm{Diag}_2^{(2h)}(p^2 \to \infty) &= g\frac{8-\eta_a}{12\pi}, \end{aligned} \tag{59}$$

while the asymptotics for the graviton three-point function, again ordered as displayed in Fig. 6, are

$$\begin{aligned} \mathrm{Diag}_1^{(3h)}(p^2 \to \infty) &= -g^{3/2}\frac{8-\eta_a}{19\pi}, \\ \mathrm{Diag}_2^{(3h)}(p^2 \to \infty) &= g^{3/2}\frac{4(8-\eta_a)}{19\pi}, \\ \mathrm{Diag}_3^{(3h)}(p^2 \to \infty) &= -g^{3/2}\frac{3(8-\eta_a)}{19\pi}. \end{aligned} \tag{60}$$

Consequently, we again have a highly nontrivial cancellation between different diagrams, which leads to the property of momentum locality. In summary, we assert

$$\lim_{p^2/k^2 \to \infty} \frac{\partial_t \Gamma^{(2h,3h)}(p^2)}{\Gamma^{(2h,3h)}(p^2)} = 0, \tag{61}$$

at the symmetric point in the transverse traceless mode. Hence, the full flows of the graviton two- and three-point functions including Yang-Mills corrections are momentum local.

## VI. ASYMPTOTIC SAFETY OF YANG-MILLS–GRAVITY

In this section, we provide a full analysis of the ultraviolet fixed point of the coupled Yang-Mills–gravity system. It is characterized by the nontrivial fixed point of Newton's coupling $g$, the coupling of the momentum-independent part of the graviton three-point function $\lambda_3$, and the graviton mass parameter $\mu$ while the minimal gauge coupling vanishes, $\alpha_s = 0$.

### A. Finite $N_c$

The fully coupled fixed point shows some remarkable features. The fixed point values are displayed in the left panel of Fig. 8. The fixed point value of the graviton mass parameter remains almost a constant as a function of $N_c$. The Newton's coupling is approaching zero, while $\lambda_3^*$ becomes slowly smaller and crosses zero at $N_c^2 \approx 166$. This behavior can be understood from the equations: the leading contribution from Yang-Mills to $\partial_t \mu$ cancels out, and only a term proportional to $\eta_a$ remains, see (51). The latter is small at the fixed point, and hence, the effect on $\partial_t \mu$ is strongly suppressed. The falloff of $g^*$ and $\lambda_3^*$ is explained by the respective contribution in the flow equations, see (57) and (58).

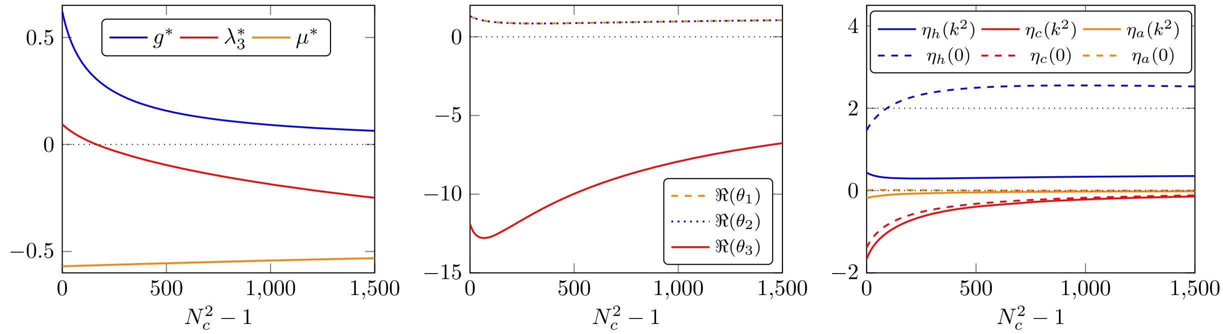


FIG. 8. Properties of the UV fixed point as a function of $N_c^2 - 1$ in the uniform approximation with one Newton's coupling. Displayed are the fixed point values (left panel), the critical exponents (central panel), and the anomalous dimensions (right panel).

The critical exponents of the fixed point, which are given by minus the eigenvalues of the stability matrix, are displayed in the central panel of Fig. 8. They remain stable over the whole investigated range. Two critical exponents form a complex conjugated pair. The real part of this pair is positive and thus corresponds to two UV attractive directions. The third critical exponent is real and negative and corresponds to a UV repulsive direction. The eigenvector belonging to the latter exponent points approximately in the direction of $\lambda_3$, which is in accordance with pure gravity results [31].

In the right panel of Fig. 8, we show the anomalous dimensions at the fixed point, evaluated at $p^2 = 0$ and $p^2 = k^2$. The ghost and gluon anomalous dimensions tend towards zero for increasing $N_c$. Most importantly, $\eta_a(k^2)$ is always negative, which is a necessary condition for asymptotic freedom in the Yang-Mills sector. The graviton anomalous dimension does not tend towards zero. At $p^2 = k^2$, it is getting smaller with an increasing $N_c$ despite the positive gluon contribution (54). The reason is that the anomalous dimension is also proportional to $g^*$, which is decreasing, and this effect dominates over the gluon contribution. At $p^2 = 0$, on the other hand, the gluon contribution is also positive but larger in value, see (53), and consequently, dominates over the decrease in $g^*$. $\eta_h(0)$ is increasing, crosses the value 2 and starts to decrease again for large $N_c$. As mentioned in (19), $\eta < 2$ is a bound on regulators that are proportional to the respective wave function renormalization. In our case, $\eta_h(0)$ exceeds the value 2 just slightly and remains far from the strict bound, which is $\eta_h < 4$, see [79] for details.

The fixed point values of the background couplings are displayed in Fig. 9. The equations for the pure gravity part are identical to the ones in [36] and the gluon part is identical to the one in [73]. In this setting, the background couplings behave very similar to the dynamical ones. The background Newton's coupling goes to zero with $1/N_c^2$ while the background cosmological constant goes to a constant for large $N_c$. Interestingly, the background coupling approach their asymptotic behavior faster than the dynamical ones.

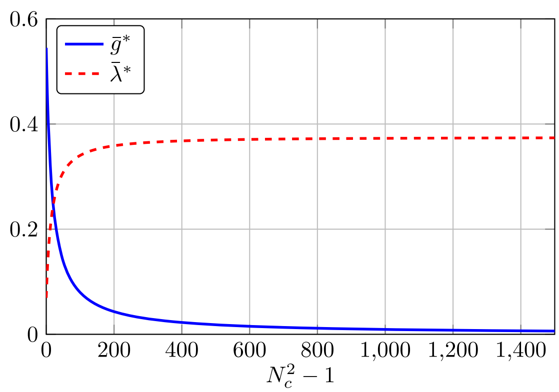


FIG. 9. Displayed are the background couplings $\bar{g}^*$ and $\bar{\lambda}^*$ as a function of $N_c^2 - 1$ evaluated at the UV fixed point displayed in Fig. 8. The coupling $\bar{g}^*$ is going to zero with $\frac{1}{N_c^2}$ and $\bar{\lambda}^*$ goes to the constant 0.38, see (64).

### B. Large $N_c$ scaling

In the limit $N_c \to \infty$, the couplings approach the fixed point values

$$g^* \to \frac{89}{N_c^2} + \frac{8.0 \times 10^4}{N_c^4}, \qquad \mu^* \to -0.45 - \frac{3.3 \times 10^2}{N_c^2},$$
$$\lambda_3^* \to -0.71 + \frac{2.4 \times 10^3}{N_c^2}. \tag{62}$$

As expected, the 't Hooft coupling $g^* N_c^2$ is going to a constant in the large $N_c$ limit. This behavior is also displayed in Fig. 10 for finite $N_c$. Remarkably, $\mu^*$ and $\lambda_3^*$ remain finite. In the $\lambda_3$ equation, this originates from a balancing of the gluon contribution with the canonical term. In the $\mu$ equation, on the other hand, all contributions go to zero in leading order and the fixed point value of $\mu$ follows from the second order contributions. The asymptotic anomalous dimensions follow as

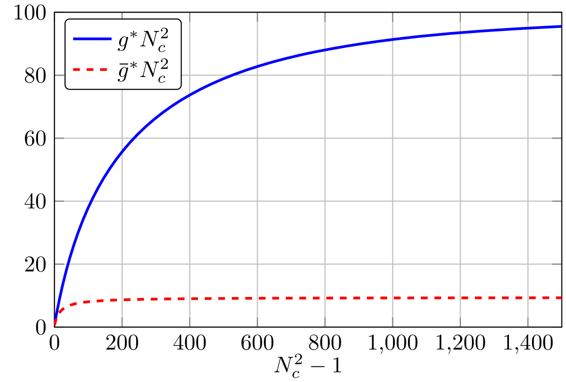


FIG. 10. Displayed are the fixed point 't Hooft couplings $g^*N_c^2$ and $\bar{g}^*N_c^2$ as a function of $N_c^2-1$. The couplings approach the asymptotic values $g^*N_c^2\to 89$ and $\bar{g}^*N_c^2\to 9.4$, see (62) and (64).

$$\eta_h(0)\to 2+\frac{2.7\times 10^3}{N_c^2},\qquad \eta_h(k^2)\to 0.36+\frac{2.9\times 10^2}{N_c^2},$$
$$\eta_c(0)\to -\frac{1.3\times 10^2}{N_c^2},\qquad \eta_c(k^2)\to -\frac{1.5\times 10^2}{N_c^2},$$
$$\eta_a(0)\to -\frac{8.7}{N_c^2},\qquad \eta_a(k^2)\to -\frac{22}{N_c^2}, \tag{63}$$

which satisfy the bounds $\eta_i\leq 2$ necessary for the consistency of the regulators that are proportional to $Z_h$, $Z_c$, $Z_a$. Note that only the graviton anomalous dimension is nonvanishing in this limit. Importantly, the gluon anomalous dimension approaches zero from the negative direction, which means that it supports asymptotic freedom in the Yang-Mills sector. The asymptotic value $\eta_h(0)=2$ follows directly from the demand that all contributions in the $\mu$ equation have to go to zero in leading order, as discussed in the last paragraph. The critical exponents are given by

$$\theta_{1,2}\to 1.2\pm 2.1i+\frac{(1.1\mp 5.6i)\cdot 10^3}{N_c^2},$$
$$\theta_3\to -2.3-\frac{14\times 10^3}{N_c^2}. \tag{64}$$

The fixed point has two attractive and one repulsive direction for all colors. Remarkably, the values of the critical exponents remain of order 1. The background couplings approach the values

$$\bar{g}^*\to\frac{9.4}{N_c^2}-\frac{1.3\times 10^2}{N_c^4},\qquad \bar{\lambda}^*\to 0.38-\frac{1.4}{N_c^2}. \tag{65}$$

Again, the background 't Hooft coupling, $\bar{g}^*N_c^2$ remains finite in the large $N_c$ limit, which is also displayed in Fig. 10.

