P_c-like pentaquarks in a hidden strange sector

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Analogous to the work of hidden charm molecular pentaquarks, we study possible hidden strange molecular pentaquarks composed of Σ (or Σ^*) and K (or K^*) in the framework of a quark delocalization color screening model. Our results suggest that the ΣK, ΣK^*, and $\Sigma^* K^*$ with $IJ^P = \frac{1}{2}\frac{1}{2}^-$ and ΣK^*, $\Sigma^* K$, and $\Sigma^* K^*$ with $IJ^P = \frac{1}{2}\frac{3}{2}^-$ are all resonance states by coupling the open channels. The molecular pentaquark $\Sigma^* K$ with quantum numbers $IJ^P = \frac{1}{2}\frac{3}{2}^-$ can be seen as a strange partner of the LHCb $P_c(4380)$ state. The possibility of identifying the resonances as nucleon resonances is proposed.

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I. INTRODUCTION

The multiquark study is essential for understanding the low-energy quantum chromodynamics (QCD), because the multiquark states can provide information unavailable for the $q\bar{q}$ meson and q^3 baryon, especially the property of hidden color structure. The pentaquark is one of the important topics of multiquark study. In 2015, the observations of two hidden-charm pentaquarks, $P_c(4380)$ and $P_c(4450)$, at LHCb [1] invoked a renewed interest in pentaquark states. The JLab also proposed to search for these two P_c states by using photo-production of J/ψ at threshold [2]. Various interpretations of the hidden-charm pentaquarks have been discussed, and many other possible pentaquarks were also proposed in the literature [3–13].

Analogous to the hidden-charm pentaquark P_c states, one may consider the existence of possible P_c-like pentaquarks in a hidden strange sector, in which the $c\bar{c}$ is replaced by the $s\bar{s}$. In fact, as early as 2001, a $\phi - N$ bound state was proposed by Gao *et al.* [14], which is an analogy to the work of Refs. [15,16], in which they suggested that the QCD van der Waals interaction, mediated by multi-gluon exchanges, will dominate the interaction between two hadrons when they have no common quarks, and this supported the prediction of a nucleon-charmonium bound state near the charm production threshold. In addition, Liska *et al.* [17] demonstrated the feasibility of searching for the $\phi - N$ bound state from ϕ meson subthreshold production; some chiral quark model calculations [18] and lattice QCD calculations [19] also support the existence of such a bound state. Very recently, Xie and Guo studied the possible ϕp resonance in the $\Lambda_c^+ \to \pi^0 \phi p$ decay by considering a triangle singularity mechanism [20]. Our group also investigated the $\phi - N$ bound state in the quark delocalization color screening model (QDCSM) [21], performed a Monte Carlo simulation of the bound state production with an electron beam and a gold target, and found it was feasible to experimentally search for the $\phi - N$ bound state through the near threshold ϕ meson production from heavy nuclei. In Ref. [21], we only focused on the $\phi - N$ bound state; however, we also found that the interaction between Σ (or Σ^*) and K (or K^*) was strong enough to form bound states, which is similar to that of Σ_c (or Σ_c^*) and D (or D^*) [13]. Since the $P_c(4380)$ and $P_c(4450)$ are close to the thresholds of the $\Sigma_c^* D$ and $\Sigma_c D^*$, many works studied two P_c states as the molecular states composed of Σ_c (or Σ_c^*) and D (or D^*) [4,5]. Therefore, we expect the existence of some molecular states consisted of Σ (or Σ^*) and K (or K^*), which are analogous to the P_c state.

