

Dilatonic imprints on exact gravitational wave signatures

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By employing the moduli space approximation, we analytically calculate the gravitational wave signatures emitted upon the merger of two extremally charged dilatonic black holes. We probe several values of the dilatonic coupling constant a , and find significant departures from the Einstein-Maxwell ($a = 0$) counterpart studied in [Phys. Rev. D **96**, 061501 (2017)]. For (low-energy) string theory black holes ($a = 1$) there are no coalescence orbits and only a memory effect is observed, whereas for an intermediate value of the coupling ($a = 1/\sqrt{3}$) the late-time merger signature becomes exponentially suppressed, compared to the polynomial decay in the $a = 0$ case without a dilaton. Such an imprint shows a clear difference between the case with and without a scalar field (as, for example, predicted by string theory) in black hole mergers.

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I. INTRODUCTION

The great discovery made by the Laser Interferometer Gravitational Wave Observatory (LIGO) on September 14, 2015 [1] provided the first direct confirmation that strong gravitational waves are emitted in the process of the coalescence of two black holes. The first event was for black holes of around 30 solar masses; other discoveries soon followed, and gravitational waves have now been detected from several binary black hole mergers over a range of masses [2–5]. The most recently announced event is from a neutron star–neutron star collision [6], marking the beginning of multimessenger astronomy.

Understanding a black hole merger (or scattering) and the associated emission of gravitational waves is a complicated problem in which strong field dynamical effects play an important role. For this reason, there is little hope for attacking this problem exactly, and various approximations [7] and/or numerical studies [8–11] have been considered. For example, a number of analytic predictions of gravitational waves have been computed within the post-Newtonian approximation (see, e.g., [7] and references therein).

In this paper we calculate the gravitational wave signature of two colliding black holes surrounded by a dilatonic field. Such a problem was recently studied numerically for weakly charged black holes where the dilatonic field vanishes at infinity [12], and in the post-Newtonian approximation for nonvanishing asymptotic values of the dilaton [13]. (See also [14] for a discussion of collisions of dilatonic black holes with angular momentum.)

We study this problem from a different perspective, analytically calculating the gravitational wave signature in an approximation that is applicable in the strong field regime and for any black hole mass ratio. To carry out this procedure it is necessary that the two black holes be extremally charged and that the system evolve adiabatically, through a series of approximately static configurations—the so-called *moduli space approximation* (MSA) [15,16]. We thereby generalize recent results for the Einstein-Maxwell case [17], finding imprints of the dilatonic field on the gravitational wavefront. As we shall see, such imprints depend crucially on the value of the dilatonic coupling constant a . Interesting analytic results can be obtained at least in two cases: i) (low energy) string theoretic black holes, characterized by $a = 1$, for which there are no coalescence orbits and only a memory effect is observed; and ii) an intermediate value $a = 1/\sqrt{3}$ of the coupling. We show that the late-time wavefront in the latter case becomes exponentially suppressed, in notable contrast to the polynomial decay in the case without a dilaton [17].

The outline of our paper is as follows. In the next section, we review the evolution of a black hole binary system in the MSA in Einstein-Maxwell theory. Following [17], the corresponding gravitational wavefront is calculated in Sec. III. The main results of the paper are gathered in Sec. IV, where the dilatonic case is studied. We conclude in Sec. V.

II. BLACK HOLE MERGER IN MODULI SPACE APPROXIMATION

To describe a black hole merger in the MSA in Einstein-Maxwell theory, we start with the static multi-black hole

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solution due to *Majumdar and Papapetrou* (MP) [18,19]. The MP solution represents a static configuration of n extremally charged black holes, each of mass m_i and position \vec{x}_i ; for $n = 1$ it reduces to the familiar extremal Reissner-Nordström spacetime. The solution reads

$$ds^2 = -\psi^{-2}dt^2 + \psi^2 d\vec{x} \cdot d\vec{x}, \quad (1)$$

$$A = -(1 - \psi^{-1})dt. \quad (2)$$

Here, A is the Maxwell vector potential, and the metric function ψ is given by

$$\psi = 1 + \sum_{i=1}^n \frac{m_i}{r_i}. \quad (3)$$

In what follows, we shall often write $d\vec{x} \cdot d\vec{x} = dr^2 + r^2 d\Omega^2$, with $r = \sqrt{\vec{x} \cdot \vec{x}} = |\vec{x}|$. We also have $r_i = |\vec{r}_i| = |\vec{x} - \vec{x}_i|$.

