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Brane with variable tension as a possible solution to the problem of the late cosmic acceleration

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Braneworld models have been proposed as a possible solution to the problem of the accelerated expansion of the Universe. The idea is to dispense the dark energy (DE) and drive the late-time cosmic acceleration with a five-dimensional geometry. We investigate a brane model with variable brane tension as a function of redshift called chrono-brane. We propose the polynomial $\lambda = (1+z)^n$ function inspired in tracker-scalar-field potentials. To constrain the n exponent we use the latest observational Hubble data from cosmic chronometers, Type Ia Supernovae from the full joint-light-analysis sample, baryon acoustic oscillations and the posterior distance from the cosmic microwave background of Planck 2015 measurements. A joint analysis of these data estimates $n \simeq 6.19 \pm 0.12$ which generates a DE-like (cosmological-constantlike at late times) term, in the Friedmann equation arising from the extra dimensions. This model is consistent with these data and can drive the Universe to an accelerated phase at late times.

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I. INTRODUCTION

The accelerated expansion of the Universe in the present epoch is supported by high-resolution observations of Supernovae Type Ia (SNIa) at high redshift [1–3], anisotropies in cosmic microwave background radiation (CMB) [4,5] and baryon acoustic oscillations (BAO) [6]. To explain this within the general relativity framework, a negative-pressure fluid, dubbed dark energy (DE), must be postulated to produce the observed gravity repulsion [7]. The most economic attempt comes from the cosmological constant [CC; 8], originated by quantum vacuum fluctuations [9,10].

Extra-dimensions scenarios have been proposed to solve the CC problems such as braneworld models, which accelerate the Universe under the assumption of a $4+1$-dimensional space-time (the bulk) containing an ordinary $3+1$-dimensional manifold (the brane). However, the majority of them (including the Randall and Sundrum (RS).[1] [11,12] models) achieve a stable late cosmic acceleration only by including DE [13,14]. Although models with variable brane tension (VBT), $\lambda(t) \propto a(t)$, sourcing from thermodynamics assumptions (Eötvös law) have been studied previously[2] [15–23], they either have not been contrasted with recent observations or still need the introduction of a DE fluid to reproduce the late acceleration.

We propose a phenomenological braneworld model based on RSII using one brane with variable tension, called *chrono-brane* (CB) hereafter, which does not only supply the late-time cosmic acceleration but it is also in agreement with observational data.

In contrast with previous studies, a double Bianchi identity is not applied, i.e. there is no matter creation into the brane. We investigate the effects of a VBT in terms of the scale factor (redshift), i.e. $\lambda(a)$ or $\lambda(z)$. We propose a polynomial function for the brane tension which is dominant in later times in the Universe, but subdominant in the early Universe to be consistent with nucleosynthesis. To probe this model, we perform a Monte Carlo Markov Chain (MCMC) analysis using SNIa, H(z), BAO, and CMB data. We also investigate the scale factor dynamics and the cosmological evolution of the different components of the Universe.

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[1]RS models are divided in the case of two (RSI) and one brane (RSII) respectively.

[2]Notice that only models with constant brane tension have been observational constrained.

II. BRANEWORLD COSMOLOGICAL FRAMEWORK

The characteristic brane parameter is encoded in the brane tension which establishes the limits of high and low energies. Following Refs. [24,25] the Einstein's field equations projected onto the four dimensional manifold with a VBT (see [26,27] for a constant brane tension) can be written as:

$$G_{\mu\nu} + \xi_{\mu\nu} = \kappa_{(4)}^2 T_{\mu\nu} + \frac{6\kappa_{(4)}^2}{\lambda}\Pi_{\mu\nu}. \tag{1}$$

We assume no matter fields in the bulk [25] and AdS_5 background [27]. Therefore, the Friedmann equation is $H^2 = (\kappa_{(4)}^2/3)\sum_i \rho_i[1+\rho_i/(2\lambda)]$, recovering the canonical form when $\lambda \to \infty$. The brane tension can be rewritten as $\lambda(a) \equiv \lambda_0 \hat{\lambda}(a)$, with λ_0 as a free parameter. The general dimensionless function, $\hat{\lambda}(a)$, gives the brane tension behavior in terms of the scale factor. This way of writing λ avoids problems with the fundamental constants, eliminating the temporary dependence. For instance, $\hat{\lambda}(a)$ has been absorbed by any of the tensors associated in (1).

