Primordial perturbations with pre-inflationary bounce

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Based on the effective field theory (EFT) of nonsingular cosmologies, we build a stable model, without the ghost and gradient instabilities, of bounce-inflation (inflation is preceded by a cosmological bounce). We perform a full simulation for the evolution of scalar perturbation, and find that the perturbation spectrum has a large-scale suppression (as expected), which is consistent with the power deficit of the cosmic microwave background (CMB) TT-spectrum at low multipoles, but unexpectedly, it also shows itself one marked lower valley. The depth of valley is relevant with the physics around the bounce scale, which is model-dependent.

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I. INTRODUCTION

Inflation [1–4] is the current paradigm of early universe. It predicts nearly scale-invariant scalar perturbation, which is consistent with the cosmic microwave background (CMB) observations [5,6], as well as the gravitational waves (GWs). However, it is not the final story of the early universe. As pointed out by Borde, Vilenkin and Guth [7,8], inflation is past-incomplete, and "*inflationary models require physics other than inflation to describe the past boundary of the inflating region of spacetime.*" [8].

This past-incompletion (singularity) of inflation has inspired radical alternatives to inflation, e.g., [9–12]. However, how to make inflation happen in a past-complete scenario is also a noteworthy issue. In certain sense, this actually requires that the pre-inflationary phase should be past-complete. One possibility is that it is slow contracting, so that the infinite past is complete Minkowski spacetime. In such a scenario, a nonsingular bounce preceding inflation must occur (so-called the bounce-inflation scenario) [13].

Recently, the Planck collaboration [14,15] has observed the power deficit of CMB TT-spectrum at large scale. This might be a hint of the pre-inflationary physics, which happens around ~60 efolds, see, e.g., [16]. The idea of bounce-inflation accounted for not only the power deficit on large angular scales [13,17,18], but also a large dipole power asymmetry [17,19] in the CMB fluctuation. Thus we conjectured that the physics hinted by the CMB anomalies might be relevant with the pre-inflationary bounce, see also [20–27].

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In physical time t, the equation of motion of scalar perturbation ζ in momentum space is

$$\ddot{\zeta}_k + \left(3H + \frac{\dot{Q}_s}{Q_s}\right)\dot{\zeta}_k + c_s^2 \frac{k^2}{a^2}\zeta_k = 0, \qquad (1)$$

where $H = \dot{a}/a$, a dot denotes d/dt, a is the scale factor, k is the comoving wave number, c_s is the sound speed and Q_s is a variable with the dimension of mass squared that depends on the behavior of scalar perturbation. Generally, $Q_s/M_p^2 \sim \epsilon_{\rm cont} = {\rm const} \gg 1$ for the contraction, while $Q_s/M_p^2 \sim \epsilon_{\rm inf} < 1$ for the inflation, where $\epsilon = -\dot{H}/H^2$. Thus Q_s inevitably shows itself a jumping around the nonsingular bounce, even if this phase lasts shortly enough. Previous studies neglected the effect of Q_s on the perturbation spectrum, since this effect is ambiguous without a fully stable (without the ghost and gradient instabilities) nonsingular bounce. Since recently, with the effective field theory (EFT) of nonsingular cosmologies [28–30], we have been able to stably manipulate the bounce [31,32], see also [33,34]. This impels us to reconsider the relevant issue.

In this paper, inspired by [28,29,31,32], we build a fully stable model of bounce-inflation, in which the universe is in the ekpyrotic contraction initially. By numerically solving Eq. (1), we find that the pre-inflationary bounce not only brings the power deficit of the CMB TT-spectrum at low multipoles (as expected in [13,17]), but unexpectedly, also provides an explanation to the dip at multipole $l \sim 20$ hinted by Planck [6].

II. THE LAGRANGIAN

Recently, it was found that the nonsingular cosmological models usually suffer from the ghost or gradient instabilities ($c_s^2 < 0$) [35,36], see also [37,38]. Based on the EFT of nonsingular cosmologies [28–30], this no-go result has been clearly illustrated (see [39] for recent discussions on the choice of gauge). The cubic Galileon interaction $\sim\Box\phi$ in Horndeski theory [40–42] only moves the period of $c_s^2 < 0$ to the outside of bounce phase, but cannot dispel it completely [43,44]. It was found first in [28,29] that the operator $R^{(3)}\delta g^{00}$ in EFT could play significant role in curing the gradient instability of scalar perturbation. Recently, we have built fully stable cosmological bounce models in Ref. [31] by applying the covariant $L_{R^{(3)}\delta g^{00}}$.

We follow Ref. [31], and after defining $\phi_\mu = \nabla_\mu\phi$, $\phi^\mu = \nabla^\mu\phi$, $\phi_{\mu\nu} = \nabla_\nu\nabla_\mu\phi$, $X = \phi_\mu\phi^\mu$ and $\Box\phi = \phi^\mu_\mu$, write the effective Lagrangian of nonsingular bounce-inflation as (ϕ is set dimensionless)

$$L \sim \underbrace{\frac{M_p^2}{2}R - \frac{M_p^2}{2}X - V(\phi)}_{\text{Contraction+Inflation}} + \underbrace{\tilde{P}(\phi, X)}_{\text{(Ghost free) Bounce}} + \underbrace{L_{\delta g^{00}R^{(3)}}}_{\text{Removing } c_s^2<0} + L_{\delta K\delta g^{00}}, \quad (2)$$

where

$$\begin{aligned} L_{\delta g^{00}R^{(3)}} &= \frac{f_1(\phi)}{2}\delta g^{00}R^{(3)} \\ &= \frac{f}{2}R - \frac{X}{2}\int f_{\phi\phi} d\ln X - \left(f_\phi + \int \frac{f_\phi}{2} d\ln X\right)\Box\phi \\ &\quad + \frac{f}{2X}[\phi_{\mu\nu}\phi^{\mu\nu} - (\Box\phi)^2] \\ &\quad - \frac{f - 2Xf_X}{X^2}[\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu - (\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu], \end{aligned} \quad (3)$$

$$\begin{aligned} L_{\delta K\delta g^{00}} &= \frac{g_1(\phi)}{2}\delta K\delta g^{00} \\ &= \frac{g}{2}\frac{1}{\sqrt{-X}}\left(\frac{\phi^\mu\phi_{\mu\nu}\phi^\nu}{X} - \Box\phi\right) - \frac{3}{2}g\cdot h, \end{aligned} \quad (4)$$

with $R^{(3)}$ being the 3-dimensional Ricci scalar on the spacelike hypersurface, $R^{(3)}\delta g^{00}$ and $\delta K\delta g^{00}$ being the EFT operators. Though the covariant expression of $L_{\delta g^{00}R^{(3)}}$ has the higher order of the second order derivative of ϕ, it is Ostrogradski ghost-free [45,46]. In fact, it is straightforward to check that the resulting covariant action $S = \int d^4x\sqrt{-g}L$ belongs to the class Ia degenerate higher order scalar-tensor (DHOST) theories [47], see Appendix A for details. We briefly review the EFT of nonsingular cosmologies in Appendix B, see (B3) for the definition of δg^{00} and δK.

In (3) and (4), f_1, g_1 and h are (arbitrary) functions of ϕ, thus are also time dependent since ϕ is a function of t. In the derivation above, the time dependences of f, g and h should satisfy

$$\begin{aligned} f[\phi(t), X(t)] &\equiv f_1(\phi(t))\left[1 + \frac{X}{f_2(\phi(t))}\right], \\ g[\phi(t), X(t)] &\equiv g_1(\phi(t))\left[1 + \frac{X}{f_2(\phi(t))}\right] \end{aligned} \quad (5)$$

and

$$h(\phi(t)) \equiv H(t), \quad (6)$$

where f_2 is also a function of ϕ and its time dependence should satisfy $f_2 \equiv \frac{X}{\delta g^{00}-1} = \dot{\phi}^2(t)$, so that f, g and $(h - H)$ vanish at background level (see Fig. 9 in Appendix C for the ϕ-dependences of f_1, g_1, f_2 and h). Hence, $L_{\delta g^{00}R^{(3)}}$ and $L_{\delta K\delta g^{00}}$ have no contribution to the background equations of motion, like their EFT forms. Therefore, we can construct background evolution without taking account of $L_{\delta g^{00}R^{(3)}}$ and $L_{\delta K\delta g^{00}}$, as we will do in Sec. III A.

One can also directly work with the covariant expressions (3) and (4) while disregarding conditions (5) and (6). In such a case, the construction of background could be more complicate since g and h do appear in the background equations. However, because f, g and h are undetermined functions, one can assume that $|g| \ll M_p^2|H|$, $|\dot{g}| \ll |\dot{H}|M_p^2$ and $|H - h| \ll |H|$, i.e., $L_{\delta K\delta g^{00}}$ has no contribution at background level, for simplicity. The particular solutions of f, g and h can be obtained once the background evolution is determined.

