$f(R, R_{\mu\nu}^2)$ at one loop

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We compute the one-loop divergences in a theory of gravity with a Lagrangian of the general form $f(R, R_{\mu\nu}R^{\mu\nu})$, on an Einstein background. We also establish that the one-loop effective action is invariant under a duality that consists of changing certain parameters in the relation between the metric and the quantum fluctuation field. Finally, we discuss the unimodular version of such a theory and establish its equivalence at one-loop order with the general case.

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I. INTRODUCTION

The calculation of the one-loop divergences in Einstein theory without cosmological constant was performed originally in [1] and established the perturbative non-renormalizability of the theory in the presence of matter. The cosmological constant was included in [2]. The non-renormalizability of pure gravity requires a two-loop calculation that was only done much later [3]. One-loop divergences in higher-derivative gravity (containing terms quadratic in the Ricci scalar, Ricci tensor and Riemann tensor) were calculated originally in [4–6]. The theory is renormalizable [7] and one obtains beta functions that in a certain regime lead to asymptotic freedom.

This calculation was redone and extended to $4+\epsilon$ dimensions in [8]. In a context of asymptotic safety, these beta functions were reproduced starting from the functional renormalization group in [9,10] and in other dimensions in [11], and pushed beyond the one-loop approximation in [12–14]. The conformal case requires a separate calculation [15,16]. Higher derivative gravity has been revisited recently [17]. The main issue with this theory is the presence of ghosts in the perturbative spectrum. Over time, several ways out have been suggested [18–21], and more recently [22–28], but this point remains unsettled, for the time being.

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Beyond the purely quadratic terms, systematic investigations have been done only in the case of $f(R)$ actions. The one-loop divergences for such a theory, on a maximally symmetric background, have been computed originally in [29]. This calculation has been extended recently to arbitrary backgrounds [30] and to Einstein spaces [31]. Functions of the Ricci scalar only can be recast as Einstein theory coupled to a scalar, so in these theories one is not probing the effects due to higher-derivative spin-two propagators. In the functional renormalization group approach, $f(R)$ gravity has been studied in [32–43] and in [44] in the unimodular setting. The classically equivalent scalar-tensor theories have been studied for example in [45]. Recently there has also been work on Lagrangians of the form $f(R_{\mu\nu}R^{\mu\nu}) + RZ(R_{\mu\nu}R^{\mu\nu})$ [46].

In this paper we will consider more general theories depending on the Ricci scalar and the square of the Ricci tensor

$$S(g) = \int d^d x \sqrt{|g|} f(R, X), \tag{1.1}$$

where

$$X = R_{\mu\nu}R^{\mu\nu}.$$

Our treatment will be limited to Einstein backgrounds, where some powerful tools allow us to reduce the calculation to that of determinants of second order operators only.

An important aspect of these calculations is that they only give universal results on shell. By "universal" we mean independent of arbitrary choices such as the choice of gauge and parametrization of the quantum field. The off-shell dependence of one-loop divergences on the choice of gauge and parametrization has been investigated in [47–49]; see also [50] for a recent account. In the context of asymptotic safety, this dependence has been explored in [51,52]. More recently, we have computed the gauge and parametrization dependence of the one-loop divergences in Einstein gravity [53] and higher-derivative gravity [54] (with four free parameters altogether). We will use this general parametrization also in this paper.

One curious feature of the result is its invariance under a certain discrete, idempotent change of the parameters, that involves the replacement of a densitized metric by a densitized inverse metric, that we called "duality." The existence of this invariance was first noticed in [53] in the case of Einstein gravity and in [54] in higher-derivative gravity on an Einstein space. The question of whether it exists also in a more general context was one of the original motivations for this work. We find that the answer is positive.

This paper is organized as follows. The long Sec. II contains most of the technical steps, including the derivation of the second variation of the action in terms of York variables and the gauge fixing. In Sec. III the main results are presented and conclusions drawn. Section IV contains the discussion of the unimodular case.

II. THE HESSIAN

A. Variations

For a one-loop calculation we need to expand the action to second order in the fluctuation field. We begin by considering the linear splitting

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \tag{2.1}$$

The function $f(R, R_{\mu\nu}^2)$ can be expanded as

$$\begin{aligned} f(R, R_{\mu\nu}^2) = \bar{f} &+ \bar{f}_R(R^{(1)} + R^{(2)}) + \frac{1}{2}\bar{f}_{RR}(R^{(1)})^2 \\ &+ \bar{f}_X(X^{(1)} + X^{(2)}) + \frac{1}{2}\bar{f}_{XX}(X^{(1)})^2 \\ &+ \bar{f}_{RX}R^{(1)}X^{(1)} + \cdots, \end{aligned} \tag{2.2}$$

where the subscripts on f denote derivatives with respect to its arguments:

$$f_R = \frac{\partial f}{\partial R}, \qquad f_X = \frac{\partial f}{\partial X}, \qquad f_{RR} = \frac{\partial^2 f}{\partial R^2},$$
$$f_{RX} = \frac{\partial^2 f}{\partial R \partial X}, \qquad f_{XX} = \frac{\partial^2 f}{\partial X^2}.$$

