

Fluctuations of the flux of energy on the apparent horizon

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Adopting the Landau-Lifshitz method of classical fluctuations, we determine the statistical average strength of the fluctuations of the energy flux on the apparent horizon of a homogeneous and isotropic universe described by Einstein gravity. We find that the fluctuations increase with the temperature of the horizon and decrease with its area, in accordance with the features of systems where gravity can be neglected. We further find, on the one hand, that the fluctuations vanish in the cosmological constant dominated de Sitter expansion and, on the other hand, that the domination of phantom fields is excluded. The reasonableness of the results we have obtained lend support to the view that the Universe behaves as a normal thermodynamic system.

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I. INTRODUCTION

Nowadays, the existence of a close connection between gravity and thermodynamics is widely acknowledged—see for instance Ref. [1] and references therein. It may be said that this interplay was first intimated by Tolman’s law for the equilibrium temperature in a medium placed in a gravitational field [2] and shortly afterward by the realization that a heat flux must run through an accelerated body in direction opposite to the acceleration [3]. Both effects are a direct consequence of the equivalence principle [4]. The understanding that the said connection is deep, and not merely coincidental, was strongly reinforced by the discovery that black holes obey the thermodynamic laws [5–8] and, later on, practically confirmed by the finding that Einstein field equations can be derived from the definition of entropy and the proportionality between the latter and the horizon area [9].

In view of the above and, on the other hand, given the huge number of degrees of freedom of the Universe, one may wonder whether the latter can be considered a thermodynamic system. Recently, this was partially answered in the affirmative by the suggestion, based on the observed evolution of the Hubble factor, that the entropy of the Universe tends to a finite maximum [10,11]—like any other macroscopic isolated physical system. However, it is not at all simple to determine experimentally the evolution of the said factor—see Ref. [12] and references therein. Therefore, if one wishes

to answer this question before more abundant data and data of much higher quality become available, it seems advisable to resort to the study the thermodynamic fluctuations of the energy flux.

As is well known, physical quantities of macroscopic systems experience small random fluctuations around their average values, because of the discontinuous nature of matter and of the thermal motion of its microscopic constituents. They are spontaneous, ubiquitous, grow in size with temperature and are at the root of the inevitable, but usually controllable, “noise” in measurement devices. Paradigmatic examples of thermal fluctuations are, for instance, the Brownian motion of a solid small particle in a fluid [13,14] and the fluctuations of the electric voltage in a resistor [14].

In this work, we consider a homogeneous and isotropic universe described at large scale by the Friedmann-Robertson-Walker (FRW) metric, and we calculate the average size of the fluctuations of the energy flux on the apparent horizon using the method of Landau and Lifshitz [15,17–19], succinctly recalled below (see though Ref. [16] for a clarification of the Refs. [15,17,18]). In principle, one might study the said fluctuations on any other closed surface as the event horizon. However, the former horizon is decidedly more suitable as it fulfills the laws of thermodynamics, while the latter does not [20].

We resort to the method of Landau and Lifshitz (LL) to determine the strength of the fluctuations which, although being based on microscopic considerations, offers a macroscopic approach to the issue and thus has the great advantage that it obviates the use of concepts stemming from a microscopic description, e.g., distribution functions,

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when the latter are unclear. As we aim at assessing the fluctuation of the energy flux across the apparent horizon of the expanding universe, the LL method does not depend on any underlying microscopic entities making up the space-time and thus avoids such an unclear issue. As it is, the problem only involves tackling the flux of matter, radiation and/or dark energy, which are the familiar components that source the gravitational field, and hence the usual Einstein equations are valid in this classical context.

We wish to emphasize that the behavior of the fluctuations of a physical system gives information about the properties of the latter. If the fluctuations of the energy flux mentioned above do behave in accordance with the fluctuations in normal systems which are not dominated by gravity, our confidence in the Universe being indeed a thermodynamic system (one that complies with the thermodynamic laws) will get significantly strengthened.

II. FLUCTUATIONS ON THE APPARENT HORIZON

At this point, it is expedient to recall the notion of apparent horizon in a FRW universe—see Refs. [21,22] for details (or see also Refs. [23–25] for additional insights). A spherically symmetric spacetime region will be called “trapped” if the expansion of ingoing and outgoing null geodesics, normal to the spatial two-sphere of radius \tilde{r} [where $\tilde{r} = a(t)r$] centered at the origin (i.e., at the comoving observer), is negative. By contrast, the region will be called “antitrapped” if the expansion of the geodesics is positive. In normal regions, outgoing null rays have positive expansion, and ingoing null rays have negative expansion. Thus, the antitrapped region is given by the condition

$$\tilde{r} > \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}, \quad (1)$$

where H and k stand for the Hubble rate and the spatial curvature index. Clearly, the surface of the apparent horizon is nothing but the boundary hypersurface of the spacetime antitrapped region. In the case of an exact de Sitter expansion, the apparent and event horizons coincide.