III. A STABLE MODEL OF BOUNCE-INFLATION

A. Background

A sketch of the bounce-inflation scenario is plotted in Fig. 1. We will show how to build its stable model with the Lagrangian (2).

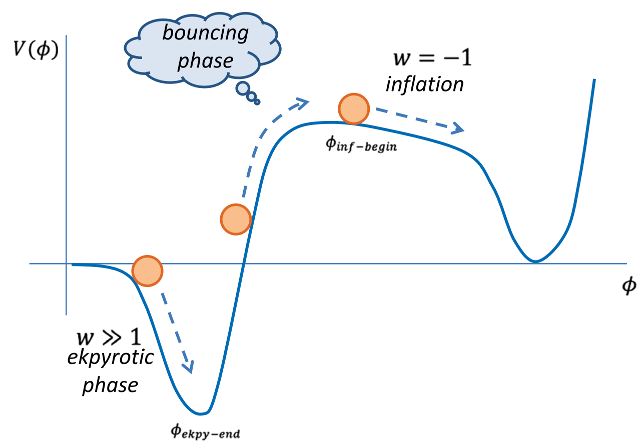


FIG. 1. A sketch of the bounce-inflation scenario.

As a specific model, we set

$$\tilde{P}(\phi, X) = \frac{\alpha_0}{(1+(\phi/\lambda_1)^2)^2} M_p^2 X/2 + \frac{\beta_0}{(1+(\phi/\lambda_1)^2)^2} X^2/4, \tag{7}$$

$$V(\phi) = -\frac{V_0}{2} e^{\sqrt{\frac{2}{q}}\phi}\left[1-\tanh\left(\frac{\phi}{\lambda_2}\right)\right] + \frac{\Lambda}{2}\left(1-\left(\frac{\phi}{\lambda_3}\right)^2\right)^2\left[1+\tanh\left(\frac{\phi}{\lambda_2}\right)\right], \tag{8}$$

with the positive constants $\lambda_{1,2,3}$ and q, α_0, β_0 being dimensionless. We have $\tilde{P}(\phi, X) \neq 0$ only around $\phi \simeq 0$ [48–50], while $\tilde{P}(\phi, X) = 0$ for $|\phi| \gg \lambda_1$. Here, the second term in $\tilde{P}(\phi, X)$ is required for avoiding ghost instability, as will be seen in the next subsection. In addition, as will be shown, the parameters α_0 and β_0 together with λ_1 are crucial for setting the duration of the bounce phase.

The first and the second terms in $V(\phi)$ set the effective potential of the field ϕ in the contracting phase and the inflation phase, respectively. The value of λ_2 sets the scale of the intermediate phase, namely, the bouncing phase, in ϕ space. The values of q and V_0 decide the evolutions of the background and ϕ in the contracting phase, as will be seen. The parameters Λ and λ_3 are responsible for the expansion rate of inflation.

Thus we have

$$3H^2 M_p^2 = \left[1-\frac{\alpha_0}{(1+(\phi/\lambda_1)^2)^2}\right] M_p^2 \dot{\phi}^2/2 + \frac{3\beta_0}{(1+(\phi/\lambda_1)^2)^2}\dot{\phi}^4/4 + V(\phi), \tag{9}$$

$$\dot{H} M_p^2 = -\left[1-\frac{\alpha_0}{(1+(\phi/\lambda_1)^2)^2}\right] M_p^2 \dot{\phi}^2/2 - \frac{\beta_0}{(1+(\phi/\lambda_1)^2)^2}\dot{\phi}^4/2. \tag{10}$$

In infinite past, the universe is almost Minkowski, which will experience the ekpyrotic contraction. In the ekpyrotic phase ($\phi \ll -\lambda_1$ and $-\lambda_2$), we have $\tilde{P} = 0$ and $V_{ekpy} = -V_0 e^{\sqrt{\frac{2}{q}}\phi}$ ($q \ll 1$). Thus we could write Eqs. (9) and (10) as

$$3H^2 = \dot{\phi}^2/2 - \frac{V_0}{M_p^2} e^{\sqrt{\frac{2}{q}}\phi}, \qquad \dot{H} = -\dot{\phi}^2/2. \tag{11}$$

By solving (11), we have

$$a \sim (-t)^{1/\epsilon}, \qquad \dot{\phi} = \sqrt{\frac{2}{\epsilon}}(-t)^{-1}, \tag{12}$$

and

$$\phi(t) = \sqrt{\frac{2}{\epsilon}} \ln\left[\frac{\sqrt{\epsilon-3}}{\epsilon\sqrt{V_0}/M_p}(-t)^{-1}\right], \tag{13}$$

where $\epsilon = -\dot{H}/H^2 = 1/q \gg 1$, which suggests $H = -\epsilon^{-1}(-t)^{-1}$.

When $\phi \simeq \lambda_1$, we could have

$$\dot{H} \simeq \left(\frac{\alpha_0}{4} - \frac{\beta_0\dot{\phi}^2}{4M_p^2} - 1\right)\dot{\phi}^2/2 > 0, \tag{14}$$

the nonsingular bounce will occur. Especially, around the bounce point, where $|\phi| \ll \lambda_1$ and V can be negligible with appropriate V_0 and Λ, we have $\dot{H} \approx \frac{(1-\alpha_0)^2}{9\beta_0}$, $\dot{\phi} \approx \sqrt{-\frac{2(1-\alpha_0)}{3\beta_0}}$ and $\ddot{\phi} \approx \frac{dV/d\phi}{1-\alpha_0}$. Thus α_0 and β_0 are crucial for setting the duration of bouncing phase. The smaller is $\dot{H}$, the longer is this duration. Therefore, we should set larger $(1-\alpha_0)$ and even much larger β_0.

While after $\phi \gg \lambda_1, \lambda_2$, the field ϕ will be canonical ($\tilde{P} = 0$) again. We have

$$3H^2 = \dot{\phi}^2/2 + \frac{\Lambda}{M_p^2}\left(1-\left(\frac{\phi}{\lambda_3}\right)^2\right)^2, \qquad \dot{H} = -\dot{\phi}^2/2. \tag{15}$$

Thus the slow-roll inflation will occur. Actually, after the nonsingular bounce, the Lagrangian (2) will reduce to $L \sim M_p^2 R/2 - M_p^2 X/2 - V_{\rm inf}$ with $V_{\rm inf}$ being the potential of slow-roll inflation.

We plot the background evolution in Fig. 2 with $\alpha_0 = 20$, $\beta_0 = 5\times10^9$, $\lambda_1 = 0.224$, $\lambda_2 = 0.0667$, $\lambda_3 = 12$, $V_0 = 5\times10^{-9}M_p^4$, $q = 0.1$, $\Lambda = 2.5\times10^{-9}M_p^4$. The initial values are set by (12) and (13).

B. Simulation for the scalar perturbation spectrum

In unitary gauge $\delta\phi = 0$, the quadratic action of scalar perturbation ζ for (2) is (see Appendix B and also our [28])

$$S_\zeta^{(2)} = \int a^3 Q_s\left(\dot{\zeta}^2 - c_s^2\frac{(\partial\zeta)^2}{a^2}\right)d^4x, \tag{16}$$

in which

$$Q_s = \frac{2\dot{\phi}^4\tilde{P}_{XX} - M_p^2\dot{H}}{\gamma^2} + 3\left(\frac{g_1}{2\gamma M_p}\right)^2, \tag{17}$$

$$c_s^2 Q_s = \frac{\dot{c}_3}{a} - M_p^2, \qquad c_3 = \frac{aM_p^2}{\gamma}\left(1+\frac{2f_1}{M_p^2}\right), \tag{18}$$

with $\gamma = H + \frac{g_1}{2M_p^2}$.

