Study of molecular ND bound states in the Bethe-Salpeter equation approach

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We study the $\Lambda_c(2595)^+$ and $\Sigma_c(2800)^0$ states as the ND bound systems in the Bethe-Salpeter formalism in the ladder and instantaneous approximations. With the kernel induced by ρ, ω and σ exchanges, we solve the Bethe-Salpeter equations for the ND bound systems numerically and find that the bound states may exist. We assume that the observed states $\Lambda_c(2595)^+$ and $\Sigma_c(2800)^0$ are S-wave ND molecular bound states and calculate the decay widths of $\Lambda_c(2595)^+ \to \Sigma_c^0\pi^+$ and $\Sigma_c(2800)^0 \to \Lambda_c^+\pi^-$.

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I. INTRODUCTION

Before 2000, most hadrons could be easily understood with the naive quark model, in which mesons are made up of a quark and an antiquark while baryons consist of three quarks, only except a few cases, e.g. the lowest lying scalar nonet, the $\Lambda(1405)$, and the Roper resonances [1]. With the discovery of the $X(3872)$ by the Belle Collaboration [2], which cannot be easily arranged into standard models of constituent quarks, this situation changed. Thereafter, many other so-called XYZ exotic states have been discovered. Various theoretical interpretations of these resonances have been proposed, including quark-gluon hybrids, tetraquark states, molecular states, and so on. The molecular state is one of the most popular ones to discuss whether the observed XYZ states can be explained with the molecule configuration. In recent years, it has been found that, somehow unexpectedly, not only the exotic states but also some states long believed to be conventional hadrons, which can be explained by the constituent quark models, turn out to contain large hadron-hadron components. Many studies of these states in various decays and reactions have been performed, and the results seem to be consistent with such a molecular picture.

For the $\Lambda(1405)$ state which has been unsuccessfully interpreted by the traditional quark model, people consider it an exotic configuration such as the $N\bar{K}$ molecular state [3,4]. This molecular configuration has been supported by recent Lattice QCD results [5]. In the charm sector, the $\Lambda_c(2595)^+$ as a ND molecular state has also been studied [6–9]. The $\Lambda_c(2595)^+$ $[I(J^P) = 0(1/2^-)]$ has much resemblance to the $\Lambda(1405)$ and can be thought of as being obtained by substituting the strange quark by a c quark. Then, we expect that the $\Lambda_c(2595)^+$ is a bound state of the D meson and a nucleon, in a way similar to the $\Lambda(1405)$. On the other hand, there are also two obvious differences between $\Lambda(1405)$ and $\Lambda_c(2595)^+$. One is that the decay width of $\Lambda_c(2595)^+$ is quite smaller (2.59 MeV) than that of $\Lambda(1405)$ (50.5 MeV). The other one is that the D meson mass is about twice a nucleon mass, while the mass of $\bar{K}$ meson is about only a half of a nucleon mass.

In 2005, the Belle Collaboration [10] observed an isotriplet of open charmed baryon states, $\Sigma_c(2800)$, decaying into $\Lambda_c\pi$, and it was tentatively assigned the quantum numbers $J^P = 3/2^-$. The same neutral state $\Sigma_c(2800)^0$ was also possibly observed in B decays by the *BABAR* Collaboration with the measured mass of 2802^{+4}_{-7} MeV and width of 61^{+28}_{-18} MeV [11], and it was pointed out that there was weak evidence that the excited Σ_c they observed had $J = 1/2$. Because the observed $\Sigma_c(2800)$ states are very close to the ND threshold, one probable explanation of the $\Sigma_c(2800)$ structure is the ND molecular bound state. In this picture, the $\Sigma_c(2800)$ has been studied in the chiral quark model [12,13], in the coupled-channel approach [14], in the effective Lagrangian approach [15] assuming $\Sigma_c(2800)$ has different quantum numbers, and in QCD sum rules [16].

The Bethe-Salpeter (BS) equation is a formally exact equation to describe the relativistic bound state [17–19] and has been applied to many theoretical studies concerning heavy mesons and heavy baryons [20–28]. In this paper, we will work in the BS equation approach, which can

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automatically include relativistic corrections comparing with the potential model which was applied in Ref. [29] to investigate the possible states of $K\bar{K}$, DK, $B\bar{K}$, and $K^- p$ in the framework of the nonrelativistic Schrödinger equation with the potential between pseudoscalar mesons being derived from the relevant Lagrangian. We will try to investigate the possibilities that the $\Lambda_c(2595)$ and $\Sigma_c(2800)^0$ are composed as the ND molecular states with quantum numbers $J^P = 1/2^-$ in the BS equation approach. We will also study the decays of $\Lambda_c(2595)^+ \to \Sigma_c \pi$ and $\Sigma_c(2800)^0 \to \Lambda_c^+ \pi^-$ in this picture.

The paper is organized as follows. In Sec. II, we establish the BS equation for the bound state of a D meson and a nucleon. In Sec. III, we discuss the normalization condition of the BS wave function and obtain the numerical results of the BS wave functions. In Sec. IV, the decays of $\Lambda_c(2595)^+ \to \Sigma_c \pi$ and $\Sigma_c(2800)^0 \to \Lambda_c^+ \pi^-$ are discussed, and we give numerical results. Finally, Sec. V is devoted to a summary and conclusion.

