Scale invariance in heavy hadron molecules

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We discuss a scenario in which the $P_c(4450)^+$ heavy pentaquark is a $\Sigma_c\bar{D}^*$-$\Lambda_c(2595)\bar{D}$ molecule. The $\Lambda_{c1}\bar{D} \to \Sigma_c\bar{D}^*$ transition is mediated by the exchange of a pion almost on the mass shell that generates a long-range $1/r^2$ potential. This is analogous to the effective force that is responsible for the Efimov spectrum in three-boson systems interacting through short-range forces. The equations describing this molecule exhibit approximate scale invariance, which is anomalous and broken by the solutions. If the $1/r^2$ potential is strong enough this symmetry survives in the form of discrete scale invariance, opening the prospect of an Efimov-like geometrical spectrum in two-hadron systems. For a molecular pentaquark with quantum numbers $\frac{3}{2}^-$ the attraction is not enough to exhibit discrete scale invariance, but this prospect might very well be realized in a $\frac{1}{2}^+$ pentaquark or in other hadron molecules involving transitions between particle channels with opposite intrinsic parity and a pion near the mass shell. A very good candidate is the $\Lambda_c(2595)\bar{\Xi}_b - \Sigma_c\bar{\Xi}_b{}'$ molecule. Independently of this, the $1/r^2$ force is expected to play a very important role in the formation of this type of hadron molecule, which points to the existence of $\frac{1}{2}^+$ $\Sigma_c D^*$-$\Lambda_c(2595)D$ and 1^+ $\Lambda_c(2595)\Xi_b - \Sigma_c\Xi_b'$ molecules and $0^+/1^-$ $\Lambda_c(2595)\bar{\Xi}_b - \Sigma_c\bar{\Xi}_b{}'$ baryonia.

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The onset of scale invariance in two-body systems is a remarkable property. It connects a series of seemingly disparate low-energy phenomena in atomic, nuclear and particle physics under the same theoretical description [1]. When the scattering length a_0 of a two-body system is much larger than any other scale, i.e. $a_0 \to \infty$, the system is invariant under the scale transformation $r \to \lambda r$ with arbitrary λ [2]. The low-energy properties of this two-body system can be fully explained independently of the underlying short-range dynamics. That is, few-body systems with a large scattering length admit a universal description. Efimov discovered that three-boson systems exhibit a characteristic three-body spectrum for $a_0 \to \infty$, where the binding energy of the states is arranged in a geometric series [3]. The continuous scale invariance of the three-body equations is anomalous and the spectrum only shows discrete scale invariance under the transformation $r \to \lambda_0 r$ where the value of λ_0 is now fixed. Conversely if E_n is the binding energy of a three-body state there is another state with binding $E_{n+1} = E_n/\lambda_0^2$, a prediction that was confirmed experimentally with Cs atoms a decade ago [4]. This type of discrete geometrical spectrum also happens in three-body systems containing at least two identical particles [5], or when the scattering is resonant in higher partial waves [6,7]. This mechanism might be responsible for the binding of the triton [8], ^{4}He [9], a series of halo nuclei [10–14] and the Hoyle state [15,16].

There is a two-body system that is intimately related to the Efimov effect, which is the $1/r^2$ potential.[1] At zero energy the reduced Schrödinger equation for the s-wave becomes

$$-u''(r) + \frac{g}{r^2}u(r) = 0, \tag{1}$$

which is obviously scale invariant (for a finite energy analysis we refer to [18]). The connection with the three-body system is apparent when one realizes that it also

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[1]While a two-body system with infinite scattering length is scale invariant, a two-body system with a $1/r^2$ potential can display at most discrete scale invariance, a possibility which will depend on the strength of the potential. Continuous scale invariance for $1/r^2$ is broken by the existence of a fundamental state with $E \neq 0$. Equivalently, as happens in the three-boson system [17], the renormalization of $1/r^2$ is nontrivial and requires the appearance of a new energy scale and henceforth that scale invariance is broken.