To source the late cosmic acceleration with a VBT without DE we only consider matter (baryonic and dark matter) and radiation as components. A dimensionless Friedmann equation, $E(z)^2 \equiv H^2(z)/H_0^2$, can be re written as

$$E(z)^2 = E_{nb}^2(z) + \frac{\mathcal{M}}{\hat{\lambda}(z)}[\Omega_{m0}^2(z+1)^6 + \Omega_{r0}^2(z+1)^8], \tag{2}$$

where $E_{nb}^2(z) = \Omega_{m0}(z+1)^3 + \Omega_{r0}(z+1)^4$, $\mathcal{M} = 3H_0^2/2\kappa_4^2\lambda_0$, and $\Omega_{m0} = \Omega_{b0} + \Omega_{DM0}$. The radiation component can be expressed as $\Omega_{r0} = 2.469\times10^{-5}(H_0/100\mathrm{kms}^{-1}\mathrm{Mpc}^{-1})^{-2}\times(1+0.2271(3.04))$. From the flatness condition we obtain

$$\mathcal{M} = \frac{1-\Omega_{m0}-\Omega_{r0}}{\Omega_{m0}^2+\Omega_{r0}^2}\hat{\lambda}(0). \tag{3}$$

The deceleration parameter can be written as $q(z) = (q_I(z) + q_{II}(z))/E^2(z)$, where:

$$q_I(z) \equiv \frac{1}{2}\Omega_{m0}(z+1)^3 + \Omega_{r0}(z+1)^4, \tag{4a}$$

$$q_{II}(z) \equiv \frac{\mathcal{M}}{\hat{\lambda}(z)}\Bigg[2\Omega_{m0}^2(z+1)^6 + 3\Omega_{r0}^2(z+1)^8 - \frac{1}{2\hat{\lambda}(z)}\frac{d\hat{\lambda}(z)}{dz}[\Omega_{m0}^2(z+1)^7 + \Omega_{r0}^2(z+1)^9]\Bigg]. \tag{4b}$$

Notice that Eq. (2) and $q(z)$ are reduced to those shown in [14] when the brane tension is constant.

In order to explore the background cosmology, we propose the following ansatz for the VBT: $\hat{\lambda}(a) = a^{-n} \to \hat{\lambda}(z) = (z+1)^n$, where $n \in \mathbb{R}$ is the free parameter and $\hat{\lambda}(1) = \hat{\lambda}(0) = 1$ for the scale factor and redshift respectively. Other authors have analyzed the case $\hat{\lambda}(a) = 1 - a^{-1} \to |\hat{\lambda}(z)| = z$ [15–23]. This form ($n = 1$) is inferred through the Eötvös law $\lambda(T) = K(T_c - T)$, where K is a constant, T_c is a critical temperature, and $T \sim a^{-1}$, is the Universe temperature (see [17] for details). Therefore, from a phenomenological point of view, our proposed $\hat{\lambda}(z)$ could be a obtained from a generalization of Eötvös law, similar to the Gauss theorem in n-dimensions.

III. OBSERVATIONAL CONSTRAINTS

The free parameters are constrained by performing a Bayesian MCMC analysis [28], with flat priors on $n:[0,20]$ and $\Omega_{m0}:[0,1]$ and a Gaussian prior on $\Omega_{b0}: 0.02202 \pm 0.00046$. The observational data used are:

(i) *H(z) measurements:* We employ the most recent observational Hubble data compiled by [29] and references therein, which contains 31 points in the range $0 \leq z \leq 1.965$. We also consider the local value H_0 given by [30] as a Gaussian prior in analysis. These $H(z)$ measurements could be overestimated up to 25%, as claimed by [31].

