

Radiative Kähler moduli stabilization

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We propose a new type of Kähler moduli stabilization mechanisms in type IIB superstring theory on Calabi-Yau manifolds with the positive Euler number. The overall Kähler modulus can be perturbatively stabilized by radiative corrections due to sparticles. Its minimum is the anti-de Sitter vacuum, where supersymmetry is broken. We can uplift it to the de Sitter vacuum by introducing anti-D-branes, keeping the modulus stabilized. Although our numerical results depend on the choice of the cutoff scale and degeneracies of sparticles, at any rate there exist the parameter spaces where the masses of Kaluza-Klein and stringy modes are larger than the cutoff scale. Furthermore, this stabilization scenario predicts an ultralight axion.

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I. INTRODUCTION

The moduli fields appear ubiquitously in four-dimensional low-energy effective field theory derived from superstring theory on six-dimensional compact space. These fields correspond to a geometrical character of compact space. Unless these moduli fields are stabilized at a high scale, it will lead to the fifth force. Hence, moduli stabilization is one of the major topics in the string cosmology and phenomenology.

So far, moduli stabilization has been well studied in the type IIB superstring theory on Calabi-Yau (CY) manifolds. The closed string moduli are categorized into the dilaton S , complex structure moduli U and Kähler moduli T .¹ Here, we denote the numbers of complex structure moduli and Kähler moduli by $h^{2,1}$ and $h^{1,1}$, which correspond to the numbers of three-cycles and four-cycles of CY, respectively. Three-form fluxes in the type IIB superstring theory can stabilize the dilaton S and all the complex structure moduli U at the compactification scale [3] and they generically generate the nonvanishing flux-induced superpotential $W_0 = \langle W_{\text{flux}} \rangle$ [4]. The remaining Kähler moduli are stabilized at an anti-de Sitter minimum in the

Kachru-Kalosh-Linde-Trivedi (KKLT) scenario [5] and large volume scenario (LVS) [6] using the nonperturbative superpotential for the Kähler moduli. Such an anti-de Sitter minimum is uplifted to a metastable de Sitter minimum by introducing anti-D3 branes [5]. The F -term uplifting scenario is another way to realize the de Sitter minimum [7,8]. In the KKLT scenario, the stabilization of Kähler moduli is achieved by tuning $|W_0| \ll M_{\text{Pl}}^3$, where M_{Pl} is the reduced Planck mass. On the other hand, even if $|W_0| \sim \mathcal{O}(M_{\text{Pl}}^3)$, the LVS works for CY manifolds with the negative Euler number, $\chi = 2(h^{1,1} - h^{2,1}) < 0$, i.e., $h^{2,1} > h^{1,1} > 1$.² Note that stringy modes and Kaluza-Klein modes should be sufficiently heavier than all the moduli fields to justify description of the low-energy effective field theory.

In this paper, along the line of Ref. [10], we propose a new type of Kähler moduli stabilization mechanisms in type IIB superstring theory on CY orientifolds with the positive Euler number $\chi > 0$, namely $h^{1,1} > h^{2,1} > 1$. In Ref. [11], it was discussed that the overall volume modulus can be stabilized perturbatively by one-loop corrections to the Kähler potential. In that scenario, one needs a certain amount of fine-tuning of complex structure moduli to realize the Kähler moduli stabilization. In contrast to Ref. [11], here we focus on the radiative corrections due to the sparticles living on D7-branes wrapping a divisor of CY. Such corrections generate the potential of overall volume modulus through the Coleman-Weinberg potential [12]. Since the soft masses are functions of only the overall volume modulus, the overall Kähler modulus can be perturbatively stabilized at a sufficiently large volume region without tuning W_0 . In this large volume region, the

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¹In this paper, we do not discuss the stabilization of open string moduli. For discussion of open string moduli in F-theory context, see, e.g., Ref. [1,2], where the open string moduli are identified with the complex structure moduli of CY fourfold.

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²For the LVS with zero or positive Euler number, see Ref. [9].

string axion associated with the overall volume modulus remains light and it could be a candidate of dark matter. Other Kähler moduli could be stabilized by the non-perturbative effects and/or moduli-dependent D -terms. (See, e.g., Refs. [13].) We find that stringy and Kaluza-Klein modes are sufficiently heavier than sparticles and Kähler moduli at the anti-de Sitter minimum. We can uplift the anti-de Sitter vacuum to the de Sitter vacuum by introducing anti-D branes, and such uplifting does not change the behavior of the moduli stabilization.

The remaining paper is organized as follows. In Sec. II, we discuss the stabilization of overall Kähler modulus without uplifting terms in type IIB superstring theory on CY orientifolds with D7-branes. We add the uplifting term in the setup of Sec. III to achieve a tiny cosmological constant. We study the modulus potential with the uplifting terms analytically and numerically in Secs. III A and III B, respectively. Other Kähler moduli can be stabilized by two scenarios. One is due to moduli-dependent D -terms as discussed in Sec. III C, and the other is due to non-perturbative effects as discussed in Sec. III D. Also, in Sec. III E, we show that the nonperturbative effects generate the mass of ultralight axion associated with the volume modulus. Finally, Sec. IV is devoted to the conclusion.

II. KÄHLER MODULI STABILIZATION WITHOUT UPLIFTING

A. Setup

Let us consider the stabilization of closed string moduli on the basis of type IIB superstring theory on CY orientifolds with D7-branes. For a general class of CY threefolds, the total Kähler potential at the leading order of α' is obtained through Kaluza-Klein reduction of type IIB supergravity action [14,15],

$$K = -2M_{\text{Pl}}^2 \ln \left(\mathcal{V} + \frac{\xi}{2} \right) + M_{\text{Pl}}^2 K(S, U) + Z_{a\bar{a}}^{(i)} |Q_a^{(i)}|^2, \quad (1)$$

where \mathcal{V} is the volume of CY manifold in Einstein frame measured by the string length $l_s = 2\pi\sqrt{\alpha'}$, $K(S, U)$ represents the Kähler potential of dilaton S and complex structure moduli U . The leading α' -correction is characterized by $\xi = -\frac{\zeta(3)\chi}{2(2\pi)^3 g_s^{3/2}}$ where χ is the Euler number

of CY and g_s is the string coupling [14]. Here, $Q_a^{(i)}$ are the matter fields living on D7-branes wrapping the divisor $D_i \in H_4(\text{CY}, \mathbb{Z})$ and their Kähler metrics $Z_{a\bar{a}}^{(i)}$ are given by

$$Z_{a\bar{a}}^{(i)} = (T_i + \bar{T}_i)^{-n_i^a}, \quad (2)$$

with n_i^a being the modular weights and $T_i = \tau_i + i\sigma_i$ with $i \in \{1, 2, \dots, h_+^{1,1}\}$ denotes the Kähler modulus.³ Also, the

³In this paper, we consider the orientifold projection to realize $h_-^{1,1} = 0$, namely $h^{1,1} = h_+^{1,1}$.