contains a similar equation with an effective $1/\rho^2$ potential in the hyper-radius ρ [19]. For $g > -1/4$ the equation above admits power-law solutions of the type

$$u(r) = c_+ r^{\frac{1}{2}+\nu} + c_- r^{\frac{1}{2}-\nu}, \tag{2}$$

with $c_\pm$ constants and $\nu = \sqrt{1/4 + g}$, where scale invariance is lost. For $g < -1/4$ we have instead solutions of the type

$$u(r) = c r^{1/2} \sin(\nu \log \Lambda_2 r), \tag{3}$$

with c a constant, $\nu = \sqrt{-1/4 - g}$ and Λ_2 an energy scale that depends on the short-range physics (it can be obtained from the energy of the fundamental state). Λ_2 is the reason why exact scale invariance is broken and its appearance resembles dimensional transmutation [20,21]. Now the solutions display discrete scale invariance with $r \to \lambda_0 r$, where $\lambda_0 = e^{\pi/\nu}$ [22,23]. In turn there is a geometric bound state spectrum where $E_{n+1} = E_n/\lambda_0^2$, with E_n and E_{n+1} the energy of two consecutive states. Incidentally this is a rare example of an anomaly in quantum mechanics [24]. Here we make the observation that the $1/r^2$ potential can appear in heavy hadron molecules, for instance the $P_c(4450)^+$ heavy pentaquark if it happens to be molecular (the only other known example of a $1/r^2$ potential is the atom-dipole interaction [25]). There might be other two-hadron systems where the potential might be attractive enough to exhibit discrete scale invariance. The ideas presented here involve long-range physics and hence only apply to molecular hadrons (i.e. nonrelativistic bound states of two-hadrons) that fulfill a series of conditions, but not to compact hadrons.

The heavy pentaquarks $P_c(4380)^+$ and $P_c(4450)^+$, P_c and P_c^* from now on, were discovered by LHCb [26] and are a recent and interesting addition to a growing family of exotic hidden charm (and bottom) hadrons that began with the $X(3872)$ more than ten years ago [27]. There is still a lot of discussion regarding the nature of the P_c and P_c^*, from the role of threshold effects [28–31], to baryocharmonia [32], a compact pentaquark [33–38], a heavy baryon-antimeson molecule [39–43] and other more exotic possibilities [44,45]. The P_c^* is an interesting molecular candidate because of the following two reasons: its width is not particularly big, $\Gamma = 35 \pm 5 \pm 19$ MeV, and it is very close to the $\Sigma_c \bar{D}^*$ threshold, see Fig. 1. As a matter of fact a series of works predicted the possibility of a heavy baryon-antimeson molecule before the discovery of the P_c^* [46–49]. The probable quantum numbers of the P_c and P_c^* are $\frac{3}{2}^-$ and $\frac{5}{2}^+$ respectively, followed by $\frac{5}{2}^+$ and $\frac{3}{2}^-$. The standard molecular explanation for the P_c^* heavy pentaquark is that of a $\Sigma_c \bar{D}^*$ bound state, which prefers the quantum number $\frac{3}{2}^-$ for the P_c^*. Here we discuss the scenario in which the molecular P_c^* also contains a $\Lambda_{c1}\bar{D}$ component in addition to $\Sigma_c \bar{D}^*$, where Λ_{c1} denotes

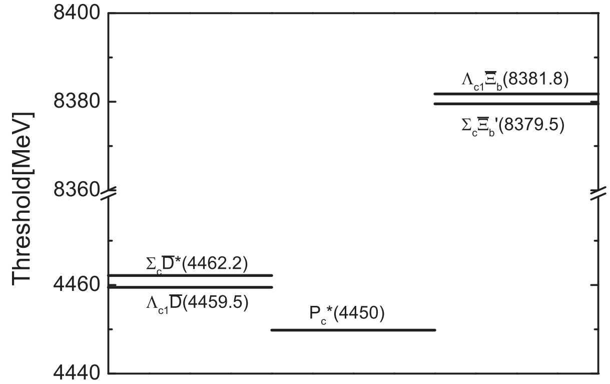

FIG. 1. Location of the thresholds for the two scale invariant molecule candidates considered in this work, the $\Lambda_{c1}\bar{D} - \Sigma_c \bar{D}^*$ and the $\Lambda_{c1}\bar{\Xi}_b - \Sigma_c \bar{\Xi}_b{}'$. We also show the location of the P_c^* for comparison.