The MP solution is static. To describe a dynamical system, we promote the black hole positions \vec{x}_i in (3) to functions of time, $\vec{x}_i = \vec{x}_i(t)$, and further employ the MSA, requiring that the system moves through configurations with small velocities, always remaining approximately static. This amounts to perturbing the solution and treating the black holes as slow-moving. To second order in velocities one obtains the *moduli space metric*, in which the motion of black holes is geodesic [15,16]. In particular, the Lagrangian

$$L = \frac{1}{2}\mu\gamma(r_{12})\vec{v} \cdot \vec{v} \quad (4)$$

describes the center-of-mass motion of two black holes, with the center-of-mass motion subtracted. Here $M \equiv m_1 + m_2$ and $\mu \equiv \frac{m_1 m_2}{M}$ are the total and reduced black hole masses, and $\vec{r}_{12} \equiv \vec{x}_1 - \vec{x}_2$ and $\vec{v} = \frac{d\vec{r}_{12}}{dt}$ are the relative black hole separation and velocity. The conformal factor $\gamma(r_{12})$ takes the form

$$\gamma(r_{12}) = \left(1 + \frac{M}{r_{12}}\right)^3 - \frac{2\mu M^2}{r_{12}^3}. \quad (5)$$

The approximation holds for [16]

$$\frac{r_{12}}{M} \gg v_\infty^2, \quad (6)$$

and so will certainly break down in the final stages of the black hole coalescence, although note that by choosing small v_∞ we can get arbitrarily close to the complete merger.

All we have to do to study the black hole merger or scattering is to solve the equations of motion

$$\dot{\phi}_{12} - \frac{bv_\infty}{r_{12}^2\gamma(r_{12})} = 0 \quad (7)$$

$$\left(\frac{dr_{12}}{dt}\right)^2 + \frac{v_\infty^2}{\gamma(r_{12})} \left(\frac{b^2}{\gamma(r_{12})r_{12}^2} - 1\right) = 0 \quad (8)$$

that follow from (4). Conservation of energy $E = \frac{1}{2}Mv_\infty^2$ and angular momentum $l = bv_\infty$ follow straightforwardly, with v_∞ the relative velocity at infinite separation of the black holes, and b the impact parameter. Without loss of generality we can confine the motion to a plane $\theta = \frac{\pi}{2}$ due to the spherical symmetry of $\gamma(r_{12})$.

These equations of motion allow for both coalescing and scattering orbits, depending on the value of the impact parameter: if $b > b_{\text{crit}}$, scattering will occur, and for $b < b_{\text{crit}}$ there will be a merger. For any mass ratio, b_{crit} is obtained by computing the degenerate positive root in the effective potential in (8), yielding

$$\frac{2b_{\text{crit}}^3}{3\sqrt{3}} - b_{\text{crit}}^2 M + 2\mu M^2 = 0, \quad (9)$$

which becomes $b_{\text{crit}} = \frac{3+\sqrt{3}}{2}M$ for equal-mass black holes.

There are two limiting cases of physical interest for which trajectories can be found: i) $M \ll r_{12}$ when the black holes are widely separated (corresponding to early times of the interaction, $t \rightarrow -\infty$, or late times of the black hole scattering, $t \rightarrow +\infty$) and ii) $r_{12} \ll M$ for late times for black hole coalescence.

In the first regime, Eqs. (7) and (8) become

$$\dot{r}_{12} = \mp v_\infty \left(1 - \frac{3}{2}\epsilon + \dots\right), \quad \dot{\phi}_{12} = \frac{bv_\infty\epsilon^2}{M^2} + \dots, \quad (10)$$

where $\epsilon = M/r_{12} \ll 1$, yielding

$$r_{12}^{\text{early/late}} = \mp v_\infty t - \frac{3}{2}M \log(\mp v_\infty t/r_0),$$

$$\phi_{12}^{\text{early/late}} = -\frac{b}{v_\infty t} + \phi_{120}. \quad (11)$$

For late-time coalescing orbits, Eqs. (7) and (8) read

$$\dot{r}_{12} = -\frac{v_\infty\epsilon^{3/2}\sqrt{M}}{\sqrt{M-2\mu}}, \quad \dot{\phi}_{12} = \frac{bv_\infty\epsilon}{M(M-2\mu)}, \quad (12)$$

where now $\epsilon = r_{12}/M \ll 1$, giving

$$r_{12}^{\text{coalescing}} = \frac{4M^2(M-2\mu)}{v_\infty^2 t^2}, \quad \phi_{12}^{\text{coalescing}} = -\frac{4b}{v_\infty t}, \quad (13)$$

disregarding the integration constants.