gaugino fields couple to the Kähler moduli through the gauge kinetic function

$$f = \frac{T_i}{2\pi}. \quad (3)$$

To stabilize the complex structure moduli U and dilaton S , we consider the following superpotential induced by imaginary self-dual three-form fluxes [4],

$$W = W_{\text{flux}}(S, U). \quad (4)$$

By the flux-induced superpotential, all the complex structure moduli and dilaton fields are stabilized at the compactification scale [3]. When $W_0 = \langle W_{\text{flux}} \rangle \neq 0$ at the minima of the complex structure moduli and dilaton, the scalar potential is nonvanishing because of the breaking of the so-called no-scale structure [14]

$$\begin{aligned} V_{\alpha'} &= e^{\langle K(S,U) \rangle} (K^{T_i \bar{T}_j} K_{T_i} K_{\bar{T}_j} - 3) \frac{|W_0|^2}{M_{\text{Pl}}^2} \\ &\simeq e^{\langle K(S,U) \rangle} \frac{3\xi}{4\mathcal{V}^3} \frac{|W_0|^2}{M_{\text{Pl}}^2}, \end{aligned} \quad (5)$$

where $K_{T_i} = \partial_{T_i} K$ and $K^{T_i \bar{T}_j}$ is the inverse of Kähler metric $K_{T_i \bar{T}_j} = \partial_{T_i} \partial_{\bar{T}_j} K$. The nonvanishing F -term of the Kähler modulus

$$F^{T_i} = -e^{K/(2M_{\text{Pl}}^2)} K^{T_i \bar{T}_j} K_{\bar{T}_j} \frac{\bar{W}_0}{M_{\text{Pl}}}, \quad (6)$$

generates the soft scalar masses and gaugino masses [16],

$$\begin{aligned} m_a^2 &= \frac{V_{\alpha'}}{M_{\text{Pl}}^2} + m_{3/2}^2 - (\partial_{T_i} \partial_{\bar{T}_j} \ln(Z_{a\bar{a}}^{(i)})) \frac{F^{T_i} \bar{F}^{\bar{T}_j}}{M_{\text{Pl}}^2} \\ &= \frac{V_{\alpha'}}{M_{\text{Pl}}^2} + m_{3/2}^2 - \frac{n_i^a}{M_{\text{Pl}}^2} \left| \frac{F^{T_i}}{T_i + \bar{T}_i} \right|^2 \\ &\simeq \frac{V_{\alpha'}}{M_{\text{Pl}}^2} + (1 - n_i^a) m_{3/2}^2 \simeq (1 - n_i^a) m_{3/2}^2, \\ M_f &= \frac{F^{T_i}}{M_{\text{Pl}}} \partial_{T_i} \ln \text{Re}(f) \\ &= M_{\text{Pl}}^{-1} \frac{F^{T_i}}{T_i + \bar{T}_i} \simeq \frac{e^{\langle K(S,U) \rangle / 2} \bar{W}_0}{\mathcal{V} M_{\text{Pl}}^2} \simeq m_{3/2}, \end{aligned} \quad (7)$$

with

$$m_{3/2} = e^{K/(2M_{\text{Pl}}^2)} W_0 M_{\text{Pl}}^{-2} \simeq \frac{e^{\langle K(S,U) \rangle / 2} W_0}{\mathcal{V} M_{\text{Pl}}^2}, \quad (8)$$

evaluated in the large volume region.⁴ Here, we evaluate $\frac{F^{T_i}}{T_i + \bar{T}_i} \simeq -e^{K/(2M_{\text{Pl}}^2)} \frac{\bar{W}_0}{M_{\text{Pl}}} \simeq -m_{3/2} M_{\text{Pl}}$, which is satisfied in a

⁴We take W_0 as a real constant for simplicity and discuss the case with $n_i^a = 0$ later.

general class of CY threefolds with a sufficiently large volume. When these sparticles contribute to the radiative corrections, the 1-loop Coleman-Weinberg (CW) potential is given by

$$\begin{aligned}
 V_{\text{CW}} &= \frac{1}{32\pi^2} \int^{\Lambda^2} dk^2 k^2 \text{STr} \ln(k^2 + M^2) \\
 &= \frac{1}{32\pi^2} \left\{ \Lambda^2 \text{STr}(M^2) + \frac{1}{2} \text{STr} M^4 \left[\ln\left(\frac{M^2}{\Lambda^2}\right) - \frac{1}{2} + \mathcal{O}\left(\frac{M^2}{\Lambda^2}\right) \right] \right\} \\
 &\simeq \frac{c_1}{32\pi^2} \Lambda^2 m_{3/2}^2 + \sum_a \frac{c_b^a}{64\pi^2} m_a^4 \left[\ln\left(\frac{m_a^2}{\Lambda^2}\right) - \frac{1}{2} \right] + \frac{-2c_f - 4}{64\pi^2} m_{3/2}^4 \left[\ln\left(\frac{m_{3/2}^2}{\Lambda^2}\right) - \frac{1}{2} \right] + \frac{32}{64\pi^2} m_{3/2}^4 \left[\ln\left(\frac{4m_{3/2}^2}{\Lambda^2}\right) - \frac{1}{2} \right],
 \end{aligned} \tag{9}$$

where M characterizes the mass matrix for canonically normalized bosons and fermions, and

$$c_1 = \sum_a c_b^a (1 - n_i^a) - 2c_f + 4, \tag{10}$$

with c_b^a and c_f being the multiplicities of bosons $Q_a^{(i)}$ ⁵ and fermions, respectively. Here, we take the limit $M^2/\Lambda^2 \ll 1$, where our description of low-energy effective theory is valid and the supertrace is defined as

$$\text{STr}f(M^2) = \text{tr}f(m_a^2) - 2\text{tr}f(M_f^2) - 4f(m_{3/2}^2) + 2f(4m_{3/2}^2), \tag{11}$$

for an arbitrary function f . We consider the contribution of ghosts in the gauge $\sum_{\mu=0}^3 \gamma^\mu \psi_\mu = 0$, where ψ_μ and γ^μ are the gravitino and four-dimensional gamma-matrices, respectively. (For more details, see Refs. [17].) Obviously, there is no contribution from sfermion fields with $n_i^a = 1$.

For example, in the minimal supersymmetric standard model (MSSM) with three generations of right-handed (s) neutrinos, those multiplicities become

$$c_b^{(\text{MSSM})} = \sum_a c_b^a = 49 + 3 = 52, \quad c_f^{(\text{MSSM})} = 12. \tag{12}$$

On the other hand, when the visible sector consists of the MSSM with singlets and multi-Higgs doublets, the multiplicities are given by

$$c_b = c_b^{(\text{MSSM})} + 4(n_H - 1) + n_S, \quad c_f^{(\text{MSSM})} = 12, \tag{13}$$

where n_H and n_S denote the numbers of multi-Higgs doublets and singlet fields. Note that the following moduli stabilization is also applicable to the scenario where sparticles living on hidden D7-branes contribute to the radiative corrections. If there exist multiple D7_i-branes

wrapping the divisor D_i , the matter fields $Q_a^{(i)}$ and gauginos living on D7_i-branes contribute to the radiative corrections through the soft masses (7). For sake of simplicity, we discuss the D7-branes wrapping only the divisor D_i with volume τ_i in the following analysis.

To justify this low-energy effective action, the soft masses and masses of Kähler moduli m_T should be smaller than those of Kaluza-Klein (KK) modes, stringy modes, complex structure and dilaton moduli, namely

$$m_T, m_{3/2} < m_U, m_S < m_{\text{KK}} < m_{\text{st}} < M_{\text{Pl}}, \tag{14}$$

and their typical masses are given by

$$m_{S,U} \simeq N m_{3/2}, \quad m_{\text{KK}} \simeq \frac{\sqrt{\pi}}{\mathcal{V}^{2/3}} M_{\text{Pl}}, \quad m_{\text{st}} \simeq \frac{\sqrt{\pi} g_s^{1/4}}{\mathcal{V}^{1/2}} M_{\text{Pl}}, \tag{15}$$

with $N > 1$ being a parameter determined by three-form fluxes. Note that we have taken the string-frame volume $\mathcal{V}_s = \mathcal{V} g_s^{3/2} \simeq (2\pi R)^6 l_s^6$, where R is the typical length of CY in string units. Since the cutoff scale is a physical quantity in the nonrenormalizable theory, there exist several options for the cutoff scale. For concreteness, we take the cutoff scale as the typical masses of KK modes in the following analysis, since there exists $N = 2$ supersymmetry above the KK scale. The CW potential is then valid below the cutoff scale. The string loop corrections to the Kähler potential was discussed in Ref. [11], and they can also be interpreted as the 1-loop CW potentials [18]. However, in this paper, we focus on the situation where sparticle contributions dominate over the 1-loop CW potential.⁶ Since the following analysis requires that a relatively large number of sparticle contribute to stabilize the CY volume, they could dominate over the string loop corrections to the Kähler potential controlled by the string coupling.