the $\Lambda_c(2595)$. Burns [50] proposed this idea on the analogy between the $D\bar{D}^* + D^*\bar{D}$ and the $Y_c^*\bar{D} + Y_c\bar{D}$ systems, i.e. the X(3872) and the P_c^*, where Y_c, Y_c^* are charmed baryons. He argued that the most natural analog to the $D\bar{D}^* + D^*\bar{D}$ system is $\Sigma_c\bar{D}^* - \Lambda_{c1}\bar{D}$ on the basis that the mass difference of the Λ_{c1} and Σ_c is very close to the D^* and D splitting. Here we will explore this possibility.

The low-energy dynamics of $\Sigma_c\bar{D}^* - \Lambda_{c1}\bar{D}$ is driven by one pion exchange (OPE) and is fascinating for two reasons. First, if the $\Lambda_{c1}\bar{D}$ pair exchanges a pion to become a $\Sigma_c\bar{D}^*$ pair the pion will be almost on the mass shell, leading to an unusual long-range potential for strong interactions. Second, the intrinsic parities of $\Lambda_{c1}(\frac{1}{2}^-)$ and $\Sigma_c(\frac{1}{2}^+)$ are different while the ones for the D and D^* are the same. As a consequence OPE will switch odd (even) waves in the $\Lambda_{c1}\bar{D}$ channel to even (odd) waves in the $\Sigma_c\bar{D}^*$ one. That is, there is a vector force analogous to the tensor, except that it carries orbital angular momentum $L = 1$ instead of $L = 2$. The tensor force behaves as $1/r^3$ for $m_\pi r < 1$, while the vector force as $1/r^2$. This short-range property becomes long range if the pion is near the mass shell.

We can compute the $\Sigma_c\bar{D}^* \to \Lambda_{c1}\bar{D}$ potential from the heavy baryon chiral Lagrangian of Cho [51]

$$\langle \Lambda_{c1}\bar{D}|V_{\rm OPE}(\vec{r})|\Sigma_c\bar{D}^*\rangle = \omega_\pi \tau \vec{\epsilon}\cdot\hat{r} W_E(r), \tag{4}$$

with $\omega_\pi = m(\Lambda_{c1}) - m(\Sigma_c)$ the energy of the pion, τ an isospin factor such that $\tau = \sqrt{3}$ for $I = 1/2$ and $\tau = 0$ for $I = 3/2$ and $\vec{\epsilon}$ the polarization vector of the incoming $\bar{D}^*$ meson. W_E reads

$$W_E(r) = \frac{g_1 h_2 \mu_\pi^2}{4\pi\sqrt{2} f_\pi^2} \frac{e^{-\mu_\pi r}}{\mu_\pi r}\left(1 + \frac{1}{\mu_\pi r}\right), \tag{5}$$

with g_1 the axial coupling for the heavy mesons, h_2 the coupling for the $\pi\Lambda_{c1}\Sigma_c$ vertex, $f_\pi \simeq 130$ MeV the pion decay constant and $\mu_\pi = m_\pi^2 - \omega_\pi^2$ is the effective pion mass. Besides, there is standard-range OPE in the $\Sigma_c \bar{D}^*$ channel while OPE vanishes in the $\Lambda_{c1} \bar{D}$ channel.

Actually $|\mu_\pi| \simeq 5$–35 MeV $\ll m_\pi$ depending on whether we exchange a charged or neutral pion; i.e. the $\Sigma_c \bar{D}^* \to \Lambda_{c1} \bar{D}$ transition potential dominates the long-range dynamics of the system for $1/m_\pi < r < 1/|\mu_\pi|$ (which is also the region of validity of the equations we will write below). We stress that scale invariance is only approximate and broken by two interrelated factors: (i) the pion is not exactly on the mass shell, and (ii) the $\Sigma_c \bar{D}^*$ and $\Lambda_{c1} \bar{D}$ thresholds are a pair of MeV away from each other. For the moment we will assume $\mu_\pi = 0$, which implies overlapping thresholds. In principle the widths of the Σ_c and Λ_{c1} baryons are another factor to consider. Yet the widths can be ignored if the time required for the formation of the state is shorter than the lifetime of its components: $\Gamma \ll m$, with Γ the width of the component and m the mass of the exchanged particle [52]. For the Σ_c and Λ_{c1} the widths are about a pair of MeVs, well below $m_\pi \sim 140$ MeV. The potential in the $I = 1/2$ channel reads