These simple expressions will allow us to find analytic approximations for the early- and late-time radiation. For intermediate times we shall solve Eqs. (7) and (8) numerically, plotting the trajectories for various values of b . We depict the solutions in Fig. 1, which provides an illustration

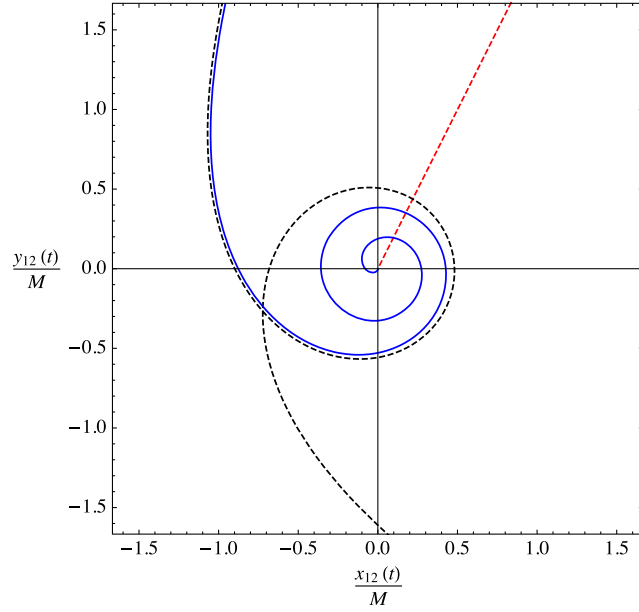


FIG. 1. Black hole trajectories for $a = 0$. Trajectories are illustrated for equal-mass black holes and various impact parameters. In red we have plotted a head-on ($b = 0$) collision. The blue solid line corresponds to a slightly below-critical collision: $b = 0.999b_{\text{crit}} = 2.36366M$, and the black dashed line to a slightly above-critical scattering: $b = 1.01b_{\text{crit}} = 2.38969M$. We set up the two near-critical collisions with otherwise identical initial conditions. Recall that $b_{\text{crit}} = \frac{3+\sqrt{3}}{2}M \approx 2.36603M$.

of trajectories just above and below the critical impact parameter for a collision.

III. GRAVITATIONAL RADIATION TO LEADING ORDER

Following closely the discussion in [17], let us now study the gravitational radiation from the binary black hole system described by the moduli space approximation.

To leading order, gravitational radiation experienced by an observer at radial coordinate r is given by the quadrupole formula

$$h^{TT} = \frac{2}{r} \frac{d^2}{dt^2} Q^{TT} \Big|_{t_{\text{ret}}}. \quad (14)$$

Here, h is the metric perturbation describing the gravity wave, Q is the mass quadrupole, TT denotes the transverse-traceless projection, and $t_{\text{ret}} = t - r$ is the retarded time. For a metric such as (1), it is easy to read off Q due to its asymptotically Cartesian mass-centered form (see Sec. XI of [20] for a definition of this form). In the center of mass frame, the expansion of g_{tt} gives¹

¹Here, the Y_l^m are the spherical harmonics normalized such that $\int Y_l^m \bar{Y}_{l'}^{m'} d\Omega = \delta_{l,l'} \delta_{m,m'}$.

$$g_{tt} = -1 + \frac{2M}{r} + \frac{3M^2}{r^2} - \frac{4M^3}{r^3} + \frac{\mu r_{12}^2}{r^3} \sqrt{\frac{6\pi}{5}} \left(e^{-2i\phi_{12}} Y_2^2 - \sqrt{\frac{2}{3}} Y_2^0 + e^{2i\phi_{12}} Y_2^{-2} \right) + \mathcal{O}\left(\frac{1}{r^4}\right), \quad (15)$$