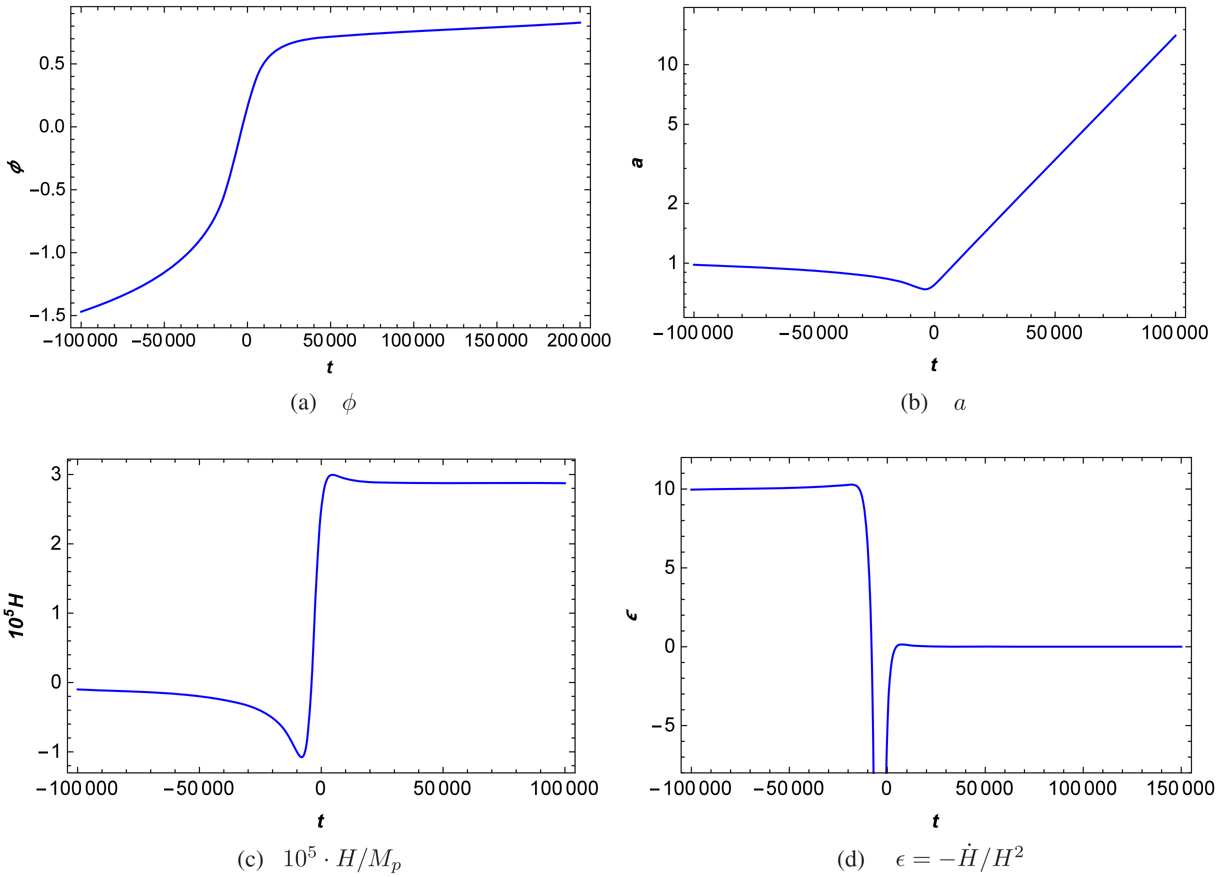


FIG. 2. The background evolution of our model with $\alpha_0 = 20$, $\beta_0 = 5 \times 10^9$, $\lambda_1 = 0.224$, $\lambda_2 = 0.0667$, $\lambda_3 = 12$, $V_0 = 5 \times 10^{-9} M_p^4$, $q = 0.1$, $\Lambda = 2.5 \times 10^{-9} M_p^4$.

The conditions $Q_s > 0$ and $c_s^2 > 0$ are required to avoid the ghost and gradient instabilities, respectively. Generally, $Q_s > 0$ can be obtained by applying $\tilde{P}(\phi, X)$. However, around the bounce point, $H \simeq 0$,

$$c_s^2 \sim -\dot{\gamma}\left(1 + \frac{2f_1}{M_p^2}\right) + \frac{2\dot{f}_1\gamma}{M_P^2} - \gamma^2. \tag{19}$$

Thus, around the bounce point, we will have $c_s^2 \sim -\dot{\gamma} - \gamma^2 < 0$ if $f_1 \equiv 0$, and $c_s^2 > 0$ only if $2f_1 < -M_p^2$, as has been clarified in Refs. [28–30]. Therefore, the gradient instability ($c_s^2 < 0$) could be cured by $L_{\delta g^{00} R^{(3)}}$ with a proper function $f_1(\phi)$. There could be infinite many different $f_1(\phi)$ that are able to guarantee $c_s^2 > 0$. From Eq. (18), we can find the solution of $f_1(\phi)$ for a given (specific) c_s^2 (see also [30]), i.e.,

$$f_1(\phi) = \frac{\gamma}{2a}\int a(Q_s c_s^2 + M_p^2)dt - \frac{M_p^2}{2}. \tag{20}$$

For simplicity, we will set $c_s^2 \equiv 1$ in the following.

In conformal time $\eta = \int dt/a$, the equation of motion of ζ is

$$u'' + \left(c_s^2 k^2 - \frac{z_s''}{z_s}\right)u = 0, \tag{21}$$

where $u = z_s \zeta$ and $z_s = \sqrt{2a^2 Q_s}$. In infinite past, the universe is almost Minkowski, and will come through the ekpyrotic phase. The perturbation modes have the wavelength $\lambda \simeq 1/k \ll \sqrt{z_s/z_s''}$ and $c_s^2 = 1$. Thus the initial state of the perturbation is

$$u \simeq \frac{1}{\sqrt{2k}} e^{-ik\eta}. \tag{22}$$

The perturbation modes will pass through the ekpyrotic phase, the bounce phase and the inflation phase, sequentially. The resulting spectrum P_ζ of ζ (at $-k\eta \ll 1$) is

$$P_\zeta = \frac{k^3}{2\pi^2}|\zeta|^2. \tag{23}$$

In physical time, the equation of motion of ζ is (1). In the ekpyrotic phase, $z_s \sim a \sim (-\eta)^{\frac{1}{\epsilon_{ekpy}-1}}$, since $Q_s/M_p^2 \sim \epsilon_{ekpy} = \text{const} \gg 1$. While in the inflationary phase, $\epsilon_{\inf} < 1$. This suggests that Q_s (or $z_s \sim a\sqrt{Q_s}$) will show itself a jump around the nonsingular bounce, which will inevitably affect P_ζ. Whether the jump of Q_s is gentle or not is model-dependent. We will simulate its effect on P_ζ by numerically solving Eq. (1), with $c_s^2 \equiv 1$ set by Eq. (20).

It should be mentioned that if $g_1 = 0$ ($L_{\delta K \delta g^{00}}$ is absent), we will have $\gamma = H = 0$ at the bounce point and $Q_s \sim 1/\gamma^2$ is divergent, see (18), so that Eq. (1) is singular. Here, in order to avoid it, we apply $g_1(\phi)$, see also [30].

For simplicity, we set

$$Q_s = \mathcal{A}_Q\left[\mathcal{B} - \tanh\left(\frac{t}{t_*}\right)\right]. \tag{24}$$

Such a steplike Q_s requires a particular $g_1(\phi)$, i.e., $g_1(\phi)$ in Lagrangian (2) should satisfy the condition

$$g_1(\phi(t)) = -\frac{2HM_p^2 Q_s - 2\sqrt{3H^2 M_p^6 Q_s + M_p^4(3M_p^2 - Q_s)(\dot{H}M_p^2 - 2\dot{\phi}^4 \tilde{P}_{XX})}}{Q_s - 3M_p^2}, \tag{25}$$

which can be derived from (18). We plot the spectrum P_ζ of scalar perturbation in Fig. 3 for the background in Fig. 2 with different values of $\mathcal{B}$ and t_*, where $P_\zeta^{\rm inf} = \frac{H_{\rm inf}^2}{8Q_s^{\rm inf}\pi^2 M_p^2}\left(\frac{k}{\mathcal{H}_{\rm inf}}\right)^{n_s - 1}$ is that of the inflation, with $Q_s^{\rm inf}$ being the value of Q_s during inflation, $n_s - 1 \simeq 0$ (but is slightly red). The evolutions of Q_s, g_1 and $|\zeta|$ with respect to t, respectively, are plotted in Figs. 7 and 8 of Appendix C.

As expected in [13], P_ζ shows itself a large-scale cutoff, but is flat (with a damped oscillation) at small scale. However, due to the steplike evolution of Q_s, the peaks and valleys of the oscillations are obviously pulled lower. Actually, after the nonsingular bounce, with Eq. (1), we shortly have the effective Hubble parameter

$$H_{\rm inf}^{\rm eff} = H_{\rm inf} + \frac{\dot{Q}_s}{3Q_s} < H_{\rm inf}, \tag{26}$$

since $\dot{Q}_s < 0$, see Figs. 7(b) and 8(b) in Appendix C. Thus P_ζ is pulled lower at the corresponding scale, since $P_\zeta \sim (H_{\rm inf}^{\rm eff})^2$. The change rate of Q_s is relevant to the physics of nonsingular bounce, as showed in Eq. (24), so the depth of valley pulled lower is actually model-dependent.

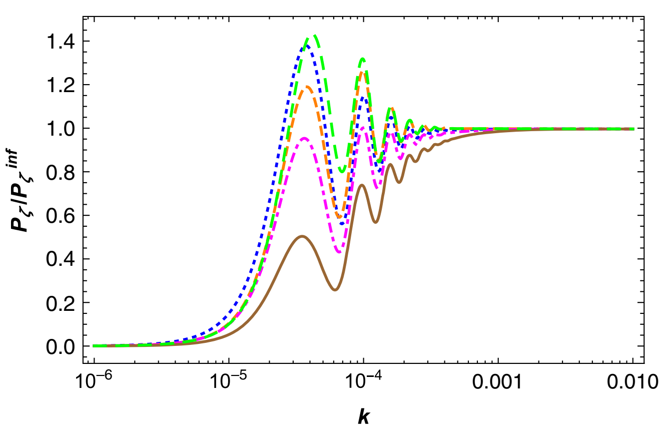


FIG. 3. $P_\zeta/P_\zeta^{\rm inf}$ with background set by Fig. 2, where the {brown solid, magenta dotdashed, orange dashed, blue dotted, green long dashed} curves correspond to $\mathcal{A}_Q = \{3,3,3,3,3\}$, $\mathcal{B} = \{1.3, 1.8, 2, 3, 2\}$, $t_* = \{4, 4, 2.5, 4, 1.5\} \times 10^4$, respectively.