II. BETHE-SALPETER FORMALISM FOR THE *ND* SYSTEM

In this section, we will review the general formalism of the BS equation and derive the BS equation for the system composed of a baryon (N) and a pseudoscalar meson (D). We will also derive the normalization condition for the BS wave function. Let us start by defining the BS wave function for the bound state $|P\rangle$ of a baryon (N) and a pseudoscalar meson (D) as the following,

$$\chi(x_1, x_2, P) = \langle 0|TN(x_1)D(x_2)|P\rangle, \tag{1}$$

where $N(x_1)$ and $D(x_2)$ are the field operators of the baryon and pseudoscalar meson at space coordinates x_1 and x_2, respectively, and P denotes the total momentum of the bound state with mass M and velocity v. In momentum space, the BS wave function can be defined as

$$\chi_P(x_1, x_2, P) = e^{-iPX} \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \chi_P(p), \tag{2}$$

where p represents the relative momentum of the two constituents.

The BS equation for the bound state can be written in the following form,

$$\chi_P(p) = S_N(p_1) \int \frac{d^4q}{(2\pi)^4} K(P, p, q) \chi_P(q) S_D(p_2), \tag{3}$$

where $S_N(p_1)$ and $S_D(p_2)$ are the propagators of the baryon N and the pseudoscalar meson D, respectively, and $K(P, p, q)$ is the kernel which contains two-particle-irreducible diagrams. For convenience, we define $p_l (= p \cdot v)$ and $p_t^\mu (= p^\mu - p_l v^\mu)$ to be the longitudinal and transverse projections of the relative momentum (p) along the bound state momentum (P). Then, the propagator of N has the form

$$S_N(\lambda_1 P + p) = \frac{i[(\lambda_1 M + p_l)\not{v} + \not{p}_t + m_1]}{(\lambda_1 M + p_l - \omega_1 - i\epsilon)(\lambda_1 M + p_l + \omega_1 + i\epsilon)}, \tag{4}$$

where $\omega_1 = \sqrt{m_1^2 - p_t^2}$.

The propagator of the D meson in the heavy quark limit can be expressed at the leading order of the $1/m_D$ expansion as follows:

$$S_D(\lambda_2 P - p) = \frac{i}{2m_2(p_l + M - m_2 + i\epsilon)}. \tag{5}$$

In general, for a baryon and a pseudoscalar meson bound state, considering $\not{v}u(v, s) = u(v, s)$ [$u(v, s)$ is the spinor of the bound state with helicity s], $\chi_P(p)$ can be written as

$$\chi_P(p) = (g_1 + g_2\gamma_5 + g_3\gamma_5\not{p}_t + g_4\not{p}_t + g_5\sigma_{\mu\nu}\varepsilon^{\mu\nu\alpha\beta}p_{t\alpha}v_\beta), \tag{6}$$

where g_i ($i = 1, \ldots, 5$) are Lorentz-scalar functions. After considering the constraints imposed by $\not{v}\chi_P(p) = \chi_P(p)$ from the heavy quark symmetry and those imposed by parity and Lorentz transformations, it is easy to prove that $\chi_P(p)$ can be simplified as

$$\chi_P(p) = f(p)u(v, s), \tag{7}$$

in which $f(p)$ is a Lorentz-scalar function of p.

As discussed in the Introduction, we will study the S-wave bound state of the ND system. The isospin field doublets $N = (N^0, N^-)^T$ and $D = (-D^+, D^0)^T$ have the following expansions in momentum space:

$$\begin{aligned}
N_1(x) &= \int \frac{d^3p}{(2\pi)^3\sqrt{2E_N^\pm}} (a_{N^-}e^{-ipx} + a_{N^+}^\dagger e^{ipx}), \\
N_2(x) &= \int \frac{d^3p}{(2\pi)^3\sqrt{2E_N^0}} (a_{N^0}e^{-ipx} + a_{\bar{N}^0}^\dagger e^{ipx}), \\
D_1(x) &= \int \frac{d^3p}{(2\pi)^3\sqrt{2E_D^\pm}} (a_{D^+}e^{-ipx} + a_{D^-}^\dagger e^{ipx}), \\
D_2(x) &= \int \frac{d^3p}{(2\pi)^3\sqrt{2E_D^0}} (a_{D^0}e^{-ipx} + a_{\bar{D}^0}^\dagger e^{ipx}).
\end{aligned} \tag{8}$$

The isospin quantum number of $\Lambda_c(2595)^+$ is 0, and the isoscalar bound state of the ND system can be written as

$$|P\rangle_{0,0} = \frac{1}{\sqrt{2}}|N^0D^+ - N^+D^0\rangle. \tag{9}$$

The isospin quantum number of $\Sigma_c(2800)$ is 1, corresponding to the following isovector bound state:

$$|P\rangle_{1,1} = |N^+D^+\rangle, \qquad |P\rangle_{1,0} = \frac{1}{\sqrt{2}}|N^+D^0 + N^0D^+\rangle,$$
$$|P\rangle_{1,-1} = |N^0D^0\rangle. \tag{10}$$

Let us now project the bound states on the field operators $N_1(x)$, $N_2(x)$, $D_1(x)$, and $D_2(x)$. Then, we have

$$\langle 0|TN_i(x_1)D_j(x_2)|P\rangle_{I,I_3} = C^{ij}_{(I,I_3)}\chi^I_P(x_1,x_2), \tag{11}$$

where $\chi_P^{(I)}$ is the common BS wave function for the bound state with isospin I which depends on I but not I_3 of the state $|P\rangle_{I,I_3}$. The isospin coefficients $C^{ij}_{(I,I_3)}$ for the isoscalar state are

$$C^{12}_{(0,0)} = -C^{21}_{(0,0)} = -\frac{1}{\sqrt{2}}, \qquad \text{else} = 0, \tag{12}$$

and for the isovector states, we have

$$C^{11}_{(1,1)} = 1, \qquad C^{21}_{(1,0)} = \frac{1}{\sqrt{2}}, \qquad C^{12}_{(1,0)} = \frac{1}{\sqrt{2}},$$
$$C^{22}_{(1,-1)} = 1, \qquad \text{else} = 0. \tag{13}$$