⁵We use the same notation of chiral superfields Q_a and their lowest components.

⁶The moduli stabilization involving the string loop corrections to the Kähler potential is discussed for CY threefolds with negative Euler number [19] and zero or positive Euler number [9].

B. Stabilization of overall volume modulus

In the following analysis, we first consider the large volume region $\mathcal{V} \rightarrow \infty$, where $\tau_i \rightarrow \infty$ with $i = 1, 2, \dots, h^{1,1}$ for all of the divisors. (In the LVS, some of the divisor volumes are not extremely large in string units [6].) When there exist the D-brane instanton effects or gaugino condensation on D7-branes, the nonperturbative superpotential for the Kähler moduli is generated as

$$W_{\text{non}} = \sum_i A_i e^{-a_i T_i}, \quad (16)$$

where A_i are functions of the complex structure moduli and $a_i = 2\pi$ for the brane instanton and $a_i = 2\pi/N$ for the gaugino condensation on N stacks of D7-branes wrapping the divisor with volume τ_i . In the moduli space $\tau_i \rightarrow \infty$, these nonperturbative effects are negligible and the total scalar potential consists of the leading α' -corrections and radiative corrections. We come back to these nonperturbative effects in Secs. III C–III E.

Including the radiative corrections due to sfermions and gauginos, we can write the scalar potential by

$$\begin{aligned} V_F &= V_{\alpha'} + V_{\text{CW}} \\ &= e^{\langle K(S,U) \rangle} \frac{3\xi}{4\mathcal{V}^3} \frac{|W_0|^2}{M_{\text{Pl}}^2} + \frac{c_1}{32\pi^2} \Lambda^2 m_{3/2}^2 \\ &\quad + \sum_a \frac{c_b^a}{64\pi^2} m_a^4 \left[\ln\left(\frac{m_a^2}{\Lambda^2}\right) - \frac{1}{2} \right] \\ &\quad + \frac{-2c_f - 4}{64\pi^2} m_{3/2}^4 \left[\ln\left(\frac{m_{3/2}^2}{\Lambda^2}\right) - \frac{1}{2} \right] \\ &\quad + \frac{32}{64\pi^2} m_{3/2}^4 \left[\ln\left(\frac{4m_{3/2}^2}{\Lambda^2}\right) - \frac{1}{2} \right]. \end{aligned} \quad (17)$$

It turns out that the F -term scalar potential is a function of only the overall volume \mathcal{V} . In the large volume regime $\mathcal{V} \gg 1$, the above scalar potential is further approximated as

$$V_F \simeq e^{\langle K(S,U) \rangle} \frac{3\xi}{4\mathcal{V}^3} \frac{|W_0|^2}{M_{\text{Pl}}^2} + \frac{c_1}{32\pi^2} \Lambda^2 \frac{e^{\langle K(S,U) \rangle} |W_0|^2}{\mathcal{V}^2 M_{\text{Pl}}^4}. \quad (18)$$

Let us fix the cutoff scale as the KK scale and consider the following units

$$\Lambda = m_{\text{KK}} = \frac{\sqrt{\pi}}{\mathcal{V}^{2/3}} M_{\text{Pl}} = 1, \quad (19)$$

and then the dimensional quantities are rewritten as

$$\begin{aligned} M_{\text{Pl}} &= \frac{\mathcal{V}^{2/3}}{\sqrt{\pi}}, \\ W_0 &\equiv \hat{W}_0 M_{\text{Pl}}^3 = \hat{W}_0 \frac{\mathcal{V}^2}{\pi^{3/2}}. \end{aligned} \quad (20)$$

In units of m_{KK} , the approximated scalar potential reduces to

$$V_F \simeq e^{\langle K(S,U) \rangle} \frac{|\hat{W}_0|^2}{\pi^2} \left[\frac{3\xi}{4\mathcal{V}^{1/3}} + \frac{c_1}{32\pi^2} \frac{\pi}{\mathcal{V}^{2/3}} \right]. \quad (21)$$

From this scalar potential, we aim to find the minimum of \mathcal{V} in the large volume regime $\mathcal{V} \gg 1$. The extremal condition of modulus field $\langle \partial_{\mathcal{V}} V_F \rangle = 0$ in the large volume limit,

$$\partial_{\mathcal{V}} V_F \simeq e^{\langle K(S,U) \rangle} \frac{|\hat{W}_0|^2}{\pi^2} \left[-\frac{\xi}{4\mathcal{V}^{4/3}} - \frac{c_1}{48\pi^2} \frac{\pi}{\mathcal{V}^{5/3}} \right] = 0, \quad (22)$$

is satisfied at

$$\langle \mathcal{V} \rangle \simeq 18\,664 \left(-\frac{c_1/\xi}{10^3} \right)^3, \quad (23)$$

in string units. In particular, when the CY volume is dominated by a single Kähler modulus $\mathcal{V} \simeq \kappa(T + \bar{T})^{3/2}$ with κ being a real positive constant, the value of overall volume modulus is evaluated as

$$\langle \tau \rangle \simeq 351.8 \left(-\frac{c_1/\xi}{10^3} \right)^2 \kappa^{-2/3}, \quad (24)$$

where $T \equiv \tau + i\sigma$. We find that the ratio c_1/ξ should be negative and larger than 10^{-3} such that the Kähler moduli space resides in the physical domain.

Furthermore, the positivity of $\langle \partial_{\mathcal{V}} \partial_{\mathcal{V}} V_F \rangle$,

$$\partial_{\mathcal{V}} \partial_{\mathcal{V}} V_F \simeq 9.14 \times 10^8 e^{\langle K(S,U) \rangle} |\hat{W}_0|^2 \frac{\xi^8}{c_1^7} > 0, \quad (25)$$

requires that c_1 should be positive. That indicates the negative ξ from Eq. (23). Obviously, the sfermions with $n_i^a = 1$ have no contribution in the potential. Thus, it turns out that the case with $n_i^a = 1$ for all sfermions is prohibited, because such a case corresponds to the negative c_1 from Eq. (10). In addition, the potential energy becomes negative at this minimum, namely anti-de Sitter minimum,

$$\langle V_F \rangle \simeq 1.4 \times 10^{-3} e^{\langle K(S,U) \rangle} |\hat{W}_0|^2 \left(-\frac{10^3}{c_1/\xi} \right) \xi < 0, \quad (26)$$

where ξ is negative.