In summary, we have found a stable UV fixed point with two attractive directions. The fixed point values, the critical exponents and the anomalous dimensions are of order 1. In Fig. 8, we display this behavior up to $N_c^2=1500$, and in this section, we have augmented this with a solution for $N_c\to\infty$. Consequently, we conclude that the system is asymptotically safe in the gravity sector and asymptotically free in the Yang-Mills sector for all $N_c$.

### C. Decoupling of gravity-induced gluon self-interactions

It has been advocated in [72] that interacting matter-gravity systems necessarily contain self-interacting matter fixed points. This has been investigated in scalar, fermionic, and Yukawa systems in, e.g., [81,82,86].

Recently, also a Yang-Mills–gravity system with an Abelian $U(1)$ gauge group has been investigated [85]. It was found that that the coupling of the fourth power of the field strength, $F^4$, takes a finite fixed point value, while the minimal coupling that enters the covariant derivative can be asymptotically free. As already mentioned before in Sec. IV, the same happens in Yang-Mills–gravity systems. In particular, we are led to

$$w_2^*(\mathrm{tr}F_{\mu\nu}^2)^2+v_4^*\mathrm{tr}F_{\mu\nu}^4, \tag{66}$$

with $w_2^*\neq 0$ and $v_4^*\neq 0$ without nontrivial cancellations. A quantitative computations of these fixed point couplings is deferred to future work. Here, we simply discuss their qualitative behavior: even if not present in the theory, the couplings $w_2$ and $v_4$ are generated by diagrams with the exchange of two gravitons, see Fig. 11. In leading order, these diagrams are proportional to

$$\frac{g^2}{(1+\mu)^3}\propto\frac{1}{N_c^4}\to 0, \tag{67}$$

and vanish in the large $N_c$ scaling of (62). It is simple to show that the further diagrams in the fixed point equations of $w_2$, $v_2$ proportional to $w_2$, $v_2$ decay even faster when using (67) for the diagrams.

Finally, we get additional gluon tadpole contributions proportional to $\omega_2^*$, $v_4^*$ for the running of the Yang-Mills beta function. In leading order, these contributions are proportional to $N_c^2$ due to a closed gluon loop. Together with the fixed point scaling of $\omega_2^*$, $v_4^*$ in (67), this leads to a $1/N_c^2$ decay of these contributions. They have the same large $N_c$ scaling as the pure gravity contributions but also

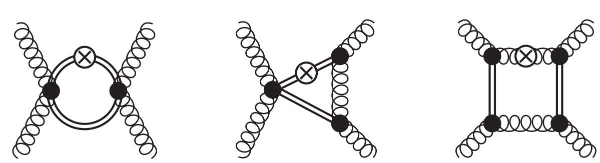

FIG. 11. Diagrammatic depiction of the graviton induced higher-order gluon interactions. Wiggly and double lines represent gluon and graviton propagators, respectively.

share the same negative sign supporting asymptotic freedom, see [85] for a study in $U(1)$ theories.

We close this chapter with a qualitative discussion of the stability for the interacting fixed point: as $\omega_2$, $v_2$ do not couple into the pure gravity subsystem, the stability matrix is skew symmetric, and the eigenvalues are computed in the respective subsystems. Both, the gravity as well as the $\omega_2$, $v_4$ subsystems are stable in the limit $g \to 0$.

This concludes our analysis of the large $N_c$ behavior of quantum gravity with the flat regulator and the identification (25). As expected, Newton's coupling $g$ shows the $1/N_c^2$-behavior discussed in Sec. II.

## VII. UV DOMINANCE OF GRAVITY

### A. Dynamical scale fixing

In Sec. VI, we used the identifications of all Newton's couplings (25). In the present chapter, we discuss the general case without this identification. We provide a comprehensive summary of results and the underlying structure, more details can be found in Appendix E. While we have argued in Sec. II that the present Yang-Mills–gravity system, as well as all free-matter–gravity systems are asymptotically safe, the interesting question is how and if at all in the present approximation this is dynamically observed.

Within the iterative procedure in Sec. II, we arrived at a fixed point action that is identical to that of the pure gravity sector with fixed point values for $g_n^*$, $\lambda_n^*$, and $\mu^*$. We also have $g_a = g_3$ due to the expansion of the metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{g_3} k^2 h_{\mu\nu}$ with $k = k_h$. Note also that in such a two-scale setting with $k_h$ and $k_a$, the latter rather is to be identified with $k_a^{\mathrm{UV}}$ and not with $k_a^{\mathrm{IR}}$. As the effect of the latter has been absorbed in a renormalization of Newton's coupling prior to the integrating out of graviton fluctuations (or rather their suppression with $k_h \to \infty$), this sets the graviton cutoff scale $k_h = k$ as the largest scale in the system. This leads to (20) that effectively induces

$$k^2 \simeq N_c^2 k_a^2, \tag{68}$$

in the large $N_c$ limit. Note that with a rescaling of our unique cutoff scale in Sec. VI with $N_c^2$ we already arrive at the $N_c$ independent fixed point values (62). The large values come from dropping the $N_c$-independent prefactor in the ratio $G/G_{\mathrm{eff}}$. The latter fact signals the unphysical nature of fixed point values, which within this two-scale setting also extends to the product $g^*\lambda^*$, typically used in the literature as a potentially rescaling-invariant observable.

Despite (20) being a natural relative scale setting, without any approximation, the full system of flow equations with $k_h = k_a$ should adjust itself dynamically to this situation with $g_a^* \sim g_c^* \sim g^*$ and with $g^* \propto 1/N_c^2$ in the large $N_c$ limit. In the present approximation, this can happen via two mechanisms that both elevate the graviton fluctuations to the same $N_c$ strength as the gluon fluctuations: the graviton propagator acquires a $N_c$ scaling

$$k^2 G_h(p^2 = 0) = \frac{1}{Z_h} \frac{1}{1+\mu} \propto N_c^2, \tag{69}$$

after an appropriate rescaling of the couplings, for more details see Appendix E. We proceed by discussing the two dynamical options that the system has to generate the $N_c$ scaling in (69):

(1) Evidently, (69) can be achieved via

$$\mu^* \propto -1 + c_+/N_c^2, \tag{70}$$

with a positive constant $c_+$. Note that (70) is not present in the fixed point results in Sec. VI. Accordingly, adding the fixed point equation for $g_a$ has to trigger this running. Below we shall investigate this possibility in more detail.

(2) The $N_c$ scaling can also be stored in $1/Z_h$. As we have chosen regulators that are proportional to $Z_h$, this leads to an effective elimination of $Z_h$ from the system; its only remnant is the anomalous dimension $\eta_h$ in the cutoff derivative. Since $1/Z_h \propto (k^2)^{\eta_h/2-1}$, the anomalous dimension $\eta_h$ has to grow large and positive in order to effectively describe the $N_c$ scaling in (69),

$$\eta_h \to \infty. \tag{71}$$

In the present setting with $R_{h,k} \propto Z_h$, this option cannot be investigated as (71) violates the bound

$$R_{h,k} \propto Z_h \quad \Rightarrow \quad \eta_h < 2, \tag{72}$$

for the regulator. For $\eta_h > 2$, the regulators of type (72) cannot be shown to suppress UV degrees of freedom anymore in the limit $k \to \infty$ as $\lim_{k\to\infty} R_k(p^2) \to 0$ for $\eta_h > 2$. This bound was introduced and discussed in [79] within the scalar-gravity system, where $\eta_h$ grows beyond this bound for the number of scalars $N_s$ getting large. It was stated there that the stability of the scalar-gravity system could not be investigated conclusively since the regulator cannot be trusted anymore. In the light of the present results and discussion, we know that the free-matter system is asymptotically safe. Then, the growing $\eta_h$ signals that the system wants to accommodate (69) with a growing $1/Z_h$.

We emphasize that the physics of both options, (1) and (2), is captured by (69) and is identical. Which part of the scaling of the propagator is captured by $\mu$ and which one by $Z_h$ is determined by the projection procedure. Note that the latter is also approximation dependent.

In summary, the coupled Yang-Mills–gravity system approaches the large $N_c$ limit via (69). Whether or not this is seen in the current approximation with the cutoff choice (72) is a technical issue. If the approximation admits option (1) then the fixed point can be approached, if (2) or a mixture of (1) and (2) is taken then the fixed point cannot be seen due to the regulator bound in our setup. We emphasize again that this does not entail the nonexistence of the fixed point, which is guaranteed by the analysis of Sec. II. The analysis here evaluates the capability of the approximation to capture this fixed point. The understanding of this structure and guaranteeing this capability of the approximation is of chief importance when evaluating the stability of more complex matter-gravity systems with genuine matter self-interaction: no conclusion concerning the stability of these systems can be drawn if the capability problem for the free-matter–gravity systems is not resolved. Moreover, even if the fixed points exist, their physics may be qualitatively biased by this problem.