Now, consider the kernel. The BS equation for the bound state can be written as

$$C^{ij}_{(I,I_3)}\chi^I_P(p) = S_N(\lambda_1P+p)\int\frac{d^4q}{(2\pi)^4}K^{ij,lk}(P,p,q) \times C^{lk}_{(I,I_3)}\chi^I_P(q)S_D(\lambda_2P-p), \tag{14}$$

where $i(j)$ and $l(k)$ refer to the components of the $N(D)$ field doublets. Explicitly, we give the isoscalar case as an example:

$$\chi^0_P(p) = S_N(\lambda_1P+p) \times \int\frac{d^4q}{(2\pi)^4}(K^{12,12}+K^{12,21})\chi^0_P(q)S_D(\lambda_2P-p). \tag{15}$$

In this work, we describe the *ND* interaction by one-particle-exchange diagrams as shown in Fig. 1. In order to perform the calculations, we use the effective Lagrangians in Refs. [30–34],

$$\mathcal{L}_{\rho NN} = g_{\rho NN}\bar{N}\left(\gamma^\mu\vec{\tau}\cdot\vec{\rho}_\mu + \frac{\kappa_\rho}{2m_N}\sigma^{\mu\nu}\vec{\tau}\cdot\partial_\mu\vec{\rho}_\nu\right)N, \tag{16}$$

$$\mathcal{L}_{\omega NN} = g_{\omega NN}\bar{N}\gamma^\mu\omega_\mu N, \tag{17}$$

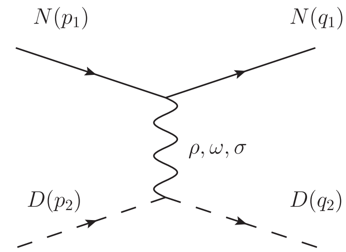


FIG. 1. The Feynman diagram for one-particle-exchange *DN* interaction where V could be ρ or ω.

$$\mathcal{L}_{\sigma NN} = g_{\sigma NN}\bar{N}\sigma N, \tag{18}$$

$$\mathcal{L}_{\rho DD} = ig_{\rho DD}[D\vec{\tau}(\partial_\mu\bar{D}) - (\partial_\mu D)\vec{\tau}\bar{D}]\cdot\vec{\rho}^\mu, \tag{19}$$

$$\mathcal{L}_{\omega DD} = ig_{\omega DD}[D(\partial_\mu\bar{D}) - (\partial_\mu D)\bar{D}]\omega^\mu. \tag{20}$$

$$\mathcal{L}_{\sigma DD} = g_{\sigma DD}\bar{D}\sigma D. \tag{21}$$

In the above equations, $\vec{\tau}$ is the Pauli spin matrix, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu]$, and $\vec{\rho}$ denotes the ρ meson isospin triplet. For the coupling constants, we use the empirical values $g_{\rho NN} = 2.6$–3.36, $\kappa_\rho = 6.1$ [30–32,35–37], $g_{\sigma NN} = 8.46$ [33], $g_{\rho DD} = g_{\omega DD} = 2.52$–$3.69$ [38–42], and $g_{\sigma DD} = g_\pi/\sqrt{6}M_D$ with $g_\pi = 3.73$ [34]. For $g_{\omega NN}$, there is considerable uncertainty. In Table I, we list the values of $g_{\omega NN}$ from various analyses.

From the above observations, at the tree level, in the t channel, we have the following kernel for the BS equation in the so-called ladder approximation (see Fig. 1),

$$K_V(P,p,q) = c_I g_{VNN}g_{VDD}(p_2+q_2)_\alpha\gamma^\mu\Delta^\alpha_\mu(k,m_V), \tag{22}$$

$$K_\sigma(P,p,q) = c_I g_{\sigma NN}g_{\sigma DD}\Delta(k,m_\sigma), \tag{23}$$

where m_V ($V=\rho$ or ω) represents the mass of the exchanged vector light meson, c_I is the isospin coefficient ($c_0^\rho = 3, c_0^\omega = 1, c_0^\sigma = 1$, and $c_1^\rho = -1, c_1^\omega = 1, c_1^\sigma = 1$, respectively), and $\Delta^{\mu\nu}(k,m_V)$ and $\Delta(k,m_\sigma)$ represent the propagators of the vector meson and σ meson.

In order to describe the phenomena in the real world, we should include a form factor at each interacting vertex of hadrons to include the finite-size effects of these hadrons. For the meson-exchange case, the form factor is assumed to take the following form [46],

TABLE I. Values of the coupling constant $g_{\omega NN}$ from different approaches: nucleon-nucleon interaction [30], nucleon electromagnetic form factors [43], $\gamma N\to\pi N$ interaction [44], *NN* collisions [35], and radiative decay $\omega\to e^+e^-$ [45].

References	[30]	[43]	[44]	[35]	[45]
$g_{\omega NN}$	15.62	20.85 ± 0.24	7–10.5	10.7	9–10.5

$$F(\mathbf{k_t}) = \frac{\Lambda^2 - m^2}{\Lambda^2 + \mathbf{k_t}^2}, \qquad \mathbf{k_t} = \mathbf{p_t} - \mathbf{q_t}, \tag{24}$$

where Λ and m represent the cutoff parameter and the mass of the exchanged meson, respectively.