In Sec. IV B, we will show that such a marked lower valley at corresponding scale helps to explain the dip around $l \simeq 20$ hinted by Planck [6].

IV. MORE ON THE SPECTRUM

A. Analytical estimation

We will attempt to analytically estimate P_ζ. The equation of motion for ζ is (21). In [26], the spectrum of primordial GWs has been calculated. Here, if the effect of Q_s is neglected, the calculation will be similar.

The bounce phase is the evolution with $\dot{H} > 0$. We define that it begins and ends at η_{B-} and η_{B+}, respectively, at which $\dot{H} = 0$. We set that $H = 0$ at η_B, which corresponds to the bounce point. Generally, $\Delta\eta_B = \eta_{B+} - \eta_{B-} \lesssim 1/\mathcal{H}_{B+}$.

In our model (Sec. III), the contracting phase ($\eta < \eta_{B-}$) is ekpyroticlike, a is almost constant for $\epsilon_{\rm ekpy} \gg 1$. Considering the continuities of a and H at η_{B-}, we have

$$a(\eta) = a_{B-}\left[\frac{x}{(\epsilon_{\rm ekpy} - 1)^{-1}\mathcal{H}_{B-}^{-1}}\right]^{\frac{1}{\epsilon_{\rm ekpy} - 1}}, \tag{27}$$

see [26] for the details, where $\mathcal{H}_{B-}$ is the comoving Hubble parameter at η_{B-} and $x = \eta - \eta_{B-} + (\epsilon_{\rm ekpy} - 1)^{-1}\mathcal{H}_{B-}^{-1}$. We have $z_s''/z_s = a''/a$, since Q_s is constant. Thus the solution of (21) is

$$u_k = \frac{\sqrt{\pi|x|}}{2} c_{1,1} H_{\nu_1}^{(1)}(-kx), \tag{28}$$

where $\nu_1 = 1/2$ for $\epsilon_{\rm ekpy} \gg 1$, and the initial condition (22) has been used.

In the nonsingular bounce phase ($\eta_{B-} < \eta < \eta_{B+}$), H should cross 0. We parametrize it as $H = \alpha(t - t_B)$ [51] with $\alpha M_P^2 \ll 1$. We have

$$a \simeq a_B e^{\frac{1}{2}\alpha(t-t_B)^2} \simeq a_B\left[1+\frac{\alpha}{2}(t-t_B)^2\right], \tag{29}$$

where $a = a_B$ at the bouncing point $t = t_B$. The continuities of a and $\mathcal{H}$ at η_{B-} and η_{B+} suggest $\mathcal{H}_{B+} = \mathcal{H}_{B-} + \alpha a_B^2(\eta_{B+} - \eta_{B-})$. In our models, $|\mathcal{H}_{B-}| \lesssim \mathcal{H}_{B+}/4$ (see Figs. 7 and 8 in Appendix C), so that we approximately have

$$\mathcal{H}_{B+} \simeq \alpha a_B^2 \Delta\eta_B. \tag{30}$$

Thus in this phase Eq. (21) is

$$u_k'' + (k^2 - \alpha a_B^2)u_k = 0, \tag{31}$$

which has a solution

$$u_k(\eta) = c_{2,1}e^{l(\eta-\eta_B)} + c_{2,2}e^{-l(\eta-\eta_B)}, \tag{32}$$

with $l = \sqrt{\alpha a_B^2 - k^2}$. Here, we have neglected the effect of Q_s. Otherwise, Eq. (21) is difficult to be solved.

In inflationary phase ($\eta \geq \eta_{B+}$), Q_s^{inf} is almost constant. Considering the continuities of a and $\mathcal{H}$ at η_{B+}, we have

$$a_{\mathrm{inf}}(\eta) = a_{B+}(-y\mathcal{H}_{B+})^{\frac{1}{\epsilon_{\mathrm{inf}}-1}}, \tag{33}$$

where $y = \eta - \eta_{B+} + 1/\mathcal{H}_{B+}$ and $H_{B+} = \mathcal{H}_{B+}/a$, $H_{\mathrm{inf}} \lesssim H_{B+}$. The solution of (21) is

$$u_k = \frac{\sqrt{\pi|y|}}{2}[c_{3,1}H_{\nu_2}^{(1)}(-ky) + c_{3,2}H_{\nu_2}^{(2)}(-ky)], \tag{34}$$

where $\nu_2 = \frac{\epsilon_{\mathrm{inf}}-3}{2(\epsilon_{\mathrm{inf}}-1)}$.

The power spectrum is

$$\begin{aligned} P_\zeta(k, \mathcal{H}_{B+}, \mathcal{H}_{B-}, \Delta\eta) &\approx \frac{H_{\mathrm{inf}}^2}{8\pi^2 Q_s^{\mathrm{inf}} M_p^2}|c_{31} - c_{32}|^2 \\ &= P_\zeta^{\mathrm{inf}}|c_{31} - c_{32}|^2, \end{aligned} \tag{35}$$

where $P_\zeta^{\mathrm{inf}} = \frac{H_{\mathrm{inf}}^2}{8\pi^2 Q_s^{\mathrm{inf}} M_p^2}$ is that of the slow-roll inflation. Requiring the continuities of ζ and $\dot{\zeta}$, we could write the coefficients as

$$\begin{pmatrix} c_{3,1} \\ c_{3,2} \end{pmatrix} = \mathcal{M}^{(3,2)} \times \mathcal{M}^{(2,1)} \times \begin{pmatrix} c_{1,1} \\ c_{1,2} \end{pmatrix}, \tag{36}$$

see Appendix E for the matrices $\mathcal{M}^{(2,1)}$ and $\mathcal{M}^{(3,2)}$.

The effects of bounce has been encoded in $\mathcal{M}^{(3,2)}$ and $\mathcal{M}^{(2,1)}$ (or $|c_{3,1} - c_{3,2}|^2$). We approximately have

$$|c_{3,1} - c_{3,2}|^2 \approx 1 - \mathcal{A}\sin\left(\frac{2k}{\mathcal{H}_{B+}}\right) - \mathcal{A}\sin\left(\frac{2k}{\mathcal{H}_{B+}} + 2k\Delta\eta_B\right) \tag{37}$$

for $k \gg \mathcal{H}_{B+}$, where

$$\mathcal{A} = \frac{\mathcal{H}_{B+}}{k}\left(1 - \frac{\alpha a_B^2}{2\mathcal{H}_{B+}}\Delta\eta_B\right) \simeq \frac{\mathcal{H}_{B+}}{2k} \tag{38}$$

and (30) is used. This result suggests that on small scale, i.e., $k \gg \mathcal{H}_{B+}$, P_ζ is flat with a rapidly damped oscillation and its maximal oscillating amplitude is around $k \simeq \mathcal{H}_{B+}$. However, if the bounce phase lasts shortly enough, $\Delta\eta_B \ll 1/\mathcal{H}_{B+}$, (37) will be

$$|c_{3,1} - c_{3,2}|^2 \approx 1 - \frac{\mathcal{H}_{B+}}{k}\sin\left(\frac{2k}{\mathcal{H}_{B+}}\right). \tag{39}$$

While on large scale, i.e., $k \ll \mathcal{H}_{B+}$, $P_\zeta \sim k^2$ will have a strongly blue tilt, since

$$|c_{3,1} - c_{3,2}|^2 \approx w(\Delta\eta_B)\left(\frac{k}{\mathcal{H}_{B+}}\right)^2, \tag{40}$$

where

$$\begin{aligned} w(\Delta\eta_B) = \Bigg[&\left(1 - \frac{l^2\Delta\eta_B}{2\mathcal{H}_{B+}}\right)\cosh(l\Delta\eta_B) \\ &+ \frac{l}{2}\left(\frac{1}{\mathcal{H}_{B+}} - \Delta\eta_B + \frac{l^2}{4\mathcal{H}_{B+}}\Delta\eta_B^2\right)\sinh(l\Delta\eta_B)\Bigg]^2, \end{aligned} \tag{41}$$

which indicates $w(\Delta\eta_B) \simeq 1$ for $\Delta\eta_B \simeq 0$.