$$\langle \Lambda_{c1} \bar{D} | V_{\rm OPE}(\vec{r}) | \Sigma_c \bar{D}^* \rangle = \frac{g_1 h_2 \omega_\pi}{4\pi f_\pi^2} \sqrt{\frac{3}{2}} \frac{\vec{\epsilon} \cdot \hat{r}}{r^2} + \mathcal{O}(\mu_\pi^2 r^2), \quad (6)$$

i.e. the $\mu_\pi = 0$ limit of Eqs. (4) and (5). If we consider $J^P = \frac{3}{2}^-$ (the standard quantum numbers for a molecular pentaquark), the partial waves contributing are $\Sigma_c \bar{D}^* ({}^2D_{3/2})$, $\Sigma_c \bar{D}^* ({}^4S_{3/2})$, $\Sigma_c \bar{D}^* ({}^4D_{3/2})$ and $\Lambda_{c1} \bar{D} ({}^2P_{3/2})$. In this partial wave basis the reduced Schrödinger equation at zero energy reads

$$-\mathbf{u}'' + \left[2\mu_{P_c^*} \mathbf{V}_{\rm OPE} + \frac{\mathbf{L}^2}{r^2} \right] \mathbf{u} = 0, \quad (7)$$

where $\mathbf{u}$ is the wave function in vector notation and $\mu_{P_c^*}$ the reduced mass of the molecule (actually there is one reduced mass for each particle channel, but here we can take their geometric mean). The combination of the vector OPE potential and the centrifugal barrier reads

$$2\mu_{P_c^*} \mathbf{V}_{\rm OPE} + \frac{\mathbf{L}^2}{r^2} = \frac{\mathbf{g}(\frac{3}{2}^-)}{r^2} = \frac{1}{r^2} \begin{pmatrix} 6 & 0 & 0 & g \\ 0 & 0 & 0 & g \\ 0 & 0 & 6 & -g \\ g & g & -g & 2 \end{pmatrix}. \quad (8)$$

That is, we have a four channel version of Eq. (1). We can diagonalize the matrix $\mathbf{g}(\frac{3}{2}^-)$, in which case we end up with four equations of the type

$$-u_i'' + \frac{g_i}{r^2} u_i = 0, \quad (9)$$

where the g_i's ($i = 1, 2, 3, 4$) are the eigenvalues of $\mathbf{g}(\frac{3}{2}^-)$. There are three positive and one negative eigenvalue

$$g_i = \{6, 2, 3 + \sqrt{9 + 3g^2}, 3 - \sqrt{9 + 3g^2}\}, \quad (10)$$

where the negative one can trigger discrete scale invariance. This will happen if $|g| > 5/(4\sqrt{3}) \simeq 0.7217$. However the value of g for the P_c^* molecule is $g = 0.60^{+0.10}_{-0.10} h_2$, where we have used $g_1 = 0.59 \pm 0.01 \pm 0.07$ from $D^* \to D\pi$ and $D^* \to D\gamma$ decays [53,54]. This requires $|h_2| > 1.21^{+0.25}_{-0.19}$, which is well above $h_2 = 0.60 \pm 0.07$ from CDF [55] or $h_2 = 0.63 \pm 0.07$ from the analysis of Ref. [56], where in both cases h_2 is extracted from $\Gamma(\Lambda_{c1} \to \Sigma_c \pi)$. That is, there is not enough attraction to achieve discrete scale invariance. For the $\frac{1}{2}^-$ molecule the matrix is different but the attractive eigenvalue is still $g_-(\frac{1}{2}^-) = 3 - \sqrt{9 + 3g^2}$, requiring $|g| > 5/(4\sqrt{3})$. The most interesting pentaquark-like molecule is the $\frac{1}{2}^+$, with partial waves $\Sigma_c \bar{D}^* ({}^2P_{1/2})$, $\Sigma_c \bar{D}^* ({}^4P_{1/2})$ and $\Lambda_{c1} \bar{D} ({}^2S_{1/2})$, where

$$\mathbf{g}\left(\frac{1^+}{2}\right) = \begin{pmatrix} 2 & 0 & g \\ 0 & 2 & -\sqrt{2}g \\ g & -\sqrt{2}g & 0 \end{pmatrix}. \quad (11)$$