To simplify the BS equation, Eq. (3), we impose the so-called covariant instantaneous approximation [21] in the kernel, $p_l = q_l$. Then, from Eqs. (4), (5), and (23), Eq. (3) becomes

$$\begin{aligned} f(p_l, p_t) = & \frac{i[(\lambda_1 M + p_l)\not{v} + \not{p}_t + m_1]}{2m_2(p_l + M - m_2 + i\epsilon)(\lambda_1 M + p_l + \omega_1 - i\epsilon)(\lambda_1 M + p_l - \omega_1 + i\epsilon)} \\ & \times \int \frac{d^4 q}{(2\pi)^4} \left[c_I^\rho g_{\rho NN} g_{\rho DD} \frac{2(\lambda_2 M - p_l)\not{v} - \not{p}_t - \not{q}_t + \frac{(p_t^2 - q_t^2)(\not{p}_t - \not{q}_t)}{m_\rho^2}}{(p_t - q_t)^2 - m_\rho^2} F_{m_\rho}^2(\mathbf{k_t}) \right. \\ & + c_I^\omega g_{\omega NN} g_{\omega DD} \frac{2(\lambda_2 M - p_l)\not{v} - \not{p}_t - \not{q}_t + \frac{(p_t^2 - q_t^2)(\not{p}_t - \not{q}_t)}{m_\omega^2}}{(p_t - q_t)^2 - m_\omega^2} F_{m_\omega}^2(\mathbf{k_t}) \\ & \left. - c_I^\sigma g_{\sigma NN} g_{\sigma DD} \frac{1}{(p_t - q_t)^2 - m_\sigma^2} F_{m_\sigma}^2(\mathbf{k_t}) \right] f(q_l, q_t), \end{aligned} \tag{25}$$

where $F_{m_\rho}^2(\mathbf{k_t})$, $F_{m_\omega}^2(\mathbf{k_t})$, and $F_{m_\sigma}^2(\mathbf{k_t})$ represent the corresponding form factors for different exchanged mesons.

In the rest frame, performing the integration over p_l on both sides through the residue theorem, we have

$$\begin{aligned} \tilde{f}(\mathbf{p_t}) = & \frac{1}{4m_2\omega_1(\lambda_2 M - \omega_1 - m_2)} \int \frac{d^3\mathbf{q_t}}{(2\pi)^3} \\ & \times \left\{ \frac{c_I^\rho g_{\rho NN} g_{\rho DD}}{(\mathbf{p_t} - \mathbf{q_t})^2 + m_\rho^2} \left[2\omega_1(M + \omega_1) - \mathbf{p_t}^2 - \mathbf{p_t}\cdot\mathbf{q_t} - \frac{(\mathbf{p_t}^2 - \mathbf{q_t}^2)(\mathbf{p_t}^2 - \mathbf{p_t}\cdot\mathbf{q_t})}{m_\rho^2} \right] F_{m_\rho}^2(\mathbf{k_t}) \right. \\ & + \frac{c_I^\omega g_{\omega NN} g_{\omega DD}}{(\mathbf{p_t} - \mathbf{q_t})^2 + m_\omega^2} \left[2\omega_1(M + \omega_1) - \mathbf{p_t}^2 - \mathbf{p_t}\cdot\mathbf{q_t} - \frac{(\mathbf{p_t}^2 - \mathbf{q_t}^2)(\mathbf{p_t}^2 - \mathbf{p_t}\cdot\mathbf{q_t})}{m_\omega^2} \right] F_{m_\omega}^2(\mathbf{k_t}) \\ & \left. + \frac{c_I^\sigma g_{\sigma NN} g_{\sigma DD} m_1}{(\mathbf{p_t} - \mathbf{q_t})^2 + m_\sigma^2} F_{m_\sigma}^2(\mathbf{k_t}) \right\} \tilde{f}(\mathbf{q_t}), \end{aligned} \tag{26}$$

where $\tilde{f}(\mathbf{p}_t) = \int dp_l f(p_l, p_t)$.

III. NORMALIZATION CONDITION AND NUMERICAL RESULTS OF BS WAVE FUNCTIONS

Following Ref. [19], the normalization condition for the BS wave function can be written as

$$i \int \frac{d^4 p d^4 q}{(4\pi)^8} \bar{\chi}(p) \frac{\partial}{\partial P^0} [I(P, p, q) + K(P, p, q)] \chi(q) = 1, \tag{27}$$

where $P^0 = E$, $I(P, p, q) = -(2\pi)^4 \delta^4(p - q) S_N^{-1}(\lambda_1 P + p) S_D^{-1}(\lambda_2 P - p)$.

In the DN bound state rest frame, the normalization condition can be written in the following form:

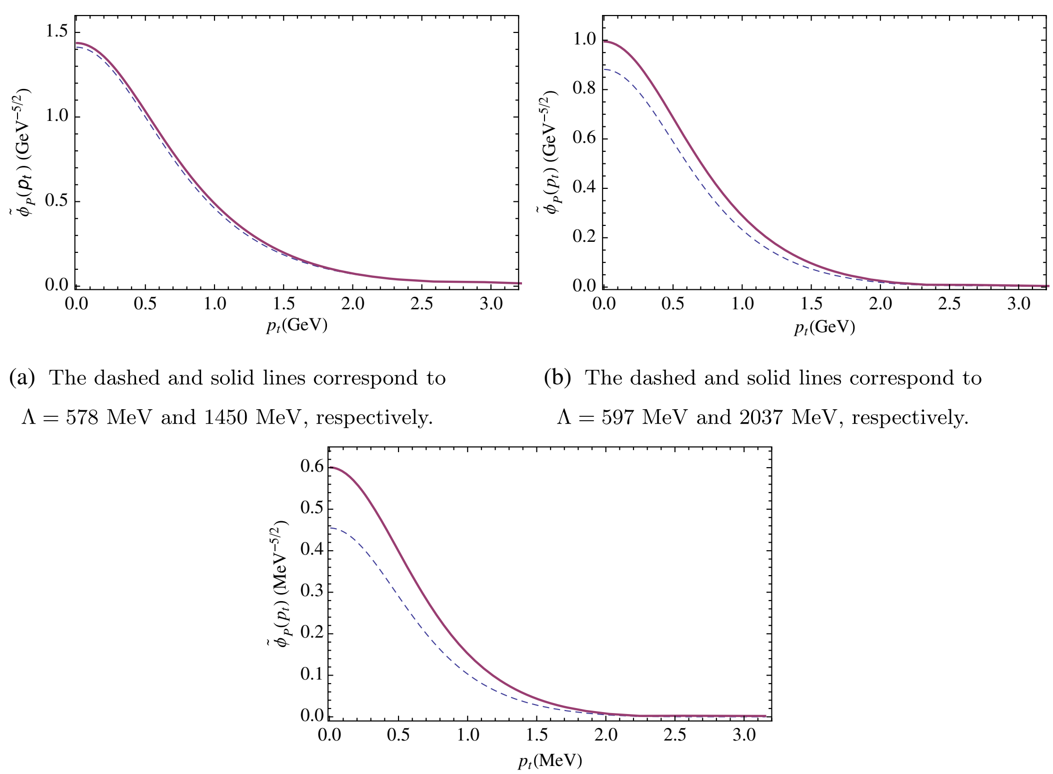
$$-8i\lambda_1 M m_2 \int \frac{d^4 p}{(2\pi)^4} (p_l + M - m_2) f^2(p) = 1. \tag{28}$$

According to Eq. (25), we have

$$f(p_l,p_t)=\frac{i}{2m_2(p_l+M-m_2+i\epsilon)(\lambda_1 M+p_l+\omega_1-i\epsilon)(\lambda_1 M+p_l-\omega_1+i\epsilon)}\int\frac{d^3\mathbf{q}_t}{(2\pi)^4}$$
$$\times\Bigg[c_I g_{\rho NN}g_{\rho DD}\frac{2(\lambda_1 M+p_l)(\lambda_2 M-p_l)+\mathbf{p_t}^2+\mathbf{p_t}\cdot\mathbf{q_t}+\frac{(\mathbf{p_t}^2-\mathbf{q_t}^2)(\mathbf{p_t}^2-\mathbf{p_t}\cdot\mathbf{q_t})}{m_\rho^2}}{-(\mathbf{p_t}-\mathbf{q_t})^2-m_\rho^2}F^2_{m_\rho}(\mathbf{k_t})$$
$$+c_I g_{\omega NN}g_{\omega DD}\frac{2(\lambda_1 M+p_l)(\lambda_2 M-p_l)+\mathbf{p_t}^2+\mathbf{p_t}\cdot\mathbf{q_t}+\frac{(\mathbf{p_t}^2-\mathbf{q_t}^2)(\mathbf{p_t}^2-\mathbf{p_t}\cdot\mathbf{q_t})}{m_\omega^2}}{-(\mathbf{p_t}-\mathbf{q_t})^2-m_\omega^2}F^2_{m_\omega}(\mathbf{k_t})$$
$$-c_I g_{\sigma NN}g_{\sigma DD}\frac{m_1}{-(\mathbf{p_t}-\mathbf{q_t})^2-m_\sigma^2}F^2_{m_\sigma}(\mathbf{k_t})\Bigg]\tilde{f}(\mathbf{q}_t). \tag{29}$$

Then, one can recast the normalization condition for the BS wave function into the form

$$\lambda_1 M\int\frac{d^3p_t}{(2\pi)^3}(\lambda_2 M-m_2-\omega_1)\frac{-\lambda_2 M+m_2+3\omega_1}{2m_2\omega_1^3(-\lambda_2 M+m_2+\omega_1)^3}$$
$$\times\Bigg\{\int\frac{d^3\mathbf{q}_t}{(2\pi)^4}\Bigg[c_I g_{\rho NN}g_{\rho DD}\frac{-2\omega_1(M+\omega_1)+\mathbf{p_t}^2+\mathbf{p_t}\cdot\mathbf{q_t}+\frac{(\mathbf{p_t}^2-\mathbf{q_t}^2)(\mathbf{p_t}^2-\mathbf{p_t}\cdot\mathbf{q_t})}{m_\rho^2}}{-(\mathbf{p_t}-\mathbf{q_t})^2-m_\rho^2}F^2_{m_\rho}(\mathbf{k_t})$$
$$+c_I g_{\omega NN}g_{\omega DD}\frac{-2\omega_1(M+\omega_1)+\mathbf{p_t}^2+\mathbf{p_t}\cdot\mathbf{q_t}+\frac{(\mathbf{p_t}^2-\mathbf{q_t}^2)(\mathbf{p_t}^2-\mathbf{p_t}\cdot\mathbf{q_t})}{m_\omega^2}}{-(\mathbf{p_t}-\mathbf{q_t})^2-m_\omega^2}F^2_{m_\omega}(\mathbf{k_t})$$
$$-c_I g_{\sigma NN}g_{\sigma DD}\frac{m_1}{-(\mathbf{p_t}-\mathbf{q_t})^2-m_\sigma^2}F^2_{m_\sigma}(\mathbf{k_t})\tilde{f}(\mathbf{p}_t)\Bigg]\Bigg\}^2=1. \tag{30}$$