The bar on any quantity means that it is evaluated on the background [for example, $\bar{f}_X = f_X(\bar{R}, \bar{X})$] and we denote by a superscript in parentheses the power of h contained in a certain term of the expansion. Thus we have

$$\begin{aligned} R &= \bar{R} + R^{(1)} + R^{(2)} + \cdots, \\ R_{\mu\nu}R^{\mu\nu} \equiv X &= \bar{X} + X^{(1)} + X^{(2)} + \cdots. \end{aligned} \tag{2.3}$$

Explicitly we have for the Ricci tensor

$$\begin{aligned} R^{(1)}_{\mu\nu} =& -\frac{1}{2}(\bar{\nabla}_\mu\bar{\nabla}_\nu h - \bar{\nabla}_\mu h_\nu - \bar{\nabla}_\nu h_\mu + \Box h_{\mu\nu}) - \bar{R}_{\alpha\mu\beta\nu}h^{\alpha\beta} \\ &+ \frac{1}{2}\bar{R}_{\mu\alpha}h^\alpha_\nu + \frac{1}{2}\bar{R}_{\nu\alpha}h^\alpha_\mu, \\ R^{(2)}_{\mu\nu} =& \frac{1}{2}\bar{\nabla}_\mu(h^{\alpha\beta}\bar{\nabla}_\nu h_{\alpha\beta}) \\ &- \frac{1}{2}\bar{\nabla}_\alpha\{h^{\alpha\beta}(\bar{\nabla}_\mu h_{\nu\beta} + \bar{\nabla}_\nu h_{\mu\beta} - \bar{\nabla}_\beta h_{\mu\nu})\} \\ &- \frac{1}{4}(\bar{\nabla}_\mu h^\beta_\alpha + \bar{\nabla}_\alpha h^\beta_\mu - \bar{\nabla}^\beta h_{\alpha\mu})(\bar{\nabla}_\beta h^\alpha_\nu + \bar{\nabla}_\nu h^\alpha_\beta - \bar{\nabla}^\alpha h_{\beta\nu}) \\ &+ \frac{1}{4}\bar{\nabla}_\alpha h(\bar{\nabla}_\mu h^\alpha_\nu + \bar{\nabla}_\nu h^\alpha_\mu - \bar{\nabla}^\alpha h_{\mu\nu}), \end{aligned} \tag{2.4}$$

where we have used the notation $h_\mu = \bar{\nabla}^\nu h_{\mu\nu}$ and $\Box = \bar{\nabla}^2$. This implies

$$\begin{aligned} X^{(1)}(h_{\mu\nu}) &= 2(\bar{R}^{\mu\nu}R^{(1)}_{\mu\nu} - \bar{R}_{\mu\nu}\bar{R}^\mu{}_\rho h^{\nu\rho}), \\ X^{(2)}(h_{\mu\nu}) &= (R^{(1)}_{\mu\nu})^2 + 2\bar{R}^{\mu\nu}R^{(2)}_{\mu\nu} - 4\bar{R}^{\mu\nu}h_\nu{}^\rho R^{(1)}_{\mu\rho} \\ &\quad + 2\bar{R}_{\mu\lambda}\bar{R}_\nu{}^\lambda(h^2)^{\mu\nu} + \bar{R}_{\mu\nu}\bar{R}_{\lambda\rho}h^{\mu\lambda}h^{\nu\rho}. \end{aligned} \tag{2.5}$$

For the Ricci scalar we have

$$\begin{aligned} R^{(1)}(h_{\mu\nu}) &= \bar{\nabla}_\mu h^\mu - \Box h - \bar{R}_{\mu\nu}h^{\mu\nu}, \\ R^{(2)}(h_{\mu\nu}) &= \frac{3}{4}\bar{\nabla}_\alpha h_{\mu\nu}\bar{\nabla}^\alpha h^{\mu\nu} + h_{\mu\nu}\Box h^{\mu\nu} - h_\mu^2 + h_\mu\bar{\nabla}^\mu h \\ &\quad - 2h_{\mu\nu}\bar{\nabla}^\mu h^\nu + h_{\mu\nu}\bar{\nabla}^\mu\bar{\nabla}^\nu h \\ &\quad - \frac{1}{2}\bar{\nabla}_\mu h_{\nu\alpha}\bar{\nabla}^\alpha h^{\mu\nu} - \frac{1}{4}\bar{\nabla}_\mu h\bar{\nabla}^\mu h \\ &\quad + \bar{R}_{\alpha\beta\gamma\delta}h^{\alpha\gamma}h^{\beta\delta}. \end{aligned} \tag{2.6}$$