Since the radius of the apparent horizon fulfills $\tilde{r}_H = 1/\sqrt{H^2 + ka^{-2}}$, the area and entropy of the horizon, in units of the Boltzmann’s constant, are [21,22]

$$\mathcal{A}_H = 4\pi\tilde{r}_H^2 = \frac{4\pi}{H^2 + \frac{k}{a^2}} \quad \text{and} \quad S_H = \frac{1}{\ell_P^2} \frac{\pi}{H^2 + \frac{k}{a^2}}, \quad (2)$$

respectively.

As the universe expands at the Hubble rate, the energy inside the horizon increases at a rate

$$-\dot{E} = \mathcal{A}_H(\rho + p)H\tilde{r}_H = -\frac{\mathcal{A}_H}{4\pi G} \left(\dot{H} - \frac{k}{a^2} \right) \frac{H}{\sqrt{H^2 + ka^{-2}}}. \quad (3)$$

In arriving at the second equality, we relied on the conservation of matter energy,

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (4)$$

alongside Friedmann’s equation

$$3 \left(H^2 + \frac{k}{a^2} \right) = 8\pi G\rho. \quad (5)$$

In (3), the pressure adds to the energy density, because it also gravitates, and thereby the energy flux is, in reality, a flux of enthalpy. Owing to Lorentz invariance, the enthalpy of the quantum vacuum, $\rho_\Lambda + p_\Lambda$, vanishes identically; thus, the enthalpy of the vacuum in any spatial three-volume must also vanish. Consequently, the fluctuations of the energy flux of the vacuum are identically zero.

To apply the LL method of calculating the fluctuations of the fluxes in a system, the latter must be at thermodynamical equilibrium or near to it. The second possibility means that the system should evolve slowly. In our case, “slowly” entails that the rate by which the horizon area increases per unit of horizon area does not exceed its expansion rate, $3H$. A brief calculation gives

$$\frac{\dot{\mathcal{A}}_H/\mathcal{A}_H}{3H} = \frac{\rho + p}{\rho}. \quad (6)$$

The right-hand side of this equation is smaller than unity for a Lambda-cold dark matter (Λ CDM) universe and a universe dominated by quintessence and pressureless matter, and equal to unity for the Einstein-de Sitter universe. By contrast, it is 4/3 for a radiation dominated universe. Thus, we can safely apply the LL method to determine the statistical average strength of the fluctuations of $-\dot{E}$ on the apparent horizon for various cases of interest.

According to this method, if the flux \dot{y}_i of a given thermodynamic quantity, which evolves in a generic dissipative process, is governed by $\dot{y}_i = \Sigma_j \Gamma_{ij} Y_j + \delta\dot{y}_i$ and the entropy rate can be written as $\dot{S} = \Sigma_i (\pm Y_i \dot{y}_i)$, where $Y_i = (\partial\dot{S}/\partial\dot{y}_i)$, then the second moments of the fluctuations of the fluxes obey $\langle \delta\dot{y}_i \delta\dot{y}_j \rangle = (\Gamma_{ij} + \Gamma_{ji}) \delta_{ij} \delta(t_i - t_j)$, where the angular brackets stand for statistical average with respect to the reference state (namely, $\Sigma_j \Gamma_{ij} Y_j$), which is taken to be steady or quasisteady and corresponds to the systematic part of the flux. Obviously, $\langle \delta\dot{y}_i \rangle = 0$.

In the case at hand, we have just one flux, $\dot{y} = -\dot{E}$, and the above equations imply

$$\dot{S}_H = \frac{1}{4\ell_P^2} \dot{\mathcal{A}}_H = 2\pi G \frac{\mathcal{A}_H^2}{\ell_P^2} H(\rho + p) \quad (7)$$

as well as

$$\langle (\delta(-\dot{E}))^2 \rangle = 2\Gamma\delta(\tau) = \frac{3\ell_P^2}{8\pi^2 G^2} H \frac{\rho + p}{\rho} \delta(\tau), \quad (8)$$

where τ is the time interval between two consecutive measurements of \dot{E} (say $t_i - t_j$ in the notation of Ref. [15] recalled above).