### B. Results in the extended approximation

In the following analysis, we concentrate on the $g_a$ fixed point equation and keep $g_c = g$. Before we extend the approximation to this case, let us reevaluate the results with $g_a = g$ in the light of the last Sec. VII A. There it has been deduced that a consistent $N_c$ scaling requires $g^* \propto 1/N_c^2$ and either (70) or (71), or both. Figure 8 shows the consistent large $N_c$ scaling for Newton's coupling but neither (70) nor (71). This comes as a surprise as the system is asymptotically safe and the large $N_c$ limit in the approximation $g = g_a$ is seemingly stable. To investigate this stability, we examine the regulator dependence of the coefficients of the flow equations. To that end, we notice that the coefficients in the $\mu$ equation (and the $g_3$, $g_a$ equations) are of crucial importance for the stability of the system. The coefficient $c_{\mu,a} = -1/(60\pi)\eta_a$ of the Yang-Mills contribution to the graviton mass parameter is proportional to the gluon anomalous dimension $\eta_a$: the leading coefficient vanishes, see (E4) and (G1). Indeed, choosing other regulators, the leading order term is non-vanishing with

$$-0.2 \lesssim c_{\mu,a}(R_k) \lesssim 0.2, \tag{73}$$

see Appendix B. Typically, it supersedes the $\eta_a$-dependent term, and the flat regulator appears to be a very special choice. If $c_{\mu,a} \gtrsim 0.013$, we indeed find a solution, which is consistent with (70), see Fig. 13 for $c_{\mu,a} = \frac{1}{24\pi} \approx 0.0133$. In turn, for $c_{\mu,a} \lesssim -0.005$, we find solutions with growing $\eta_h$, hence in the class (71). Accordingly, this solution is not trustworthy with $\eta_h$ beyond the bound (72). Its failure simply is one of the approximation (within this choice of regulator) rather than that of asymptotic safety.

In summary, this leads us to a classification of the regulators according to the large $N_c$ limit: they either induce the dynamical readjustment of the scales via (70) or via (71) or they fall in between such as the flat cutoff. Within the current approximation it is required that the readjustment happens via (70).

Now we are in the position to discuss the general case with $g_a \neq g$. An optimal scenario would be that the inclusion of the $g_a$ equation already stabilizes the system such that it enforces the dynamical readjustment via (70) for all regulators proportional to $Z_h$. However, as we shall see, the general scheme from the uniform approximation persists with this upgrade of the approximation.

#### 1. No apparent $N_c$ scaling for $\mu$ and $\eta_h$

In the uniform approximation with one Newton's coupling (25), this scenario was taken with regulators with $-0.005 \lesssim c_{\mu,a} \lesssim 0.013$. A typical regulator in this class is the flat regulator used in the present work. This scenario does not enhance the graviton propagator and hence, does not fulfil (69). The stability of the results in the large $N_c$ limit in the uniform approximation must thus rather be considered a mere coincidence. Indeed in the extended truncation with $g \neq g_a$, the enhancement of the graviton propagator is not triggered by the included $g_a$ equation, and consequently, the flat regulator does not have a stable large $N_c$ limit anymore. The fixed point values, critical exponents, and the anomalous dimensions in this approximation are shown in Fig. 14. The fixed point values show a marginal $N_c$ dependence up to the point where the fixed point vanishes into the complex plane at $N_c^2 \approx 13.5$, which is signaled by one of the critical exponents going towards zero. The vanishing critical exponent can be associated with $g_a$. Typically, this is interpreted as a sign for the failure of asymptotic safety. Here, it is evident that the truncation cannot accommodate the dynamical readjustment of the scales that takes place in the full system. This could also signal an overcomplete system: $g$ and $g_a$ are related by diffeomorphism invariance. In any case, the failure of the approximation can either lead to the divergence of the couplings [related to (71)], or in complex parts of the fixed point values. For the flat regulator, the latter scenario is taken.

#### 2. Scenario with $1+\mu \propto 1/N_c^2$

This scenario requires regulators with $c_+ < c_{\mu,a} < c_{\max}$. A typical regulator in this class is the sharp regulator, see (A3) and Fig. 12. Here, we do not present a full analysis of this case but only change the coefficient $c_{\mu,a}$ accordingly. This is justified in terms of linear small perturbations of the system: $c_{\mu,a}$ is the only leading order coefficient in the system that exhibits a qualitative change when changing the regulator away from the flat regulator. Note however, that this change ceases to be small for large $N_c$ as $c_{\mu,a}$ is multiplied by $N_c^2$. If accompanied by a respective change of the relative cutoff scales $k_h/k_a$, this factor could be

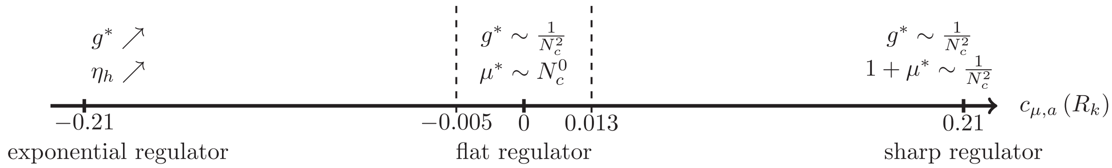


FIG. 12. Schematic picture of the dynamical scale readjustment mechanisms as a function of the coefficient $c_{\mu,a}(R_k)$.

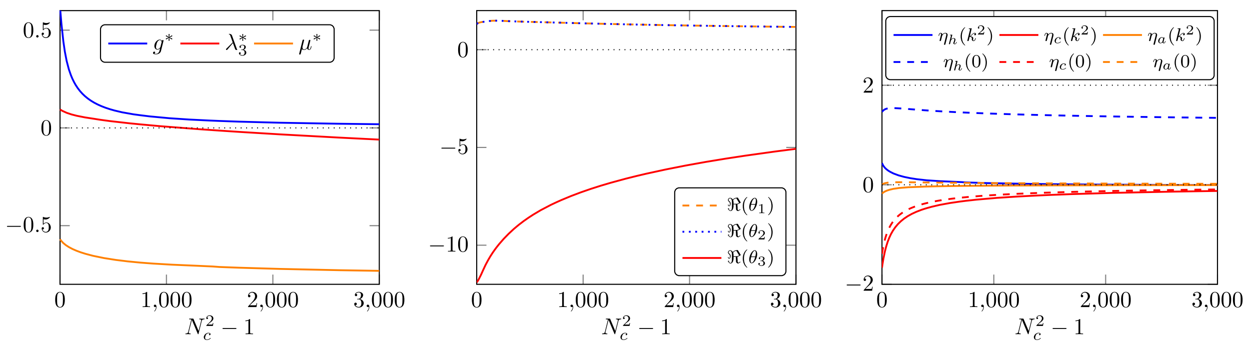


FIG. 13. Properties of the UV fixed point as a function of $N_c^2 - 1$ in the uniform approximation with one Newton's coupling and with $c_{\mu,a} = \frac{1}{24\pi} \approx 0.0133$. Displayed are the fixed point values (left panel), the critical exponents (central panel), and the anomalous dimensions (right panel).

compensated. Then, however, we are directly in the stable regulator choice with (20). Here, we are more interested in the dynamical stabilization, and we refrain from the rescaling. The system exhibits the $1/N_c^2$ scaling in the Newton's couplings, $g^*$ and $g_a^*$, as well as the mass parameter $\mu^*$, see Fig. 15 for $c_{\mu,a} \approx 0.08$. However, with this choice, the critical exponents of the fixed point become rather large. We determined the constant $c_+ \approx 0.07$.

#### 3. Scenario with $\eta_h$ growing large

This scenario requires regulators with $-c_{\min} < c_{\mu,a} < -c_-$. A typical regulator in this class is the exponential regulator, see (A2) and Fig. 12. For this class of regulators, both couplings grow large, and we have the scenario with (71) bound to fail to provide fixed point solutions beyond a maximal $N_c$ due to the failure of the approximation scheme.

### C. Resumé: Signatures of asymptotic safety of Yang-Mills–gravity systems

In summary, with the choice of the regulator, we can dial the different scenarios that all entail the same physics: the dynamical readjustment of the respective scales in the gauge and gravity subsystems and the asymptotic safety of the combined system. The two different scenarios are described in Sec. VII B 2 and Sec. VII B 3. Both scenarios entail the same physics mechanism: the enhancement of the graviton propagator, see (69). This triggers the dominance of gravity in the ultraviolet, which is clearly visible in the consecutive integrating out of degrees of freedom discussed in Sec. II. The crucial property for the validity of this structure is the asymptotic freedom of the Yang-Mills system, and hence, the existence of the gauge system in a given background. This property is trivially present in systems with free

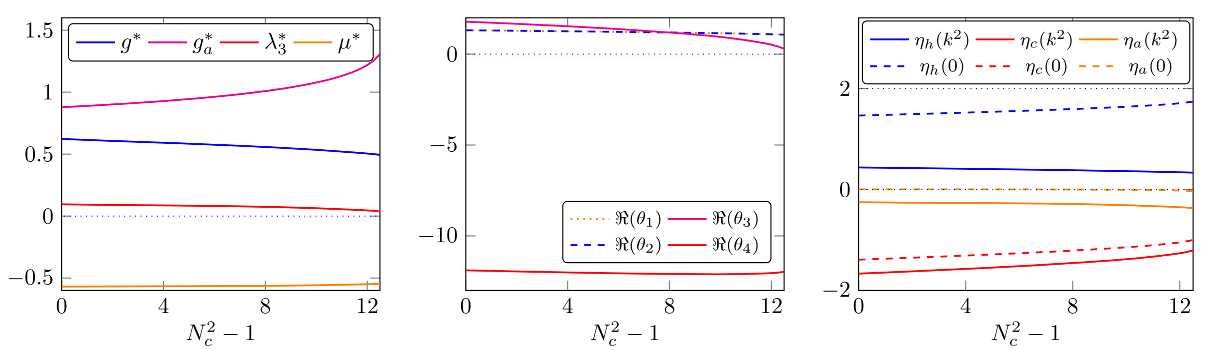


FIG. 14. Properties of the UV fixed point as a function of $N_c^2 - 1$ in the approximation with two Newton's couplings and with the flat regulator, $c_{\mu,a} = 0$. Displayed are the fixed point values (left panel), the critical exponents (central panel), and the anomalous dimensions (right panel).