We will assume that the background is Einstein:

$$\bar{R}_{\mu\nu} = \frac{\bar{R}}{d}\bar{g}_{\mu\nu}. \tag{2.7}$$

Then, the covariant derivatives of the Ricci tensor vanish and the Ricci scalar is constant. When $R^{(2)}$ appears linearly in the expansion of the action, we can make partial integration and use a simpler expression:

$$\begin{aligned} R^{(2)}(h_{\mu\nu}) = \frac{1}{4}(&h_{\mu\nu}\Box h^{\mu\nu} + h\Box h + 2h_\mu^2 \\ &+ 2\bar{R}_{\mu\nu}h^{\mu\alpha}h^\nu_\alpha + 2\bar{R}_{\alpha\beta\gamma\delta}h^{\alpha\gamma}h^{\beta\delta}). \end{aligned} \tag{2.8}$$

For the same reason, when $R^{(2)}_{\mu\nu}$ appears in the action linearly, we can neglect the first two lines of (2.4).

The last ingredient for the linear expansion is the expansion of the measure

$$\sqrt{|g|} = \sqrt{|\bar{g}|}\left(1 + \frac{1}{2}h + \frac{h^2 - 2h_{\mu\nu}^2}{8} + \cdots\right). \tag{2.9}$$

As in [53,54], we will calculate the Hessian for a more general parametrization of the fluctuations, namely

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \tag{2.10}$$

where the fluctuation is expanded:

$$\delta g_{\mu\nu} = \delta g_{\mu\nu}^{(1)} + \delta g_{\mu\nu}^{(2)} + \delta g_{\mu\nu}^{(3)} + \cdots, \tag{2.11}$$

where $\delta g_{\mu\nu}^{(n)}$ contains n powers of $h_{\mu\nu}$. We will parametrize the first two terms of the expansion as follows:

$$\delta g_{\mu\nu}^{(1)} = h_{\mu\nu} + m\bar{g}_{\mu\nu}h,$$
$$\delta g_{\mu\nu}^{(2)} = \omega h_{\mu\rho}h^{\rho}{}_{\nu} + mhh_{\mu\nu} + m\left(\omega - \frac{1}{2}\right)\bar{g}_{\mu\nu}h^{\alpha\beta}h_{\alpha\beta} + \frac{1}{2}m^2\bar{g}_{\mu\nu}h^2. \tag{2.12}$$

As explained in [53], the parameter m is related to the weight (in the sense of tensor density) of the full quantum field:

$$\gamma_{\mu\nu} = g_{\mu\nu}(\det(g_{\mu\nu}))^{-\frac{m}{1+dm}},$$
$$\gamma^{\mu\nu} = g^{\mu\nu}(\det(g_{\mu\nu}))^{\frac{m}{1+dm}}. \tag{2.13}$$

We consider four main types of expansion: linear expansion of the densitized metric or its inverse,

$$\gamma_{\mu\nu} = \bar{\gamma}_{\mu\nu} + \hat{h}_{\mu\nu} \quad \text{or} \quad \gamma^{\mu\nu} = \bar{\gamma}^{\mu\nu} - \hat{h}^{\mu\nu}, \tag{2.14}$$

and exponential expansion of the densitized metric or its inverse,

$$\gamma_{\mu\nu} = \bar{\gamma}_{\mu\rho}(e^{\hat{h}})^{\rho}{}_{\nu} \quad \text{or} \quad \gamma^{\mu\nu} = (e^{-\hat{h}})^{\mu}{}_{\rho}\bar{\gamma}^{\rho\nu}. \tag{2.15}$$

The parameter ω discriminates among the linear expansion of the covariant metric (for $\omega = 0$); the exponential expansion (for $\omega = 1/2$, independent of whether one expands the metric or its inverse); and the linear expansion of the inverse metric (for $\omega = 1$).

We found in [53,54] that the divergences of Einstein theory and of higher-derivative gravity on an Einstein background are invariant under the following change of the parameters:

$$(\omega, m) \mapsto \left(1 - \omega, -m - \frac{2}{d}\right). \tag{2.16}$$

Since this maps the expansion of the metric to the expansion of the inverse metric, we referred to it as duality. We also observed that the case $\omega = 1/2$, $m = -1/d$, which corresponds to the exponential expansion of unimodular gravity, is a fixed point of the duality and is a point of minimum sensitivity for the parameter dependence of the off-shell divergences.