Notice that, due to the presence of the square of the Planck length in the numerator of the latter equation, the average strength of the fluctuations is minute, as expected. [Assuming the Universe is well described at the background level by the Λ CDM model, currently $\langle(\delta(-\dot{E}))^2\rangle^{1/2}$ and $(-\dot{E})$ are, in natural units, of the order of 10^{-21} and 10^7 , respectively]. In accordance with our previous comment, they vanish, for an exact de Sitter universe, as they should. This is quite reasonable, since the quantum vacuum is continuous at the classical level, and therefore it does not source classical fluctuations. Furthermore, because the right-hand side of (8) cannot be negative, and the Universe is expanding, the null energy condition must be satisfied. This directly excludes the dominance of phantom fields. This agrees with the fact that these exotic fields are disfavored by the second law of thermodynamics [26], the occurrence of quantum instabilities and other inherent maladies [27–29]. Likewise, a restriction arises on the product $a^2\dot{H}$. The latter must fulfill $a^2\dot{H} < k$. This is guaranteed when the spatial sections are either spherical or flat; when they are hyperbolic, each particular case should be studied on a single basis. Moreover, the fact that the statistical averaged size of the fluctuations grows with H implies that the lower the scale factor, the lower the area of the apparent horizon and the larger the strength of the fluctuations. (For instance, in the Λ CDM universe, $\rho = \rho_\Lambda + \rho_{m0}a^{-3}$, while $\mathcal{A}_H \sim [\rho_\Lambda + \rho_{m0}a^{-3} + ka^{-2}]^{-1}$). This result could have been anticipated on physical grounds. It parallels the behavior of the fluctuations of the fluxes in fluids (the smaller the volume of the fluid under consideration, the stronger the fluctuations of the fluxes [15]). Further, since the temperature of the horizon is proportional to its surface gravity and this increases with the Hubble factor [30], so does the size of the fluctuations. They behave, also in this regard, similarly to the statistical fluctuations of normal systems not dominated by gravity.

Clearly, the intensity of the random fluctuations of $(-\dot{E})$ should be fairly lower than $(-\dot{E})$ itself. In other words,

$$\eta = \frac{3\ell_p^2}{8\pi^2 G^2} \frac{H}{\mathcal{A}_H^2 \rho(\rho + p)} \delta(\tau) < 1. \quad (9)$$

At late times, the Universe must approach a state of maximum entropy; this means that it will get steadily dominated by the cosmological constant, with $(\rho + p) \rightarrow 0$. Accordingly, η will grow. However, the reasonable condition (9) sets a generous upper bound on the fluctuations. This is rather sensible because, as said above, in an exact de Sitter expansion, the energy flux vanishes identically and so do its fluctuations.

The aforesaid bound can be recast as a lower limit on the energy flux,

$$(-\dot{E}) > \frac{3\ell_p^2}{8\pi^2 G^2} \frac{H}{\mathcal{A}_H \rho} \delta(\tau). \quad (10)$$

A stronger bound, not yet found, must hold in the quantum regime.

Note that (10) does not apply to the case of the quantum vacuum itself. Indeed, as mentioned above, the latter content—unlike matter and radiation—is neither discontinuous nor presents thermal motion.

At early times (i.e., well before the vacuum energy started to dominate), the energy density could (in principle) be so high that the bound (9) would be violated—recall that $\mathcal{A}_H \propto \rho^{-1}$ and that $(\rho + p) \sim -\dot{H} > 0$. However, if this were to occur, it would be very likely to happen before the beginning of the matter era, whence the near-equilibrium condition $(\rho + p)/\rho \leq 1$ would not be met at that epoch, and the LL method would not apply, according to Eq. (6).

The analysis carried out here can be readily extended to the case in which there are sources of matter and/or radiation creation. Then, the Universe should be treated as an open system “à la Prigogine” [31]. In this instance, the continuity equation reads

$$\dot{\rho} + 3H(\rho + p) = \Gamma_c \rho, \quad (11)$$

where Γ_c is the rate of creation of energy. It obeys $0 \leq \Gamma_c/3H < 1$ [32–35]. Then,

$$\begin{aligned} -\dot{E} &= \mathcal{A}_H(\rho + p)H\tilde{r}_H \\ &= \frac{\mathcal{A}_H}{3H} \left[\Gamma_c \rho - \frac{3}{4\pi G} H(\dot{H} - ka^{-2}) \right] H\tilde{r}_H. \end{aligned} \quad (12)$$

Associated to this rate, there is an extra, negative, pressure, $p_c = -(\rho + p)\Gamma_c/3H$. However, as it is easy to realize, when there is creation of particles, the energy flux is augmented accordingly. However, as can be checked, the expression for the statistical fluctuations of $(-\dot{E})$ formally coincide with (8), though ρ , p and H will differ from the situation where $\Gamma_c = 0$.

III. CONCLUSIONS

We have evaluated the statistical average strength of the classical fluctuations of energy flux on the apparent horizon of an expanding FRW universe, namely, Eq. (8). This equation has a number of desirable features (e.g., the fluctuations increase with the temperature of the horizon and decrease with its area), thus showing clear similarities with the typical fluctuations in systems in which gravity does not play a main role, and they identically vanish when the expansion is purely de Sitter. This indicates to us that the Universe, governed by Einstein gravity, behaves as a normal thermodynamic system. It should be interesting to study these fluctuations assuming the Universe is described by any other reasonable theory of gravity [36,37], which is left for a subsequent work.

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