where the mass quadrupole moments I_2^m are

$$I_2^{\pm 2} = 2\sqrt{\frac{2\pi}{5}} \mu r_{12}^2 e^{\mp 2i\phi_{12}}; \quad I_2^0 = -4\sqrt{\frac{\pi}{15}} \mu r_{12}^2, \quad (16)$$

obtained by comparing with equation (11.4a) of [20]. The transverse traceless projection of Q^{TT} is

$$Q^{TT} = \frac{1}{4} (I_2^2 {}_{-2}Y_2^2 + I_2^0 {}_{-2}Y_2^0 + I_2^{-2} {}_{-2}Y_2^{-2}) \hat{e}_R + \text{c.c.}, \quad (17)$$

where c.c. stands for complex conjugate, \hat{e}_R is the circular polarization tensor

$$\hat{e}_R = \frac{1}{\sqrt{2}} (\hat{e}_+ + i\hat{e}_\times), \quad (18)$$

and ${}_{-2}Y_l^m$ are the spin-weighted spherical harmonics of spin-weight -2 :

$$\begin{aligned} {}_{-2}Y_2^2 &= \frac{1}{2} \sqrt{\frac{5}{\pi}} e^{2i\phi} \cos^4\left(\frac{\theta}{2}\right), & {}_{-2}Y_2^0 &= \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2\theta, \\ {}_{-2}Y_2^{-2} &= \frac{1}{2} \sqrt{\frac{5}{\pi}} e^{-2i\phi} \sin^4\left(\frac{\theta}{2}\right). \end{aligned} \quad (19)$$

(θ, ϕ) are the angular coordinates of the observer. To simplify matters, we can choose an observer on the north pole $(\theta, \phi) = (0, 0)$ (so ${}_{-2}Y_2^0 = 0 = {}_{-2}Y_2^{-2}$) and

$$h^{TT} = \frac{\mu}{\sqrt{2}r} \frac{d^2}{dt^2} (r_{12}^2 e^{-2i\phi_{12}}) \hat{e}_R + \text{c.c.} \quad (20)$$

All that remains to calculate the gravitational radiation is to solve (8) for r_{12} and ϕ_{12} . This can easily be done numerically, or, for early and late times, analytically, using the results of the previous section. Using (11) we find

$$h_{\text{early/late}}^{TT} = \frac{\sqrt{2}\mu v_\infty^2}{r} \left(1 \pm \frac{3}{2} \frac{M}{v_\infty t} \right) e^{-2i\phi_{12}} \hat{e}_R + \text{c.c.}, \quad (21)$$

where the upper/lower signs correspond to early-/late-time scattering orbits. As noted in [17], (21) provides a clear illustration of the *gravitational memory effect*: h^{TT} takes different values at early and late times and we have

$$\Delta h^{TT} = \frac{\sqrt{2}\mu v_\infty^2}{r} (e^{-2i\phi_{12}^i} - e^{-2i\phi_{12}^f}) \hat{e}_R + \text{c.c.}, \quad (22)$$

where ϕ_{12}^i and ϕ_{12}^f are the respective initial and final angular separations. For coalescing orbits at late times (13) we recover

$$h_{\text{coalescing}}^{TT} = \frac{160\sqrt{2}\mu M^4 (M - 2\mu)^2}{rt^6 v_\infty^4} e^{-2i\phi_{12}} \hat{e}_R + \text{c.c.}, \quad (23)$$

and we note that, at late times of a coalescence, the t -dependence of ϕ_{12} is too small to appear at this order in h^{TT} . Note also the t^{-6} fall-off, characteristic of Einstein-Maxwell theory. As we shall see in the next section, this becomes very different in the presence of a dilaton.