At this minimum, the mass scales of typical modes become

$$\begin{aligned}
 m_a &\simeq M_f \simeq m_{3/2} = \frac{e^{\langle K(S,U) \rangle / 2} |\hat{W}_0|}{\mathcal{V}} M_{\text{Pl}} \\
 &\simeq 2.1 \times 10^{-2} \left(-\frac{10^3}{c_1/\xi} \right) e^{\langle K(S,U) \rangle / 2} |\hat{W}_0| m_{\text{KK}}, \\
 m_{U,S} &= N m_{3/2} = 2.1 \times 10^{-2} N \left(-\frac{10^3}{c_1/\xi} \right) e^{\langle K(S,U) \rangle / 2} |\hat{W}_0| m_{\text{KK}}, \\
 m_{\text{KK}} &= \frac{\sqrt{\pi}}{\mathcal{V}^{2/3}} M_{\text{Pl}} = 1, \\
 m_{\text{st}} &= \frac{\sqrt{\pi} g_s^{1/4}}{\mathcal{V}^{1/2}} M_{\text{Pl}} = 2.9 \left(\frac{g_s}{10^{-1}} \right)^{1/4} \left(-\frac{10^3}{c_1/\xi} \right)^{1/2} m_{\text{KK}}, \\
 M_{\text{Pl}} &= 397 \left(-\frac{c_1/\xi}{10^3} \right)^2 m_{\text{KK}}. \tag{27}
 \end{aligned}$$

Especially, when the CY volume is dominated by a single Kähler modulus, the mass scale of canonically-normalized overall Kähler modulus is

$$m_\tau \simeq 1.1 \times 10^{-3} e^{\langle K(S,U) \rangle / 2} |\hat{W}_0| \left(-\frac{10^3}{c_1/\xi} \right)^{3/2} (-\xi)^{1/2} m_{\text{KK}}. \tag{28}$$

In this way, we find that the volume modulus can be perturbatively stabilized at the anti-de Sitter minimum, where four-dimensional supergravity description is reliable. The nonperturbative effects for the overall Kähler modulus are suppressed enough by the value of Kähler modulus, $e^{-2\pi\tau} \ll 1$, but it generates the potential of remaining massless axion as discussed in Sec. III E. However, the negative ξ indicates that we have to consider the CY threefolds with the positive Euler number, namely a large number of Kähler moduli compared with that of complex structure moduli.⁷ (In the LVS, the negative Euler number is required to realize the large CY volume.) Since such CY threefolds with the positive Euler number account for half of the whole CY threefolds in the sense of mirror symmetry, the CYs with the positive Euler number are not so restricted manifolds, but generic. The O7-plane contribution will also help us to change the number of CY Euler number in a weak coupling limit [21]. In Secs. III C and III D, we discuss the stabilization of other Kähler moduli. Although for a generic value of $e^{\langle K(S,U) \rangle / 2} |\hat{W}_0|$, the overall volume modulus can be stabilized at the anti-de Sitter minimum in a way similar to the LVS, we require the relatively large c_1 to realize the large volume of CY, e.g., $c_1 \simeq \mathcal{O}(10^2)$ for $|\xi| \simeq \mathcal{O}(10^{-1})$. Such a situation is easily realized by the models with multi-Higgs doublets and

⁷Our result is similar to Ref. [20], where the higher-derivative corrections are taken into account.

singlets via Eq. (13), and such models often appear from concrete string model constructions. (For the string model building, see e.g., Refs. [22,23].)

Finally, we comment on the case with no leading α' -corrections, which corresponds to the CY manifold with the vanishing Euler number, i.e., $\chi = 0$. Then, the scalar potential consists of only the CW potential,

$$V_{\text{CW}} = \frac{c_1}{32\pi^2} \Lambda^2 m_{3/2}^2 + \frac{c_2}{64\pi^2} m_{3/2}^4 \left[\ln \left(\frac{m_{3/2}^2}{\Lambda^2} \right) \right] + \frac{c_3}{64\pi^2} m_{3/2}^4, \tag{29}$$

with

$$\begin{aligned}
 c_2 &\equiv \sum_a c_b^a (1 - n_i^a)^2 - 2c_f - 4 + 32, \\
 c_3 &\equiv \sum_a \left[c_b^a (1 - n_i^a)^2 \left(-\frac{1}{2} + \ln(1 - n_i^a) \right) \right] + c_f \\
 &\quad + 2 + 32 \left(\ln 4 - \frac{1}{2} \right). \tag{30}
 \end{aligned}$$

It can be rewritten in units of $\Lambda = m_{\text{KK}} = 1$,

$$\begin{aligned}
 V_{\text{CW}} &\simeq e^{\langle K(S,U) \rangle} \frac{|\hat{W}_0|^2}{(32\pi^2)\pi^2} \left[c_1 \frac{\pi}{\mathcal{V}^{2/3}} + c_2 \frac{e^{\langle K(S,U) \rangle} |\hat{W}_0|^2}{\mathcal{V}^{4/3}} \right. \\
 &\quad \left. \times \ln \left(\frac{e^{\langle K(S,U) \rangle / 2} |\hat{W}_0|}{\mathcal{V}^{1/3} \sqrt{\pi}} \right) + c_3 \frac{e^{\langle K(S,U) \rangle} |\hat{W}_0|^2}{2\mathcal{V}^{4/3}} \right] \\
 &\simeq e^{\langle K(S,U) \rangle} \frac{|\hat{W}_0|^2}{(32\pi^2)\pi^2} \left[c_1 \frac{\pi}{\mathcal{V}^{2/3}} - \frac{c_2 e^{\langle K(S,U) \rangle} |\hat{W}_0|^2}{3 \mathcal{V}^{4/3}} \ln(\mathcal{V}) \right], \tag{31}
 \end{aligned}$$

in the large volume regime $\mathcal{V} \gg 1$ and $e^{\langle K(S,U) \rangle / 2} |\hat{W}_0| \sim \mathcal{O}(1)$. When c_2 is almost the same order of c_1 , we cannot find the minimum with a sufficiently large CY volume.

C. Numerical estimation

In this section, we numerically analyze the stabilization of moduli fields. The unapproximated scalar potential of the overall volume modulus (17) in units of $\Lambda = m_{\text{KK}}$ is drawn as functions of \mathcal{V} and τ in Fig. 1, where we set the following parameters,

$$\begin{aligned}
 \xi &= -0.1, & e^{\langle K(S,U) \rangle / 2} |\hat{W}_0| &= 1, & n_i^a &= 0, \\
 \kappa &= 1, & c_b &= 120, & c_f &= 12. \tag{32}
 \end{aligned}$$

At this vacuum in Fig. 1, the CY volume is determined by

$$\langle \mathcal{V} \rangle \simeq 18433, \tag{33}$$

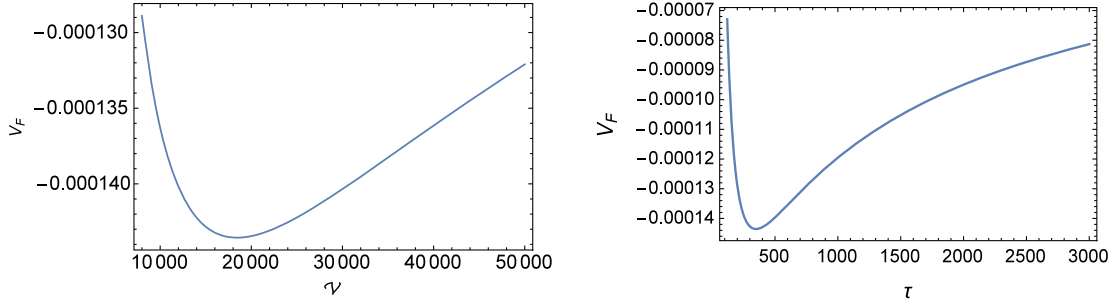


FIG. 1. The scalar potential (17) as functions of CY volume \mathcal{V} and noncanonically normalized modulus τ , where the CY volume is approximated as $\mathcal{V} = (T + \bar{T})^{3/2}$ with $T = \tau + i\sigma$ in the right panel.