(ii) *SNIa data:* we choose the full JLA sample by [32] containing 740 observations in the interval $0.01 < z < 1.2$. The sources of systematics identified in SN Ia analysis [33–37] are already considered in the covariance matrix [32].

(iii) *BAO data:* Referencing [14], we use: $d_z \equiv r_s(z_d)/D_V(z) = 0.336 \pm 0.015$ at redshift $z = 0.106$ [6dFGS, 38], $d_z = (0.0870 \pm 0.0042, 0.0672 \pm 0.0031, 0.0593 \pm 0.0020)$ at $z = (0.44, 0.6, 0.73)$ [WiggleZ, 39,40], $d_z = 0.2239 \pm 0.0084$ at $z = 0.15$ [SDSS-DR7, 41], $d_z = (0.1181 \pm 0.0022, 0.0726 \pm 0.0007)$ at $z = (0.32, 0.57)$ [BOSS-DR11, 42] and $D_H/r_d = 9.07 \pm 0.31$ at $z = 2.33$ [43].

(iv) *CMB from Planck 2015 measurements:* We use the acoustic scale, $l_A = 301.787 \pm 0.089$, the shift parameter, $R = 1.7492 \pm 0.0049$, and the decoupling redshift, $z_* = 1089.99 \pm 0.29$ obtained for a flat w-cold dark matter model [5,44]. Although this method could lead to biased constraints when used in modified gravity models (see discussion in [45]), this is a first approach.

Table I gives the chi-square and the mean values for the free parameters using each data set. We obtain consistent Ω_{m0} mean values, within the 1σ of confidence level (CL), which are also in agreement with those estimated for the standard scenario (ΛCDM). The goodness-of-fit test for the joint analysis (H(z) + SNIa + BAO + CMB) indicates that our scenario fits the data with a 95% of reliability. We obtain consistent values for the exponent n within the range $\sim[5.5 - 7.5]$ at 1σ CL. Figure 1 presents the $1D$ marginalized posterior distributions and $2D$ at 68%, 95%, 99.7% of CL for the brane parameters. Notice that there is a strong correlation between n and Ω_{m0} ($\mathrm{corr}(n, \Omega_{m0}) = 0.912$),

TABLE I. Mean values for the model parameters (Ω_{m0}, h, n) derived from each data set and joint analysis.

Data set	$\chi^2_{\rm min}$	Ω_{m0}	h	n	$\lambda_0(10^{-12}\ \mathrm{eV}^4)$
H(z)	14.46	$0.318^{+0.039}_{-0.042}$	$0.730^{+0.017}_{-0.017}$	$7.400^{+1.100}_{-0.926}$	$3.20^{+1.05}_{-0.95}$
BAO	9.49	$0.297^{+0.031}_{-0.028}$	$0.718^{+0.016}_{-0.016}$	$6.730^{+0.287}_{-0.289}$	$2.62^{+0.77}_{-0.57}$
CMB	3.64	$0.288^{+0.014}_{-0.013}$	$0.732^{+0.017}_{-0.017}$	$6.420^{+0.185}_{-0.185}$	$2.52^{+0.19}_{-0.17}$
SN Ia	691.10	$0.231^{+0.114}_{-0.120}$	$0.731^{+0.017}_{-0.017}$	$5.580^{+0.815}_{-0.568}$	$1.48^{+2.40}_{-1.16}$
Joint	716.43	$0.31^{+0.008}_{-0.008}$	$0.706^{+0.009}_{-0.009}$	$6.190^{+0.121}_{-0.120}$	$2.81^{+0.12}_{-0.11}$

i.e. fluctuations on Ω_{m0} within 1σ could yield n values larger or smaller than 6.

Figure 2 shows the 68% and 99.7%λ_0/ρ_c-n confidence contours. Notice that they overlap at $\lambda_0/\rho_c \sim 0.06$, confirming that the λ_0 constrains are consistent among them, solving the tension between the observables found in [14]. Furthermore, the joint analysis value corresponds to a brane tension of $\sim 10^{40}\ \mathrm{eV}^4$ in the nucleosynthesis epoch, in concordance with other observations [46,47]. We also infer, from Eq. (3), a strong positive correlation of $\lambda_0 - \Omega_{m0}$.

