Unruh effect for mixing neutrinos

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(Received 24 March 2018; published 29 May 2018)

Recently, the inverse β-decay rate calculated with respect to uniformly accelerated observers (experiencing the Unruh thermal bath) was revisited. Concerns have been raised regarding the compatibility of inertial and accelerated observers' results when neutrino mixing is taken into account. Here, we show that these concerns are unfounded by discussing the properties of the Unruh thermal bath with mixing neutrinos and explicitly calculating the decay rates according to both sets of observers, confirming thus that they are in agreement. The Unruh effect is perfectly valid for mixing neutrinos.

DOI: 10.1103/PhysRevD.97.105022

I. INTRODUCTION

The Unruh effect, which states that uniformly accelerated observers with proper acceleration a perceive the Minkowski vacuum as a thermal state with temperature $T_U \equiv a/2\pi$, was initially derived assuming free quantum fields [1]. Later, it was shown to be valid also for interacting ones [2–5]. Surprisingly, perhaps, only recently the Unruh effect has been discussed in the context of mixing neutrinos, with disturbing conclusions being drawn. In Ref. [6], it is claimed that the inverse β-decay rate for an accelerated proton as calculated with respect to inertial and uniformly accelerated observers (experiencing the Unruh thermal bath) would disagree with each other when taking into account the existence of multiple families of mixing neutrinos. We claim that this is impossible because calculations of observables must necessarily yield the same answer regardless of the frame used in intermediate steps. Thus, either the Unruh effect is wrong (contradicting several previous results [7], including what we consider to be a virtual observation of it [8]) or some mistake was made in the previously mentioned analysis. A similar criticism of [6] has been made in [9]. While finding no difference between inertial and accelerated frames, those authors declared the Unruh effect for mixing neutrinos to be in some sense nonthermal.

The purpose of this paper is to discuss the Unruh thermal bath for mixing neutrino fields and also to revisit the inverse β decay for accelerated protons with the aim of showing that the Unruh effect is perfectly valid in this setting. As we will argue below, working with mixing neutrinos in quantum field theory is notoriously subtle (see, e.g., Refs. [10,11]). Although working with flavor neutrinos is useful in many situations, we must recall that they only make physical sense in particular regimes. Disregarding this fact leads one astray. The point is that the neutrinos of definite mass are stable particles, and in the absence of nucleons they (and the other leptons) can be represented by standard free fermion fields (such processes as neutrino-neutrino scattering and nucleon-nucleon pair creation being very rare). Therefore, their thermal theory should be routine, both for ordinary gases and for Unruh thermal baths. States of definite flavor become relevant only when the thermal state is detected, or observed, by interaction with something else, such as a nucleon. The flavor structure of the weak interaction may well leave its traces on the amplitudes for such processes, but those traces will be qualitatively the same for ordinary thermal baths and Unruh ones (with differences in the details, stemming from the different spectral decompositions of the respective Hamiltonians). We cannot agree with [9] that any correct result in this connection can be construed as a "nonthermal signature"; at most, it can be a surprising aspect of a thermal phenomenon observed by an interaction that is

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off-diagonal in the mass basis. We shall argue that the introduction of canonical field operators for flavor fields is, at best, unnecessary.

The paper is organized as follows. In Sec. II, we discuss the Unruh thermal bath for the case of mixing neutrinos with particular attention to when we can consider flavor particle states as legitimate quantum states and how this is reflected in measurements made involving the Unruh thermal bath. In Sec. III we set the stage for calculating the inverse β-decay rate for accelerated protons. Section IV concerns the calculation of the inverse β-decay rate from the inertial point of view. In Sec. V, we calculate independently the inverse β-decay rate from the point of view of uniformly accelerated observers and show that the result is in full agreement with the one previously obtained in Sec. IV. Our closing remarks appear in Sec. VI.

Throughout this work we use $(+,-,-,-)$ signature for the Minkowski metric, $\eta_{\mu\nu}$, and natural units, $\hbar = c = k_B = 1$, unless stated otherwise. The same conventions as in Ref. [12] are followed for the Dirac matrices and normal modes.

II. THE UNRUH EFFECT FOR MIXING NEUTRINOS

In this section, we will analyze some properties of the Unruh thermal bath assuming the existence of mixing neutrino fields $\hat{\nu}_i$, $i \in \{1,2,3\}$, each with mass m_i. For this purpose, let us begin by setting our notation and briefly reviewing some relevant features of the Unruh effect for fermionic fields.

A. The Unruh effect for fermionic fields

Consider a fermionic field $\hat{\psi}$ with mass m satisfying Dirac's equation. Inertial observers following the orbits of the timelike Killing field ∂_t, where (t,x,y,z) are usual Cartesian coordinates covering Minkowski spacetime, expand $\hat{\psi}$ in terms of positive- and negative-frequency modes (with respect to ∂_t), $u^{+\omega}_{\vec{k},\sigma}$ and $u^{-\omega}_{\vec{k},\sigma}$, respectively, as

$$\hat{\psi} = \sum_{\sigma=\pm} \int d^3k (\hat{a}_{\vec{k},\sigma} u^{+\omega}_{\vec{k},\sigma} + \hat{b}^{\dagger}_{\vec{k},\sigma} u^{-\omega}_{\vec{k},-\sigma}), \tag{1}$$

where $\omega \equiv \sqrt{|\vec{k}|^2 + m^2}$, $\vec{k} = (k^x, k^y, k^z) \in \mathbb{R}^3$, and $\sigma \in \{+,-\}$. The modes $u^{\pm\omega}_{\vec{k},\sigma}$ are given by

$$u^{\pm\omega}_{\vec{k},\sigma} = \frac{e^{\mp i k_\mu x^\mu}}{(2\pi)^{3/2}} v^{\pm\omega}_\sigma(\vec{k}), \tag{2}$$

where $k^\mu = (\omega, \vec{k})$, $x^\mu = (t,x,y,z)$, and

$$v^{\pm\omega}_\sigma(\vec{k}) = \frac{(k_\mu \gamma^\mu \pm m\mathbb{I})}{\sqrt{[2\omega(\omega \pm m)]}} \hat{v}_\sigma, \tag{3}$$

with $\gamma^\mu = (\gamma^0, \gamma^1, \gamma^2, \gamma^3)$ being the Dirac matrices and

$$\hat{v}_+ \equiv \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \hat{v}_- \equiv \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}. \tag{4}$$