We are going to use again the parametrization defined above and check whether the results are duality invariant in the case of the theory $f(R, X)$. To this end we insert the preceding parametrization in the second order linear expansion of the metric and we obtain

$$\begin{aligned}
&\bar{f}_R(R^{(1)}|_{h_{\mu\nu}\to\delta g_{\mu\nu}^{(2)}} + R^{(2)}|_{h_{\mu\nu}\to\delta g_{\mu\nu}^{(1)}}) + \frac{1}{2}\bar{f}_{RR}(R^{(1)}|_{h_{\mu\nu}\to\delta g_{\mu\nu}^{(1)}})^2 \\
&\quad + \bar{f}_X(X^{(1)}|_{h_{\mu\nu}\to\delta g_{\mu\nu}^{(2)}} + X^{(2)}|_{h_{\mu\nu}\to\delta g_{\mu\nu}^{(1)}}) + \frac{1}{2}\bar{f}_{XX}(X^{(1)}|_{h_{\mu\nu}\to\delta g_{\mu\nu}^{(1)}})^2 \\
&\quad + \bar{f}_{RX}(R^{(1)}X^{(1)})|_{h_{\mu\nu}\to\delta g_{\mu\nu}^{(1)}} + \frac{1}{2}\bar{f}\delta g^{(2)} + \frac{1}{8}\bar{f}((\delta g^{(1)})^2 - 2(\delta g_{\mu\nu}^{(1)})^2) \\
&\quad + \frac{\delta g^{(1)}}{2}(\bar{f}_R R^{(1)} + \bar{f}_X X^{(1)})|_{h_{\mu\nu}\to\delta g_{\mu\nu}^{(1)}} \\
&= \frac{1}{8}\bar{f}(4\delta g^{(2)} + (\delta g^{(1)})^2 - 2(\delta g_{\mu\nu}^{(1)})^2) + \bar{f}_R\left(R^{(1)}(\delta g_{\mu\nu}^{(2)}) + R^{(2)}(\delta g_{\mu\nu}^{(1)}) + \frac{\delta g^{(1)}}{2}R^{(1)}(\delta g_{\mu\nu}^{(1)})\right) \\
&\quad + \frac{1}{2}\bar{f}_{RR}(R^{(1)}(\delta g_{\mu\nu}^{(1)}))^2 + \bar{f}_X\left(X^{(1)}(\delta g_{\mu\nu}^{(2)}) + X^{(2)}(\delta g_{\mu\nu}^{(1)}) + \frac{\delta g^{(1)}}{2}X^{(1)}(\delta g_{\mu\nu}^{(1)})\right) \\
&\quad + \frac{1}{2}\bar{f}_{XX}(X^{(1)}(\delta g_{\mu\nu}^{(1)}))^2 + \bar{f}_{RX}R^{(1)}(\delta g_{\mu\nu}^{(1)})X^{(1)}(\delta g_{\mu\nu}^{(1)}),
\end{aligned} \tag{2.17}$$

where $\delta g^{(1)} = \bar{g}^{\mu\nu}\delta g_{\mu\nu}^{(1)}$ and $\delta g^{(2)} = \bar{g}^{\mu\nu}\delta g_{\mu\nu}^{(2)}$.

B. York decomposition

The structure of the Hessian on an Einstein space is much clearer when one separates the spin two, one and zero components of the fluctuation field:

$$h_{\mu\nu} = h_{\mu\nu}^{TT} + \bar{\nabla}_\mu\xi_\nu + \bar{\nabla}_\nu\xi_\mu + \bar{\nabla}_\mu\bar{\nabla}_\nu\sigma - \frac{1}{d}\bar{g}_{\mu\nu}\bar{\nabla}^2\sigma + \frac{1}{d}\bar{g}_{\mu\nu}h, \tag{2.18}$$

where

$$\bar{\nabla}^\mu h^{TT}_{\mu\nu} = 0; \qquad \bar{g}^{\mu\nu} h^{TT}_{\mu\nu} = 0; \qquad \bar{\nabla}_\mu \xi^\mu = 0.$$

In the path integral this change of variables gives rise to a Jacobian that will be discussed later. When (2.18) is squared, we get

$$\int d^d x \sqrt{|\bar{g}|}\left[h^{TT}_{\mu\nu} h^{TT\mu\nu} + 2\xi_\mu \left(\Delta_{L1} - \frac{2}{d}\bar{R}\right)\xi^\mu + \frac{d-1}{d}\sigma \Delta_{L0}\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right)\sigma + \frac{1}{d}h^2\right], \tag{2.19}$$

where Δ_{L2}, Δ_{L1} and Δ_{L0} are the Lichnerowicz Laplacians defined as

$$\begin{aligned} \Delta_{L2} T_{\mu\nu} &= -\bar{\nabla}^2 T_{\mu\nu} + \bar{R}_\mu{}^\rho T_{\rho\nu} + \bar{R}_\nu{}^\rho T_{\mu\rho} - \bar{R}_{\mu\rho\nu\sigma} T^{\rho\sigma} - \bar{R}_{\mu\rho\nu\sigma} T^{\sigma\rho}, \\ \Delta_{L1} V_\mu &= -\bar{\nabla}^2 V_\mu + \bar{R}_\mu{}^\rho V_\rho, \\ \Delta_{L0} S &= -\bar{\nabla}^2 S. \end{aligned} \tag{2.20}$$