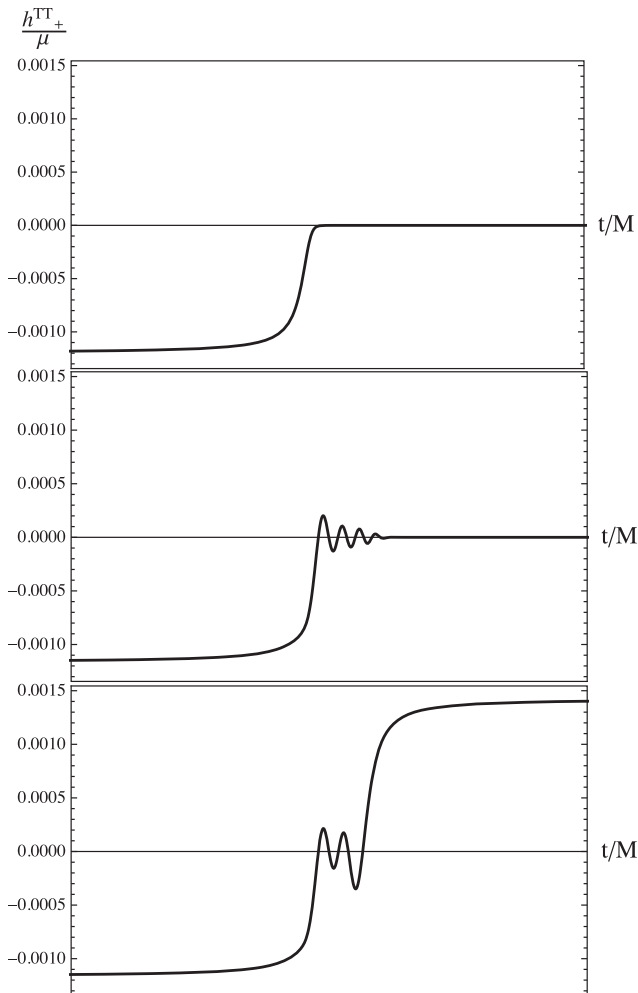


FIG. 2. Gravitational wave signatures for $a = 0$. We have plotted the h_+^{TT} wavefronts for the three different orbits depicted in Fig. 1. The top graph illustrates the wavefront emitted upon the head-on ($b = 0$) merger, the middle graph the wavefront emitted upon the below-critical coalescence, and the bottom graph the wavefront emitted upon the scattering interaction.

The h_+^{TT} signatures can be seen in Fig. 2, where we have plotted the numerically calculated signatures for orbits with impact parameters $b = 0$, $b = 0.999b_{\text{crit}}$, and $b = 1.01b_{\text{crit}}$. In Sec. IVB we shall plot the logarithm of the numerically calculated wavefront for a head-on and a near-critical merger and include the early- and late-time analytic expressions for comparison purposes; the analytic predictions are followed closely.

IV. COUPLING TO A DILATON

Let us now consider the following generalization of the Einstein-Maxwell theory [21,22]:

$$S = \int d^4x \sqrt{-g} (-R + 2(\nabla\phi)^2 + e^{-2a\phi} F^2), \quad (24)$$

with ϕ a dilatonic scalar field and a the corresponding coupling constant. This action describes a broad range of fundamental theories: $a = 0$ yields Einstein-Maxwell theory, $a = 1$ gives the low energy action of string theory, and $a = \sqrt{3}$ corresponds to Kaluza-Klein theory. The corresponding static multi-black hole solution [23] is given by

$$ds^2 = -\psi_a^{\frac{2}{1+a^2}} dt^2 + \psi_a^{\frac{2}{1+a^2}} d\vec{x} \cdot d\vec{x},$$

$$A = \frac{1}{\sqrt{1+a^2}} \psi_a^{-1} dt, \quad e^{-2a\phi} = \psi_a^{\frac{2a^2}{1+a^2}}, \quad (25)$$

where

$$\psi_a = 1 + (1+a^2) \sum_{i=1}^n \frac{m_i}{r_i}, \quad (26)$$

and reduces to the MP solution (3) for $a = 0$.

The dilatonic multi-black hole solutions are smooth in the conformal frame² with metric $\tilde{g}_{ab} = e^{-2a\phi} g_{ab}$ but are singular at the horizon in the Einstein frame with metric g_{ab} in (25), a point noted previously [24]. However, in the Einstein frame both the moduli space approximation [25] and an effective field theory [26] can be fully worked out for general a . These approximations are valid provided the black holes are sufficiently separated [Eq. (6)]; within this context the singular behavior at the horizons does not affect the motion of these extremal objects.

In order to find the quadrupole moment for this metric, we need to perform an expansion of g_{tt} , similar to (15), obtaining in general a -dependent coefficients of expansion.

²Such a conformal frame is different from the Jordan frame considered typically in string theory for $a = 1$, which would be obtained by $g_{ab}^{(s)} = e^{2a\phi} g_{ab}$. Contrary to the extremal electrically charged multi-black hole solutions [23] that are regular in the frame \tilde{g}_{ab} , the extremal magnetically charged multi-black hole solutions [22] are regular in the string frame.