The modes are orthonormalized according to the inner product

$$(\psi, \phi) \equiv \int_\Sigma d\Sigma_\mu \bar{\psi} \gamma^\mu \phi, \tag{5}$$

where $d\Sigma_\mu \equiv d\Sigma n_\mu$, $d\Sigma$ is the proper-volume element on the Cauchy surface Σ, n^μ is a future-pointing unit vector field orthogonal to Σ, and $\bar{\psi} \equiv \psi^\dagger \gamma^0$. The fermionic annihilation and antifermionic creation operators, $\hat{a}_{\vec{k},\sigma}$ and $\hat{b}^\dagger_{\vec{k},\sigma}$, respectively, satisfy the usual anticommutation relations:

$$\{\hat{a}_{\vec{k},\sigma}, \hat{a}^\dagger_{\vec{k}',\sigma'}\} = \delta^3(\vec{k} - \vec{k}')\delta_{\sigma,\sigma'}, \tag{6}$$

$$\{\hat{b}_{\vec{k},\sigma}, \hat{b}^\dagger_{\vec{k}',\sigma'}\} = \delta^3(\vec{k} - \vec{k}')\delta_{\sigma,\sigma'}, \tag{7}$$

with all the other anticommutators vanishing. We define the Minkowski vacuum state, $|0_M\rangle$, as the state annihilated by all annihilation operators, i.e.,

$$a_{\vec{k},\sigma}|0_M\rangle = b_{\vec{k},\sigma}|0_M\rangle = 0, \quad \forall\ \vec{k}, \sigma. \tag{8}$$

On the other hand, uniformly accelerating (Rindler) observers covering the right Rindler wedge portion of Minkowski spacetime, $z > |t|$, quantize the field using a different set of normal modes more appropriate to them. In order to describe this quantization, it is convenient to cover the right Rindler wedge with coordinates (v, x, y, u) in which case the line element is written as

$$ds^2 = u^2 dv^2 - dx^2 - dy^2 - du^2, \tag{9}$$

where $v \in (-\infty, \infty)$ and $u \in (0, \infty)$ are given by

$$v = \tanh^{-1}(t/z), \tag{10}$$

$$u = \sqrt{z^2 - t^2}. \tag{11}$$

Rindler observers, which are labeled by constant values of u, x, and y, expand $\hat{\psi}$ (in the right Rindler wedge) in terms of positive- and negative-frequency modes (with respect to ∂_v), $g^{+\varpi}_{\vec{k}_\perp,\sigma}$ and $g^{-\varpi}_{\vec{k}_\perp,\sigma}$, respectively, as

$$\hat{\psi} = \sum_{\sigma=\pm} \int_0^\infty d\varpi \int d^2k_\perp [\hat{c}_{\varpi,\vec{k}_\perp,\sigma} g^{+\varpi}_{\vec{k}_\perp,\sigma} + \hat{d}^\dagger_{\varpi,\vec{k}_\perp,\sigma} g^{-\varpi}_{\vec{k}_\perp,-\sigma}], \tag{12}$$

where $\varpi \in [0,\infty)$ stands for the Rindler frequency and $\vec{k}_\perp \equiv (k^x, k^y) \in \mathbb{R}^2$ labels the transverse momentum quantum number. The modes $g^{\pm\varpi}_{\vec{k}_\perp,\sigma}$, orthonormalized according to Eq. (5) (using the appropriate set of gamma matrices for Rindler observers), have the form

$$g^{\pm\varpi}_{\vec{k}_\perp,\sigma} = \frac{e^{\mp i\varpi v/a - i\vec{k}_\perp\cdot\vec{x}_\perp}}{(2\pi)^{3/2}} h_\sigma(\pm\varpi, \vec{k}_\perp), \tag{13}$$

where $\vec{x}_\perp = (x, y)$ and

$$h_\sigma(\pm\varpi, \vec{k}_\perp) = \left[\frac{\cosh(\varpi\pi/a)}{\pi a l}\right]^{1/2} \times \gamma^0[(-\vec{k}_\perp\cdot\vec{\gamma}_\perp + m\mathbb{I})K_{\pm i\varpi/a+1/2}(lu) + il\gamma^3 K_{\pm i\varpi/a-1/2}(lu)]\hat{h}_\sigma, \tag{14}$$

with

$$\hat{h}_+ \equiv \begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \qquad \hat{h}_- \equiv \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}, \tag{15}$$