The attractive eigenvalue is $g_-(\frac{1}{2}^+) = 1 - \sqrt{1 + 3g^2}$, which requires $|g| > \sqrt{3}/4 \simeq 0.4330$ and $|h_2| > 0.73^{+0.11}_{-0.06}$, i.e. overlapping with current estimations of h_2. Finally the $\frac{3}{2}^+$, $\frac{5}{2}^+$ and $\frac{5}{2}^-$ cases require $|g| > 7\sqrt{3}/4$, $|g| > 7\sqrt{3}/4$ and $|g| > 15\sqrt{3}/4$. That is, the strength of the vector force is in general too weak in the pentaquark-like molecules to achieve discrete scale invariance, with the notable exception of $\frac{1}{2}^+$ which lies on the limit.

Yet the P_c^* is not the only system where this can happen. The general conditions for a $H_1 H_2 - H_1' H_2'$ hadronic molecule to have scale invariance are (i) the hadrons are particularly long lived; (ii) the mass difference of the hadrons in each vertex is similar to that of a pseudo-Goldstone boson $m(H_1)' - m(H_1) \simeq m(H_2) - m(H_2)' \simeq m_P$; (iii) the intrinsic parity of H_2 and H_2' is the same, while that of H_1 and H_1' is different; and (iv) H_1 and H_1' have the same spin for the pseudo-Goldstone boson to be emitted in the s-wave. This applies as well if we substitute hadrons for antihadrons in one of the vertices: the vector force will change sign but the eigenvalues of the $1/r^2$ potential matrix will remain the same. Notice that it is not strictly necessary to exchange a pion near the mass shell to have a long-range $1/r^2$ force. A kaon near the mass shell will also generate this type of force.

If we have the Λ_{c1}-Σ_c on the one side, besides the D-D^*, the $\Xi_b - \Xi_b'$ bottom baryon combination also fulfills the previous conditions; see Fig. 1 for the threshold location. In this regard the $\Lambda_{c1}\bar{\Xi}_b$-$\Sigma_c\bar{\Xi}_b{}'$ system seems to be the best candidate for a scale invariant molecule in the heavy sector. The $\Lambda_{c1}\bar{\Xi}_b \to \Sigma_c\bar{\Xi}_b{}'$ potential for $I = 1/2$ reads

$$\langle \Sigma_c\bar{\Xi}_b{}'|V_{\mathrm{OPE}}(\vec{r})|\Lambda_{c1}\bar{\Xi}_b\rangle = \frac{g_3 h_2 \omega_\pi}{8\pi f_\pi^2}\frac{\sigma_2\cdot\hat{r}}{r^2} + \mathcal{O}(\mu_\pi^2 r^2), \quad (12)$$

where g_3 is the axial coupling for the $\bar{\Xi}_b{}'\bar{\Xi}_b\pi$ vertex and σ_2 is the Pauli matrix for that vertex. If we consider states in which the $\Sigma_c\bar{\Xi}_b{}'$ is in the s-wave or alternatively in the p-wave where the tensor force is attractive, we have

$$0^+ = \Sigma_c\bar{\Xi}_b{}'({}^3P_0) - \Lambda_{c1}\bar{\Xi}_b({}^1S_0), \quad (13)$$

$$0^- = \Sigma_c\bar{\Xi}_b{}'({}^1S_0) - \Lambda_{c1}\bar{\Xi}_b({}^3P_0), \quad (14)$$

$$1^- = \Sigma_c\bar{\Xi}_b{}'({}^3S_1 - {}^3D_1) - \Lambda_{c1}\bar{\Xi}_b({}^1P_1 - {}^3P_1). \quad (15)$$