In fact, the study of pentaquarks composed of light quarks has a very long history. The $\Lambda(1405)$ resonance has been explained as a $N\bar{K}$ molecular state since the 1960s [22–28]. The quantities of nucleon resonances near 2 GeV were still unclear both in theory and experiment. Some nucleon resonances were investigated by coupling with pentaquark channels. One peculiar state is the $N^*(1535)$ resonance with spin parity $J^P = 1/2^-$, which is found to couple strongly to the pentaquark channels with strangeness [29–34]. Another $J^P = 1/2^-$ nucleon resonance is the $N^*(1895)$, which is a two-star state in the compilation of the Particle Data Group (PDG) [35]. However, its existence

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is supported by the analysis of the new η photo production data [36,37], which showed that the $N^*(1895)$ is crucial to describe the cusp observed in the η photo production around 1896 MeV. Moreover, Refs. [36,37] suggested that this $N^*(1895)$ had strong coupling to the $N\eta$ and $N\eta'$ channels. In our previous work, we found a $J^P = 1/2^-$ bound state with a mass varying from 1873 to 1881 MeV, and the main component is $N\eta'$ [21], which could correspond to the resonance $N^*(1895)$. Four $J^P = 3/2^-$ nucleon resonances, $N^*(1520)$, $N^*(1700)$, $N^*(1875)$, and $N^*(2120)$, are listed in new versions of the PDG [35], of which the three-star $N^*(1875)$ and two-star $N^*(2120)$ still have various interpretations about their internal structures [38–40]. J. He investigated both $N^*(1875)$ and $N^*(2120)$. He interpreted the $N^*(1875)$ as a hadronic molecular state from the $\Sigma^* K$ interaction [40] and showed that the $N^*(2120)$ in the $K\Lambda(1520)$ photo-production was assigned as a naive three-quark state in the constituent quark model [39,41]. Besides, the structure near 2.1 GeV in the ϕ photo-production showed an enhancement in the same energy region as that of $N^*(2120)$ [42–44]. A recent analysis suggested that it has a mass of 2.08 ± 0.04 GeV and quantum number of $J^P = 3/2^-$ [45–48]. Reference [49] denoted this state as $N^*(2100)$ and investigated it from the ΣK^* interaction on the hadron level in a quasipotential Bethe-Saltpeter equation approach. So it is also interesting to study the Σ (or Σ^*) and K (or K^*) interactions on the quark level to investigate the possibility of interpreting these nucleon resonances as hadronic molecular states.

Generally, one of the important ways to generate and identify multiquark states is the hadron-hadron scattering process. The multiquark state will appear as a resonance state in the scattering process. Therefore, to provide the necessary information for experiment to search for the multiquark states, we should not only calculate the mass spectrum but also study the corresponding scattering process. By using the constituent quark models and the resonating group method (RGM) [50], we have obtained the d^* resonance in the NN scattering process, and we have found that the energy and the partial decay width to the D wave of NN are consistent with experiment data [51]. Extending to the pentaquark system, we investigated the $N\phi$ state in the different scattering channels: $N\eta'$, ΛK, and ΣK [14]. Both the resonance mass and decay width were obtained, which provided the necessary information for experimental searching at JLab. Therefore, it is interesting to extend such study to the molecular states composed of Σ (or Σ^*) and K (or K^*). In this work, we will investigate the scattering process of the corresponding open channels to search for any possible resonance states composed of Σ (or Σ^*) and K (or K^*).

It is a general consensus that quantum chromodynamics (QCD) is the fundamental theory of the strong interaction in the perturbative region. However, it is difficult to use QCD directly to study complicated systems in the low-energy region. The QCD-inspired models, incorporating the properties of low-energy QCD, color confinement and chiral symmetry breaking, are still powerful tools to obtain physical insights into many phenomena of the hadronic world. Among these phenomenological models, the quark delocalization color screening model (QDCSM), which was developed in the 1990s with the aim of explaining the similarities between nuclear (hadronic clusters of quarks) and molecular forces [52], has been quite successful in reproducing the energies of the baryon ground states, the properties of the deuteron, and the nucleon-nucleon (NN) and the hyperon-nucleon (YN) interactions [53]. In this model, quarks confined in one cluster are allowed to delocalize to a nearby cluster, and the delocalization parameter is determined by the dynamics of the interacting quark system, which allows the quark system to choose the most favorable configuration through its own dynamics in a larger Hilbert space. Besides, the confinement interaction between quarks in different cluster orbits is modified to include a color screening factor, which is a model description of the hidden color channel coupling effect [54]. Recently, this model has been used to study the hidden-charm pentaquarks [13]. We found that the interaction between Σ_c (or Σ_c^*) and D (or D^*) was strong enough to form some bound states, and $P_c(4380)$ can be interpreted as the molecular state $\Sigma_c^* D$ with quantum numbers $IJ^P = \frac{1}{2}\frac{3}{2}^-$.