$l \equiv \sqrt{|\vec{k}_\perp|^2 + m^2}$, $\vec{\gamma}_\perp \equiv (\gamma^1, \gamma^2)$, and a being the proper acceleration of fiducial observers labeled by $u = 1/a$, with respect to whom the quantization is performed. The Rindler fermionic annihilation, $\hat{c}_{\varpi,\vec{k}_\perp,\sigma}$, and antifermionic creation, $\hat{d}^\dagger_{\varpi,\vec{k}_\perp,\sigma}$, operators satisfy the anticommutation relations:

$$\{\hat{c}_{\varpi,\vec{k}_\perp,\sigma}, \hat{c}^\dagger_{\varpi',\vec{k}'_\perp,\sigma'}\} = \delta(\varpi-\varpi')\delta^2(\vec{k}_\perp - \vec{k}'_\perp)\delta_{\sigma,\sigma'}, \tag{16}$$

$$\{\hat{d}_{\varpi,\vec{k}_\perp,\sigma}, \hat{d}^\dagger_{\varpi',\vec{k}'_\perp,\sigma'}\} = \delta(\varpi-\varpi')\delta^2(\vec{k}_\perp - \vec{k}'_\perp)\delta_{\sigma,\sigma'}, \tag{17}$$

with all the other anticommutators vanishing. The Rindler vacuum is the state $|0_\mathrm{R}\rangle$ defined by

$$\hat{c}_{\varpi,\vec{k}_\perp,\sigma}|0_\mathrm{R}\rangle = \hat{d}_{\varpi,\vec{k}_\perp,\sigma}|0_\mathrm{R}\rangle = 0, \quad \forall\ \varpi, \vec{k}_\perp, \sigma. \tag{18}$$

By relating Minkowski and Rindler modes, Eqs. (2) and (13), respectively, in the usual manner via a Bogolubov transformation [13], the Minkowski vacuum state, $|0_\mathrm{M}\rangle$, as seen by Rindler observers restricted to the right Rindler wedge can be written as

$$\hat{\rho}_{\beta_U} = \bigotimes_{\varpi,\vec{k}_\perp,\sigma,J} Z_\varpi \sum_{n_J=0}^{1} \exp(-2\pi n_J \varpi/a) \times |n_J; \varpi, \vec{k}_\perp, \sigma\rangle\langle n_J; \varpi, \vec{k}_\perp, \sigma|, \tag{19}$$

where $Z^{-1}_\varpi = 1 + \exp(-2\pi\varpi/a)$, $J = c, d$ label particles (c) and antiparticles (d):

$$|n_c; \varpi, \vec{k}_\perp, \sigma\rangle \equiv c^{\dagger n_c}_{\varpi,\vec{k}_\perp,\sigma}|0_\mathrm{R}\rangle,$$

and

$$|n_d; \varpi, \vec{k}_\perp, \sigma\rangle \equiv d^{\dagger n_d}_{\varpi,\vec{k}_\perp,\sigma}|0_\mathrm{R}\rangle.$$

We see that $\hat{\rho}_{\beta_U}$ is a thermal state at inverse temperature $\beta_U = 2\pi/a$, clearly showing the Unruh effect.

Now, let us take our fermionic field $\hat{\psi}$ to be one of the massive neutrino fields $\hat{\nu}_i$. (No mixing appears at this point because we are considering neither interactions nor flavor neutrinos yet.) Then, it follows directly from Eq. (19) that the mean flux of neutrinos with well-defined mass m_i, energy ϖ, transverse momentum $\vec{k}_\perp$, and spin σ, as seen by Rindler observers, is given by

$$\bar{n}(\varpi, \vec{k}_\perp, \sigma) \equiv \frac{d}{d\tau}\frac{d^2}{d^2x_\perp} \lim_{\substack{\varpi'\to\varpi \\ \vec{k}'_\perp\to\vec{k}_\perp}} \left\langle c^{\dagger i}_{\varpi',\vec{k}'_\perp,\sigma} c^{i}_{\varpi,\vec{k}_{i\perp},\sigma}\right\rangle_{\hat{\rho}_{\beta_U}} = (2\pi)^{-3} n_F(\varpi), \tag{20}$$

where

$$n_F(\varpi) \equiv (1 + e^{\beta_U \varpi})^{-1}, \tag{21}$$

and $\tau = v/a$ is the proper time of fiducial Rindler observers (at $u = 1/a$). As expected, the result is proportional to the Fermi-Dirac factor $n_F(\varpi)$ and only depends on the Rindler energy ϖ. In particular, there is no dependency on the neutrino mass. This does not mean, however, that detectors carried by a given Rindler observer and sensitive to neutrinos ν_i with different masses m_i, would respond in the same way to the Unruh thermal bath.

B. Fermionic particle detector

To illustrate this point, let us define a generalization of the Unruh-DeWitt detector which couples to the neutrino field $\hat{\nu}_i$ through the interaction action

$$\hat{S}_D \equiv \lambda \sum_i \int d\tau [\hat{\bar{m}}(\tau)\hat{\nu}_i[x^\mu_D(\tau)] + \mathrm{H.c.}], \tag{22}$$

where λ is a (dimensional) constant, $x^\mu_D(\tau)$ is the detector's trajectory, and

$$\hat{m}(\tau) = \hat{m}_0(\tau)\begin{pmatrix}\hat{\eta}\\ \hat{\xi}\end{pmatrix}, \tag{23}$$

with $\hat{m}_0(\tau)$ being a monopole operator such that

$$\langle e|\hat{m}_0(\tau)|g\rangle = e^{i\Delta E\tau}\langle e|\hat{m}_0(0)|g\rangle. \tag{24}$$

Here, $\hat{\eta}$ and $\hat{\xi}$ are arbitrary bispinors satisfying the normalization conditions $\hat{\eta}^\dagger\hat{\eta} = \hat{\xi}^\dagger\hat{\xi} = 1$ and ΔE is the energy gap between the detector's excited, $|e\rangle$, and unexcited, $|g\rangle$, states. (In Appendix A, we analyze the behavior of this detector in the simpler setting of an inertial thermal bath and highlight its nice features).