Note that we can freely insert the covariant derivatives inside the above expression. We also have the useful formulas

$$\begin{aligned} \bar{\nabla}_\mu h^\mu_\nu &= -\left(\Delta_{L1} - \frac{2}{d}\bar{R}\right)\xi_\nu - \frac{d-1}{d}\bar{\nabla}_\nu \left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right)\sigma + \frac{1}{d}\bar{\nabla}_\nu h, \\ \bar{\nabla}_\mu \bar{\nabla}_\nu h^{\mu\nu} &= \frac{d-1}{d}\Delta_{L0}\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right)\sigma - \frac{1}{d}\Delta_{L0} h. \end{aligned} \tag{2.21}$$

We then find that the Hessian is[1]

$$S^{(2)} = \int d^d x \sqrt{\bar{g}}[h^{TT}_{\mu\nu} H^{TT} h^{TT\mu\nu} + \xi_\mu H^{\xi\xi}\xi^\mu + \sigma H^{\sigma\sigma}\sigma + \sigma H^{\sigma h} h + h H^{h\sigma}\sigma + h H^{hh} h], \tag{2.22}$$

where

$$H^{TT} = \frac{1}{4}\left[\left\{\bar{f}_X\left(\Delta_{L2} - \frac{4\bar{R}}{d}\right) - \bar{f}_R\right\}\left(\Delta_{L2} - \frac{2\bar{R}}{d}\right) - (1-2\omega)(1+md)\tilde{E}\right], \tag{2.23}$$

$$H^{\xi\xi} = -\frac{(1-2\omega)(1+md)}{2}\left(\Delta_{L1} - \frac{2\bar{R}}{d}\right)\tilde{E}, \tag{2.24}$$

$$\begin{aligned} H^{\sigma\sigma} = \frac{1}{2}\left(\frac{d-1}{d}\right)^2 \Bigg[& P\Delta_{L0}\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right) + Q\Delta_{L0} \\ & - \frac{d(1-2\omega)(1+md)}{2(d-1)}\tilde{E}\Bigg]\Delta_{L0}\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right), \end{aligned} \tag{2.25}$$

$$H^{\sigma h} = \left(\frac{d-1}{d}\right)^2 \frac{1+md}{2}\left[P\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right) + Q\right]\Delta_{L0}\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right), \tag{2.26}$$

$$\begin{aligned} H^{hh} = \left(\frac{d-1}{d}\right)^2 \frac{(1+md)^2}{2} \Bigg[& P\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right)^2 + Q\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right) \\ & + \frac{d[(1+md)d - 2(1-2\omega)]}{4(d-1)^2(1+md)}\tilde{E}\Bigg], \end{aligned} \tag{2.27}$$

where we used the shorthands

$$P = \bar{f}_{RR} + \frac{4}{d^2}\bar{R}^2 \bar{f}_{XX} + 4\bar{R}\bar{f}_{RX} + \frac{d}{2(d-1)}\bar{f}_X \tag{2.28}$$

[1]From here on we always consider the Euclidean case.

$$Q = \frac{d-2}{2(d-1)}\bar{f}_R + \frac{3d^2-10d+8}{2d(d-1)^2}\bar{R}\bar{f}_X \quad (2.29)$$

and

$$\tilde{E} \equiv \bar{f} - \frac{2}{d}\bar{R}\bar{f}_R - \frac{4\bar{R}^2}{d^2}\bar{f}_X = 0 \quad (2.30)$$

is the field equation evaluated on the Einstein space (2.7).

C. Gauge fixing

We now use a trick that has proven convenient in the case of general relativity (GR) and more generally of $f(R)$ theories. A cleaner separation of physical and gauge degrees of freedom can be achieved by changing variables in the scalar sector. Instead of σ and h we define

$$s = \Delta_{L0}\sigma + (1+dm)h, \quad (2.31)$$

$$\chi = \sigma + \frac{b}{(d-1-b)\Delta_{L0} - \bar{R}}s = \frac{(d-1)\Delta_{L0} - \bar{R}}{(d-1-b)\Delta_{L0} - \bar{R}}\sigma + \frac{b(1+dm)}{(d-1-b)\Delta_{L0} - \bar{R}}h, \quad (2.32)$$

where s is gauge invariant and $b = \bar{b}/(1+md)$ is a gauge parameter that will be defined below. A short calculation shows that the Jacobian of the transformation $(\sigma, h) \to (s, \chi)$ is 1. It is easy to see that the whole scalar part of the Hessian can be rewritten in the simple form $\int d^dx\sqrt{\bar{g}}sH^{ss}s$, where, on shell,

$$H^{ss} = \frac{1}{2}\left(\frac{d-1}{d}\right)^2\left[P\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right) + Q\right]\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right). \quad (2.33)$$