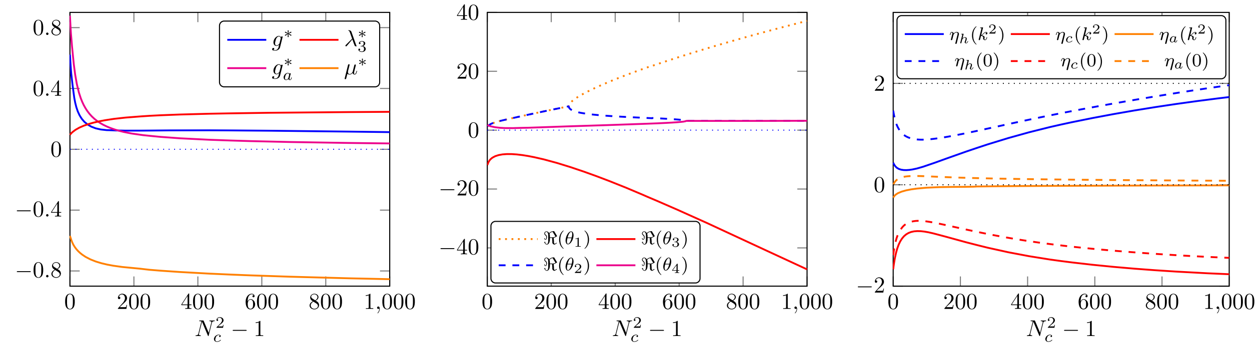


FIG. 15. Properties of the UV fixed point as a function of $N_c^2 - 1$ in the approximation with two Newton's couplings and with $c_{\mu,a} = \frac{1}{4\pi} \approx 0.08$. Displayed are the fixed point values (left panel), the critical exponents (central panel), and the anomalous dimensions (right panel).

matter coupled to gravity, and hence the present analysis extends to these cases.

This leaves us with the question of how to reevaluate the existing results on matter-gravity system in the light of the present findings. We first notice that the helpful peculiarity of the Yang-Mills–gravity system that allowed us to easily access all the different scenarios, is the possibility to choose the sign of $c_{\mu,a}$ with the choice of the regulator. Clearly, the gauge contribution to the running of the graviton mass parameter plays a pivotal role for how the enhancement of the graviton propagator in (69) is technically achieved. In the other matter-gravity system, this parameter has a definite sign, which is why one sees a specific scenario for typical regulators. Collecting all the results and restricting ourselves to truncations that resolve the difference between fluctuation and background fields, [79], we find the following:

(1) Fermion-gravity systems: they fall into the class Sec. VII B 2, and the asymptotic safety of the system can be accessed in the approximation. The required large flavor $N_f$ pattern with (70) is visible in the results.

(2) Scalar-gravity systems: they fall into the class Sec. VII B 3, and for large enough number of scalars $N_s$, the fixed point seemingly disappears due to the fixed point coupling $g^*$ and anomalous dimension $\eta_h$ growing too large.

(3) Vector-gravity/Yang-Mills–gravity systems: this system has been discussed here, and it falls into all classes, Secs. VII B 1, VII B 2, and VII B 3. This also includes the $U(1)$ system.

(4) Self-interacting gauge-matter–gravity systems: these systems only fall into the pattern described in Secs. VII B 1, VII B 2, and VII B 3 if the gauge-matter system is itself ultraviolet stable. For example, one flavor QED exhibits a UV-Landau pole and is stabilized by gravity, which makes the combined system asymptotically safe, for a comprehensive analysis see [85,88]. Adding more flavors potentially destabilizes the system; however, such an analysis has to avoid the interpretation of the seeming failure of asymptotic safety described here. One possibility to take this into account is the scale adjustment (20). This discussion also carries over to general gauge-matter–gravity systems including the Standard Model and its extensions.

In summary, this explains the results obtained in gravitationally interacting gauge-matter–gravity systems, which are the basis of general gauge-matter–gravity system. While it suggests the use of relative cutoff scales such as (20), it still leaves us with the task of devising approximations that are capable of capturing the dynamical readjustment of scales that happens in gravitationally interacting gauge-matter–gravity systems. In particular, the marginal operator $R^2 \ln(1 + R/k_a^{\mathrm{IR}\,2})$, cf. (17), has to be included as discussed in Sec. II B.

Besides this task, the present analysis also requires a careful reanalysis of phenomenological bounds on ultraviolet fixed point couplings. It is well-known that the values of the latter are subject to rescalings and only dimensionless products of couplings such as $g^*\lambda^*$ possibly have a direct physical interpretation. We have argued here that the dynamically adjusted or explicitly adjusted relative cutoff scales ask for a reassessment also of these dimensionless products.

## VIII. SUMMARY AND CONCLUSIONS

We have investigated the prospect for asymptotic safety of gravity in the presence of general matter fields. A main new addition are general arguments, which state that if matter remains sufficiently weakly coupled in the UV, or even free, asymptotic safety for the combined matter-gravity theory follows, in essence, from asymptotic safety of pure gravity (Sec. II). Ultimately, the UV dominance of gravitons relates to the fact that the integrating out of UV-free matter fields only generates local counter terms in the gravitational sector.

Our reasoning has been tested comprehensively for Yang-Mills theory coupled to gravity. Using identical cutoffs for gravity and matter, we invariably find that asymptotic safety arises at a partially interacting fixed point with asymptotic freedom in the Yang-Mills and asymptotic safety in the gravity sector. Fluctuations of the gravitons dominate over those by matter fields including in the asymptotic limit of infinite $N_c$ (Fig. 8). Interestingly, the UV dominance of gravity can materialize itself in different manners (Figs. 13–15), strongly depending on technical parameters of the theory such as the gauge, the regularization, and the momentum cutoff. The overall physics, however, is not affected (Fig. 12). This pattern is reminiscent of how confinement arises in gauge-fixed continuum formulations of QCD. It is also worth noting that the observed $N_c$ independence with identical cutoffs follows automatically, if, instead, "relative cutoffs" for matter and gravity fluctuations are adopted, following (20). This may prove useful for practical studies of gravity-matter systems in set approximations. The necessity for "relative cutoffs" is well-understood in condensed matter systems, albeit for other reasons [112,113].

There are several points that would benefit from further study in the future. While we explained in general terms how findings extend to more general matter sectors (Sec. VII), it would seem useful to further substantiate this in explicit studies. Also, our study highlighted the appearance of logarithmic terms such as $R^2 \ln R$, and similar (Sec. II). These classically marginal terms are of relevance for the question of unitarity of asymptotically safe gravity. It remains to be seen whether they affect the observed $N_c$ independence of gravity-matter fixed points in any significant manner (Sec. VII). Finally, our findings offer a natural reinterpretation of earlier results. It is important to confirm whether this is sufficient to remove a tension amongst previous findings based on different implementations of the renormalization group. Understanding these aspects opens a door towards reliable conclusions for UV completions of the Standard Model or its extensions.

## ACKNOWLEDGMENTS

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## APPENDIX A: REGULATORS

In the present work, we use the optimized or flat regulator [117,118,123,124] for all field modes. Specifically, the superfield regulator at $\bar{g} = 1\!\!1$ and $\bar{A} = 0$ with flat Euclidean background metric is given by

$$R_k^{ij}(p) = \delta^{ij}\Gamma^{(\phi_i\phi_i^*)}(p)|_{\mu=0} r_{\phi_i}(p^2/k^2),$$
$$r(x) = \left(\frac{1}{x} - 1\right)\theta(1-x). \tag{A1}$$

Here, $\phi^*$ is the dual superfield with $\phi^* = (h_{\mu\nu}, -\bar{c}_\mu, c_\mu, A_\mu, -\bar{c}, c)$. The regulator (A1) is diagonal in field space keeping in mind the symplectic metric and allows for analytic expressions of the flow [13]. For the general scaling analysis, we also discuss more general regulators, in particular, we refer to the exponential regulator with

$$r(x) = \frac{1}{\exp(x) - 1}, \tag{A2}$$

and to the sharp cutoff regulator with

$$r(x) = \frac{1}{\theta(x-1)} - 1. \tag{A3}$$

These regulators and variants thereof can be used to scan the space of cutoff functions [125,126].