The worldline of a uniformly accelerated detector with proper acceleration a in (v, x, y, u) coordinates is given by

$$x^\mu_D(\tau) = (a\tau, 0, 0, 1/a). \tag{25}$$

For such a detector, the excitation rate, i.e., the excitation probability (with absorption of a Rindler neutrino with mass m_i) per detector proper time, when the field is in the Minkowski vacuum is

$$\frac{dP_{\text{exc},i}}{d\tau} = \frac{d}{d\tau}\sum_{\sigma=\pm}\int_0^\infty d\varpi \int d^2k_\perp |\mathcal{A}_{\text{exc}}|^2 n_F(\varpi), \tag{26}$$

where

$$\mathcal{A}_{\text{exc}} = -i\langle e| \otimes \langle 0_{\text{R}}|\hat{S}_D|\nu_i;\varpi,\vec{k}_\perp,\sigma\rangle \otimes |g\rangle. \tag{27}$$

Using Eqs. (22), (25), and (27) in Eq. (26) yields

$$\frac{dP_{\text{exc},i}}{d\tau} = \Lambda^2\int_0^\infty d\varpi\delta(\varpi-\Delta E)e^{-\pi\varpi/a} \times \int d^2k_\perp (l_i/a)|K_{i\varpi/a+1/2}(l_i/a)|^2, \tag{28}$$

where the constant $\Lambda^2 = |\lambda|^2|\langle e|\hat{m}_0(0)|g\rangle|^2/2\pi^3$ depends on the detector's specifics and we recall that $l_i = \sqrt{|\vec{k}_\perp|^2 + m_i^2}$. Note that the detector is only sensitive to particles with Rindler energy $\varpi = \Delta E$.

The excitation rate should be proportional to the local neutrino density. We can confirm this by extending the scalar-field construction of the finite-volume particle number operator given in Ref. [14] (borrowed from quantum optics) to spinor fields. By making use of the inner product (5), we define the neutrino density operator in an infinitesimal volume around the detector as

$$\hat{n}_{i,D} \equiv d(\nu_i^+,\nu_i^+)/dud^2x_\perp|_{x^\mu(\tau)=x^\mu_D(\tau)} \tag{29}$$

$$= \bar{\nu}_i^+\gamma^0\nu_i^+|_{x^\mu(\tau)=x^\mu_D(\tau)}, \tag{30}$$

where ν_i^+ denotes the (Rindler) positive-frequency part of Eq. (12) and Eq. (29) was evaluated over a $v = \text{const}$ surface. The expectation value of this density operator in the Minkowski vacuum $|0_{\text{M}}\rangle$ is given by

$$\langle 0_{\text{M}}|\hat{n}_{i,D}|0_{\text{M}}\rangle = \frac{1}{2\pi^4}\int_0^\infty d\varpi e^{-\pi\varpi/a} \times \int d^2k_\perp (l_i/a)|K_{i\varpi/a+1/2}(l_i/a)|^2. \tag{31}$$

Clearly, Eq. (28) is proportional to Eq. (31) when restricted to particles with $\varpi = \Delta E$, confirming that the excitation rate is proportional to the local neutrino density. We also note from Eq. (28) that Rindler observers will have a harder time detecting more massive neutrinos since they concentrate closer to the horizon. This is in accordance with previous results obtained for the scalar field case [14,15].

C. On flavor neutrinos and the Unruh thermal bath

Let us consider now the properties of the Unruh thermal bath in terms of flavor neutrinos. It is possible to define *phenomenologically* flavor states $|\nu_\alpha;\vec{k},\sigma\rangle$ and $|\nu_\alpha;\varpi,\vec{k}_\perp,\sigma\rangle$ for inertial and Rindler observers, respectively, in the realm of quantum field theory, where $\alpha \in \{e,\mu,\tau\}$ labels the leptonic flavor. Their usual form

$$|\nu_\alpha;\vec{k},\sigma\rangle \equiv \sum_i U^*_{\alpha,i}|\nu_i;\vec{k},\sigma\rangle, \tag{32}$$

$$|\nu_\alpha;\varpi,\vec{k}_\perp,\sigma\rangle \equiv \sum_i U^*_{\alpha,i}|\nu_i;\varpi,\vec{k}_\perp,\sigma\rangle, \tag{33}$$

with $U_{\alpha,i}$ being the PMNS matrix [16], arise as a useful approximation when it is possible to disregard the mass differences in neutrino production and detection processes [17,18]. This is achieved when $\Delta m^2_{ij} \equiv |m_i^2 - m_j^2|$ are much smaller than the intrinsic uncertainties in the neutrino momenta, where by "intrinsic" we mean the uncertainties as calculated in the reference frame where they acquire their minimal values. [Hence, the states on the right-hand side of Eqs. (32) and (33) must be seen as wave-packets peaked at momentum $\vec{k}$ and $\vec{k}_\perp$, respectively.] In what follows we will denote this situation by $\Delta m^2_{ij} \sim 0$. For more details, we refer the reader to Appendix B. (See also the next-to-last paragraph of Sec. III.)