as shown in the left panel and the vacuum expectation value of overall Kähler modulus is

$$\langle \tau \rangle \simeq 348.9, \quad (34)$$

as shown in the right panel. The mass scales of typical modes become

$$\begin{aligned} m_i = M_f = m_{3/2} &= 2.1 \times 10^{-2} m_{\text{KK}}, \\ m_{U,S} = N m_{3/2} &= 2.1 \times 10^{-2} N m_{\text{KK}}, \\ m_{\text{KK}} &= 1, \\ m_{\text{st}} &= 2.9 \left(\frac{g_s}{10^{-1}} \right)^{1/4} m_{\text{KK}}, \\ M_{\text{Pl}} &= 394 m_{\text{KK}}. \end{aligned} \quad (35)$$

In addition, the mass scale of canonically-normalized overall Kähler modulus is

$$m_\tau \simeq 3.5 \times 10^{-4} m_{\text{KK}}, \quad (36)$$

when the volume of CY is dominated by the single Kähler modulus $T = \tau + i\sigma$. Hence, even when we involve the next leading terms in the scalar potential, which are volume-suppressed, the numerical result is in agreement with the analytical one in Sec. II B.

III. KÄHLER MODULI STABILIZATION WITH UPLIFTING

In this section, we introduce the uplifting term to achieve a tiny cosmological constant. In particular, we focus on the anti-D3 branes to uplift the anti-de Sitter minimum to the de Sitter minimum.

A. Analytical estimation

When there exist anti-D3 branes at certain moduli space of CY manifold, the uplifting term is given by

$$V_{\text{up}} = \frac{\epsilon}{\mathcal{V}^2} M_{\text{Pl}}^4, \quad (37)$$

where ϵ is a real constant suppressed by the warp factor. Hence, the total scalar potential is a sum of V_F in Eq. (17) and V_{up} .

Similar to the analysis in Sec. II, we aim to find the minimum of modulus field analytically. In the large volume regime, the above scalar potential in units of m_{KK} is simplified as

$$\begin{aligned} V &= V_F + V_{\text{up}} \\ &\simeq e^{\langle K(S,U) \rangle} \frac{|\hat{W}_0|^2}{\pi^2} \left[\frac{3\xi}{4\mathcal{V}^{1/3}} + \frac{c_1}{32\pi^2} \frac{\pi}{\mathcal{V}^{2/3}} \right] + \frac{\epsilon}{\pi^2} \mathcal{V}^{2/3}, \end{aligned} \quad (38)$$

from which the minimum of overall volume modulus is determined by solving $\partial_{\mathcal{V}} V = 0$,

$$\langle \mathcal{V} \rangle \simeq 5530 \left(-\frac{c_1/\xi}{10^3} \right)^3, \quad (39)$$

and ϵ is chosen such that $V \simeq 0$,

$$\epsilon \simeq 3.4 \times 10^{-5} e^{\langle K(S,U) \rangle / 2} |\hat{W}_0|^2 (-\xi) \left(-\frac{10^3}{c_1/\xi} \right)^3. \quad (40)$$

In particular, when the CY volume is dominated by a single Kähler modulus $\mathcal{V} \simeq \kappa (T + \bar{T})^{3/2}$ with κ being a real positive constant, the modulus value at this minimum is evaluated as

$$\langle \tau \rangle \simeq 156 \left(-\frac{c_1/\xi}{10^3} \right)^2 \kappa^{-2/3}. \quad (41)$$

It turns out that the deviation from the SUSY-breaking anti-de Sitter minimum to the uplifted minimum is estimated as

$$\frac{\langle \tau \rangle|_{\text{anti-de Sitter}} - \langle \tau \rangle|_{\text{de Sitter}}}{\langle \tau \rangle|_{\text{anti-de Sitter}}} \simeq 0.56. \quad (42)$$

Our scalar potential still resides in the supergravity-controlled regime, since the mass scales of typical modes have a desirable hierarchical structure,

TABLE I. The volume of CY (\mathcal{V}) and divisor (τ), and typical scales of several modes, setting $e^{\langle K(S,U) \rangle / 2} |\hat{W}_0| = 1$, $g_s = 0.1$, and $\xi = -0.1$.

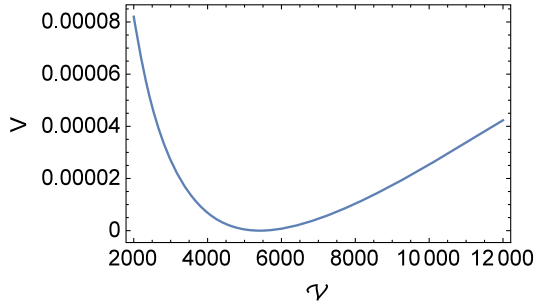
Scale	$c_1 = 50$	$c_1 = 100$	$c_1 = 1000$
\mathcal{V}	691	5530	5.5×10^6
τ	$39\kappa^{-2/3}$	$156\kappa^{-2/3}$	$1.6 \times 10^4 \kappa^{-2/3}$
m_τ [GeV]	1.7×10^{14}	1.5×10^{13}	4.8×10^9
$m_{3/2}$ [GeV]	3.5×10^{15}	4.4×10^{14}	4.4×10^{11}
$m_{U,S}$ [GeV]	$3.5N \times 10^{15}$	$4.4N \times 10^{14}$	$4.4N \times 10^{11}$
m_{KK} [GeV]	5.5×10^{16}	1.4×10^{16}	1.4×10^{14}
m_{st} [GeV]	9.1×10^{16}	3.2×10^{16}	1.0×10^{15}
M_{Pl} [GeV]	2.4×10^{18}	2.4×10^{18}	2.4×10^{18}

$$\begin{aligned}
 m_a \simeq M_f \simeq m_{3/2} &= \frac{e^{\langle K(S,U) \rangle / 2} |\hat{W}_0|}{\langle \mathcal{V} \rangle} M_{\text{Pl}} \\
 &\simeq 3.2 \times 10^{-2} \left(-\frac{10^3}{c_1/\xi} \right) e^{\langle K(S,U) \rangle / 2} |\hat{W}_0| m_{\text{KK}}, \\
 m_{U,S} &= N m_{3/2} = 3.2 \times 10^{-2} N \left(-\frac{10^3}{c_1/\xi} \right) e^{\langle K(S,U) \rangle / 2} |\hat{W}_0| m_{\text{KK}}, \\
 m_{\text{KK}} &= \frac{\sqrt{\pi}}{\langle \mathcal{V} \rangle^{2/3}} M_{\text{Pl}} = 1, \\
 m_{\text{st}} &= \frac{\sqrt{\pi} g_s^{1/4}}{\langle \mathcal{V} \rangle^{1/2}} M_{\text{Pl}} = 2.4 \left(\frac{g_s}{10^{-1}} \right)^{1/4} \left(-\frac{10^3}{c_1/\xi} \right)^{1/2} m_{\text{KK}}, \\
 M_{\text{Pl}} &= 176 \left(-\frac{c_1/\xi}{10^3} \right)^2 m_{\text{KK}}. \tag{43}
 \end{aligned}$$

Furthermore, when the CY volume is dominated by a single Kähler modulus, the mass scale of canonically-normalized overall Kähler modulus is

$$m_\tau \simeq 3.5 \times 10^{-3} e^{\langle K(S,U) \rangle / 2} |\hat{W}_0| \left(-\frac{10^3}{c_1/\xi} \right)^{3/2} (-\xi)^{1/2} m_{\text{KK}}. \tag{44}$$

We find that the volume modulus is still stabilized at the supergravity-reliable de Sitter minimum, even after the anti-de Sitter minimum is uplifted to the de Sitter vacuum.