Figure 3 illustrates a good fit to H(z) (top panel) and the reconstructed $q(z)$ (bottom panel) for the model using the mean values and the joint analysis. From the latter, we obtain that the Universe starts an accelerated stage at $z = 0.641 \pm 0.018$. The $q(z)$ behavior for the model is consistent with the CC within 1σ of CL, with $q(0) \simeq -0.60$ at $z = 0$.

We compare the Akaike [48,49] and the Bayesian (BIC) [49,50] information criteria between CB and ΛCDM models using each data set. When the joint constraints are considered, we obtain $\Delta\mathrm{AIC} \sim 5.6$ and $\Delta\mathrm{BIC} \sim 0.95$, i.e., weak evidence in favor and not enough evidence against of CB. We obtain a Bayes factor [51] of 1.6 that gives a weak support of CB over ΛCDM as well.

As a main conclusion, considering that the data prefer constraints consistent with $n = 6$, we suggest that *a brane with VBT* $\lambda(z) = \lambda_0(1+z)^{6.19\pm0.12}$ *can mimic the DE dynamics*. Although at first glance this result seems trivial, the origin of the acceleration is different to the one in the standard scenario. In extra dimensional models the topology influence the acceleration, obtaining in some cases phantomlike DE. Our results are also consistent with those explored in [52].

We also performed new parameter estimations using a different Gaussian prior on h of [4]. We obtain that the λ_0/ρ_c-n contours from $H(z)$ data shifts towards smaller values of n ($6.17^{+1.00}_{-0.83}$) and larger values of λ_0/ρ_c, but they are still consistent at 3σ. In the SNIa analysis, the difference in the λ_0/ρ_c-n confidence contours is negligible. A deviation of 3% on the n value ($6.00^{+0.10}_{-0.09}$) and a shift down of the contours was found in the joint analysis. We also consider smaller errors (0.75% of the original ones) on the $H(z)$ measurements, as suggested by [31], yielding to smaller confidence contours and a deviation of 1% ($7.32^{+0.82}_{-0.73}$) from our value of n. We use different priors

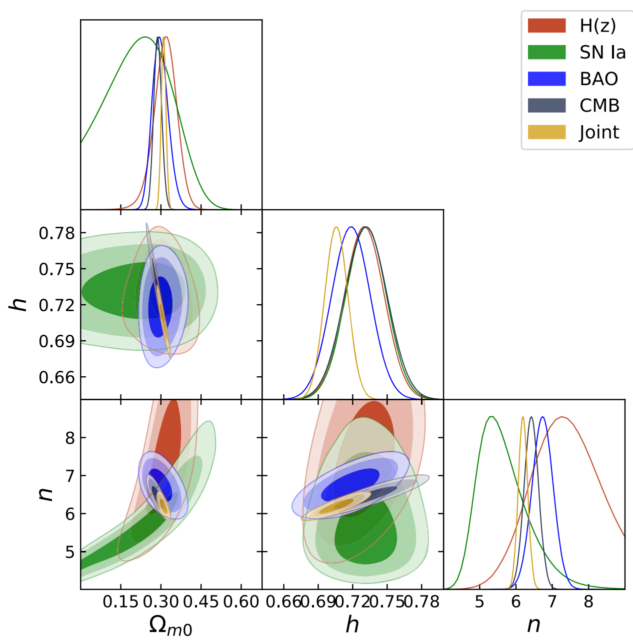

FIG. 1. 1D marginalized posterior distributions and the 2D 68%, 95%, 99.7% of CL for the Ω_{m0}, h, and n parameters of the brane model.