The structures of Eqs. (25) and (26) ensure that the quadrupole moment is a -independent and is still given by (15).

Let us now promote the static metric to a dynamical setting, using the MSA approximation. The corresponding moduli space metric for general a [25] yields a description of the motion of two black holes via the Lagrangian (4) where now³

$$\gamma_a(r_{12}) = 1 + M \left(\frac{3 - a^2}{4\pi} \right) \int d^3x \left(\psi_a^{\frac{2(1-a^2)}{1+a^2}} \right) \frac{\vec{r}_1 \cdot \vec{r}_2}{r_1^3 r_2^3}, \quad (27)$$

and is nontrivial to integrate for generic a . Of course, for $a = 0$ we obtain (5). For the Kaluza–Klein case, $a = \sqrt{3}$, the moduli space metric vanishes, and there is no interaction between the black holes at this order of expansion—to get nontrivial results one would have to go to a higher order in velocities.

In what follows, we focus on two cases where we can perform analytically the integration in (27): the string-theoretic case $a = 1$, for which [25]

$$\gamma_{a=1} = 1 + \frac{2M}{r_{12}}, \quad (28)$$

and the case $a = \frac{1}{\sqrt{3}}$, where we find

$$\gamma_{a=\frac{1}{\sqrt{3}}} = 1 + \frac{8}{3} \left(\frac{M}{r_{12}} + \frac{2M^2}{3r_{12}^2} \right). \quad (29)$$

Note that these are both independent of μ , in contrast to what happens in the Einstein–Maxwell case. In other words, in our approximation and for these two special cases the gravitational wave signature will depend only on the total mass of the system, not on the binary mass ratio.

A. String theory black holes: $a = 1$

When $a = 1$, ψ does not contribute at all to the integral in (27). Interestingly, there is no value of b for which the black holes merge. At least within the MSA, all trajectories are scattering, including the head-on collision [25] (although it is not unreasonable to suspect that mergers could happen when the approximation is taken to higher order in v^2). As such, no oscillatory waveforms exist, and we only observe a memory effect, according to

$$\phi_{12 \text{ early/late}} = -\frac{b}{v_\infty t} + \dots, \quad (30)$$

$$r_{12 \text{ early/late}} = \mp v_\infty t - M \log(\mp v_\infty t) + \dots, \quad (31)$$

and so

³Note that this reduces to equation (IV.9) in Ref. [13] for the weak field $\psi_a \rightarrow 1$ approximation.

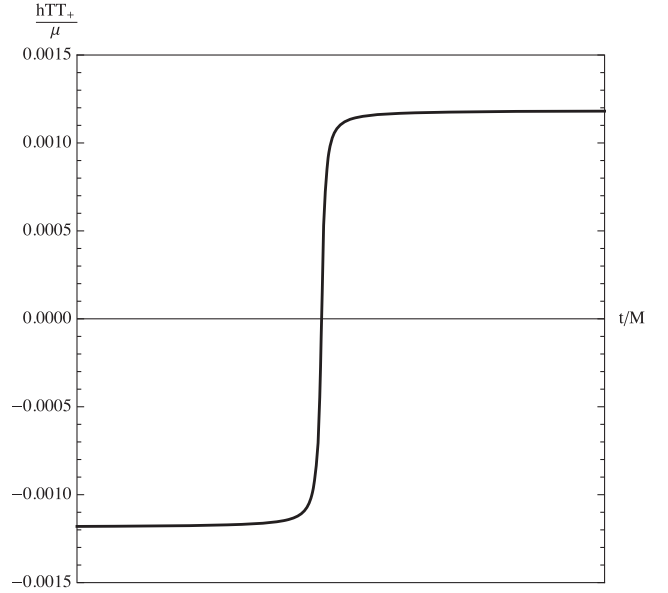


FIG. 3. The memory effect for $a = 1$. For this case there are no merging orbits and no oscillatory behavior in h^{TT} . However, we do see very clearly a memory effect.

$$h_{\text{early/late}}^{TT} = \frac{\sqrt{2}\mu v_\infty^2}{r} \left(1 \pm \frac{M}{v_\infty t} \right) e^{-2i\phi_{12}} \hat{e}_R + \text{c.c.} \quad (32)$$

The memory effect can be seen in Fig. 3.