By setting $e^{\langle K(S,U) \rangle / 2} |\hat{W}_0| = 1$, $g_s = 0.1$, and $\xi = -0.1$, we list the volume of CY, the typical scales of gravitino, moduli fields, KK and stringy modes for several values of c_1 in Table I.

B. Numerical estimation

Next, let us numerically estimate the unapproximated scalar potential $V = V_F + V_{\text{up}}$ setting the same parameters as ones in Sec. II C and

$$\epsilon \simeq 3.437 \times 10^{-6}, \tag{45}$$

under which the scalar potential with the uplifting term is drawn in Fig. 2.

In the left panel in Fig. 2, the CY volume is determined as

$$\langle \mathcal{V} \rangle \simeq 5424, \tag{46}$$

and in the right panel, the vacuum expectation value of overall Kähler modulus is

$$\langle \tau \rangle \simeq 154.4. \tag{47}$$

The mass scales of typical modes become

$$\begin{aligned}
 m_i &= M = m_{3/2} = 3.2 \times 10^{-2} m_{\text{KK}}, \\
 m_{U,S} &= N m_{3/2} = 3.2 \times 10^{-2} N m_{\text{KK}}, \\
 m_{\text{KK}} &= 1, \\
 m_{\text{st}} &= 2.4 \left(\frac{g_s}{10^{-1}} \right)^{1/4} m_{\text{KK}}, \\
 M_{\text{Pl}} &= 174 m_{\text{KK}}, \tag{48}
 \end{aligned}$$

and in addition the mass scale of canonically-normalized overall Kähler modulus is

$$m_\tau \simeq 1.1 \times 10^{-3} m_{\text{KK}}. \tag{49}$$

Hence, even when we involve the next leading terms in the scalar potential, which are volume-suppressed, the numerical result is in agreement with the analytical one.

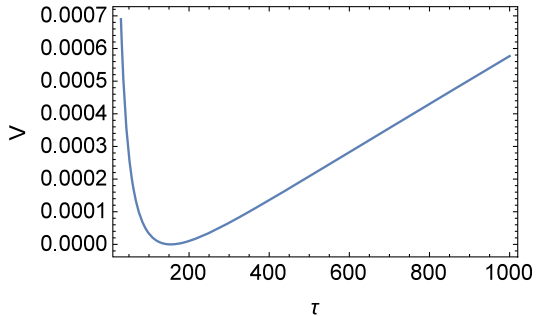


FIG. 2. The scalar potential in units of $\Lambda = m_{\text{KK}}$ as functions of CY volume \mathcal{V} and noncanonically normalized modulus τ , where the CY volume is approximated as $\mathcal{V} = (T + \bar{T})^{3/2}$ with $T = \tau + i\sigma$ in the right panel.

C. Inclusion of the D -terms

So far, we have not considered the nonperturbative effects for the Kähler moduli, since those are negligible in the large volume regime $\mathcal{V} \rightarrow \infty$, where $\tau_i \rightarrow \infty$ for all the volumes of divisors. To stabilize the Kähler moduli fields except for the overall volume modulus, we consider the D -term potential induced by the Fayet-Iliopoulos (FI) term [13]. When the anomalous $U(1)$ symmetries appear on $D7_i$ -branes wrapping the divisors τ_i , the D -term potential is written by

$$V_D^{D7} = \sum_i \frac{1}{\text{Re}(f_{D7_i})} \left(q_{T_i}^{(D7_i)} K_{T_i} - \sum_m q_{\varphi^m}^{(D7_i)} |\phi_m|^2 \right)^2, \quad (50)$$

where f_{D7_i} are the gauge kinetic functions of $D7_i$ -branes, $q_{T_i}^{(D7_i)}$ represent the gauge fluxes and ϕ_m with $U(1)$ charges $q_{\varphi^m}^{(D7_i)}$ are the canonically normalized matter fields living on $D7_i$ -branes. In addition to these $D7$ -brane contributions, anomalous $U(1)$ symmetries on the fractional $D3$ -branes located at the singularities of CY also induce the D -terms

$$V_D^{D3} = \sum_k \frac{1}{\text{Re}(f_{D3_k})} \left(q_{T_k}^{(D3_k)} K_{T_k} - \sum_m q_{\varphi^m}^{(D3_k)} |\varphi_m|^2 \right)^2, \quad (51)$$

where f_{D3_k} are the gauge kinetic functions of $D3_k$ -branes with k being the number of singularities, $q_{T_k}^{(D3_k)}$ represent the $U(1)$ charges of moduli fields and φ_m with $U(1)$ charges $q_{\varphi^m}^{(D3_k)}$ are the canonically normalized matter fields living on the fractional $D3_k$ -branes. In both cases, the anomalous $U(1)$ gauge bosons eat the linear combination of string axions through the Stückelberg couplings and become massive at the compactification scale. In order not to spoil the stabilization of F -term potential, we require the vanishing D -term potential, $V_D^{D7} = V_D^{D3} = 0$, namely

$$q_{T_i}^{(D7_i)} K_{T_i} = \sum_m q_{\varphi^m}^{(D7_i)} |\phi_m|^2, \quad q_{T_k}^{(D3_k)} K_{T_k} = \sum_m q_{\varphi^m}^{(D3_k)} |\varphi_m|^2, \quad (52)$$

which fix the linear combination of closed string moduli and matter fields (open string moduli). In addition, we consider the F -term potential of matter fields,

$$V_F^{\text{matter}} = m_\Phi^2 |\Phi|^2 + A_\Phi |\Phi|^3 + \lambda_\Phi |\Phi|^4, \quad (53)$$

where Φ denotes ϕ_m and φ_m , m_Φ^2 , A_Φ , λ_Φ are T_i -dependent functions. When Φ develops a nonvanishing value, the D -term and F -term contributions are possible to stabilize all the moduli fields except for the overall volume modulus, although it depends on a topology of CY manifold. We leave an explicit moduli stabilization for a future work, since it is difficult to analyze the topology of CY threefold with huge $h^{1,1}$. (For the model building in the LVS with small $h^{1,1}$, see Ref. [24].)

D. Inclusion of the nonperturbative effects

In contrast to the previous section, we study alternative stabilization scenario for Kähler moduli other than the overall volume modulus. We consider the large volume regime $\mathcal{V} \rightarrow \infty$, but the volume of one divisor τ_s is not extremely larger than the other Kähler moduli τ_i , namely $\tau_i \gg \tau_s > 1$ in string units. Note that we focus on the Kähler cone where τ_s is larger than the string length to suppress the KK and stringy corrections.