Next we consider the gauge-fixing term

$$S_{\mathrm{GF}} = \frac{1}{2a}\int d^dx\sqrt{\bar{g}}\bar{g}^{\mu\nu}F_\mu F_\nu, \quad (2.34)$$

with

$$F_\mu = \bar{\nabla}_\alpha h^\alpha{}_\mu - \frac{\bar{b}+1}{d}\bar{\nabla}_\mu h, \quad (2.35)$$

and a and $\bar{b}$ are gauge parameters. In terms of the new variables, the gauge-fixing action is

$$S_{\mathrm{GF}} = -\frac{1}{2a}\int d^dx\sqrt{\bar{g}}\left[\xi_\mu\left(\Delta_{L1} - \frac{2\bar{R}}{d}\right)^2\xi^\mu + \frac{(d-1-b)^2}{d^2}\chi\Delta_{L0}\left(\Delta_{L0} - \frac{\bar{R}}{d-1-b}\right)^2\chi\right]. \quad (2.36)$$

We see that on shell there is a perfect separation of gauge and physical degrees of freedom: the Hessian of the action only depends on $h_{\mu\nu}^{TT}$ and s, whereas the gauge-fixing term only depends on ξ_μ and χ.

As seen in [54], the use of the new variables also makes the gauge-fixing sector manifestly invariant under the duality.

The ghost action contains a nonminimal operator

$$S_{gh} = i\int d^dx\sqrt{\bar{g}}\bar{C}^\mu\left(\delta_\mu^\nu\bar{\nabla}^2 + \left(1 - 2\frac{b+1}{d}\right)\bar{\nabla}_\mu\bar{\nabla}^\nu + \bar{R}_\mu{}^\nu\right)C_\nu. \quad (2.37)$$

Let us decompose the ghost into transverse and longitudinal parts,

$$C_\nu = C_\nu^T + \bar{\nabla}_\nu C^L = C_\nu^T + \bar{\nabla}_\nu\frac{1}{\sqrt{\Delta_{L0}}}C'^L, \quad (2.38)$$

and the same for $\bar{C}$. This change of variables has unit Jacobian. Then, the ghost action splits into two terms:

$$S_{gh} = i\int d^dx\sqrt{\bar{g}}\left[\bar{C}^{T\mu}\left(\Delta_{L1} - \frac{2\bar{R}}{d}\right)C_\mu^T + 2\frac{d-1-b}{d}\bar{C}'^L\left(\Delta_{L0} - \frac{\bar{R}}{d-1-b}\right)C'^L\right]. \quad (2.39)$$

We are now ready to calculate the one-loop divergences.

III. RESULTS

A. Duality and parametrization independence

These properties can be established already at the level of the Hessian, which has been given in Sec. II B. In the Hessian the parameters ω and m appear only in the combinations $1-2\omega$ and $1+md$. These combinations change sign under the duality transformation (2.16). In H^{TT}, $H^{\xi\xi}$ and $H^{\sigma\sigma}$, they appear in the combination $(1-2\omega)(1+md)$, which is invariant. Next we observe that in the scalar sector the two terms on the diagonal are invariant and the two off-diagonal terms change sign. Thus the determinant and the trace of the Hessian are both invariant, and so are its eigenvalues. This is all that matters for the quantum theory.

We observe that all dependence on ω is proportional to the equation of motion. The determinant has an on-shell dependence on m, but only through an overall prefactor $(1+md)^2$, which can be absorbed in a redefinition of h. Thus all dependence on the parameters m and ω goes away on shell, as it must.

B. One-loop, on-shell effective action

Unless we (1) set $\omega = \frac{1}{2}$, or (2) $m = -\frac{1}{d}$ or (3) go on shell, the effective action is gauge dependent. We choose to impose the on-shell condition (2.30). As we have already mentioned, the result is then independent of ω and m.