## APPENDIX B: REGULATOR DEPENDENCE OF THE GLUON CONTRIBUTION TO THE GRAVITON MASS PARAMETER

The coefficient $c_{\mu,a}$, which parameterizes the gluon contribution to the graviton mass parameter, is given by

$$c_{\mu,a} = -\frac{\mathrm{Flow}_a^{(2h)}(p^2=0)}{g(N_c^2-1)}$$
$$= \frac{1}{3\pi}\int \frac{\mathrm{d}x x \dot{r}_h(x)}{(1+r_h(x))^2}\left(\frac{4}{1+r_h(x)} - 3\right), \tag{B1}$$

with $x = \frac{q^2}{k^2}$, $\eta_a = 0$ on the right-hand side and where the angular integration was already performed. We now use that

$$k\partial_k r_h(k,x) = k\frac{\partial x}{\partial k}\partial_x r_h(k,x) = -2x\partial_x r_h(k,x), \tag{B2}$$

and consequently, we get

$$c_{\mu,a} = -\frac{2}{3\pi}\int \mathrm{d}x x^2\left(\partial_x\left(\frac{2}{(1+r_h(x))^2} - 2\right)\right.$$
$$\left. - \partial_x\left(\frac{3}{1+r_h(x)} - 3\right)\right), \tag{B3}$$

where we added zeros in order to perform the partial integration without boundary terms. The result after partial integration is

TABLE I. Gluon contribution to the graviton mass parameter for different regulators. Remarkably, the contribution does not only change in size but also its sign.

| Regulator | $c_{\mu,a}$ |
|---|---|
| $r(x)=\frac{1}{\exp(x)-1}$ | −0.21 |
| $r(x)=\frac{1}{x}\exp(-x^2)$ | −0.027 |
| $r(x)=(\frac{1}{x}-1)\Theta(1-x)$ | 0 |
| $r(x)=\frac{1}{x}\Theta(1-x)$ | 0.034 |
| $r(x)=\frac{10}{x}\Theta(1-x)$ | 0.17 |
| $r(x)=\frac{1}{\Theta(x-1)}-1$ | $\frac{2}{3\pi}\approx 0.21$ |

$$c_{\mu,a}=\frac{4}{3\pi}\int \mathrm{d}x x\frac{r_h(x)(r_h(x)-1)}{(1+r_h(x))^2}. \tag{B4}$$

We have evaluated this integral for different types of regulator shape functions. The results are displayed in Table I. The flat regulator evaluates this integral to zero, while exponential regulators give a positive sign and steplike or sharp regulators even give a negative sign. The usual expectation is that the regulator changes the size of a contribution but not its sign. In this case, however, two diagrams cancel each other approximately and by changing the regulator, we shift the weights between these two diagrams. Thus, any sign of this contribution is possible.

## APPENDIX C: INHOMOGENEOUS FREDHOLM INTEGRAL EQUATIONS OF THE SECOND KIND

In this Appendix, we discuss methods to solve Fredholm integral equations on the example of the gluon anomalous dimension

$$\eta_a(p^2)=f(p^2)+g\int\frac{\mathrm{d}^4q}{(2\pi)^4}K(p,q,\mu,\eta_h)\eta_a(q^2), \tag{C1}$$

see Sec. IV B. Fredholm integral equations of the second kind are a well-known topic in pure and applied mathematics, and there are several methods in order to solve such equations. A straightforward numerical solution is the so-called Nystroem method that is based on discretization of the integral operator with quadratures on $N$ points. By doing so, one obtains Riemann sums that reduce to a system of $N$ linear equations. Moreover, if there exist a solution to (C1), it can be shown by the general theory of such equations that it is unique and the discretized version converges towards this solution in the limit $N\to\infty$. Another method that comes along with less numerical effort are iterative solutions based on the resolvent formalism and the Liouville-Neumann series. The basic idea of this approach is as follows. In order to get a feeling for such integral equations, we observe that for $g=0$, the unique solution to (C1) is trivially given by the inhomogeneity $f(p^2)$. Hence, if $g$ is small in some sense, it seems reasonable that $f(p^2)$ is at least a good zeroth order approximation to the full solution $\eta_a(p^2)$, i.e. $\eta_a(p^2)\approx\eta_{a,0}(p^2)\equiv f(p^2)$. In a first iteration step, we substitute $\eta_{a,0}(q^2)$ for $\eta_a(q^2)$ under the integral on the right-hand side of the integral equation (C1),

$$\eta_{a,1}(p^2)=f(p^2)+g\int\frac{\mathrm{d}^4q}{(2\pi)^4}K(p,q,\mu,\eta_h)\eta_{a,0}(q^2). \tag{C2}$$

In this spirit, we can construct iteratively a sequence $(\eta_{a,i}(p^2))_{i\in\mathbb{N}}$ with

$$\eta_{a,i+1}(p^2)=f(p^2)+g\int\frac{\mathrm{d}^4q}{(2\pi)^4}K(p,q,\mu,\eta_h)\eta_{a,i}(q^2). \tag{C3}$$

The convergence properties depend on the kernel $K$ and the coupling constant $g$. We observe that due to the regulator structure, the kernel $K$ is proportional to $r_a(q^2)$. Therefore, the kernel is integrable with respect to the loop momentum $q$. For the sake of simplicity, we will assume in the following a flat regulator $r_a(q^2)\sim\theta(1-q^2)$, where $q$ is the dimensionless momentum. The discussion can be generalized straightforwardly to arbitrary regulators. With a flat regulator, we write $K(p,q)=:\theta(1-q^2)\check{K}(p,q)$. As a consequence, the integral in the Fredholm equation is defined on the domain [0, 1], and in all equations, $K$ is substituted by $\check{K}$. Moreover, we define the angular averaged kernel

$$\langle\check{K}\rangle_\Omega(p,q,\mu,\eta_h)\coloneqq\int_{S^3}\frac{\mathrm{d}\Omega}{(2\pi)^4}\check{K}(p,q,x,\mu,\eta_h), \tag{C4}$$

where $\mathrm{d}\Omega$ is the canonical measure on the three sphere. The kernel $\langle\check{K}\rangle_\Omega$ can be normed, in particular, it exists its 2-norm with respect to the first two arguments

$$||\langle\check{K}\rangle_\Omega||_2\coloneqq\left(\int_0^1\int_0^1\mathrm{d}q\mathrm{d}p|\langle\check{K}\rangle_\Omega(p,q,\mu,\eta_h)|^2\right)^{1/2}. \tag{C5}$$

It can then be shown that the sequence $(\eta_i(p^2))_{i\in\mathbb{N}}$ converges towards the full solution, i.e.,

$$\lim_{i\to\infty}\eta_{a,i}(p^2)=\eta_a(p^2), \tag{C6}$$

if the kernel is bounded as

$$|g|||\langle\check{K}\rangle_\Omega||_2<1. \tag{C7}$$

The solution can then be written as a Liouville-Neumann series according to

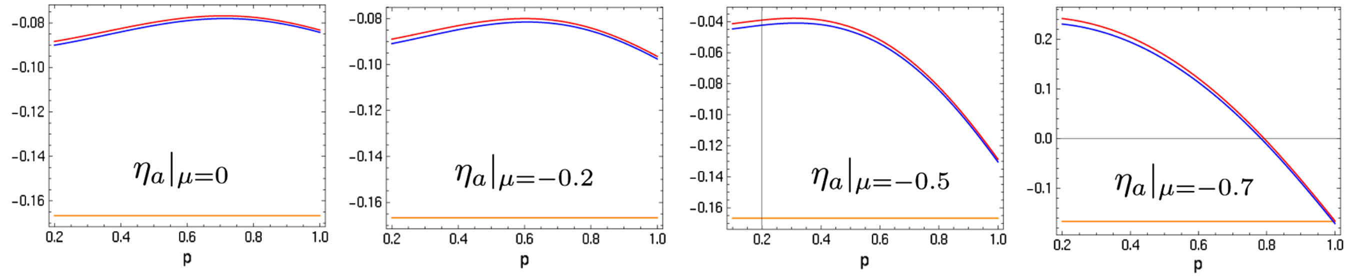


FIG. 16. Shown is the momentum dependence of the graviton contribution to the gluon anomalous dimension $\eta_a$ for different values of the graviton mass parameter $\mu = 0, -0.2, -0.5$, and $-0.7$ (from left to right). In each case, starting with a flat trial function (orange), a fast convergence from first (blue) to second (red) order in the iteration (C3) is observed ($g = 0.5$ and $\eta_h = 0.5$).

$$\eta_a(p^2) = f(p^2) + g\int_{\mathbb{R}^4} \frac{d^4q}{(2\pi)^4} R(p,q,\mu,\eta_h,g) f(q^2), \tag{C8}$$

with the resolvent kernel

$$R(p,q,\mu,\eta_h,g) = \sum_{i=1}^{\infty} g^{i-1} K_i(p,q,\mu,\eta_h), \tag{C9}$$

where $K_i$ are the iterated kernels given by

$$\begin{aligned} K_i(p,q,\mu,\eta_h) = &\int\int \dots \int \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\cdots\frac{d^4q_{i-1}}{(2\pi)^4} \\ &\times K(p,q_1,\mu,\eta_h)K(q_1,q_2,\mu,\eta_h)\times\dots \\ &\times K(q_{i-1},q,\mu,\eta_h). \end{aligned} \tag{C10}$$

By truncating the resolvent series at some finite order $i_0$, one obtains an approximate solution to the integral equation. If the bound (C7) is satisfied, the Liouville-Neumann series converges for any smooth initial choice $\eta_{a,0}$. One can also choose zeroth iterations that are different from the inhomogeneity $f(p^2)$. It is clear that convergence properties depend on the initial choice. For instance, if one has the correct guess for the full solution and uses this as a starting point for the iteration, then one finds $\eta_{a,0} = \eta_{a,1}$, and one can conclude that the exact solution has been found. Additionally, there are improved iteration schemes that increase the radius of convergence significantly. In [127], it has been proven that it exists a parameter $c \in \mathbb{R}$, such that the iteration prescription

$$\begin{aligned} \eta_{a,i+1}(p^2) = &(1-c)f(p^2) + c\eta_{a,i}(p^2) \\ &+ (1-c)g\int \frac{d^4q}{(2\pi)^4} K(p,q,\mu,\eta_h)\eta_{a,i}(q^2) \end{aligned} \tag{C11}$$

has a radius of convergence that is larger than the one of the standard Liouville-Neumann series, which is obtained from the improved iterations with $c = 0$.