In these partial wave bases the **g** matrices read

$$\mathbf{g}(0^+) = \begin{pmatrix} 2 & g \\ g & 0 \end{pmatrix}, \quad (16)$$

$$\mathbf{g}(0^-) = \begin{pmatrix} 0 & g \\ g & 2 \end{pmatrix}, \quad (17)$$

$$\mathbf{g}(1^-) = \begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{3}}g & -\sqrt{\frac{2}{3}}g \\ 0 & 6 & -\sqrt{\frac{2}{3}}g & -\frac{1}{\sqrt{3}}g \\ \frac{1}{\sqrt{3}}g & -\sqrt{\frac{2}{3}}g & 2 & 0 \\ -\sqrt{\frac{2}{3}}g & -\frac{1}{\sqrt{3}}g & 0 & 2 \end{pmatrix}. \quad (18)$$

For $|g| > 3/4$ the attractive eigenvalue of the matrices above will trigger discrete scale invariance. The evaluation of g depends on the axial coupling g_3, which can be extracted from the $\Sigma_c \to \Lambda_c\pi$ decay, yielding $g_3 = 0.973^{+0.019}_{-0.042}$ [56]. This translates into $g = 1.12^{+0.03}_{-0.05}h_2$, requiring $|h_2| > 0.67^{+0.03}_{-0.02}$, which is within the error of $h_2 = 0.63 \pm 0.07$ [56].

The approximate scale invariance of the Schrödinger equation describing these hadronic molecules has long- and short-range consequences, where the former—the appearance of a geometric spectrum—depends on how far these systems are from $\mu_\pi = 0$. For $\mu_\pi \neq 0$ scale invariance holds for

$$R_s < r < \frac{1}{|\mu_\pi|}, \quad (19)$$

with R_s the short-range scale, $1/m_\pi$ in this case[2] (this also applies to three-boson systems after the substitution $\mu_\pi \to 1/a_0$). The existence of a geometric excited state requires the relative size of the scale invariant window to be bigger than the discrete scaling factor. For P_c^*-like molecules this window is $1/(R_s\mu_\pi) \sim 10$–20, requiring the coupling $|g_-|$ to be about 1 at least, which is considerably larger than $1/4$. That is, the observation of geometric states in hadron and atomic physics shares a similar difficulty: the fine-tuning of the pion mass (hadrons) or the scattering length (atoms). For atoms near a Feschback resonance this is solved with a magnetic field [1]. The equivalent for hadrons will be to fine-tune the pion mass in the lattice. There is also the possibility of increasing $|g_-|$, for instance by having a larger reduced mass (i.e. two bottom hadrons) or if the exchanged particle is a kaon. This can happen naturally in the heavy sector where there are still plenty of hadrons to be discovered, of which a few might be candidates for a long-range vector force.

Concerning the short-range consequences, even if the vector force is not enough to trigger discrete scale invariance it will still play a remarkable role in binding. This is indeed analogous to the conjectured importance of Efimov physics in light nuclei [9] (despite the glaring absence of Efimov states). If the binding mechanism is s-wave short-range attraction, a way to see this is the following: for $r \leq R_s$ we will assume that OPE is not valid and model the short-range interaction with a delta shell,

$$V(r) = V_{\mathrm{OPE}}(r)\theta(r - R_s) + \frac{C_0(R_s)}{4\pi R_s^2}\delta(r - R_s), \quad (20)$$

where R_s is the short-range radius. Then we calculate the relative strength of the coupling C_0 required to have a bound state at zero energy in the presence/absence of a vector force. In the one-channel problem of Eq. (1) for $g > -1/4$ and in the absence of tensor OPE, the relative strength of C_0 is $(1/2 + \nu)$ of that required to bind if $g = 0$ (for $\mu_\pi R_s < 1$), while for $g < -1/4$ it always binds (for $\mu_\pi = 0$). Owing to scale invariance this happens independently of R_s. Thus if $\nu \to 0$ ($g \to -1/4$) the short-range potential only has to be half the normal strength to be able to bind the system. If there is standard-range OPE or other intermediate-range physics this binding enhancement will change. Taking $R_s = 1$ fm, $\mu_\pi = 0$ and $h_2 = 0.63$, the $\Sigma_c\bar{D}^* - \Lambda_{c1}\bar{D}$ $P_c^*(\frac{3}{2}^-)$ requires 70% of the attraction of a standard $\Sigma_c\bar{D}^*$ P_c^* to bind (for the $\Sigma_c\Sigma_c\pi$ axial coupling we use $g_3 = -1.38$ [56]). For the heavy baryonium the numbers are 46% (0^-) and 53% (1^-) respectively. The probability of binding is enhanced but dependent on unknown short-range physics.