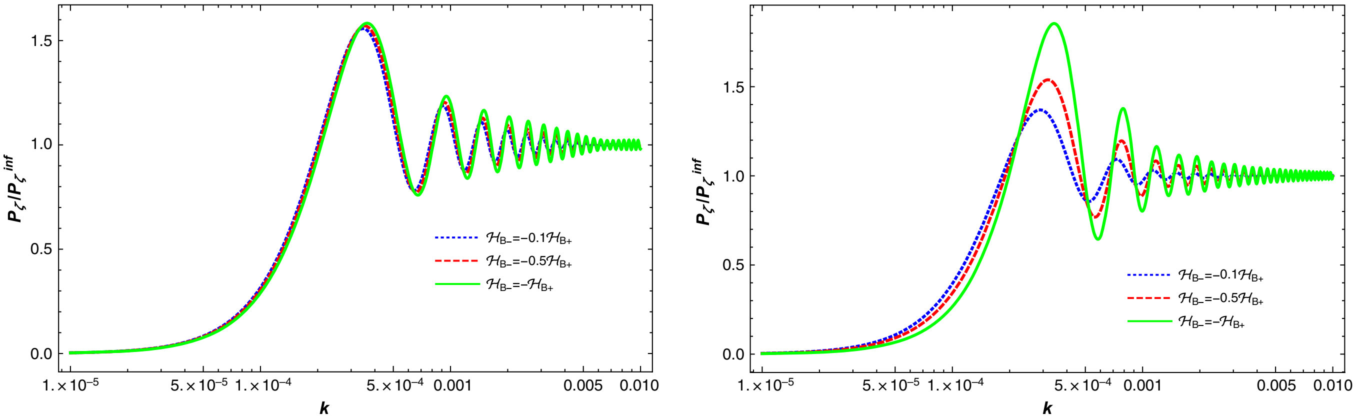
We plot P_ζ for (35) in Fig. 4 for the different values of $\Delta\eta$ and $\mathcal{H}_{B-}$. We see that for $k > \mathcal{H}_{B+}$, $P_\zeta \sim k^0$ but has a damped oscillation, while for $k < \mathcal{H}_{B+}$, $P_\zeta \sim k^2$ shows itself a large-scale cutoff. Thus (35) is consistent with our simulation result at large and small scales, respectively (see Fig. 7 in Sec. III).

However, since we have neglected the steplike evolution of Q_s, the pull-lower around $k \simeq \mathcal{H}_{B+}$ in Fig. 7(d) cannot be reflected in (35).

B. Template

To conveniently fit the observational data, a simple *"Template"* capturing the essential shape of P_ζ is indispensable. Based on the simulation in Sec. III and the analytical estimate in Sec. IV A, we write the power spectrum as

$$P_\zeta = F(k, \mathcal{H}_{B+}, A_d, \omega_d) \cdot P_\zeta^{\mathrm{inf}}, \tag{42}$$



(a) $\Delta\eta = 0.1/\mathcal{H}_{B+}$ (b) $\Delta\eta = 0.5/\mathcal{H}_{B+}$

FIG. 4. The power spectrum with different $\Delta\eta$ and different $\mathcal{H}_{B-}/\mathcal{H}_{B+}$.

where $P_\zeta^{\rm inf} = A_{\rm inf}(\frac{k}{k_*})^{n_{\rm inf}-1}$ is the spectrum predicted by slow-roll inflation, $A_{\rm inf}$ is the amplitude at the pivot scale k_*, $n_{\rm inf}$ is its tilt and

$$F(k,\mathcal{H}_{B+},A_d,\omega_d) = \left\{1 + e^{-(k/\mathcal{H}_{B+})^2}\left(\frac{k}{\mathcal{H}_{B+}}\right)^2 + e^{-(k/\mathcal{H}_{B+})^2} - \frac{\sin(2k/\mathcal{H}_{B+})}{k/\mathcal{H}_{B+}}\right\}\cdot\left[1 - A_d\cdot e^{-\omega_d(\frac{k}{\mathcal{H}_{B+}}-\pi)^2}\right]. \tag{43}$$

Here, the parameter set $(\mathcal{H}_{B+},A_d,\omega_d)$ reflects the effect of pre-inflationary bounce on the spectrum. Around $k\gtrsim\mathcal{H}_{B+}$, we have

$$F(k,\mathcal{H}_{B+},A_d,\omega_d) \simeq 1 - A_d e^{-\mathcal{O}(1)\omega_d}, \tag{44}$$

so A_d and ω_d (related with the parameter $\Delta\eta < 1/\mathcal{H}_{B+}$ in Sec. IVA) depict the width and depth of valley around $k\gtrsim H_{B+}$, respectively. Here, A_d is related with the change rate of Q_s (which was neglected in Sec. IVA). With Eq. (26), we have approximately

$$A_d \simeq \frac{2|\dot{Q}_s|_{\rm max}}{3H_{\rm inf}Q_s} \tag{45}$$

noting $\dot{Q}_s < 0$. In (43), we have

$$F(k,\mathcal{H}_{B+},A_d,\omega_d) \sim 1 - \frac{\sin(2k/\mathcal{H}_{B+})}{k/\mathcal{H}_{B+}} \tag{46}$$

for $k\gg\mathcal{H}_{B+}$, which equals to (39), while for $k\ll\mathcal{H}_{B+}$, we approximately have $F(k,\mathcal{H}_{B+},A_d,\omega_d)\simeq(\frac{k}{\mathcal{H}_{B+}})^2$, which is consistent with (40). P_ζ for the *"Template"* (43) is plotted in Fig. 5. We see that (43) has effectively captured the essential shape of P_ζ showed in Fig. 3.

C. Data fitting

We modify the CAMB and CosmoMC code package and perform a global fitting with Planck2015 data. The parameter set of the lensed-ΛCDM model is $\{\Omega_b h^2, \Omega_c h^2, 100\theta_{\rm MC}, \tau, \ln(10^{10}A_{\rm inf}), n_{\rm inf}\}$, with $\Omega_b h^2$ the baryon density, $\Omega_c h^2$ the cold dark matter density, $\theta_{\rm MC}$ the angular size of the sound horizon at decoupling and τ the reionization optical depth. We also include the parameter set $\{\mathcal{H}_{B+}, A_d, \omega_d\}$ (the so-called 3-parameters of bounce) defined in (43), which captures the physics of pre-inflationary bounce,

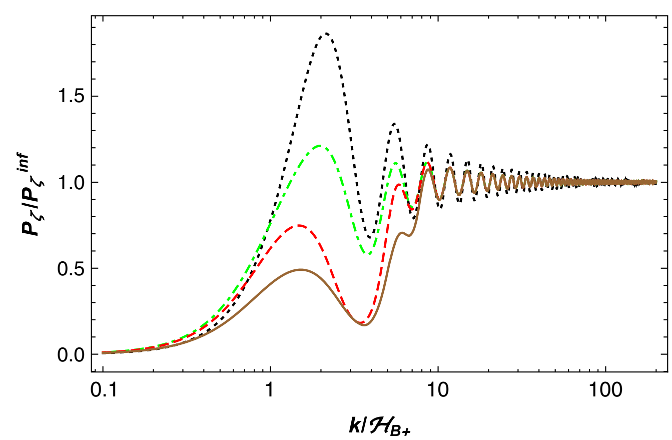
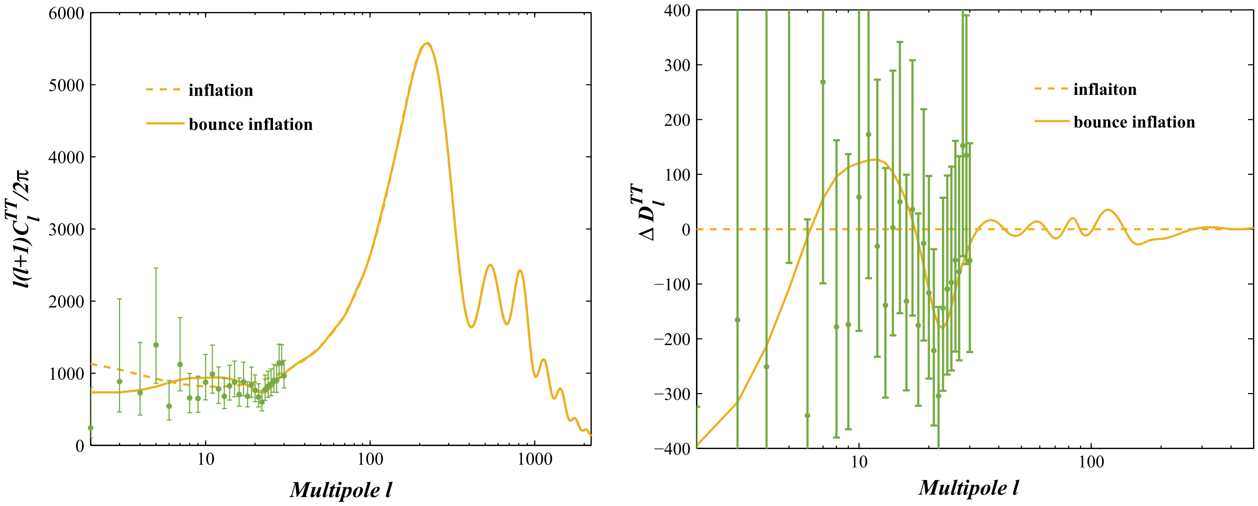


FIG. 5. The black dotted curve is the spectrum $P_T/P_T^{\rm inf}$ of the primordial GWs in bounce-inflation scenario, see [26], while the {green dot-dashed, red dashed, brown solid} curves are those of the primordial scalar perturbation based on the results of *"Template"* (43) with $A_d = \{0.25, 0.8, 0.8\}$, $d = \{\pi,\pi,\pi\}$ and $\omega_d = \{0.25, 0.25, 0.1\}$, which are consistent with those in Fig. 3.