The one-loop effective action is given by the determinants coming from different components of the fluctuation, and the ghosts and Jacobians. The contributions are

$$\mathrm{Det}\left(\Delta_{L2}-\frac{4\bar{R}}{d}-\frac{\bar{f}_R}{\bar{f}_X}\right)^{-1/2}\mathrm{Det}\left(\Delta_{L2}-\frac{2\bar{R}}{d}\right)^{-1/2}, \quad (3.1)$$

from $h^{TT}_{\mu\nu}$,

$$\mathrm{Det}\left(\Delta_{L1}-\frac{2\bar{R}}{d}\right)^{-1}, \quad (3.2)$$

from ξ_μ,

$$\mathrm{Det}\left(\Delta_{L0}-\frac{\bar{R}}{d-1}\right)^{-1/2}\mathrm{Det}\left(\Delta_{L0}-\frac{\bar{R}}{d-1}+\frac{Q}{P}\right)^{-1/2}, \quad (3.3)$$

from s,

$$\mathrm{Det}\Delta_{L0}^{-1/2}\mathrm{Det}\left(\Delta_{L0}-\frac{\bar{R}}{d-1-b}\right)^{-1}, \quad (3.4)$$

from χ, and finally

$$\mathrm{Det}\left(\Delta_{L1}-\frac{2\bar{R}}{d}\right)\mathrm{Det}\left(\Delta_{L0}-\frac{\bar{R}}{d-1-b}\right), \quad (3.5)$$

from the ghost. In addition the York decomposition has the Jacobian

$$\mathrm{Det}\left(\Delta_{L1}-\frac{2}{d}\bar{R}\right)^{1/2}\mathrm{Det}[\Delta_{L0}]^{1/2}\mathrm{Det}\left(\Delta_{L0}-\frac{\bar{R}}{d-1}\right)^{1/2}. \quad (3.6)$$

The effective action is then given by

$$\Gamma=\frac{1}{2}\log\mathrm{Det}\left(\Delta_{L2}-\frac{4\bar{R}}{d}-\frac{\bar{f}_R}{\bar{f}_X}\right)+\frac{1}{2}\log\mathrm{Det}\left(\Delta_{L2}-\frac{2\bar{R}}{d}\right)$$
$$+\frac{1}{2}\log\mathrm{Det}\left(\Delta_{L0}-\frac{\bar{R}}{d-1}+\frac{Q}{P}\right)$$
$$-\frac{1}{2}\log\mathrm{Det}\left(\Delta_{L1}-\frac{2\bar{R}}{d}\right). \quad (3.7)$$

If $\bar{f}_X=0$, the first contribution is absent. The gauge independence of the result is manifest.

C. The logarithmic divergence

The divergent part of the effective action can be computed by standard heat kernel methods. On an Einstein background in four dimensions, the logarithmically divergent part is

$$\Gamma_{\log}(\bar{g})=\frac{1}{720(4\pi)^2}\int d^4x\sqrt{\bar{g}}\log\left(\frac{\Lambda^2}{\mu^2}\right)\Bigg[-826\bar{R}^2_{\mu\nu\rho\sigma}$$
$$+509\bar{R}^2-\frac{300\bar{R}\bar{f}_R}{\bar{f}_X}-\frac{900\bar{f}_R^2}{\bar{f}_X^2}$$
$$+\frac{240\bar{R}(3\bar{f}_R+2\bar{R}\bar{f}_X)}{8\bar{f}_X+12\bar{f}_{RR}+48\bar{R}\bar{f}_{RX}+3\bar{R}^2\bar{f}_{XX}}$$
$$-\frac{320(3\bar{f}_R+2\bar{R}\bar{f}_X)^2}{(8\bar{f}_X+12\bar{f}_{RR}+48\bar{R}\bar{f}_{RX}+3\bar{R}^2\bar{f}_{XX})^2}\Bigg], \quad (3.8)$$

where Λ stands for a cutoff and we introduced a reference mass scale μ. Note that on an Einstein space $\bar{X}=\bar{R}^2/4$, so in this formula $\bar{f}_R(\bar{R},\bar{X})$, $\bar{f}_X(\bar{R},\bar{X})$ etc., have to be interpreted as $\bar{f}_R(\bar{R},\bar{R}^2/4)$, $\bar{f}_X(\bar{R},\bar{R}^2/4)$ etc., so all the terms except the first are functions of $\bar{R}$ only.

We can check this expression against two existing results in the literature. If we put

$$f(R,X)=\alpha R^2+\beta X,$$

it reduces to

$$\Gamma_{\log}(\bar{g})=\frac{1}{(4\pi)^2}\int d^4x\sqrt{\bar{g}}\log\left(\frac{\Lambda^2}{\mu^2}\right)\Bigg[-\frac{413}{360}\bar{R}^2_{\mu\nu\rho\sigma}$$
$$-\frac{1200\alpha^2+200\alpha\beta-183\beta^2}{240\beta^2}\bar{R}^2\Bigg], \quad (3.9)$$

which is the standard universal result in higher derivative gravity.

On the other hand if we put

$$f(R,X)=f(R),$$

we obtain

$$\Gamma_{\log}(\bar{g})=\frac{1}{(4\pi)^2}\int d^4x\sqrt{\bar{g}}\log\left(\frac{\Lambda^2}{\mu^2}\right)\Bigg[-\frac{71}{120}\bar{R}^2_{\mu\nu\rho\sigma}$$
$$+\frac{433}{1440}\bar{R}^2+\frac{\bar{f}_R\bar{R}}{12\bar{f}_{RR}}-\frac{\bar{f}_R^2}{36\bar{f}_{RR}^2}\Bigg], \quad (3.10)$$

which agrees with the recent results of [30,31].