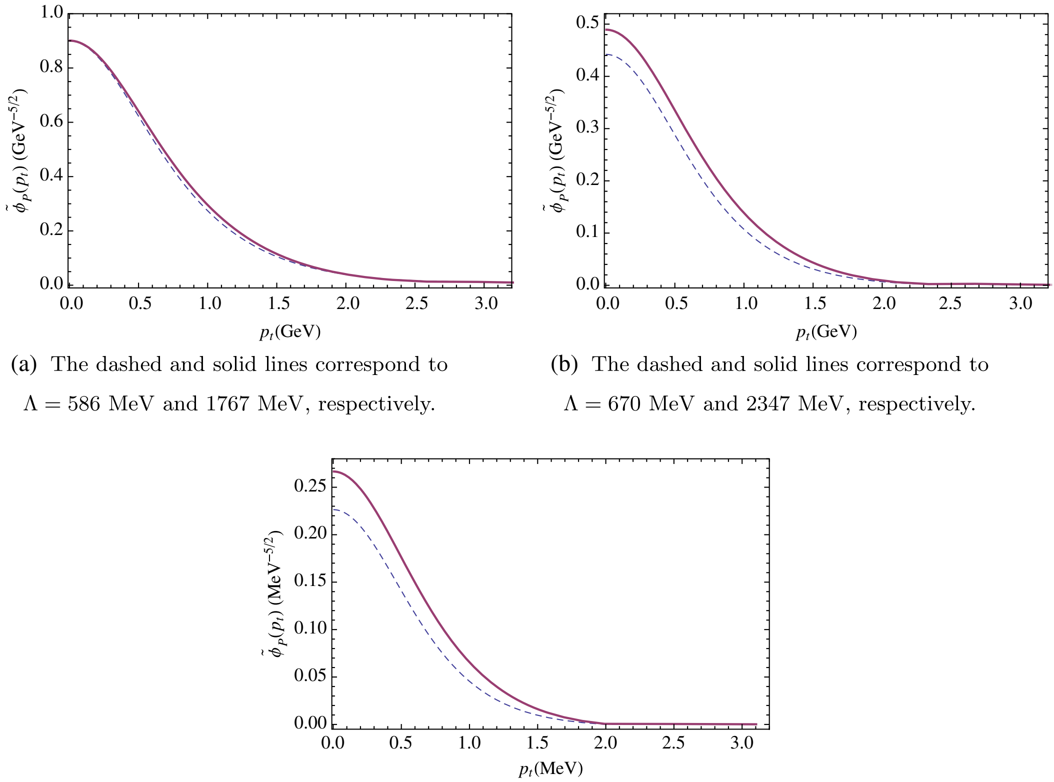


(a) The dashed and solid lines correspond to $\Lambda = 578$ MeV and 1450 MeV, respectively.

(b) The dashed and solid lines correspond to $\Lambda = 597$ MeV and 2037 MeV, respectively.

(c) The dashed and solid lines correspond to $\Lambda = 613$ MeV and 2515 MeV, respectively.

FIG. 2. Numerical results for the BS wave function $\tilde{\phi}_P(p_t)$ for the bound state of $\Lambda_c(2595)^+$ corresponding to (a) $g_{\rho NN}=2.60$, $g_{\omega NN}=7.00$, $g_{\rho DD}=g_{\omega DD}=2.52$; (b) $g_{\rho NN}=2.98$, $g_{\omega NN}=14.00$, $g_{\rho DD}=g_{\omega DD}=3.11$; and (c) $g_{\rho NN}=3.36$, $g_{\omega NN}=21.09.00$, $g_{\rho DD}=g_{\omega DD}=3.69$, respectively.



(a) The dashed and solid lines correspond to $\Lambda = 586$ MeV and 1767 MeV, respectively.

(b) The dashed and solid lines correspond to $\Lambda = 670$ MeV and 2347 MeV, respectively.

(c) The dashed and solid lines correspond to $\Lambda = 785$ MeV and 2463 MeV, respectively.

FIG. 3. Numerical results for the BS wave function $\tilde{\phi}_P(p_t)$ for the bound state of $\Sigma_c(2800)^0$ corresponding to (a) $g_{\rho NN} = 2.60$, $g_{\omega NN} = 7.00$, $g_{\rho DD} = g_{\omega DD} = 2.52$; (b) $g_{\rho NN} = 2.98$, $g_{\omega NN} = 14.00$, $g_{\rho DD} = g_{\omega DD} = 3.11$; and (c) $g_{\rho NN} = 3.36$, $g_{\omega NN} = 21.09.00$, $g_{\rho DD} = g_{\omega DD} = 3.69$, respectively.

It can be seen from Eq. (26) that there is only one free parameter in our model, the cutoff Λ, which contains the information about the nonpoint interaction due to the structure of hadrons at the interaction vertices. Although the value of Λ cannot be exactly determined and depends on the specific process, it should be typically the scale of low-energy physics, which is about 1 GeV. In Ref. [9], the authors found there existed $\Lambda_c(2593)$ resonance in the isospin zero DN channel with $\Lambda = 727$ and 787 MeV. In Ref. [14], the authors found that the $\Lambda_c(2595)$ could be better reproduced by the nonlocal model with $\Lambda = 903$ MeV. Dong *et al.* [15] fixed Λ with a mean value 1 GeV in the study of $\Sigma_c(2800)$ as a DN hadronic molecule. The authors of Ref. [47] studied the ND interaction from the meson exchange and found that Λ varies from 0.8 to 3.5 GeV for different processes. In this work, we treat the cutoff Λ in the form factor as a parameter varying in a reasonable range 0.5–4.8 GeV, in which we will try to search for possible solutions of the ND bound states. The BS wave function in Eq. (26) satisfies a homogeneous integral equation, and we can discretize the integration region $(0, \infty)$ into n pieces (n is large enough) by the n-point Gauss quadrature rule. The BS equation then becomes an eigenvalue equation. In the calculation, we choose to work in the rest frame of the bound state in which $P = (M, 0)$. We use $M_{\Lambda_c(2595)^+} = 2592.25$ MeV, $M_{\Sigma_c(2800)^0} = 2802.00$ MeV, $M_N = 938.92$ MeV, and $M_D = 1867.21$ MeV [1]. From our calculations, we find there exist bound states corresponding to $\Lambda_c(2595)^+$ and $\Sigma_c(2800)^0$. The numerical results for the BS wave function $\tilde{\phi}_P(p_t)$ for the bound states of ND, $\Lambda_c(2595)^+$ and $\Sigma_c(2800)^0$, are plotted in Figs. 2 and 3, respectively, for some different values of the coupling constants.