(a) $l(l+1)C_l^{TT}/2\pi$ (b) ΔD_l^{TT}

FIG. 6. The green points show the Planck2015 data with 1σ errors. The best-fit values of parameters are $\ln(10^{10}A_{\rm inf}) = 3.091$, $n_{\rm inf} = 0.966$, $\ln(\mathcal{H}_{B+}) = -7.51$, $A_d = 0.87$, $\omega_d = 5.47$.

as has been argued. We set the pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$, roughly in the middle of the logarithmic range of scales probed by Planck.

With (43), we plot the CMB TT-spectrum $D_l^{TT} \equiv l(l+1)C_l^{TT}/2\pi$ and ΔD_l^{TT} in Fig. 6 with the best-fit parameter set $\{\Omega_b h^2, \Omega_c h^2, 100\theta_{\rm MC}, \tau, \ln(10^{10}A_{\rm inf}), n_{\rm inf}, \mathcal{H}_{B+}, A_d, \omega_d\}$. Since WMAP and Planck, some models aimed at explaining the anomalies of CMB at large scale (but not solving the initial singularity problem) were proposed [52–57]. We see that the spectrum (43) of scalar perturbation predicted by our model could be consistent with not only the power deficit of the CMB TT-spectrum at low multipoles, but also the dip at $l \sim 20$. The best-fit values of the parameters in our bounce-inflation model are displayed in Table I. More details associate with other interesting numerical results will be presented an upcoming work [58].

TABLE I. The best-fit values of parameters in bounce-inflation model.

Parameters	Inflation	Bounce-inflation
$\Omega_b h^2$	0.02222	0.02213
$\Omega_c h^2$	0.1198	0.1199
H_0	67.31	67.05
τ	0.078	0.079
$\ln(10^{10}A_s)$	3.089	3.091
n_s	0.9655	0.9662
$\ln(h_k\,\mathrm{Mpc})$	...	−7.51
A_d	...	0.87
ω_d	...	5.47

V. CONCLUSION

In bounce-inflation scenario, the inflation is singularity-free (past-complete). However, its pathology-free model has been still lacking. Here, we showed such a model. The nonsingular bounce is implemented by applying $\tilde{P}(\phi, X)$, see (7), which is ghost-free, while $c_s^2 < 0$ is dispelled by $L_{\delta g^{00}R^{(3)}}$ [31].

We perform a full simulation for the evolution of scalar perturbation, and find that the spectrum P_ζ has a suppression at large scale ($k \ll \mathcal{H}_{B+}$) but is flat (with a damped oscillation) at small scale ($k \gg \mathcal{H}_{B+}$), which confirms the earlier results showed in [13,17] and is consistent with the power deficit of the CMB TT-spectrum at low multipoles $l \lesssim 30$; but unexpectedly, P_ζ also shows itself one marked lower valley at $k \gtrsim \mathcal{H}_{B+}$, though the depth is model-dependent. We show that this lower valley actually provides an explanation to the dip at $l \sim 20$ hinted by Planck [6]. Based on the simulation and the analytical estimation for the perturbation spectrum, we also offer a "*Template*" of P_ζ (effectively capturing the physics of bounce) to fit data.

The equation of motion of GWs mode γ_{ij} for (2) is

$$\ddot{\gamma}_k + \left(3H + \frac{\dot{Q}_T}{Q_T}\right)\dot{\gamma}_k + c_T^2 \frac{k^2}{a^2}\gamma_k = 0, \qquad (47)$$

which is unaffected by the operators $R^{(3)}\delta g^{00}$ and $\delta K \delta g^{00}$, where $Q_T = M_p^2$. We plot the primordial GWs spectrum P_T in Fig. 5 (the black dot curve) with $P_T^{\rm inf} = \frac{2H_{\rm inf}^2}{\pi^2 M_p^2}$, see also [26]. It should be mentioned that if $Q_T \neq M_p^2$ around the nonsingular bounce (the gravity is modified completely), P_T will be different. It is also possible that the corresponding gravity has a large parity violation [59], which might be imprinted in CMB.

Our work highlight the conjecture again that the physics hinted by the large-scale anomalies of CMB is relevant to the pre-inflationary bounce. The nonsingular cosmological bounce also has been implemented in some models of modified gravity [60–73], see also [74,75] for reviews. Confronting the corresponding models with the CMB data will be interesting.

Additionally, in the scenario considered here, it is actually not required that the contracting phase is ekpyrotic scenario. In our scenario, the large-scale cutoff in perturbation spectrum is induced by the contracting phase, while the valley in perturbation spectrum is induced by the step-like evolution of Q_s/M_p^2 around the bounce phase. Consider other than the ekpyrotic bounce, we have $\epsilon_{\rm cont} \simeq \mathcal{O}(1)$ in the contracting phase, so $Q_s/M_p^2 \sim \epsilon_{\rm cont} = {\rm const} \simeq 1$ for the contraction, while $Q_s/M_p^2 \sim \epsilon_{\rm inf} \ll 1$ for the inflation. Thus our results on the perturbation spectrum would be also applicable for the corresponding scenario, though the details might be slightly different. In such a case, the model might be simpler than that with the ekpyrotic scenario, which is worth further studying.

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APPENDIX A: CORRESPONDENCE WITH A SUBCLASS OF DHOST THEORIES

According to the classification of degenerate theories in [47], the covariant action $S = \int d^4x\sqrt{-g}L$ based on (2) belongs to class Ia of DHOST theories. In order to clearly demonstrate this point, we will use the notation of [47] but will rewrite the $f(\phi, X)$ in [47] into $F(\phi, X)$ in this Appendix. From (2) to (4), we find that our action

$$S = \int d^4x\sqrt{-g}L = S_g + S_\phi + S_{\rm other}, \tag{A1}$$

where

$$S_g = \int d^4x\sqrt{-g}F(\phi, X)R, \tag{A2}$$

$$\begin{aligned} S_{\rm other} = \int d^4x\sqrt{-g}\Bigg[& -\frac{M_P^2}{2}X - V(\phi) + \tilde{P}(\phi, X) \\ & -\frac{X}{2}\int f_{\phi\phi} d\ln X - \left(f_\phi + \int \frac{f_\phi}{2} d\ln X\right)\Box\phi \\ & + \frac{g(\phi, X)}{2}\frac{1}{\sqrt{-X}}\left(\frac{\phi^\mu\phi_{\mu\nu}\phi^\nu}{X} - \Box\phi\right) - \frac{3}{2}g(\phi, X)h(\phi)\Bigg], \end{aligned} \tag{A3}$$

$$S_\phi = \sum_{I=1}^{5}\int d^4x\sqrt{-g}\alpha_I L_I^\phi, \tag{A4}$$

$$F(\phi, X) = \frac{M_p^2 + f}{2}, \tag{A5}$$

$$\alpha_1 = -\alpha_2 = \frac{f}{2X}, \qquad \alpha_3 = -\alpha_4 = \frac{f - 2Xf_X}{X^2}, \qquad \alpha_5 = 0, \tag{A6}$$

with

$$L_1^\phi = \phi_{\mu\nu}\phi^{\mu\nu}, \qquad L_2^\phi = (\Box\phi)^2, \qquad L_3^\phi = (\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu,$$
$$L_4^\phi = \phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu, \qquad L_5^\phi = (\phi^\mu\phi_{\mu\nu}\phi^\nu)^2. \tag{A7}$$

It is straightforward to check that α_1 to α_5 and $F(\phi, X)$ satisfy the degenerate conditions given by Eqs. (3.17) to (3.19) in [47]. Thus, our action belongs to the class Ia DHOST theories and does not contain any extra degree of freedom.

APPENDIX B: THE EFT OF NONSINGULAR COSMOLOGIES

In this Appendix, we briefly review the EFT of nonsingular cosmologies, see [28] for the details.