Once we have established the conditions where Eqs. (32) and (33) are valid, we use our ν_i-neutrino detector [see Eq. (22)] to calculate the probability per proper time of finding an α-flavor neutrino, $dP_\alpha/d\tau$, in the Unruh thermal bath. We note from Eq. (33) that the probability of a ν_i neutrino (reaching the detector) to collapse as a ν_α neutrino is $|U_{\alpha,i}|^2$. Then, we find

$$\left.\frac{dP_\alpha}{d\tau}\right|_{\Delta m_{ij}^2\sim 0} = \sum_i |U_{\alpha,i}|^2 \left.\frac{dP_{\text{exc},i}}{d\tau}\right|_{\Delta m_{ij}^2\sim 0} \approx \left.\frac{dP_{\text{exc},i}}{d\tau}\right|_{m_i\approx\text{const}}, \quad (34)$$

where we have used the unitarity of the PMNS matrix in the last step. From Eq. (34) we see that detectors of flavor neutrinos will behave as if they were immersed in the Unruh thermal bath of a fermionic field with mass $m_1 \approx m_2 \approx m_3$ (which can be taken to be approximately zero in many physical situations).

We proceed now to evaluate the inverse β-decay rate for accelerated protons with neutrino mixing from the point of view of both inertial and accelerated observers and show that they agree. This illustrates how the Unruh effect is perfectly consistent with neutrino mixing.

III. SEMICLASSICAL INVERSE β DECAY WITH NEUTRINO MIXING

For the sake of our purposes, we adopt the same approach as [6,19], where the proton, $|p\rangle$, and neutron, $|n\rangle$, are seen as unexcited and excited states of a two-level system with the corresponding (proper) Hamiltonian $\hat{H}$ satisfying

$$\hat{H}|p\rangle = m_p|p\rangle, \quad (35)$$

$$\hat{H}|n\rangle = m_n|n\rangle, \quad (36)$$

where $m_{p(n)}$ is the proton (neutron) mass. The proton-neutron system is assumed to have a well-prescribed spacetime trajectory described by the semi-classical current:

$$\hat{j}^\mu = \frac{\hat{q}(\tau)}{\sqrt{-g}u^0} u^\mu(\tau)\delta^3(\vec{x}-\vec{x}_0(\tau)), \quad (37)$$

where $g = \det(\eta_{\mu\nu})$, $u^\mu(\tau)$ is the four velocity of the linearly accelerated proton-neutron system with proper time τ, proper acceleration $a = \text{const}$, and $\vec{x}_0(\tau)$ is its spatial trajectory. The monopole operator $\hat{q}(\tau)$ is defined via the Hamiltonian by

$$\hat{q}(\tau) = e^{i\hat{H}\tau}\hat{q}(0)e^{-i\hat{H}\tau}, \quad (38)$$

where the Fermi constant, G_F, will be given by $G_F \equiv |\langle n|\hat{q}(0)|p\rangle|$. The leptonic fields, in turn, will be treated as quantum fields.

The effective weak interaction action considered here is

$$\hat{S}_I = \int d^4x\sqrt{-g}\left(\sum_\alpha \hat{\bar{\nu}}_\alpha\gamma^\mu\hat{P}_L\hat{l}_\alpha\hat{j}_\mu + \text{H.c.}\right) = \int d^4x\sqrt{-g}\left(\sum_{\alpha,i} U^*_{\alpha,i}\hat{\bar{\nu}}_i\gamma^\mu\hat{P}_L\hat{l}_\alpha\hat{j}_\mu + \text{H.c.}\right), \quad (39)$$

where

$$\hat{P}_L \equiv \frac{(\mathbb{I}-\gamma^5)}{\sqrt{2}}, \quad (40)$$

$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$, and we recall that $\hat{j}^\mu$ is given by Eq. (37), $\alpha \in \{e,\mu,\tau\}$ labels the leptonic flavor,

$$\hat{\nu}_\alpha \equiv \sum_i U_{\alpha,i}\hat{\nu}_i, \quad (41)$$

and $\hat{l}_\alpha$ stand for all electrically charged leptonic fields, $\{e^-,\mu^-,\tau^-\}$.

We should view the neutrino fields $\hat{\nu}_i$ with well defined mass as the fundamental ones with the PMNS matrix elements contributing to the interaction coupling constants between the mass neutrinos and other fields. We note that the usual canonical quantization procedure is perfectly valid for these fields. In contrast, we stress that $\hat{\nu}_\alpha$ should be viewed only as a *shorthand notation* for the particular combination of massive neutrino fields [given by Eq. (41)]. Attempts to canonically quantize the $\hat{\nu}_\alpha$ fields in terms of positive (and negative) norm modes give annihilation (and creation) operators whose physical meaning is unclear [20], precluding us from constructing the associated Fock space (see, e.g., Refs. [10,21]). Overlooking this fact, as in Ref. [6], leads to contradictory results.

For the reasons outlined above, we focus only on states associated with the fundamental $\hat{\nu}_i$ fields, for which the Unruh effect must be valid and the inverse β-decay rate calculated in both inertial and accelerated frames must coincide as shown next.

IV. INERTIAL CALCULATION

The inverse β-decay process, as seen by Minkowski observers, can be generically cast in the form

$$p \to n\bar{l}_\alpha\nu_i, \quad (42)$$

where $l_\alpha = \{e^-,\mu^-,\tau^-\}$ and $\nu_i = \{\nu_1,\nu_2,\nu_3\}$. The transition amplitude associated with Eq. (42) is

$$\mathcal{A}^I_{\alpha,i} = -i\langle n| \otimes \langle\bar{l}_\alpha\nu_i|\hat{S}_I|0_{\text{M}}\rangle \otimes |p\rangle, \quad (43)$$

where the charged leptons l_α and neutrinos ν_i have quantum numbers $\sigma_{\alpha(i)} \in \{+,-\}$ and $\vec{k}_{\alpha(i)} = (k^x_{\alpha(i)}, k^y_{\alpha(i)}, k^z_{\alpha(i)})$, and $\hat{S}_I$ is given in Eq. (39).