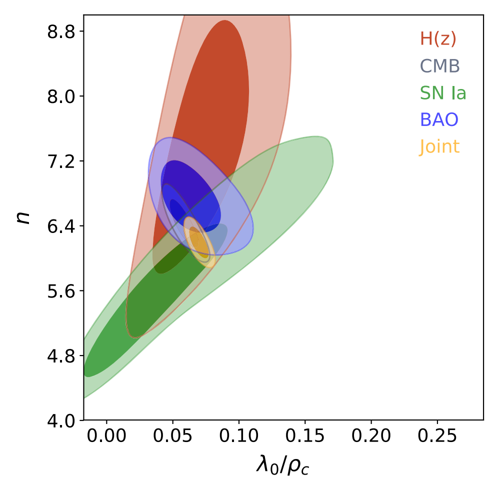

FIG. 2. Confidence contours of the λ_0/ρ_c-n parameters within the 1σ and 3σ of CL for each cosmological data with $\rho_c = 8.070 \times 10^{-11} h^2\ \mathrm{eV}^4$.

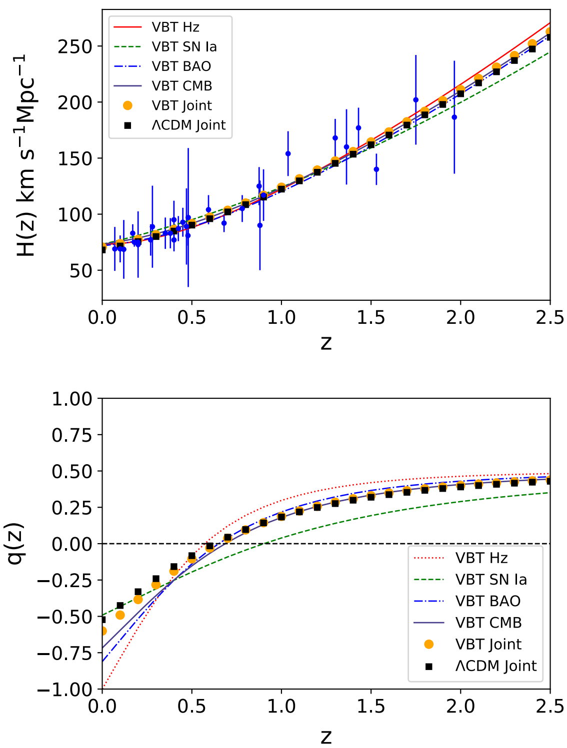

FIG. 3. The CB fitting to H(z) and reconstruction of the $q(z)$ (top and bottom panels respectively) using data constraints. The ΛCDM dynamics is plotted for comparison.

on the SNIa parameters and find that n is mainly affected by the Ω_{m0} estimation. Therefore, although the different systematics in the data introduce different bias in the estimated constraints, the final results are all consistent within the 3σ CL.

IV. SCALE FACTOR DYNAMICS

The Friedmann equation can be written as:

$$H_0(t-t_0) = \int_{a_0}^{a} \left\{ \frac{\Omega_{m0}}{a} + \frac{\Omega_{r0}}{a^2} + \mathcal{M}\left[\frac{\Omega_{m0}^2}{a^{4-n}} + \frac{\Omega_{r0}^2}{a^{6-n}} \right] \right\}^{-1/2} da. \tag{5}$$

The solution of Eq. (5), using the mean value constraints, is shown in Fig. 4 which reveals late times singularities for the values presented in Table I. Future singularities at $t_{\rm sing}$ times are computed as $t_{\rm sing} - t_{\rm today} \simeq 2H_0^{-1}(n-6)^{-1}\mathcal{M}^{-1/2}\Omega_{0m}^{-1}$. Particularly, we observe singularities for the constraints obtained from observations, being for the joint analysis $t_{\rm sing} = 13.57H_0^{-1}$. Notice that constraints relying only on SNIa analysis do not predict future singularities.

The approximate analytical solution of the scale factor as a function of time reads

$$a(t) \simeq \begin{cases} \left[\left(3 - \frac{n}{2}\right)\alpha t + a_0^{(6-n)/2}\right]^{2/(6-n)}, & \text{for } n \neq 6, \\ a_0 \exp(\alpha t), & \text{for } n = 6, \end{cases} \tag{6}$$