IV. DECAYS OF $\Lambda_c(2595)^+$ AND $\Sigma_c(2800)^0$

A. Decay $\Lambda_c(2595)^+ \to \Sigma_c^0\pi^+$

After obtaining the BS wave function, we can calculate some physical properties of the molecular bound state which can be measured in experiments. One of the most important properties is the decay width. The bound state $\Lambda_c(2595)^+$ can decay to $\Sigma_c^0\pi^+$ via the Feynman diagram in Fig. 4. In the following, we will write down the decay amplitude and calculate the decay width using the solution of the one-dimensional BS equation obtained in the previous section. From the formalism described in Refs. [31,32], the effective Lagrangians for the decay $DN \to \Sigma_c\pi$ are

$$\mathcal{L}_{D^*D\pi} = ig_{D^*D\pi}D^*_\mu\vec{\tau}\cdot(\bar{D}\partial^\mu\vec{\pi} - \partial^\mu\bar{D}\,\vec{\pi}) + \text{H.c.},$$
$$\mathcal{L}_{D^*N\Sigma_c} = g_{D^*N\Sigma_c}(\bar{N}\gamma^\mu\vec{\tau}\cdot\vec{\Sigma}_c\bar{D}^*_\mu + D^*_\mu\vec{\tau}\cdot\vec{\bar{\Sigma}}_c\gamma^\mu N), \tag{31}$$

where $g_{D^*D\pi}$ and $g_{D^*N\Sigma_c}$ are coupling constants and we will use the empirical value $g_{D^*D\pi} = 5.56$ and the value $g_{D^*N\Sigma_c} = -3.23$ obtained using $SU(4)$ relations [31,32].

In the rest frame, we define $p'_1 = (E'_1, -\mathbf{p}')$ and $p'_2 = (E'_2, -\mathbf{p}')$ to be the momenta of π and Σ_c, respectively. The masses of π and Σ_c are m'_1 and m'_2, respectively. According to the kinematics in the rest frame of the two-body decay, one has

$$E'_1 = \frac{M^2 - m'^2_2 + m'^2_1}{2M}, \quad E'_2 = \frac{M^2 - m'^2_1 + m'^2_2}{2M}, \tag{32}$$

$$|\mathbf{p}'| = \frac{\sqrt{[M^2-(m'_1+m'_2)^2][M^2-(m'_1-m'_2)^2]}}{2M}, \tag{33}$$

and

$$d\Gamma = \frac{1}{32\pi^2}|\mathcal{M}|^2\frac{|\mathbf{p}'|}{E^2}d\Omega, \tag{34}$$

where $|\mathbf{p}'|$ is the norm of the 3-momentum of the particles in the final state in the rest frame of the initial bound state and $\mathcal{M}$ is the Lorentz-invariant decay amplitude of the process.

From Fig. 4, we can write down the amplitude as

$$\mathcal{M} = -\frac{g_{D^*N\Sigma_c}g_{D^*D\pi}}{2} \times \int\frac{d^4p}{(2\pi)^4}\bar{u}_{\Sigma_c}\gamma^\mu(p_1+p'_1)^\nu\Delta_{\mu\nu}(k, m_{D^*})F^2(k)\chi_P(p). \tag{35}$$

In the calculation, we use the following input parameters: $M_{\Sigma_c} = 2453.76$ MeV, $M_\pi = 139.57$ MeV, and $M_{D^*} = 2006.96$ MeV. Using the numerical solution for the BS wave function, we calculate the decay width of the decay $\Lambda_c(2595)^+ \to \Sigma_c^0\pi^+$ and obtain the following result in the ranges of the parameters in our model:

$$\Gamma = 0.103 - 44.038 \text{ KeV}. \tag{36}$$

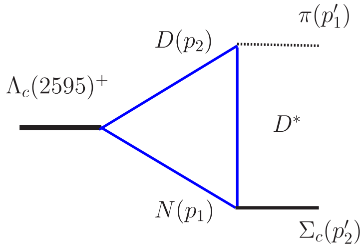


FIG. 4. The Feynman diagram for the *DN* bound state $\Lambda_c(2595)^+$ decaying into $\Sigma_c\pi$.