In usual inertial coordinates, $x^\mu = (t,x,y,z)$, current (37) is written as

$$\hat{j}^\mu = \frac{\hat{q}(\tau)}{az} u^\mu(\tau)\delta(x)\delta(y)\delta\left(z-\sqrt{t^2-a^{-2}}\right), \quad (44)$$

where $u^\mu = (az(\tau), 0, 0, at(\tau))$ with $t(\tau) = a^{-1}\sinh(a\tau)$ and $z(\tau) = a^{-1}\cosh(a\tau)$.

The differential decay probability per momentum-space volume $dV_k = d^3k_\alpha d^3k_i$ is given by

$$\frac{dP^{p\to n\bar{l}_\alpha\nu_i}}{dV_k} = \sum_{\sigma_\alpha,\sigma_i} |\mathcal{A}^I_{\alpha,i}|^2, \tag{45}$$

allowing us to define the decay rate per momentum-space volume as

$$\frac{d\Gamma^{p\to n\bar{l}_\alpha\nu_i}}{dV_k} = \frac{1}{\Delta\tau}\frac{dP^{p\to n\bar{l}_\alpha\nu_i}}{dV_k}, \tag{46}$$

where $\Delta\tau$ is the total proper time along the trajectory of the proton-neutron system. Inserting Eqs. (39), (44), and (1) into Eq. (43), we write Eq. (46) as

$$\begin{aligned}\frac{d\Gamma^{p\to n\bar{l}_\alpha\nu_i}}{dV_k} = {}& \frac{2G_F^2|U_{\alpha,i}|^2}{(2\pi)^6} \\ &\times \int_{-\infty}^{\infty} d\xi \exp(2i[\Delta m\xi + a^{-1}(\omega_\alpha\omega_i)\sinh(a\xi)]) \\ &\times \frac{1}{\omega_\alpha\omega_i}[k_i^z k_\alpha^z + \omega_i\omega_\alpha + F(k^x_{i,\alpha}, k^y_{i,\alpha})],\end{aligned} \tag{47}$$

where $\Delta m \equiv m_n - m_p$, we have made an inverse boost in the z-direction [to factor out the proper time integral implicitly contained in Eq. (45)], and $F(k^x_{i,\alpha}, k^y_{i,\alpha})$ is an odd function of its arguments whose form is not important here since it will not contribute to the decay rate when integrated over dV_k.

By integrating over momenta, we obtain the total decay rate

$$\begin{aligned}\Gamma^{p\to n\bar{l}_\alpha\nu_i} = {}& \frac{G_F^2|U_{\alpha,i}|^2}{\pi^4 a} e^{\frac{-\pi\Delta m}{a}} \int_0^\infty dk_\alpha k_\alpha^2 \int_0^\infty dk_i k_i^2 \\ &\times K_{2i\Delta m/a}(2(\omega_\alpha + \omega_i)/a),\end{aligned} \tag{48}$$

where $k_{\alpha(i)} \equiv |\vec{k}_{\alpha(i)}|$. Now, by using the same complex integration procedure employed in Ref. [12] we can rewrite the expression above as a double integral over the complex plane, i.e.,

$$\begin{aligned}\Gamma^{p\to n\bar{l}_\alpha\nu_i} = {}& \frac{G_F^2 a^5|U_{\alpha,i}|^2}{32\pi^{7/2}} e^{-\pi\Delta m/a} \\ &\times \int_{C_t}\frac{dt}{2\pi i}\int_{C_s}\frac{ds}{2\pi i}|\Gamma(3-s-t+i\Delta m/a)|^2 \\ &\times \frac{\Gamma(-s)\Gamma(-t)\Gamma(2-t)\Gamma(2-s)}{\Gamma(3-s-t)\Gamma(7/2-s-t)}\left[\frac{m_\alpha}{a}\right]^{2t}\left[\frac{m_i}{a}\right]^{2s},\end{aligned} \tag{49}$$

where C_s and C_t are integration contours containing all poles of the Γ functions both in the s and t planes.

Although apparently unwieldy, this expression for the decay rate is convenient for our purposes of analytically confirming the equality between Eq. (49) and the analogous result obtained with respect to Rindler observers, to which we proceed now.

V. RINDLER CALCULATION

Uniformly accelerated observers see the single inverse β-decay process considered by inertial observers, Eq. (42), as a set of three processes, namely

(i) $p + l_\alpha \to n + \nu_i$,
(ii) $p + \bar{\nu}_i \to n + \bar{l}_\alpha$,
(iii) $p + l_\alpha + \bar{\nu}_i \to n$,

i.e, protons lying at rest with the Rindler observers would decay into neutrons by the absorption (and possible emission) of leptons from (to) the Unruh thermal bath.

In the (v, x, y, u) coordinate system, current (37) is expressed as

$$\hat{j}^\mu = \hat{q}(\tau)u^\mu(\tau)\delta(x)\delta(y)\delta(u - a^{-1}) \tag{50}$$

with $u^\mu = (a, 0, 0, 0)$.