The convergence in the present system is analyzed in Fig. 16. We plot $\eta_a(p^2)$ for some specific parameter values. All these plots are obtained for $g = 0.5$; however, we stress that the sign of $\eta_a$ does not depend on this choice as the result is a power series in $g$. We investigate the iterations, where we have always assumed a constant function $\eta_{a,0} = \text{const}$ as a first approximation. We then plot the first, second, and third order and find rapid convergence in all cases, which is expected as we have checked that the kernel in (41) generates a very large radius of convergence. The third iteration is for this choice of $\eta_{a,0}$ not even visible any more, since the corresponding curve lies exactly on top of the second iteration.

## APPENDIX D: SIGN OF THE GLUON ANOMALOUS DIMENSION

In this appendix, we discuss the stability of the sign of the gluon anomalous dimension. As discussed in Sec. IV, we need a negative sign in order to obtain asymptotic freedom in the gauge sector. This directly corresponds to the demand that the gravity contributions to the gluon anomalous dimension should be negative. In the Appendix C, we discussed the full momentum dependent solution of $\eta_a(p^2)$. We further argued in Sec. IV that the sign at $p^2 = k^2$ is the decisive one for the Yang-Mills beta function. In the following sections, we present different approximations to the gluon anomalous dimension, and how stable the sign is within these approximations.

### 1. Derivative at vanishing momentum

The simplest approximation is to assume a momentum independent anomalous dimension and to obtain an equation for $\eta_a$ with a derivative at $p^2 = 0$. The equation for $\eta_a$ is then given by

$$\eta_{a,h} = -\partial_{p^2}\text{Flow}_h^{(AA)}\Big|_{p^2=0}. \tag{D1}$$

We obtain the analytic result

$$\eta_{a,h} = -\frac{g}{8\pi}\left(\frac{8-\eta_a}{1+\mu} - \frac{4-\eta_h}{(1+\mu)^2}\right), \tag{D2}$$

which is identical to the $\eta_a$ in the UV if the gauge sector is asymptotically free. Therefore, assuming a fixed point in the gravitational sector, we are left with the ultraviolet limit

$$\eta_a^* = \frac{g^*}{1-\frac{g^*}{8\pi(1+\mu^*)}}\left(\frac{4+8\mu^*+\eta_h^*}{8\pi(1+\mu^*)^2}\right). \tag{D3}$$

This function changes sign at the critical value

$$\mu_{\text{crit}}^* = -\frac{1}{8}(4+\eta_h^*). \tag{D4}$$

Moreover, there is a pole at $\mu^* = -1 + \frac{g^*}{8\pi}$ with another sign change for the regimes to the left and to the right of the pole. However, this sign change at the pole can be neglected, as usual fixed point values of $g$ are $\mathcal{O}(1)$. For fixed point values of this order, the pole is located at $\mu^* \approx -0.96$, which in turn is a fixed point value that is very unusual. Therefore, we assume the overall prefactor in (D3) to be positive. Then, $\eta_a^* \gtrless 0$ for $\mu^* \lessgtr -\frac{1}{8}(4+\eta_h^*)$. This agrees with previous computations in the background field approximation, where $\eta_h^* = -2$ and $\mu = -2\lambda$, and consequently, $\lambda_{\text{crit}}^* = \frac{1}{8}$ [68]. In our more general case, the anomalous dimension of the graviton is not fixed by the fixed point condition for Newton's coupling. The fixed point value for the graviton mass parameter where the gravitational contribution changes sign is plotted against the graviton anomalous dimension in the left panel of Fig. 17. There are some bounds on anomalous dimensions for well-defined theories. From previous results [24,26,31,36,61,79], we know that typical fixed point values are roughly given by $\eta_h \approx 1$ and $\mu \approx -0.6$, which is just at the critical value where asymptotic freedom is lost.

We conclude that in this simplest approximation the stability of asymptotic freedom is not guaranteed, but depends strongly on subtle effects in the gravity sector. In the following, we investigate how this picture changes in more elaborate approximations and specifications.

### 2. Derivative at nonvanishing momentum

We now generalize the procedure from the previous section and use a derivative at finite momentum. The equation for $\eta_a$ is then given by

$$\eta_{a,h} = -\partial_{p^2}\text{Flow}_h^{(AA)}|_{p=\alpha k}. \tag{D5}$$

For such derivatives, the results are only numerical. In Fig. 17, we show the results for $\alpha = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$. We again display the sign of the gluon anomalous dimension in the $(\mu^*, \eta_h^*)$ plane. We find the encouraging result that the area, which does not support asymptotic freedom in the gauge sector is getting smaller with an increasing $\alpha$. With a derivative at $p^2 = k^2$, the region has completely disappeared from the investigated area. We conclude that with this generalized derivation of the gluon anomalous dimension, asymptotic freedom is supported in the whole important parameter region of gravity.

### 3. Finite differences

A further generalization of the procedure from the previous sections is to derive the gluon anomalous dimension by a finite difference. In this case, we define $\eta_a$ to be momentum dependent. It is then given by

$$\eta_{a,h}(p^2) = -\frac{\text{Flow}_h^{(AA)}(p^2) - \text{Flow}_h^{(AA)}(0)}{p^2}. \tag{D6}$$

The corresponding results are presented in Fig. 18 for $p = \frac{1}{2}, \frac{3}{4}, 1$. The results are very similar to the ones with the derivative definition at nonvanishing momentum. The gluon anomalous dimension is negative and supports asymptotic freedom if we evaluate it at $p^2 = k^2$. This is also the approximation for $\eta_a$ that we utilize throughout this work and also Fig. 4 is computed with this approximation.

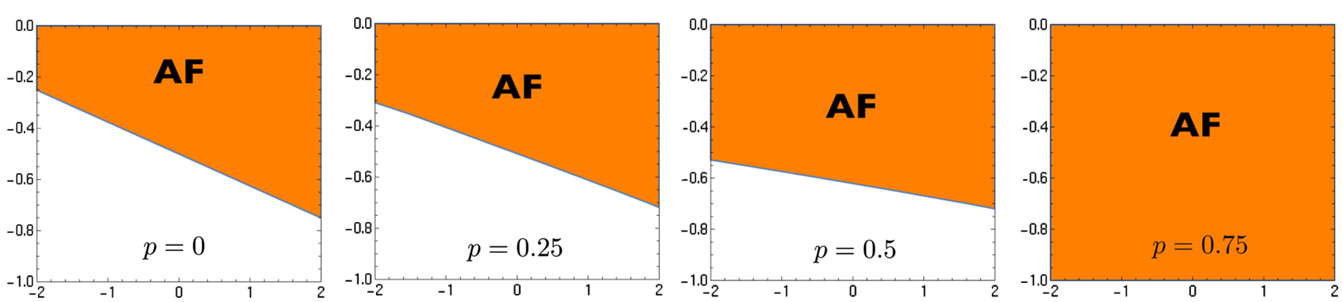


FIG. 17. In the plane of the graviton anomalous dimension ($\eta_h^*$, lower axis) and the graviton mass parameter $\mu^*$, the region with asymptotic freedom (AF) is colored (orange) corresponding to a positive sign of the gluon anomalous dimension $\eta_a$. Moreover, the gluon anomalous dimension is determined from a momentum derivative evaluated at different momenta $p = 0, 0.25, 0.5$, and 0.75 (from left to right). The domain with asymptotic freedom consistently grows as soon as momenta of order of the RG scale are adopted.

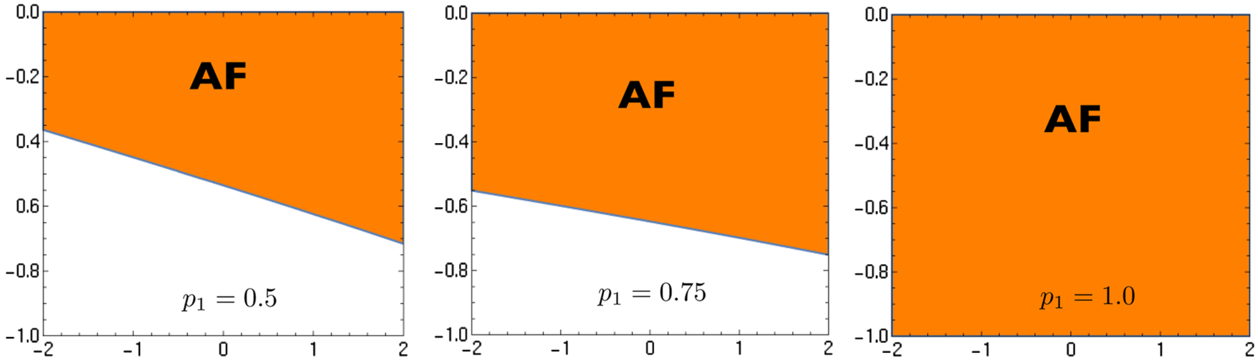


FIG. 18. Same as Fig. 17, except that the gluon anomalous dimension is determined from a finite difference derivative (D6) with $p_2 = 0$ and various momenta $p_1 = \frac{1}{2}, \frac{3}{4}, 1$ (from left to right). The domain with asymptotic freedom consistently grows with growing $p_1 - p_2$ of the order of the RG scale, fully consistent with Fig. 16.