B. Decay $\Sigma_c(2800)^0 \to \Lambda_c^+\pi^-$

The bound state $\Sigma_c(2800)^0$ can decay to $\Lambda_c^+\pi^-$ via the Feynman diagrams in Fig. 5. In the following, we will write down the decay amplitude and calculate the decay width using the solution of the one-dimensional BS equation obtained in Sec. III. From the formalism described in Refs. [31,32], the effective Lagrangians for the radiative decay $\Sigma_c(2800)^0 \to \Lambda_c^+\pi^-$ are

$$\mathcal{L}_{\pi NN} = -ig_{\pi NN}\bar{N}\gamma_5\vec{\tau}N\cdot\vec{\pi},$$
$$\mathcal{L}_{DN\Lambda_c} = ig_{DN\Lambda_c}(\bar{N}\gamma_5\Lambda_c\bar{D} + D\bar{\Lambda}_c\gamma_5 N),$$
$$\mathcal{L}_{\pi DD^*} = ig_{\pi DD^*}\bar{D}^{*\mu}\vec{\tau}\cdot(D\partial_\mu\vec{\pi} - \partial_\mu D\vec{\pi}) + \text{H.c.},$$
$$\mathcal{L}_{D^*N\Lambda_c} = g_{D^*N\Lambda_c}(\bar{N}\gamma_\mu\Lambda_c\bar{D}^{*\mu} + D^{*\mu}\bar{\Lambda}_c\gamma_\mu N), \tag{37}$$

where $g_{\pi NN}$, $g_{DN\Lambda_c}$, $g_{\pi DD^*}$, and $g_{D^*N\Lambda_c}$ are coupling constants and from empirical values and $SU(4)$ relations $g_{\pi NN} = 13.5$, $g_{DN\Lambda_c} = 13.5$, $g_{\pi DD^*} = 5.56$, and $g_{D^*N\Lambda_c} = -5.6$ [31,32].

According to the above interactions, the decay $\Sigma_c \to \Lambda_c^+\pi^-$ induced by N and D^* exchanges is shown in Fig. 5. We can write down the amplitudes as the following for Figs. 5(a) and 5(b), respectively:

$$\mathcal{M}_a = -\frac{g_{\pi NN}g_{DN\Lambda_c}}{2}\int\frac{d^4p}{(2\pi)^4}\bar{u}_{\Lambda_c}S(k, m_N)\chi_P(p), \tag{38}$$

$$\mathcal{M}_b = -\frac{g_{\pi DD^*}g_{D^*N\Lambda_c}}{2} \times \int\frac{d^4p}{(2\pi)^4}\bar{u}_{\Lambda_c}(p_1+p'_1)_\mu\Delta^{\mu\nu}(k, m_{D^*})\gamma_\nu\chi_P(p). \tag{39}$$

In the calculation, we use the following input parameters: $M_{\Lambda_c} = 2286.46$ MeV, $M_\pi = 139.57$ MeV, and $M_{D^{*0}} = 2006.96$ MeV. We use the numerical solution for the BS wave function to calculate the decay width of the decay $\Sigma_c(2800)^0 \to \Lambda_c^+\pi^-$ and obtain the following numerical result in the ranges of the parameters in our model:

$$\Gamma = 15.568\text{–}219.473 \text{ KeV}. \tag{40}$$

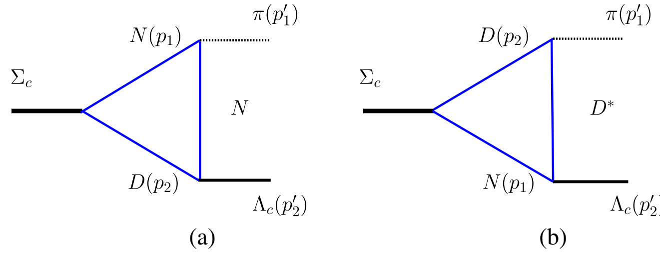


FIG. 5. Diagrams contributing to the $\Sigma_c(2800)^0 \to \Lambda_c^+\pi^-$ decay.

V. SUMMARY AND CONCLUSION

In this paper, we derive the BS equation for the S-wave DN bound state system, study the possibility that $\Lambda_c(2595)^+$ and $\Sigma_c(2800)^0$ are DN bound states with the quantum numbers $J^P = 1/2^-$, and calculate their decay widths in the BS formalism. Considering the interaction kernel based on ρ, ω, and σ mesons-exchange diagrams, we study the BS equation for the DN system in the ladder and instantaneous approximations. Since the constituent particles and the exchanged particles in the DN system are not pointlike, we introduce a form factor including a cutoff Λ which reflects the effects of the structure of these particles. Since Λ is controlled by nonperturbative QCD and cannot be determined at present, we let it vary in a reasonable range within which we examine whether $\Lambda_c(2595)^+$ and $\Sigma_c(2800)^0$ could be the DN bound states by solving the BS equations. From the numerical results, we find that there exist DN bound states which can be attributed to $\Lambda_c(2595)^+$ and $\Sigma_c(2800)^0$, respectively.

We apply the numerical solutions for the BS wave functions to calculate the decay widths of $\Lambda_c(2595)^+ \to \Sigma_c^0\pi^+$ and $\Sigma_c(2800)^0 \to \Lambda_c^+\pi^-$ induced by D^* exchange and N and D^* exchanges, respectively. We obtain that the decay width of $\Lambda_c(2595)^+ \to \Sigma_c^0\pi^+$ is in the range 0.103–44.038 KeV (the experimental data are 2.59 MeV) and that of $\Sigma_c(2800)^0 \to \Lambda_c^+\pi^-$ is in the range 15.568–219.473 KeV (the experimental data are 61^{+28}_{-18} MeV) in the ranges of our model parameters. From these decay widths, we can see that the uncertainties of the coupling constants and the value of Λ lead to large uncertainties in our results. Since the decay widths obtained from our model are much smaller than the experimental data, we conclude that the ND molecular structure should contribute to the $\Lambda_c(2595)^+$ and $\Sigma_c(2800)^0$ states, but there should be other structures besides the ND molecule in $\Lambda_c(2595)^+$ and $\Sigma_c(2800)^0$ states. Obviously, to resolve this problem, further investigations are required.

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