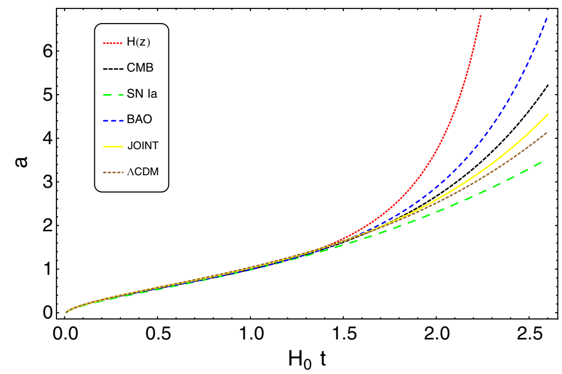

FIG. 4. Evolution of the scale factor, assuming a non singular initial condition, using each data constraints. ΛCDM is shown for comparison.

where $\alpha \equiv \Omega_{m0}\mathcal{M}^{1/2}H_0$. Notice that the d'Sitter expansion for $n = 6$, behaves like a CC.

An effective equation of state (EoS) is

$$\omega_{\rm eff}(z) = \frac{2q(z) - 1}{3[1 + 2\mathcal{M}E(z)_{nb}^2(z+1)^{-n}]} + \frac{\mathcal{M}E(z)_{nb}^2(z+1)^{-n}[2q(z) - (4-n)]}{3[1 + 2\mathcal{M}E(z)_{nb}^2(z+1)^{-n}]}. \tag{7}$$

Therefore, the Universe accelerates when $\omega_{\rm eff}$ satisfies

$$\omega_{\rm eff}(z) < -\frac{1 + \mathcal{M}E(z)_{nb}^2(z+1)^{-n}(4-n)}{3[1 + 2\mathcal{M}E(z)_{nb}^2(z+1)^{-n}]}. \tag{8}$$

Notice that $\omega_{\rm eff}(z)$ from GR is not valid anymore in this particular case.

Figure 5 shows the effective EoS evolution using the joint constraints. Notice that $\omega_{\rm eff} \to 1/3$ at high redshifts and $\omega_{\rm eff} \to 0$ at $z = 0$. The inset shows the region where the condition (8) is satisfied (at $z \lesssim 0.65$ when $w_{\rm eff} < 0.00025$), i.e., when the Universe accelerates. This transition redshift

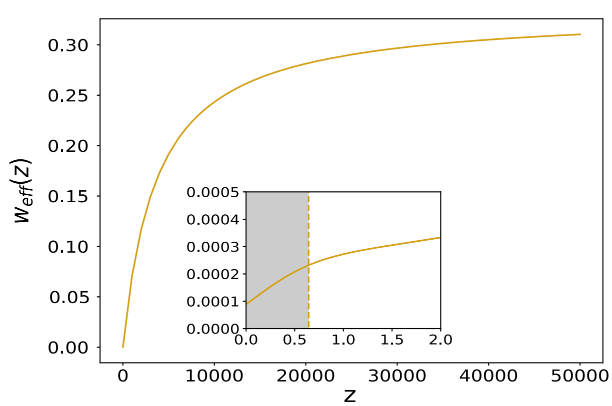

FIG. 5. Reconstruction of the effective EoS using the joint constraints. The inset shows the current $\omega_{\rm eff}$ behavior. The vertical dashed line marks the redshift where the condition of Eq. (8) is satisfied, i.e., Universe acceleration for $z \lesssim 0.65$.

in the $w_{\rm eff}$ is consistent with the one obtained in the $q(z)$ reconstruction.

V. COSMOLOGICAL EVOLUTION

Proposing the following dimensionless variables:

$$x^2 \equiv \Omega_m = \left(\frac{\kappa_{(4)}^2 \rho_m}{3H^2}\right), \qquad y^2 \equiv \Omega_r = \left(\frac{\kappa_{(4)}^2 \rho_r}{3H^2}\right),$$
$$z^2 \equiv \frac{3H^2}{2\kappa_{(4)}^2 \lambda_0 \hat{\lambda}(a)}, \tag{9}$$

where $\Omega_\lambda = z^2(x^4+y^4) = 1-\Omega_m-\Omega_r$ and $1 = x^2+y^2+z^2(x^4+y^4)$, allows us to construct the following dynamical system