## APPENDIX E: SCALING EQUATIONS

In this appendix, we augment the analysis from Sec. VII by providing scaling equations for all couplings. In particular, we are lifting the identification (25). Here, we extract the fixed point scaling from a flat regulator choice and utilize a reparameterization of the flow equations that minimizes the occurrence of factors of $1+\mu$. Moreover, in the previous chapter, we have utilized projections on gravitational couplings $g_n$ and $g_{aah}$ within a finite difference construction. In the literature, projections with derivatives at vanishing momentum, $p^2 = 0$, are often used. It has been argued in [24,26,31,36,61,79,85] that this definition has large ambiguities at $p^2 = 0$, which limits its applicability. Still, it has the charm of providing analytic flows and fixed point equations and hence facilitating the access to the current analysis.

The structure of the flow and fixed point equations is more apparent if we absorb $1/(1+\mu)$-factors in the gravitational couplings with

$$\bar{g}_n = g_n \left(\frac{1}{1+\mu}\right)^{\gamma_n}, \qquad \bar{g}_{\bar{c}ch^n} = g_{\bar{c}ch^n} \left(\frac{1}{1+\mu}\right)^{\gamma_c},$$
$$\bar{g}_{a^n h^m} = g_{a^n h^m} \left(\frac{1}{1+\mu}\right)^{\gamma_a}, \tag{E1a}$$

with the scaling coefficients

$$\gamma_n = \frac{n}{n-2}, \qquad \gamma_a = \gamma_c = 1, \tag{E1b}$$

and $\mu, \lambda_n$ are not rescaled. This removes all potentially singular factors $1/(1+\mu)$-factors in the diagrams that stem from the respective powers of the graviton propagators in the loops. It still leaves us with contributions proportional to $1/(1+\mu)$ due to the projection procedure with derivatives at $p^2 = 0$ and due to regulator insertions. The rescaling power of $1/(1+\mu)$ varies between $1/(1+\mu)^3$ for the lowest coupling $g_3$ and $1/(1+\mu)$ for $g_{n\to\infty}$.

In the following equations, we identify blocks of gravitational couplings: as before all gravitational self-couplings $\bar{g}_n$, $\bar{g}_{\bar{c}ch^n}$ are identified with $\bar{g}_3$ and all $\lambda_n$ are identified with $\lambda_3$. Additionally, we identify all Yang-Mills–gravity interactions $\bar{g}_{aah^n}$ with $\bar{g}_{aah}$. This leads us to

$$\bar{g}_n = \bar{g}_3 = \bar{g}, \qquad \lambda_{n>2} = \lambda_3, \tag{E2a}$$

for the pure gravity couplings and

$$\bar{g}_{\bar{c}ch^n} = \bar{g}_c, \qquad \bar{g}_{aah^n} = \bar{g}_a, \tag{E2b}$$

for the ghost-graviton and gluon-graviton couplings. We emphasize that (E2) and (E1a) imply

$$g_n = g_3 (1+\mu)^{\gamma_3 - \gamma_n}, \tag{E3}$$

with $\gamma_3 > \gamma_n$. Equation (E3) seemingly entails the irrelevance of the lower order couplings $g_n$ for $\mu \to -1$. However, the lower order couplings contribute to diagrams with more graviton propagators. In combination, this leads to a uniform scaling of all diagrams as expected in a scaling limit. Note that the scaling analysis can also be performed if removing the approximation (E2). It leads to an identical scaling $\bar{g}_n \sim \bar{g}_3$ and $\bar{g}_{aah^n} \sim \bar{g}_{aah}$. The discussion of such a full analysis is deferred to future work.

Here, we are only interested in the relative scaling between the pure gravity and Yang-Mills gravity diagrams, and simply discuss the structure of these equations. To that end, we use the analytic pure gravity equations derived in [31,36] expressed with the rescaled couplings (E1). We also use the identification (E2), and additionally, we suppress the ghost contribution for simplicity. The ghost contribution comes with the same power in $1+\mu$ as the gluon contribution. The analysis is facilitated by only using positive coefficients $c_i$, $d_i$, making the relative signs of the different terms apparent. In general, the sign of some of these coefficients depends on $\lambda_3$, and we define them such that they are positive at $\lambda_3 = 0$. The explicit values for the coefficients is provided in Appendix G. Within this notation, all factors $1/(1+\mu)$ in the loops are absorbed in the couplings except the one, which comes from external momentum derivatives of propagators, $\partial_{p^2} G$, due to the projection procedure or from regulator insertions. In summary, we are led to

$$\dot{\mu} = -(2-\eta_h)\mu - \bar{g}a\left[c_{\mu,h} + (1+\mu)(N_c^2-1)c_{\mu,a}\frac{\bar{g}_a}{\bar{g}}\right],$$

$$\begin{aligned}\dot{\bar{g}} &= (2+3\bar{\eta}_h)\bar{g}\\ &\quad - \bar{g}^2\left[\frac{c_{\bar{g},h}}{1+\mu} + \frac{d_{\bar{g},h}}{(1+\mu)^2} + (N_c^2-1)c_{\bar{g},a}\left(\frac{\bar{g}_a}{\bar{g}}\right)^{\frac{3}{2}}\right],\\ \dot{\lambda}_3 &= -\left(1+\frac{\partial_t\bar{g}}{2\bar{g}} - \frac{3}{2}\bar{\eta}_h\right)\lambda_3\\ &\quad + \bar{g}\left[\frac{c_{\lambda_3,h}}{1+\mu} + (N_c^2-1)c_{\lambda_3,a}\left(\frac{\bar{g}_a}{\bar{g}}\right)^{\frac{3}{2}}\right],\end{aligned} \tag{E4a}$$

for the pure gravity couplings. Here, the term $d_{\bar{g},h}/(1+\mu)^2$ stems from the $\partial_{p^2}G$ contributions, and all coefficients $c$, $d$ from graviton loops depend on $\lambda_3$ with $c(0)$, $d(0) > 0$. The ghost-graviton and the gauge-graviton coupling have the flows

$$\begin{aligned}\dot{\bar{g}}_a &= (2+2\eta_a+\bar{\eta}_h)\bar{g}_a - \bar{g}_a^2\left[-c_{\bar{g}_a,a} + \frac{d_{\bar{g}_a,a}}{1+\mu}\right.\\ &\quad \left. + \left(c_{\bar{g}_a,h} - \frac{d_{\bar{g}_a,h}}{1+\mu}\right)\left(\frac{\bar{g}}{\bar{g}_a}\right)^{\frac{1}{2}}\right].\\ \dot{\bar{g}}_c &= (2+2\eta_c+\bar{\eta}_h)\bar{g}_c - \bar{g}_c^2\left[c_{\bar{g}_c,c} + \frac{d_{\bar{g}_c,c}}{1+\mu}\right.\\ &\quad \left. + \left(c_{\bar{g}_c,h} + \frac{d_{\bar{g}_c,h}}{1+\mu}\right)\left(\frac{\bar{g}}{\bar{g}_c}\right)^{\frac{1}{2}}\right].\end{aligned} \tag{E4b}$$

Here, the $d$ terms originate from the diagram with a regularized graviton line, $(G\partial_t R_k G)^{(hh)}$. The coefficients $c_{i,h}$ and $d_{i,h}$ are $\lambda_3$ dependent as they receive contributions from the diagram with a three-graviton vertex. The signs are chosen such that $c_{i,h}(0), d_{i,h}(0) > 0$. The coefficients and the signs in the flow equation for $\bar{g}_c$ were not derived in this work.

The rescaled graviton anomalous dimension $\bar{\eta}_h$ reads

$$\bar{\eta}_h = -\frac{\partial_t[Z_h(1+\mu)]}{Z_h(1+\mu)} = \eta_h - \frac{\dot{\mu}}{1+\mu}, \tag{E5}$$

which includes the scale dependence of the full dressing of the graviton propagator including the mass parameter. The set of anomalous dimensions is given by

$$\begin{aligned}\eta_h &= \bar{g}\left[c_{\eta_h,h} + \frac{d_{\eta_h,h}}{1+\mu} + (N_c^2-1)c_{\eta_h,a}\frac{\bar{g}_a}{\bar{g}}\right],\\ \eta_c &= -\bar{g}\left[c_{\eta_c,h} + \frac{d_{\eta_c,h}}{1+\mu}\right],\\ \eta_a &= -\bar{g}_a\left[c_{\eta_a,h} - \frac{d_{\eta_a,h}}{1+\mu}\right],\end{aligned} \tag{E6}$$

and completes the set of flow equations. Again, the graviton contributions to $\eta_h$ have a $\lambda_3$ dependence with $c_{\eta_h,h}(0)$, $d_{\eta_h,h}(0) > 0$. All other coefficients do not carry a $\lambda_3$ dependence. Note also that the $\partial_t\mu/(1+\mu)$ terms in the scaling terms on the right-hand side of (E4) come from the normalization of the $\bar{g}$'s with powers of $1/(1+\mu)$. In the $\bar{g}_n$ flows, this term is $n/(n-2)\partial_t\mu/(1+\mu)$ derived from the rescaling (E1a). For the ghost-gravity and gauge gravity couplings, it is always the term $\partial_t\mu/(1+\mu)$ derived from (E1).