B. Intermediate coupling: $a = \frac{1}{\sqrt{3}}$

For $a = \frac{1}{\sqrt{3}}$ the μ -independence of $\gamma(r_{12})$ in (29) implies that wavefronts emitted by binary pairs of arbitrary mass ratio yield the same gravitational wave signature, albeit rescaled by μ ; this is not true for $a = 0$, for which the equations of motion explicitly depend on μ . The critical impact parameter b_{crit} is $b_{\text{crit}} = \frac{4}{3}M$.

Using (29) to solve (8) yields

$$r_{12 \text{ early/late}} = \mp v_\infty t - \frac{4M}{3} \log(\mp v_\infty t) + \dots, \quad (33)$$

$$\phi_{12 \text{ early/late}} = -\frac{b}{v_\infty t} + \dots, \quad (34)$$

for the separation at early and late times, when $r_{12} \gg M$. Hence

$$h_{\text{early/late}}^{TT} = \frac{\sqrt{2}\mu v_\infty^2}{r} \left(1 \pm \frac{4}{3} \frac{M}{v_\infty t} \right) e^{-2i\phi_{12}} \hat{e}_R + \text{c.c.} \quad (35)$$

For coalescing orbits at late times we find

$$r_{12 \text{ coalescing}} = r_0 \exp\left(-\frac{3q v_\infty t}{16 M}\right), \quad (36)$$

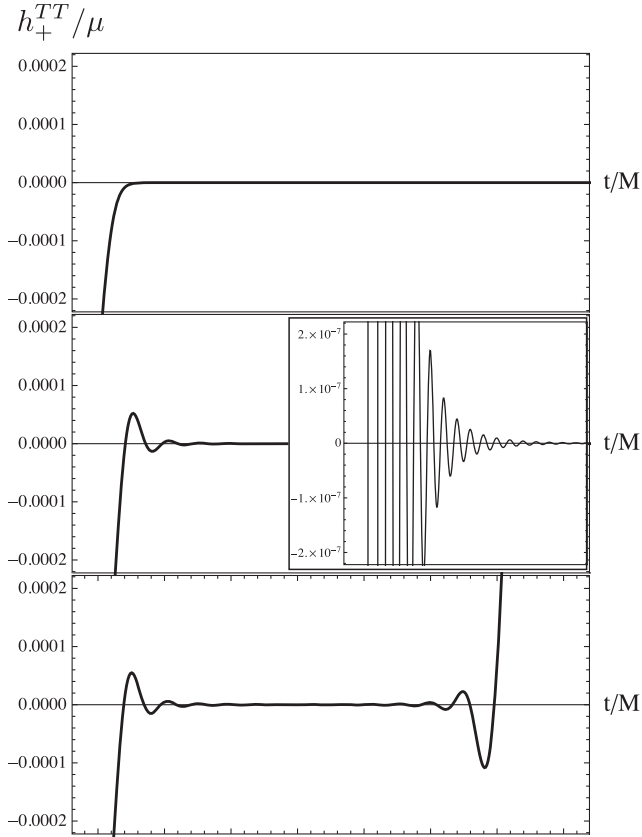


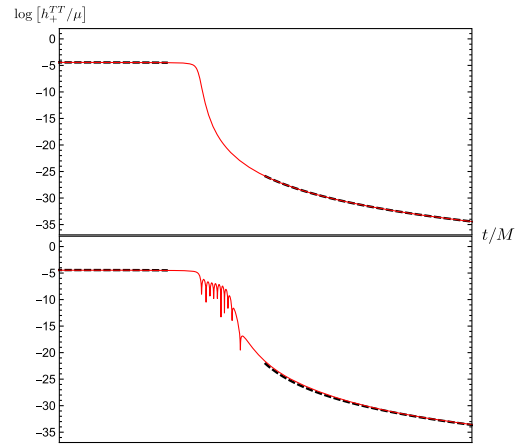
FIG. 4. Gravitational wave signatures for $a = \frac{1}{\sqrt{3}}$. We plot here graphs analogous to those in Fig. 2. The top graph illustrates the wavefront emitted upon a head-on collision ($b = 0$), the middle a subcritical case ($b = 0.999b_{\text{crit}} = 1.332M$), and the bottom a scattering event ($b = 1.01b_{\text{crit}} = 1.34667M$). The inset in the middle depicts near-critical coalescence to make the exponentially decaying behavior more explicit.