$$\frac{x'}{x} = -\frac{3}{2}+\frac{3}{2}x^2+2y^2-\frac{1}{2}\Pi, \tag{10a}$$

$$\frac{y'}{y} = -2+\frac{3}{2}x^2+2y^2-\frac{1}{2}\Pi, \tag{10b}$$

$$\frac{z'}{z} = \frac{n}{2}-\frac{3}{2}x^2-2y^2+\frac{1}{2}\Pi, \tag{10c}$$

where $\Pi \equiv [(n-6)x^4+(n-8)y^4]z^2$. Choosing as initial condition the joint constraints, we obtain the evolution of the density parameters (Fig. 6). At early times, the Universe is dominated by the radiation component, afterwards, the matter becomes the dominant. At late times, the Universe is dominated by the CB dynamics. This scenario predicts the same cosmological evolution as the standard one.

VI. CONCLUSIONS AND DISCUSSION

We constructed a brane world model which produces an accelerated Universe without a DE component. We present a new way of building RS models using a VBT $\lambda(z)$, called chrono-brane. We introduce the ansatz $\lambda(z) = (z+1)^n$, inspired by tracker-scalar-field potentials, arising from the space-time structure. To constrain the n exponent, the matter content, and the dimensionless Hubble parameters we used cosmological observations. We found consistent mean values for the different parameters using each set of observational data. From the joint analysis we estimated $n \sim 6.19 \pm 0.12$, i.e. it provides a term in the Friedmann equation which mimics the DE dynamics very close to a CC. In addition, Ω_{m0} and h are in agreement with the standard values. Our model also alleviates the tension among the λ_0 constraints obtained from the cosmological data and those from high-energy-regime. For example, we obtain from the joint analysis $\lambda = 8.35 \times 10^{40}$ eV^4 at $z \sim 3 \times 10^8$ for nucleosynthesis epoch, that would not affect well-established primordial processes. For $z = 0$, we have $\lambda = 2.81 \times 10^{-12}$ eV^4.

All of our cosmological constraints give a good fit to H(z) data and predict a phase of accelerated expansion at $z \sim 0.6$. Our results on the scale factor evolution exhibits a future singularity. We reconstructed the cosmological behavior of an effective EoS and found that the Universe accelerates when $\omega_{\rm eff} < 0.00025$ at $z < 0.65$, obtaining $q(0) \simeq -0.60$. We studied the density parameter for each component and recovered a value that is the same as the standard one. This is a key result because a CB successfully reproduces the concordance model and provides clues to the DE nature and the late cosmic acceleration.

Reference [53] explore the consequences of a simple brane model with constant brane tension on the CMB spectrum. The authors show that at large scales the temperature anisotropy caused by Sachs-Wolfe (SW) effect is the same as the canonical one. They also claim that at very small scales the effects of branes are negligible. Nevertheless, on scales up to the first CMB acoustic peak, the brane terms considerably modify the peak amplitude and position (see [54] for the effect of dynamical DE perturbations on the integrated SW effect). This implies a change in the CMB distance posteriors and in the brane constraints that we have obtained. It is important to notice that these results could also be applicable for the case of variable brane tension. However, to assess the impact of the brane perturbations, a full CMB analysis should be carried out, which is beyond of the scope of this article.

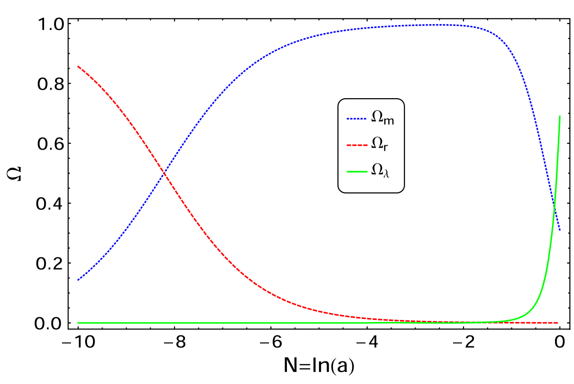

FIG. 6. Evolution of the density parameters under the CB scenario using as initial conditions the mean values of the joint analysis.

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