To obtain the total decay rate, we must sum incoherently processes (i)–(iii). Let us outline now the procedure to calculate the decay rate specifically for process (i), since processes (ii) and (iii) will be similar. First, we calculate the interaction amplitude by using Eqs. (12) and (39),

$$\begin{aligned}\mathcal{A}^{R,(\mathrm{i})}_{\alpha,i} &= -i\langle n| \otimes \langle \nu_i|\hat{S}_I|l_\alpha\rangle \otimes |p\rangle \\ &= \frac{-iG_F}{(2\pi)^2\sqrt{2}} U^*_{\alpha,i}\delta(\varpi_\alpha - \varpi_i - \Delta m) \\ &\quad\times [\bar{g}^{+\varpi_i}_{\vec{k}_{i\perp},\sigma_i}\gamma^0(\mathbb{I} - \gamma^5)g^{+\varpi_\alpha}_{\vec{k}_{\alpha\perp},\sigma_\alpha}],\end{aligned} \tag{51}$$

where the neutrino ν_i has quantum numbers $(\sigma_i, \varpi_i, \vec{k}_{i\perp})$ and the charged lepton has quantum numbers $(\sigma_\alpha, \varpi_\alpha, \vec{k}_{\alpha\perp})$ and we recall that $\bar{g} = g^\dagger\gamma^0$. We square it to obtain the differential probability of decay per Rindler momentum-space volume $dV_{k,R} = d\varpi_\alpha d^2k_{\alpha\perp}d\varpi_i d^2k_{i\perp}$.

The interaction rate,

$$\Gamma^{(\mathrm{i},R)} = \sum_{\sigma_\alpha,\sigma_i}\int dV_{k,R}\frac{|\mathcal{A}^{R,(\mathrm{i})}_{\alpha,i}|^2}{\Delta\tau} n_F(\varpi_\alpha)[1 - n_F(\varpi_i)], \tag{52}$$

is obtained by dividing the differential probability by the total proper time $\Delta\tau$, multiplying it by the relevant fermionic thermal factors, and integrating over $dV_{k,R}$.

Following a similar recipe for processes (ii) and (iii) we can write the total interaction rate, according to Rindler observers, as

$$\Gamma^{p\to n\bar{l}_\alpha\nu_i,R} = \sum_j \Gamma^{(j,R)}$$
$$= \frac{G_F^2|U_{\alpha,i}|^2}{8a^2\pi^7} e^{-\pi\Delta m/a} \int_{-\infty}^{\infty} d\varpi_i \iint dk_i^x dk_\alpha^x$$
$$\times \iint dk_i^y dk_\alpha^y l_i l_\alpha |K_{1/2+i\varpi_\alpha/a}(l_\alpha/a)|^2$$
$$\times |K_{1/2+i(\varpi_\alpha-\Delta m)/a}(l_i/a)|^2. \quad (53)$$

Now, following the reasoning of Ref. [12] we use Eqs. (6.412) and the definition of the Meijer G-function [Eq. (9.301)] of Ref. [22], along with Eq. (5.6.66) of Ref. [23], to write the total interaction rate according to Rindler observers as

$$\Gamma^{p\to n\bar{l}_\alpha\nu_i,R} = \frac{G_F^2 a^5|U_{\alpha,i}|^2}{32\pi^{7/2}} e^{-\pi\Delta m/a}$$
$$\times \int_{C_t}\frac{dt}{2\pi i}\int_{C_s}\frac{ds}{2\pi i}|\Gamma(3-s-t+i\Delta m/a)|^2$$
$$\times \frac{\Gamma(-s)\Gamma(-t)\Gamma(2-t)\Gamma(2-s)}{\Gamma(3-s-t)\Gamma(7/2-s-t)}\left[\frac{m_\alpha}{a}\right]^{2t}\left[\frac{m_i}{a}\right]^{2s}, \quad (54)$$

which can be seen to be exactly equal to Eq. (49), proving our assertion that the two rates coincide.

It is worthwhile to note that in Ref. [6] different neutrino asymptotic states are used when calculating decay rates in the two frames. Thus, the two calculations have no obligation to coincide at all (as in fact they do not).

VI. CONCLUSIONS

We have discussed the properties of the Unruh thermal bath for mixing neutrinos and in which conditions we can legitimately speak about flavor states. Also, we have shown through an explicit calculation the equality of the decay rates for the inverse β decay of protons as calculated by Minkowski and Rindler observers. This is not surprising, being the expected result from the general covariance of quantum field theory, but it explicitly demonstrates the importance of using the appropriate mode expansion (i.e., those that are eigenfunctions of an appropriate time-like isometry) for the neutrino fields, where all calculations are well defined and it is completely meaningful to talk of particles. Finally, as previously stated, this also shows that there is no incompatibility between the Unruh effect and neutrino mixing, contrary to previous claims.

ACKNOWLEDGMENTS

We are indebted to Vicente Pleitez for insightful comments. G. C. and A. L., G. M., D. V. were fully and partially supported by São Paulo Research Foundation (FAPESP) under Grants No. 2016/08025-0 and No. 2017/15084-6–2017/50388-6, No. 2015/22482-2, No. 2013/12165-4, respectively. A. L. and G. M. were also partially supported by the Institute for Quantum Science and Engineering at Texas A&M University and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), respectively.