$$\phi_{12 \text{ coalescing}} = \frac{9}{16} \frac{bv_{\infty}t}{M^2} + \dots \quad (37)$$

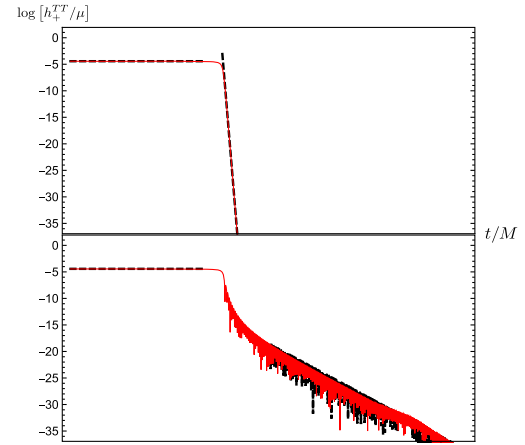
where r_0 is the separation at some t_0 , and we abbreviated $q \equiv \sqrt{16 - 9b^2/M^2}$; in particular, note that $\dot{\phi}_{12}$ is no longer small at late times. This implies an exponentially decaying signature:

$$h_{\text{coalescence}}^{TT} = \frac{9}{64} \frac{\mu\sqrt{2}r_0^2v_{\infty}^2}{rM^4} (8M^2 - 9b^2 + 3ibMq) \times \exp\left(-\frac{3}{8} \frac{v_{\infty}t}{M^2} (Mq + 3ib)\right) \hat{e}_R + \text{c.c.} \quad (38)$$

We plot the results in Figs. 4 and 5. We show the logarithm of the wavefront of coalescing orbits for different values of b in Fig. 5(b), where the b -dependence of the fall-off is seen. The exponential fall-off behavior is also clearly shown, in contrast to the t^{-6} behavior evident in Fig. 5(a) for the Einstein-Maxwell case. Note that the electromagnetic radiation would also be expected to have an



(a) $a = 0$, with (above) $b = 0$ and (below) $b = 0.999b_{\text{crit}}$.



(b) $a = \frac{1}{\sqrt{3}}$, with (above) $b = 0$ and (below) $b = 0.999b_{\text{crit}}$.

FIG. 5. Comparison of analytic expressions with numerical results for $a = 0$. Above we have plotted the behavior $\log(|h_+^{TT}|)$ as a function of time for head-on and below-critical collisions, with the numerically calculated solution in red and the analytical predictions for large and small $\frac{m}{r_{12}}$ in black; (a) is for $a = 0$ and (b) is for $a = \frac{1}{\sqrt{3}}$. We notice a number of things on the log plot that are difficult to see on the previous graphs: the t^{-6} for $a = 0$ behavior can be directly contrasted with the e^{-t} behavior for $a = \frac{1}{\sqrt{3}}$. We also notice the lack of b -dependence for the $a = 0$ case, as predicted, and the obvious b -dependence for $a = \frac{1}{\sqrt{3}}$.

exponential fall-off, as it takes a similar form as gravitational radiation (see [17]).

V. CONCLUSION

The presence of a dilaton can make a significant imprint on the gravitational waveforms emitted in black hole collisions and scattering events. By analytically computing expressions for the gravitational wavefronts emitted by the collision of two extremally charged dilatonic black holes, we have been able to compare the general relativistic

(Einstein-Maxwell) wavefronts with those occurring in a string-theoretic case ($a = 1$) and a more general dilatonic theory ($a = 1/\sqrt{3}$). In the latter case the gravitational waveforms are exponentially suppressed in time, whereas in general relativity the wavefronts decay with t^{-6} . However the gravitational memory effect for scattering is the same for all values of a , including the $a = 0$ Einstein-Maxwell case.

Our results complement those of recent studies of dilatonic black hole mergers [12,13], and illustrate a qualitative difference between cases with and without a dilaton.

It would be interesting to develop this technique to spacetimes with general coupling constant a between the dilaton and the Maxwell field, as we have only been able to do this so far for the specific values of $a = 0, \frac{1}{\sqrt{3}}, 1, \sqrt{3}$; we

leave this question for future study. Likewise, more detailed studies of nonextremal dilatonic black holes over a broad range of parameter space need to be carried out to see where the most interesting phenomenological possibilities lie.

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