## APPENDIX F: FLOW EQUATIONS

Here, we recall the results for the pure gravity flow for $\mu$, $g_3$, and $\lambda_3$ derived in [31,36], add the derived gluon contributions, and formulate them in terms of the rescaled couplings

$$\begin{aligned}\bar{g}_n &= g_n\left(\frac{1}{1+\mu}\right)^{\frac{n}{n-2}}, &\quad \bar{g}_c &= g_c\left(\frac{1}{1+\mu}\right),\\ \bar{g}_a &= g_a\left(\frac{1}{1+\mu}\right), &\quad \bar{\eta}_h &= \eta_h - \frac{\dot{\mu}}{1+\mu},\end{aligned} \tag{F1}$$

see Appendix E and (E1) for details. In order to show the interrelation of the different couplings, we keep all dependences on the higher couplings $\bar{g}_n$. The flow equations are given by

$$\begin{aligned}\partial_t\mu &= -(2-\eta_h)\mu + \frac{\bar{g}_3}{180\pi}[21(10-\eta_h) - 120\lambda_3(8-\eta_h) + 320\lambda_3^2(6-\eta_h)]\\ &\quad - \frac{\bar{g}_4}{12\pi}[3(8-\eta_h) - 8\lambda_4(6-\eta_h)] - (1+\mu)\frac{\bar{g}_c}{5\pi}(10-\eta_c) + (1+\mu)(N_c^2-1)\frac{\bar{g}_a\eta_a}{60\pi},\\ \partial_t\lambda_3 &= -\left(1+\frac{\partial_t\bar{g}_3}{2\bar{g}_3} - \frac{3}{2}\bar{\eta}_h\right)\lambda_3 + \bar{g}_3\left\{-\frac{1}{1+\mu}\frac{1}{240\pi}[11(12-\eta_h) - 72\lambda_3(10-\eta_h) + 120{\lambda_3}^2(8-\eta_h) - 80\lambda_3^3(6-\eta_h)]\right.\\ &\quad + \frac{1}{6\pi}\frac{1}{1+\mu}\frac{\bar{g}_4}{\bar{g}_3}[3\lambda_4(8-\eta_h) - 16\lambda_3\lambda_4(6-\eta_h)] + \frac{1}{8\pi}\frac{1}{1+\mu}\left(\frac{\bar{g}_5}{\bar{g}_3}\right)^{\frac{3}{2}}[(8-\eta_h) - 4\lambda_5(6-\eta_h)]\\ &\quad \left. + \frac{1}{10\pi}\left(\frac{\bar{g}_c}{\bar{g}_3}\right)^{\frac{3}{2}}(12-\eta_c) + \frac{1}{60\pi}(N_c^2-1)\left(\frac{\bar{g}_a}{\bar{g}_3}\right)^{\frac{3}{2}}(3-\eta_a)\right\},\end{aligned}$$

$$
\begin{aligned}
\partial_t \bar{g}_3 &= (2+3\bar{\eta}_h)\bar{g}_3 - \frac{\bar{g}_3^2}{19\pi}\left\{\frac{1}{(1+\mu)^2}\frac{2}{15}[229-1780\lambda_3+3640\lambda_3^2-2336\lambda_3^3]\right.\\
&\quad -\frac{1}{1+\mu}\frac{1}{180}[147(10-\eta_h)-1860\lambda_3(8-\eta_h)+3380\lambda_3^2(6-\eta_h)+25920\lambda_3^3(4-\eta_h)]\\
&\quad -\frac{1}{1+\mu}\frac{\bar{g}_4}{\bar{g}_3}\left[\frac{1}{18}[45(8-\eta_h)-8(30\lambda_3-59\lambda_4)(6-\eta_h)-360\lambda_3\lambda_4(4-\eta_h)]+\frac{16}{1+\mu}(1-3\lambda_3)\lambda_4\right]\\
&\quad \left.+\frac{1}{1+\mu}\frac{47}{6}\left(\frac{\bar{g}_5}{\bar{g}_3}\right)^{\frac{3}{2}}(6-\eta_h)+\left(\frac{\bar{g}_c}{\bar{g}_3}\right)^{\frac{3}{2}}\left[\frac{50-53\eta_c}{10}\right]+(N_c^2-1)\left(\frac{\bar{g}_a}{\bar{g}_3}\right)^{\frac{3}{2}}\left[\frac{133+\eta_a}{30}\right]\right\},\\
\partial_t \bar{g}_a &= (2+2\eta_a+\bar{\eta}_h)\bar{g}_a - \frac{\bar{g}_a^2}{30\pi}\left\{-\frac{100-13\eta_a}{2}+\frac{13(5-\eta_h)}{\mu+1}\right.\\
&\quad \left.+\left(\frac{\bar{g}_3}{\bar{g}_a}\right)^{\frac{1}{2}}\left(\frac{330-640\lambda_3-\eta_a(33-80\lambda_3)}{12}+\frac{-15+400\lambda_3-\eta_h(80\lambda_3-6)}{3(\mu+1)}\right)\right\},
\end{aligned}
\tag{F2}
$$

and the anomalous dimension read

$$
\begin{aligned}
\eta_h &= \frac{\bar{g}_3}{4\pi}\left(\frac{\bar{g}_4}{\bar{g}_3}(6-\eta_h)\frac{6(8-\eta_h)+8(6-\eta_h)\lambda_3-36(4-\eta_h)\lambda_3^2}{9}+\frac{17+8\lambda_3(9\lambda_3-8)}{3(1+\mu)}-\frac{\bar{g}_c}{\bar{g}_3}\eta_c+(N_c^2-1)\frac{\bar{g}_a}{\bar{g}_3}\frac{1+\eta_a}{3}\right),\\
\eta_c &= -\frac{\bar{g}_c}{9\pi}\left(\frac{8-\eta_h}{1+\mu}+8-\eta_c\right), \qquad \eta_a = -\frac{\bar{g}_a}{8\pi}\left(8-\eta_a-\frac{4-\eta_h}{1+\mu}\right).
\end{aligned}
\tag{F3}
$$

The two terms in the flow equation for $\bar{g}_3$ proportional to $1/(1+\mu)^2$ and the term in $\eta_h$ proportional to $1/(1+\mu)$ signal the derivative expansion at $p^2=0$. This is the price to pay for an analytic flow equation. On the other hand, the terms proportional to $1/(1+\mu)$ in $\bar{g}_a$, $\eta_a$ and $\eta_c$ come from a regulator insertion in a graviton propagator compared to a ghost or gluon propagator.

The computation of these flow equations involves contractions of very large tensor structures. These contractions are computed with the help of the symbolic manipulation system FORM [128,129]. We furthermore employ specialized MATHEMATICA packages. In particular, we use *xPert* [130] for the generation of vertex functions, and the *FormTracer* [131] to trace diagrams.

## APPENDIX G: COEFFICIENTS IN THE SCALING EQUATIONS

The coefficients in the scaling equations in Appendix E are given here in the approximation (E2). We assume that the anomalous dimensions satisfy $|\eta| \leq 2$: they should not dominate the scaling of the regulator. While the upper bound $\eta \leq 2$ is a (weak) consistency bound for the regulator, for a detailed discussion, see [79]; the lower one can be seen as a (weak) consistency bound on the propagators. For $\eta < -2$, they cease to be well-defined as Fourier transforms of space-time correlations functions (if they scale universally down to vanishing momenta). For simplicity, we display the coefficients with $\lambda_3 = 0$. Note that all coefficients are defined such that they are always positive. All coefficients can be directly read off from the Eqs. (F2) and (F3).

We get the coefficients $c_{\mu,h}$ and $c_{\mu,a}$ in the fixed point equation of the mass parameter $\mu$ are given by

$$
c_{\mu,h} = \frac{17}{6\pi} - \frac{2}{15\pi}\eta_h - \frac{1}{5\pi}\eta_c, \qquad c_{\mu,a} = -\frac{1}{60\pi}\eta_a. \tag{G1}
$$

Note that the second coefficient is positive since $\eta_a < 0$. The coefficients $c_{\bar{g},h}$ and $c_{\bar{g},a}$ in the fixed point equation of the pure gravity coupling $\bar{g}$ read

$$
\begin{aligned}
c_{\bar{g},h} &= \frac{47}{57\pi} - \frac{53}{190\pi}\eta_h - \frac{37}{190\pi}\eta_c, \qquad d_{\bar{g},h} = \frac{598}{285\pi},\\
c_{\bar{g},a} &= \frac{7}{30\pi} + \frac{1}{570\pi}\eta_a,
\end{aligned}
\tag{G2}
$$

while the coefficients $c_{\lambda_3,h}$ and $c_{\lambda_3,a}$ in the fixed point equation of the coupling $\lambda_3$ are given by

$$
\begin{aligned}
c_{\lambda_3,h} &= \frac{33}{20\pi} - \frac{19}{240\pi}\eta_h - \frac{1}{10\pi}\eta_c,\\
c_{\lambda_3,a} &= \frac{3}{60\pi} - \frac{1}{60\pi}\eta_a.
\end{aligned}
\tag{G3}
$$

Furthermore, the coefficient $c_{\bar{g}_a}$ in the fixed point equation for the two-gluon–graviton coupling $\bar{g}_a$ reads

$$c_{\bar{g}_a,a} = \frac{5}{3\pi} - \frac{13}{60\pi}\eta_a, \qquad d_{\bar{g}_a,a} = \frac{13}{6\pi} - \frac{13}{30\pi}\eta_h, \qquad c_{\bar{g}_a,h} = \frac{11}{12\pi} - \frac{11}{120\pi}\eta_a, \qquad d_{\bar{g}_a,h} = \frac{1}{6\pi} - \frac{1}{15\pi}\eta_h. \tag{G4}$$

We also summarize the coefficients of the anomalous dimensions, to wit

$$c_{\eta_h,h} = \frac{1}{6\pi} - \frac{1}{12\pi}\eta_h - \frac{1}{4\pi}\eta_c, \qquad d_{\eta_h,h} = \frac{17}{12\pi}, \qquad c_{\eta_h,a} = \frac{1}{12\pi} + \frac{1}{12\pi}\eta_a, \qquad c_{\eta_c} = \frac{8}{9\pi} - \frac{1}{9\pi}\eta_c, \qquad d_{\eta_c} = \frac{8}{9\pi} - \frac{1}{9\pi}\eta_h,$$
$$c_{\eta_a} = \frac{1}{\pi} - \frac{1}{8\pi}\eta_a, \qquad d_{\eta_a} = \frac{1}{2\pi} - \frac{1}{8\pi}\eta_h. \tag{G5}$$


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