[2]On a related note, a purely imaginary μ_π triggers a repulsive correction at second order perturbation theory. The effect is small and is suppressed as $|\mu_\pi r|^3/3$ in the scale invariant region.

If binding happens for distances in which the present picture is valid, short-range physics will not be necessary. The radius below which the $P_c^*(\frac{3}{2}^-)$ binds is 0.94 fm while for the 0^- (1^-) baryonia we have 0.40 fm (0.84 fm) respectively. For the 1^+ $\Sigma_c \Xi_b' - \Lambda_{c1} \Xi_b$ molecule we have 0.87 fm instead. For $r < 1/2m_\pi$ (∼0.7 fm) two-pion exchange and hadron finite-size effects dominate, setting the limits of the OPE description and providing a criterion for binding. From this we can be confident about the existence of the 1^- baryonium and the 1^+ $\Sigma_c \Xi_b' - \Lambda_{c1} \Xi_b$ molecule, while the 0^- baryonium is contingent on the unknown short-range physics. But the more interesting cases are those of the $\Sigma_c D^*$ $(\frac{1}{2}^+)/\Sigma_c \bar{\Xi}_b{}'$ (0^+) which bind in p-wave. Here the vector force effectively induces the existence of a channel behaving much like an s-wave. For the $1/2^+$ $\Sigma_c D^* - \Lambda_c D$ system binding happens for 0.92 fm while for the 0^+ baryonium we have 0.86 fm. These radii points towards the existence of these states. The bottom line is that the vector force induces a series of binding mechanisms which do not require the ratio m_π/μ_π to be particularly large (a factor of 2–3 is probably enough) and which in a few cases lead to predictions of new molecules.

Scale invariant hadron molecules are an intriguing theoretical possibility. They are the two-body realization of a type of universality that is usually only found in three-body atomic and nuclear systems. There are clear theoretical requirements for a hadron molecule to show scale invariance at long distances, where the most natural mechanism is the exchange of a pion almost on the mass shell between initial and final two-hadron states with opposite intrinsic parities. If we consider heavy hadrons, the candidates include $\Lambda_{c1}\bar{D} - \Sigma_c \bar{D}^*$, i.e. the molecular interpretation of the recently discovered P_c^* pentaquark state, while the most likely scale invariant molecule is probably the $\Lambda_{c1}\bar{\Xi}_b - \Sigma_c \bar{\Xi}_b{}'$ baryonium. Discrete scale invariance requires that the couplings have a minimal strength, a condition that a $\frac{1}{2}^+$ heavy pentaquark and a $\Lambda_{c1}\bar{\Xi}_b - \Sigma_c \bar{\Xi}_b{}'$ molecule can meet. The same ideas apply to the $\Lambda_{c1} D - \Sigma_c D^*$ and $\Lambda_{c1} \Xi_b - \Sigma_c \Xi_b'$ molecules as the vector force attraction is independent of whether we have hadrons or antihadrons. The appearance of a geometrical spectrum actually requires the effective mass of the pion to be considerably smaller than the other hadronic scales in the molecule. This condition is not likely to be met in nature, but could very well be realized in the lattice. Even if there is no geometrical spectrum in these molecules, the long-range attraction provided by the vector force plays an important role as a binding mechanism, which cannot be ignored, and in a few cases guarantees binding. The vector force is indeed a new type of long-range dynamics that has not been previously considered either in the P_c^* pentaquark or in other hadronic molecules where it is present and can be relevant. An illustrative example is the enhancement of P-wave interactions, as happens in the $\frac{1}{2}^+$ $\Sigma_c D^*$ system after we include the $\Lambda_{c1} D$ channel. In this type of hadronic molecule the role of scale invariance is analogous to that in the triton, ^{4}He, a few halo nuclei and a series of cold atoms systems, to name a few examples.

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