Let us assume the nonperturbative superpotential with respect to τ_s ,

$$W = W_0 + A_s e^{-a_s T_s}, \quad (54)$$

where $W_0 = \langle W_{\text{flux}} \rangle$, A_s and a_s are the constants depending on the origin of nonperturbative effects, e.g., $a_s = 2\pi$ for the brane instanton and $a_s = 2\pi/N$ for the gaugino condensation on N stacks of $D7$ -branes wrapping the divisor with volume τ_s . To simplify the following analysis, we take both the W_0 and A_s as real constants. From the Kähler potential (1) and superpotential (54), the F -term scalar potential is simply given by

$$\begin{aligned} V_F M_{\text{Pl}}^2 &= e^K (K^{T_i \bar{T}_j} D_{T_i} W D_{\bar{T}_j} \bar{W} - 3) |W|^2 \\ &\simeq \frac{e^{K(S,U)}}{\mathcal{V}^2} \left[K^{T_s \bar{T}_s} a_s^2 |A_s|^2 e^{-2a_s \tau_s} - K^{T_s \bar{T}_j} (\partial_{\bar{T}_j} K) a_s A_s e^{-a_s T_s} \bar{W} - K^{T_i \bar{T}_s} (\partial_{T_i} K) a_s \bar{A}_s e^{-a_s \bar{T}_s} W \right] \\ &\quad + \frac{3\xi}{4\mathcal{V}^3} e^{K(S,U)} |W|^2 + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right) \\ &\simeq e^{K(S,U)} \left[\frac{4}{\mathcal{V}} a_s^2 |A_s|^2 (-\kappa_{ssi} t^i) e^{-2a_s \tau_s} - \frac{4}{\mathcal{V}^2} a_s \tau_s |A_s W| e^{-a_s \tau_s} + \frac{3\xi}{4\mathcal{V}^3} |W|^2 \right] + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right), \end{aligned} \quad (55)$$

in the large volume regime, where $D_{T_i}W = W_{T_i} + K_{T_i}W$ with $W_{T_i} = \partial_{T_i}W$ and the CY volume is taken as $\mathcal{V} = \sum_{ijk} \frac{\kappa_{ijk}}{6} t^i t^j t^k$ with κ_{ijk} being the intersection numbers among the two-cycles t^i of CY. Now, the imaginary part of T_s is set as its minimum and we use $K^{T_i \bar{T}_j} K_{\bar{T}_j} = -2\tau_i + \mathcal{O}(\mathcal{V}^{-1})$ and $K^{T_i \bar{T}_j} = -4\mathcal{V}\kappa_{ijk} t^k + 4\tau_i \tau_j + \mathcal{O}(\mathcal{V}^{-1})$ [25], where the divisor volume is defined as $\tau_i = \frac{1}{2}\kappa_{ijk} t^j t^k$. Note that the first term in Eq. (55) is positive, since it is originating from the positive definite term. The simplified above scalar potential is a well-known form as discussed in the LVS, where the coefficient of α' -correction ξ is positive, namely $h^{2,1} > h^{1,1} > 1$. In LVS, the ‘‘small’’ modulus τ_s and volume modulus \mathcal{V} are stabilized at

$$\mathcal{V} \sim e^{a_s \tau_s}, \quad \tau_s \sim \xi^{2/3}, \quad (56)$$

because of the positivity of ξ .

On the contrary, in this paper, we proceed to discuss the opposite sign of ξ , namely $h^{1,1} \geq h^{2,1} > 1$. The scalar potential is given by a sum of the F -term and leading radiative corrections,

$$\begin{aligned} V \simeq e^{K(S,U)} M_{\text{Pl}}^4 & \left[\frac{4}{\mathcal{V}} a_s^2 |\hat{A}_s|^2 (-\kappa_{ssi} t^i) e^{-2a_s \tau_s} \right. \\ & \left. - \frac{4}{\mathcal{V}^2} a_s \tau_s |\hat{A}_s \hat{W}| e^{-a_s \tau_s} + \frac{3\xi}{4\mathcal{V}^3} |\hat{W}|^2 + \frac{c_1}{32\pi^2} \Lambda^2 \frac{|\hat{W}|^2}{\mathcal{V}^2 M_{\text{Pl}}^2} \right] \\ & + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right), \end{aligned} \quad (57)$$

where $A_s = \hat{A}_s M_{\text{Pl}}^3$ and $W = \hat{W} M_{\text{Pl}}^3$. In units of $\Lambda = m_{\text{KK}} = \sqrt{\pi} M_{\text{Pl}} / \mathcal{V}^{2/3} = 1$, the scalar potential reduces to

$$\begin{aligned} V \simeq \frac{e^{K(S,U)}}{\pi^2} & \left[4\mathcal{V}^{5/3} a_s^2 |\hat{A}_s|^2 (-\kappa_{ssi} t^i) e^{-2a_s \tau_s} \right. \\ & \left. - 4\mathcal{V}^{2/3} a_s \tau_s |\hat{A}_s \hat{W}| e^{-a_s \tau_s} + \frac{3\xi}{4\mathcal{V}^{1/3}} |\hat{W}|^2 + \frac{c_1}{32\pi} \frac{|\hat{W}|^2}{\mathcal{V}^{2/3}} \right] \\ & + \mathcal{O}\left(\frac{1}{\mathcal{V}^{4/3}}\right). \end{aligned} \quad (58)$$

The extremal condition of τ_s reads

$$\begin{aligned} \frac{\partial V}{\partial \tau_s} \simeq & 4\mathcal{V}^{5/3} a_s^2 |\hat{A}_s|^2 e^{-2a_s \tau_s} \left[\frac{\partial t^j}{\partial \tau_s} (-\kappa_{ssj}) - 2a_s (-\kappa_{ssi} t^i) \right] \\ & - 4\mathcal{V}^{2/3} a_s |\hat{A}_s \hat{W}| e^{-a_s \tau_s} [1 - a_s \tau_s] \\ \simeq & 4\mathcal{V}^{5/3} a_s e^{-2a_s \tau_s} \left[-2a_s^2 |\hat{A}_s|^2 (-\kappa_{ssi} t^i) \right. \\ & \left. + \frac{e^{a_s \tau_s}}{\mathcal{V}} |\hat{A}_s \hat{W}| a_s \tau_s \right] = 0, \end{aligned} \quad (59)$$

with $W_0 = \hat{W}_0 M_{\text{Pl}}^3$, where we use $\partial t^j / \partial \tau_s < 1$ and $a_s \tau_s \gg 1$ to suppress the higher instanton effects. In this way, we obtain

$$\frac{e^{a_s \tau_s}}{\mathcal{V}} |\hat{A}_s \hat{W}_0| a_s \tau_s = 2a_s^2 |\hat{A}_s|^2 (-\kappa_{ssi} t^i). \quad (60)$$

Another extremal condition of \mathcal{V} gives rise to

$$\begin{aligned} \frac{\partial V}{\partial \mathcal{V}} \simeq & \frac{20\mathcal{V}^{2/3}}{3} a_s^2 |\hat{A}_s|^2 (-\kappa_{ssi} t^i) e^{-2a_s \tau_s} - \frac{8a_s \tau_s}{3\mathcal{V}^{1/3}} |\hat{A}_s \hat{W}_0| e^{-a_s \tau_s} \\ & - \frac{\xi}{4\mathcal{V}^{4/3}} |\hat{W}_0|^2 - \frac{c_1}{48\pi} \frac{|\hat{W}_0|^2}{\mathcal{V}^{5/3}} \\ \simeq & \frac{2a_s \tau_s}{3\mathcal{V}^{1/3}} |\hat{A}_s \hat{W}_0| e^{-a_s \tau_s} - \frac{\xi}{4\mathcal{V}^{4/3}} |\hat{W}_0|^2 - \frac{c_1}{48\pi} \frac{|\hat{W}_0|^2}{\mathcal{V}^{5/3}} \\ \simeq & \frac{1}{3\mathcal{V}^{3/4}} \frac{(\tau_s)^2}{(-\kappa_{ssi} t^i)} |\hat{W}_0|^2 - \frac{\xi}{4\mathcal{V}^{4/3}} |\hat{W}_0|^2 - \frac{c_1}{48\pi} \frac{|\hat{W}_0|^2}{\mathcal{V}^{5/3}} \\ \equiv & -\frac{\hat{\xi}}{4\mathcal{V}^{4/3}} |\hat{W}_0|^2 - \frac{c_1}{48\pi} \frac{|\hat{W}_0|^2}{\mathcal{V}^{5/3}}, \end{aligned} \quad (61)$$

where

$$\hat{\xi} \equiv \xi - \frac{4}{3} \frac{(\tau_s)^2}{(-\kappa_{ssi} t^i)}. \quad (62)$$