With the ADM 3 + 1 decomposition, we have

$$g_{\mu\nu} = \begin{pmatrix} N_kN^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix},$$
$$g^{\mu\nu} = \begin{pmatrix} -N^{-2} & \frac{N^j}{N^2} \\ \frac{N^i}{N^2} & h^{ij} - \frac{N^iN^j}{N^2} \end{pmatrix}, \tag{B1}$$

and $\sqrt{-g} = N\sqrt{h}$, where $N_i = h_{ij}N^j$. The induced metric on 3-dimensional hypersurface is $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$, where $n_\mu = n_0(dt/dx^\mu) = (-N, 0, 0, 0)$, $n^\nu = g^{\mu\nu}n_\mu = (1/N, -N^i/N)$ is orthogonal to the spacelike hypersurface, and $n_\mu n^\mu = -1$. Thus

$$h_{\mu\nu} = \begin{pmatrix} N_kN^k & N_j \\ N_i & h_{ij} \end{pmatrix}, \qquad h^{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & h^{ij} \end{pmatrix}. \tag{B2}$$

The EFT is [28]

$$
\begin{aligned}
S = \int d^4x\sqrt{-g}\bigg[&\frac{M_p^2}{2}f(t)R - \Lambda(t) - c(t)g^{00} \\
&+ \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K\delta g^{00} \\
&- m_4^2(t)(\delta K^2 - \delta K_{\mu\nu}\delta K^{\mu\nu}) + \frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00} \\
&- \bar{m}_4^2(t)\delta K^2 + \frac{\bar{m}_5(t)}{2}R^{(3)}\delta K + \frac{\bar{\lambda}(t)}{2}(R^{(3)})^2 + \cdots \\
&- \frac{\tilde{\lambda}(t)}{M_p^2}\nabla_i R^{(3)}\nabla^i R^{(3)} + \cdots\bigg],
\end{aligned} \tag{B3}
$$

where $\delta g^{00} = g^{00} + 1$, $R^{(3)}$ is the 3-dimensional Ricci scalar, $K_{\mu\nu} = h_\mu^\sigma \nabla_\sigma n_\nu$ is the extrinsic curvature, $\delta K_{\mu\nu} = K_{\mu\nu} - h_{\mu\nu}H$.

Here, we focus on building a stable model of bounce inflation. We only consider the coefficients set $(f, c, \Lambda, M_2, m_3, \tilde{m}_4)$, and set other coefficients in (B3) equal to 0. We always could set $f = 1$, which suggests $c(t) = -M_p^2\dot{H}$ and $c(t) + \Lambda(t) = 3M_p^2H^2$.

As pointed out in Ref. [33], the $R^{(3)}\delta K$ operator in EFT could play similar role as $R^{(3)}\delta g^{00}$, which we will consider elsewhere. Mapping (2) into the EFT (B3), we have $M_2^4(t) = X^2\tilde{P}_{XX}$, $m_3^3(t) = -g_1(\phi)$ and $\tilde{m}_4^2 = f_1(\phi)$. Only with $(M_2, m_3, \tilde{m}_4) \neq 0$, the quadratic action of scalar perturbation ζ is (see, e.g., our [28])

$$S_\zeta^{(2)} = \int d^4x a^3 Q_s\left(\dot{\zeta}^2 - c_s^2\frac{(\partial\zeta)^2}{a^2}\right), \tag{B4}$$

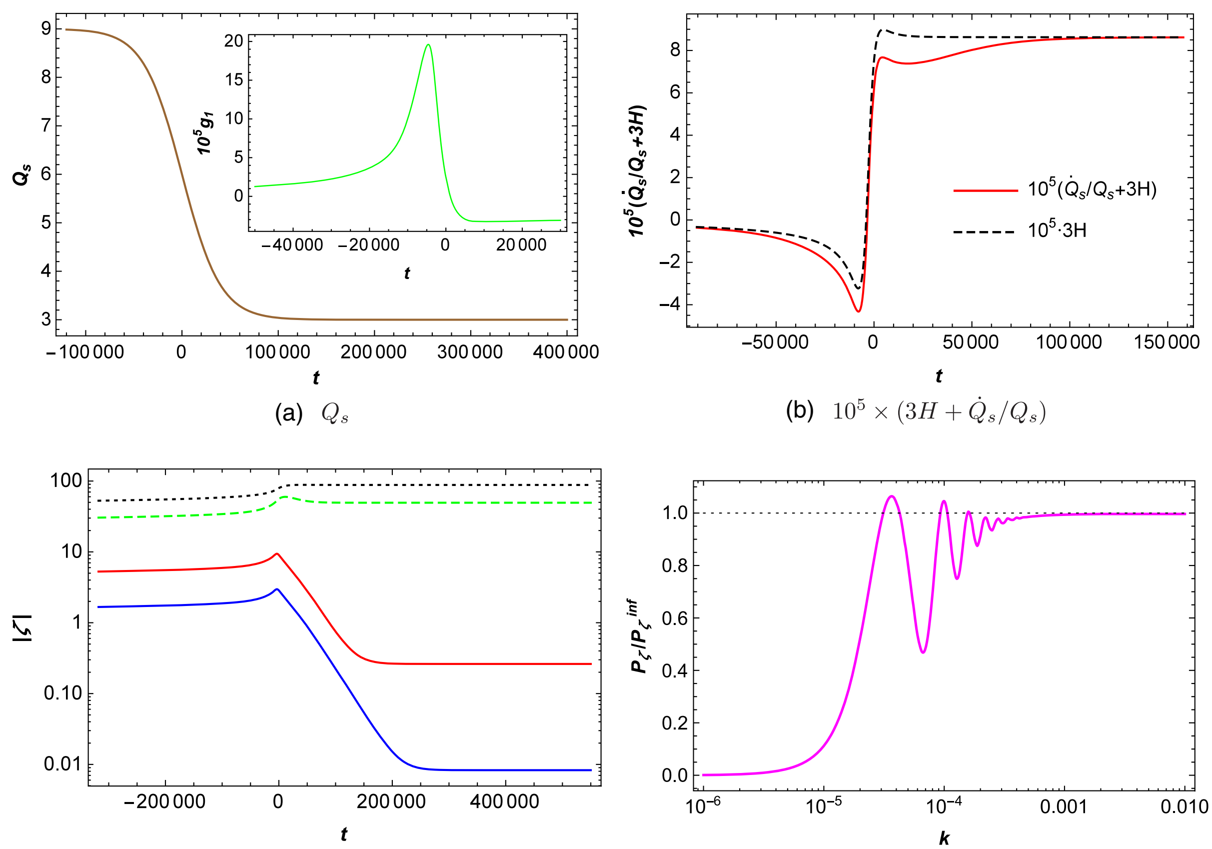
where

$$Q_s = \frac{2M_2^4}{\gamma^2} + \frac{3m_3^6}{4M_p^2\gamma^2} - \frac{\dot{H}M_p^2}{\gamma^2}, \tag{B5}$$

$$c_s^2 Q_s = \frac{\dot{c}_3}{a} - M_p^2 \tag{B6}$$

$$c_3 = \frac{aM_p^2}{\gamma}\left(1 + \frac{2\tilde{m}_4^2}{M_p^2}\right), \tag{B7}$$

where $\gamma = H - m_3^3/(2M_p^2)$. Only if $Q_s > 0$ and $c_s^2 > 0$, the nonsingular cosmological model is healthy. In models with