IV. UNIMODULAR VERSION

It is of some interest to consider the unimodular version of the same theory, by which we mean the theory with the same Lagrangian density but with the metric constrained to satisfy

$$\sqrt{|g|}=\bar{\omega},$$

where $\bar{\omega}$ is some fixed scalar density (usually chosen to be 1 in a class of coordinate systems). This condition breaks the diffeomorphism group down to the subgroup $SDiff$ of "special" diffeomorphisms, whose infinitesimal generators are transverse vector fields ϵ_μ^T, satisfying

$$\nabla^\mu \epsilon_\mu^T = 0.$$

The action is then

$$S(g) = \int d^d x \bar{\omega} f(R, X), \tag{4.1}$$

and in the variations one has to keep the determinant of g fixed. The simplest way of achieving this is to parametrize the metric exponentially as in (2.15) and to assume that the fluctuation field $h_{\mu\nu}$ is traceless:

$$\bar{g}^{\mu\nu} h_{\mu\nu} = 0.$$

The second order expansion of the action is then identical to the one in Sec. II, except for the absence of all the terms containing h and for setting the parameter $\omega = 1/2$. The final Hessian is very simple and has only two contributions:

$$H^{TT} = \frac{1}{4}\left[\bar{f}_X\left(\Delta_{L2} - \frac{4\bar{R}}{d}\right) - \bar{f}_R\right]\left(\Delta_{L2} - \frac{2\bar{R}}{d}\right), \tag{4.2}$$

$$H^{\sigma\sigma} = \frac{1}{2}\left(\frac{d-1}{d}\right)^2\left[P\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right) + Q\right] \times \Delta_{L0}^2\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right). \tag{4.3}$$

In the definition of the path integral one has to factor the volume of the gauge group $SDiff$. The standard procedure is the Faddeev-Popov construction. However, we will follow a somewhat more elegant (and ultimately equivalent) procedure that has been discussed for GR in [55], and adapted to the unimodular case in [56]. In this approach, the gauge degrees of freedom are isolated by the York decomposition, and the ghosts emerge from the Jacobian of the change of variables. The path integral of the unimodular theory at one loop is

$$Z = \int (dh^{TT} d\xi d\sigma) J_1 e^{-S^{(2)}}, \tag{4.4}$$

where J_1 is the Jacobian (3.6). From the transformation properties of ξ_μ under an infinitesimal special diffeomorphism ϵ_μ^T,

$$\delta\xi_\mu = \epsilon_\mu^T, \tag{4.5}$$

we see that we can identify ξ_μ as the coordinate along the orbits of the gauge group in the space of metrics. Thus in the path integral we can replace $(d\xi)$ with the integral over the gauge group $(d\epsilon^T)$.

The determinants are (3.1), coming from $h_{\mu\nu}^{TT}$;

$$\mathrm{Det}\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right)^{-1/2} \mathrm{Det}\left(\Delta_{L0} - \frac{\bar{R}}{d-1} + \frac{Q}{P}\right)^{-1/2} \mathrm{Det}\Delta_{L0}^{-1}, \tag{4.6}$$

coming from the scalar σ; and finally the Jacobian (3.6). Putting them all together one finds

$$Z = \int d\epsilon^T \frac{\mathrm{Det}(\Delta_{L1} - \frac{2\bar{R}}{d})^{1/2}}{\mathrm{Det}(\Delta_{L2} - \frac{4\bar{R}}{d} - \frac{\bar{f}_R}{\bar{f}_X})^{1/2} \mathrm{Det}(\Delta_{L2} - \frac{2\bar{R}}{d})^{1/2} \mathrm{Det}(\Delta_{L0} - \frac{\bar{R}}{d-1} + \frac{Q}{P})^{1/2} \mathrm{Det}\Delta_{L0}^{1/2}}. \tag{4.7}$$

Now, as explained in [56], the invariant measure on the group of special diffeomorphisms is

$$\int d\epsilon^T \mathrm{Det}\Delta_{L0}^{-1/2},$$

so that the last determinant in the denominator gets absorbed and then factored out in the overall volume of the gauge group. Alternatively, one can reach the same conclusion via the standard Faddeev-Popov procedure, which has been described for the unimodular case in [57].

The (Euclidean) effective action is $-\log Z$ and we see that it coincides with the result (3.7). From here, the calculation of the divergences proceeds in the same way, leading to (3.8). We have thus shown explicitly that the unimodular theory has the same effective action as the full theory.

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