Consequently, the volume modulus is stabilized at

$$\mathcal{V} \simeq 18\,664 \left(-\frac{c_1 / \hat{\xi}}{10^3} \right)^3, \quad (63)$$

by replacing ξ of Eq. (23) into $\hat{\xi}$. Although the $\hat{\xi}$ depends on the topology of CY manifold as in Eq. (23), the stabilization of volume modulus \mathcal{V} and τ_s is achieved inside the Kähler cone with $\mathcal{V} \gg \tau_s > 1$. Indeed, when $\kappa_{ssi} t^i \sim -\tau_s^{1/2}$, $\hat{A}_s = \hat{W}_0 = 1$, $c_1 = 200$, $\xi = -10^{-1}$ and $a_s = 2\pi$, Eqs. (60) and. (63) give

$$\mathcal{V} \simeq 4098, \quad \tau_s \simeq 1.69. \quad (64)$$

Furthermore, even if the Euler number of CY is vanishing, i.e., $\xi = 0$, both the moduli fields can be stabilized at $\mathcal{V} \simeq 3444$, $\tau_s \simeq 1.66$, where we set the same parameters of Eq. (64) except for ξ . Following the same procedure of Sec. III A, we can achieve the de Sitter vacuum by including the anti-D3 branes.

E. Ultralight axion

Finally, we take a closer look at the mass of axion associated with the overall volume modulus. To simplify our analysis, we take into account the CY volume dominated by the single Kähler modulus T , i.e., $\mathcal{V} = \kappa(T + \bar{T})^3/2$ with κ being a positive real constant. When there exist the

TABLE II. Axion mass in units of gravitino mass and its decay constant.

Scale	$n = 1$	$n = 3$	$n = 5$	$n = 7$	$n = 9$
$m_\theta/m_{3/2}$	2.3×10^{-209}	7.4×10^{-68}	9.7×10^{-40}	9.1×10^{-28}	3.7×10^{-21}
$f_\theta[\text{GeV}]$	3×10^{15}	9×10^{15}	1.5×10^{16}	2×10^{16}	2.7×10^{16}

D-brane instanton effects or gaugino condensation on D7-branes, the axion potential can be extracted from the superpotential

$$W = W_0 + A^{(T)} e^{-\frac{2\pi}{n}T}, \quad (65)$$

where $W_0 = \langle W_{\text{flux}} \rangle$ and $A^{(T)}$ are assumed to be real constants, and $n = 1$ for the brane instanton and $n = N$ for the gaugino condensation on N stacks of D7-branes wrapping the divisor with volume τ . Such a nonperturbative superpotential term generates the axion potential,

$$\begin{aligned} V &= e^{K/M_{\text{Pl}}^2} K^{T\bar{T}} K_T \left(\frac{W_T \bar{W}}{M_{\text{Pl}}^2} + \frac{\bar{W}_{\bar{T}} W}{M_{\text{Pl}}^2} \right) \\ &\simeq \frac{e^{K(S,U)} A^{(T)} W_0}{\mathcal{V}^2 M_{\text{Pl}}^2} \frac{(T + \bar{T})^2}{3} \frac{3}{T + \bar{T}} \frac{2\pi}{n} (e^{-\frac{2\pi}{n}T} + e^{-\frac{2\pi}{n}\bar{T}}) \\ &\simeq \frac{e^{K(S,U)} A^{(T)} W_0}{\mathcal{V}^2 M_{\text{Pl}}^2} \left(\frac{\mathcal{V}}{\kappa} \right)^{2/3} \frac{4\pi}{n} e^{-\frac{2\pi}{n}\tau} \cos \left(\frac{2\pi}{n}\sigma \right), \end{aligned} \quad (66)$$

where $T = \tau + i\sigma$. After canonically normalizing the axion $\theta = \sqrt{2K_{T\bar{T}}}\sigma$, we obtain the tiny mass of axion and its decay constant,

$$\begin{aligned} m_\theta &\simeq m_{3/2} \frac{(|A^{(T)}|/M_{\text{Pl}}^3)^{1/2}}{\sqrt{3}\kappa} \left(\frac{2\pi}{n} \right)^{3/2} \left(\frac{\mathcal{V}}{\kappa} \right) e^{-\frac{\pi}{2n}(\frac{\mathcal{V}}{\kappa})^{2/3}}, \\ f_\theta &= \frac{n\sqrt{2K_{T\bar{T}}}}{2\pi} = \frac{\sqrt{6}n}{2\pi} \left(\frac{\kappa}{\mathcal{V}} \right)^{2/3} M_{\text{Pl}}, \end{aligned} \quad (67)$$

where the CY volume and gravitino mass are given in Eqs. (39) and (43). Thus, large volume of CY results in the ultralight axion associated with the volume modulus in a way similar to the LVS.

Although the axion mass highly depends on the value of n , we list the typical axion mass in Table II setting the same parameters of Sec. II C and

$$\kappa = 1, \quad |A^{(T)}| = M_{\text{Pl}}^3, \quad |c_1/\xi| = 10^3. \quad (68)$$

It is interesting to discuss the astrophysical and cosmological physics of such an ultralight axion, which will be studied in a separate work.

IV. CONCLUSION

We have discussed the stabilization of Kähler moduli using the leading α' -corrections and the radiative

corrections due to the sparticles within the framework of the type IIB superstring theory on Calabi-Yau orientifolds with D7-branes. When all the volumes of the divisors in CY threefold are sufficiently large in string units, the non-perturbative effects for the Kähler moduli are suppressed enough compared with α' -corrections and the radiative corrections. We find that these perturbative corrections give rise to the stabilization of the overall Kähler modulus for a general class of CY threefolds, only if the Euler number of CY is positive. Since the volume of CY scales as the number of sparticles, we require that the relatively large number of sparticles contributes to the Coleman-Weinberg potential through the radiative corrections to achieve the large CY volume. Such a large number of sparticle contributions could dominate over the string loop corrections to the Kähler potential discussed in Ref. [11] where one needs a certain amount of fine-tuning of the complex structure moduli to realize the large volume of CY. Furthermore, our scenario of moduli stabilization does not require the tuning of flux-induced superpotential in contrast to the KKL scenario. The Kaluza-Klein and stringy modes are sufficiently heavier than Kähler moduli and sparticles. In this reason, the low-energy effective action is controllable in the four-dimensional $N = 1$ supergravity. The vacuum in our scenario of moduli stabilization is the anti-de Sitter minimum. The structure of moduli stabilization is still maintained even after we uplift the anti-de Sitter minimum to de Sitter vacuum by introducing anti-D3 branes. Note that a requirement of the positive Euler number is different from the conventional large volume scenario, but CY threefolds with the positive Euler number are accounted for half of the CY threefolds in the sense of mirror symmetry.

However, the positive Euler number of CY threefold indicates that we have to take into account a lot of Kähler moduli compared with the complex structure moduli. We expect that the D -term contribution from fractional D3-branes and magnetized D7-branes would lead to the stabilization of these Kähler moduli except for the overall volume modulus as discussed in Sec. III C. The nonperturbative effects also allow us to stabilize some Kähler moduli without spoiling the stabilization of volume modulus as shown in Sec. III D. In this moduli stabilization scenario, the axion associated with the volume modulus remains light and it could be a target of astrophysical and cosmological observations. It is interesting to show explicit moduli stabilization in a detailed setup, but we leave it for a future work.

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