APPENDIX A: THE FERMIONIC DETECTOR

In order to better understand the behavior of the fermionic detector defined in Sec. II, we apply it to a usual inertial thermal bath (at inverse temperature β) of massive neutrinos, described by the density matrix

$$\hat{\rho}_\beta = \bigotimes_{\vec{k},\sigma_i,i,J}\frac{1}{1+\exp(-\beta\omega_i)}$$
$$\times \sum_{n_{i,J}=0}^{1}\exp(-n_{i,J}\beta\omega_i)|n_{i,J};\vec{k},\sigma\rangle\langle n_{i,J};\vec{k},\sigma|, \quad (A1)$$

where ω_i satisfies the usual dispersion relation $\omega_i = \sqrt{|\vec{k}|^2+m_i^2}$ and $J=a,\ b$ label particles (a) and antiparticles (b):

$$|n_{i,a};\vec{k},\sigma\rangle \equiv a^{\dagger n_{i,a}}_{\vec{k},\sigma}|0_{\rm M}\rangle,$$

and

$$|n_{i,b};\vec{k},\sigma\rangle \equiv b^{\dagger n_{i,b}}_{\vec{k},\sigma}|0_{\rm M}\rangle,$$

with $a^\dagger_{\vec{k},\sigma}$ and $b^\dagger_{\vec{k},\sigma}$ being the fermionic and antifermionic creation operators, respectively. By using Eq. (22), we compute the mean excitation rate of an inertial detector with worldline $x_D^\mu(\tau)=(\tau,0,0,0)$ (in (t,x,y,z) coordinates) due to the absorption of a neutrino with mass m_i giving

$$\frac{dP_{{\rm exc},i}}{d\tau} = \pi^{-1}|\lambda|^2|\langle e|\hat{m}_0(0)|g\rangle|^2\int_{m_i}^{\infty}d\omega\delta(\omega-\Delta E)$$
$$\times\frac{\omega\sqrt{\omega^2-m_i^2}}{e^{\beta\omega}+1}. \quad (A2)$$

We see that the above excitation rate is, disregarding the phase-space volume factor, proportional to the mean number of particles with energy ΔE, as it should be. We also note that this detector satisfies the detailed balance condition, relating excitation and absorption rates [7]. Moreover, the excitation rate in this case is also proportional to the expectation value of the (inertial) neutrino density operator, constructed similarly as in Eq. (29), in the state (A1):

$$\langle\hat{n}_{i,D}\rangle_{\hat{\rho}_\beta} = \frac{1}{\pi^2}\int_{m_i}^{\infty}d\omega_i\frac{\omega_i\sqrt{\omega_i^2-m_i^2}}{e^{\beta\omega_i}+1}. \quad (A3)$$

APPENDIX B: FLAVOR NEUTRINOS ONE-PARTICLE STATES

We elaborate here our statement that the usual neutrino flavor states

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha,i}|\nu_i\rangle, \tag{B1}$$

are only defined in particular situations.

As said before, a physical Fock space for flavor neutrinos cannot be constructed [10,21]. However, the usual flavor states have a well-defined meaning in the limit $\Delta m^2_{ij} \sim 0$. To show this, here we reproduce an abridged derivation of the "*weak states*" given in [10,17].

Weak states are phenomenological states in the sense that their definition depends on the specific process being considered. As a particular example, we consider the inverse β decay $p^+ \to n + e^+ + $"$\nu_e$", where our goal is to give a meaning to the corresponding ν_e state given that we do not have a legitimate creation operator of flavor particles.

We assume here that (i) the flavor neutrino resulting from the decay can be described as a superposition of massive neutrino states (which are well-defined) and (ii) the corresponding massive neutrinos have some uncertainty in their momenta (otherwise energy-momentum conservation would single out a specific massive neutrino as the result of the decay, see Ref. [24]).

The decay final state will be cast as

$$|f\rangle \equiv \mathsf{S}|p^+\rangle, \tag{B2}$$

where S is the S matrix of the theory. We write $|f\rangle$ as

$$|f\rangle = \sum_i A_{ei}|ne^+\nu_i\rangle + \cdots, \tag{B3}$$

where "…" stands for all other possible decay channels, containing states orthogonal to the ones singled out above. Here, $|\nu_i\rangle$ stands as a shorthand for either $|\nu_1 00\rangle$, $|0\nu_2 0\rangle$, or $|00\nu_3\rangle$. From Eqs. (B2) and (B3) we obtain for the amplitudes:

$$A_{ei} = \langle ne^+\nu_i|\mathsf{S}|p^+\rangle. \tag{B4}$$

By projecting the state $|f\rangle$ over $|ne^+\rangle$ and normalizing the resulting state, we *define*

$$|\nu_e\rangle \equiv \sum_i \frac{A_{ei}}{\sqrt{\sum_j |A_{ej}|^2}}|\nu_i\rangle. \tag{B5}$$

In order to reach Eq. (B1), we expand the S matrix of the theory as

$$\mathsf{S} \approx \mathbb{I} - i\hat{S}_I, \tag{B6}$$

where $\hat{S}_I$ is given by Eq. (39), and plug it in Eq. (B4), which allows us to write A_{ei} as

$$A_{ei} = U^*_{ei}B_{ei}, \tag{B7}$$

where

$$B_{ei} = -i\int d^4x\sqrt{-g}\langle ne^+\nu_i|(\hat{\bar{\nu}}_i\gamma^\mu\hat{P}_L\hat{l}_e\hat{j}_\mu)|p^+\rangle. \tag{B8}$$

We note that the neutrino masses only appear in B_{ei} inside the mode expansion of the $\hat{\nu}_i$ fields. Finally, assuming $\Delta m^2_{ij} \sim 0$, we obtain the flavor neutrino states as

$$|\nu_e\rangle = \sum_i \frac{U^*_{ei}B_{ei}}{\sqrt{\sum_j |U^*_{ei}B_{ei}|^2}}|\nu_i\rangle \tag{B9}$$

$$\approx \frac{B_e}{\sqrt{|B_e|^2}}\sum_i U^*_{ei}|\nu_i\rangle, \tag{B10}$$

where in the last equation we have defined that

$$B_e \equiv B_{e1} \approx B_{e2} \approx B_{e3}.$$

Therefore, Eq. (B10) agrees with Eq. (B1) (up to a global phase).

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