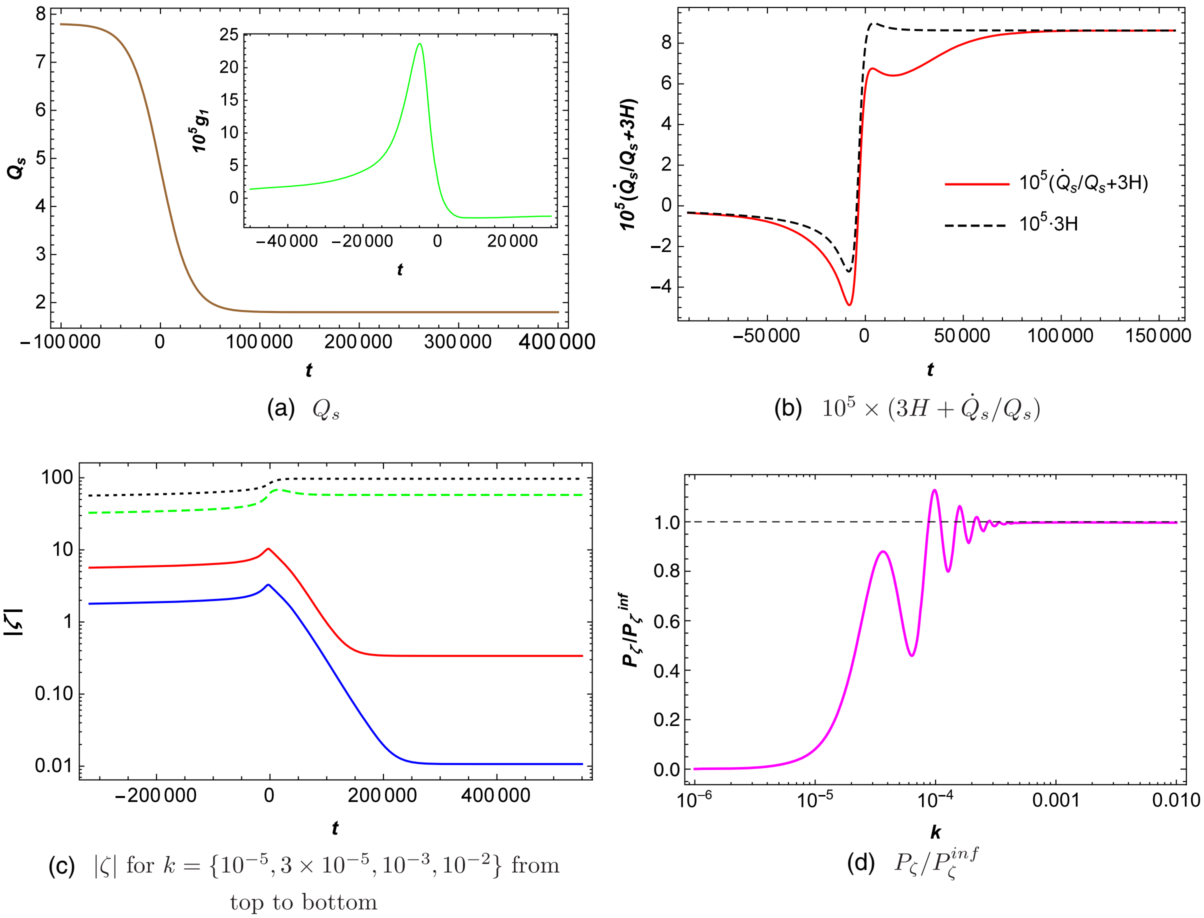
(a) Q_s

(b) $10^5 \times (3H + \dot{Q}_s/Q_s)$

(c) $|\zeta|$ for $k = \{10^{-5}, 3\times10^{-5}, 10^{-3}, 10^{-2}\}$ from top to bottom

(d) P_ζ/P_ζ^{inf}

FIG. 7. We set $\mathcal{A}_Q = 3$, $\mathcal{B} = 2$, $t_* = 4\times10^4$ and the background is given by Fig. 2.



(a) Q_s

(b) $10^5 \times (3H + \dot{Q}_s/Q_s)$

(c) $|\zeta|$ for $k = \{10^{-5}, 3\times10^{-5}, 10^{-3}, 10^{-2}\}$ from top to bottom

(d) P_ζ/P_ζ^{inf}

FIG. 8. We set $\mathcal{A}_Q = 3$, $\mathcal{B} = 1.6$, $t_* = 3\times10^4$ and the background is given by Fig. 2.

the operator $(\delta g^{00})^2$, $Q_s > 0$ always can be obtained, since $(\delta g^{00})^2$ contributes $\dot{\zeta}^2$. While $c_s^2 > 0$ requires $\dot{c}_3 > aM_p^2$, which is

$$c_3|_{t_f} - c_3|_{t_i} > M_p^2 \int_{t_i}^{t_f} a dt. \tag{B8}$$

The inequality (B8) suggests that c_3 must cross 0 ($\tilde{m}_4^2 = -M_p^2/2$ or γ is divergent), since the integral $\int adt$ is infinite. Thus if the $R^{(3)}\delta g^{00}$ operator is absent, $c_s^2 > 0$ throughout is impossible. We can set $c_s^2 \simeq 1$ by

$$m_4^2 = \frac{\gamma}{2a}\int a(Q_s c_s^2 + M_p^2)dt - \frac{M_p^2}{2}. \tag{B9}$$

APPENDIX C: MORE ON THE SIMULATION

We plot the evolutions of Q_s, g_1, $|\zeta|$ with respect to t, and also $P_\zeta(k)$ for the background in Fig. 2, with different values of $\mathcal{B}$ and t_* in this Appendix. We see how $|\zeta|$ evolves with a in different phases. Theoretically, $\zeta \sim 1/a$ for the perturbation modes with $k \gg \sqrt{z_s''/z_s}$, while $\zeta \sim$ const. for the perturbation modes with $k \ll \sqrt{z_s''/z_s}$, which is consistent with our Figs. 7(c) and 8(c).

APPENDIX D: EVOLUTIONS OF f_1, g_1, f_2, AND h WITH RESPECT TO ϕ

We plotted f_1, g_1, f_2, and h with respect to ϕ in Fig. 9. Here, $f_1(\phi)$ guarantees $c_s^2 \equiv 1$ [see, Eq. (20)], while $g_1(\phi)$ gives the particular time dependence of Q_s given by Eq. (24) [see, Eq. (25)]. The particular evolutions of f_2

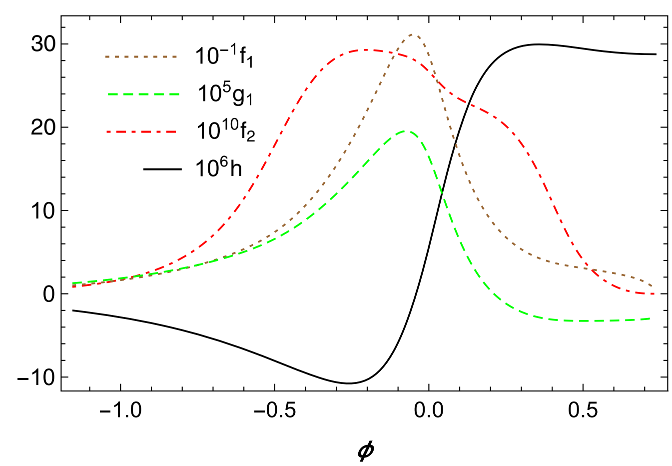


FIG. 9. The evolutions of f_1, g_1, f_2, and h with respect to ϕ while we set $c_s^2 \equiv 1$, $\mathcal{A}_Q = 3$, $\mathcal{B} = 2$ and $t_* = 4\times10^4$.

and h guarantee that $L_{\delta g^{00}R^{(3)}}$ and $L_{\delta K\delta g^{00}}$ has no contribution at background level.

APPENDIX E: THE MATRICES ELEMENTS OF $\mathcal{M}^{(2,1)}$ AND $\mathcal{M}^{(3,2)}$

We define $l=\sqrt{\alpha a_B^2-k^2}$, $x_1=1/|\mathcal{H}_{B-}|$, $x_2=\mathcal{H}_{B+}$, $y_{1,2}=(\eta_{B\mp}-\eta_B)$, and have

$$\mathcal{M}_{11}^{(2,1)}=\frac{\sqrt{\pi x_1}}{4l}[(l+\alpha a_B^2y_1)H_{\nu_1}^{(1)}(kx_1)-kH_{\nu_1-1}^{(1)}(kx_1)]e^{-ly_1}, \quad (E1)$$

$$\mathcal{M}_{12}^{(2,1)}=\frac{\sqrt{\pi x_1}}{4l}[(l+\alpha a_B^2y_1)H_{\nu_1}^{(2)}(kx_1)-kH_{\nu_1-1}^{(2)}(kx_1)]e^{-ly_1}, \quad (E2)$$

$$\mathcal{M}_{21}^{(2,1)}=\frac{\sqrt{\pi x_1}}{4l}[(l-\alpha a_B^2y_1)H_{\nu_1}^{(1)}(kx_1)-kH_{\nu_1-1}^{(1)}(kx_1)]e^{ly_1}, \quad (E3)$$

$$\mathcal{M}_{22}^{(2,1)}=\frac{\sqrt{\pi x_1}}{4l}[(l-\alpha a_B^2y_1)H_{\nu_1}^{(2)}(kx_1)-kH_{\nu_1-1}^{(2)}(kx_1)]e^{ly_1}, \quad (E4)$$

$$\mathcal{M}_{11}^{(3,2)}=\frac{i\sqrt{\pi x_2}}{2}[(l-\alpha a_B^2y_2)H_{\nu_2}^{(2)}(kx_2)+kH_{\nu_2-1}^{(2)}(kx_2)]e^{ly_2}, \quad (E5)$$

$$\mathcal{M}_{12}^{(3,2)}=\frac{i\sqrt{\pi x_2}}{2}[-(l+\alpha a_B^2y_2)H_{\nu_2}^{(2)}(kx_2)+kH_{\nu_2-1}^{(2)}(kx_2)]e^{-ly_2}, \quad (E6)$$

$$-\mathcal{M}_{21}^{(3,2)}=\frac{i\sqrt{\pi x_2}}{2}[(l-\alpha a_B^2y_2)H_{\nu_2}^{(1)}(kx_2)+kH_{\nu_2-1}^{(1)}(kx_2)]e^{ly_2}, \quad (E7)$$

$$-\mathcal{M}_{22}^{(3,2)}=\frac{i\sqrt{\pi x_2}}{2}[-(l+\alpha a_B^2y_2)H_{\nu_2}^{(1)}(kx_2)+kH_{\nu_2-1}^{(1)}(kx_2)]e^{-ly_2}. \quad (E8)$$

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