In this work, we study the molecular states of Σ (or Σ^*) and K (or K^*), calculate both the mass and decay widths of these states, analyze the possibility of the P_c-like pentaquarks in a hidden strange sector, and interpret some nucleon resonances as hadronic molecular states. In the next section, the framework of the QDCSM is briefly introduced. Section III is devoted to the numerical results and discussions. The summary is shown in the last section.

II. THE QUARK DELOCALIZATION COLOR SCREENING MODEL (QDCSM)

The quark delocalization color screening model has been widely described in the literature [52,53], and we refer the reader to those works for details. Here, we just present the salient features of the model. The model Hamiltonian is

$$H = \sum_{i=1}^{5}\left(m_i + \frac{p_i^2}{2m_i}\right) - T_{CM} + \sum_{j>i=1}^{5}(V_{ij}^C + V_{ij}^G + V_{ij}^\chi), \tag{1}$$

$$V_{ij}^C = \begin{cases} -a_c \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c (r_{ij}^2 + v_0), & \text{if } i, j \text{ in the same baryon orbit} \\ -a_c \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \left(\frac{1 - e^{-\mu_{ij} \mathbf{r}_{ij}^2}}{\mu_{ij}} + v_0 \right), & \text{otherwise} \end{cases} \tag{2}$$

$$V_{ij}^G = \frac{1}{4} \alpha_s \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \left[\frac{1}{r_{ij}} - \frac{\pi}{2} \delta(\mathbf{r}_{ij}) \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}{3 m_i m_j} \right) - \frac{3}{4 m_i m_j r_{ij}^3} S_{ij} \right] \tag{3}$$

$$V_{ij}^{\chi} = V_\pi(\boldsymbol{r}_{ij}) \sum_{a=1}^{3} \lambda_i^a \cdot \lambda_j^a + V_K(\boldsymbol{r}_{ij}) \sum_{a=4}^{7} \lambda_i^a \cdot \lambda_j^a + V_\eta(\boldsymbol{r}_{ij}) [(\lambda_i^8 \cdot \lambda_j^8) \cos\theta_P - (\lambda_i^0 \cdot \lambda_j^0) \sin\theta_P] \tag{4}$$

$$V_\chi(\boldsymbol{r}_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_\chi^2}{12 m_i m_j} \frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2} m_\chi \left\{ (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} Y(\Lambda_\chi r_{ij}) \right] + \left[H(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} H(\Lambda_\chi r_{ij}) \right] S_{ij} \right\},$$
$$\chi = \pi, K, \eta, \tag{5}$$

$$S_{ij} = \left\{ 3 \frac{(\boldsymbol{\sigma}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{\sigma}_j \cdot \boldsymbol{r}_{ij})}{r_{ij}^2} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right\}, \tag{6}$$

$$H(x) = (1 + 3/x + 3/x^2) Y(x), \qquad Y(x) = e^{-x}/x. \tag{7}$$

Where S_{ij} is the quark tensor operator, $Y(x)$ and $H(x)$ are standard Yukawa functions; T_c is the kinetic energy of the center of mass; α_s is the quark-gluon coupling constant; g_{ch} is the coupling constant for the chiral field, which is determined from the $NN\pi$ coupling constant through

$$\frac{g_{ch}^2}{4\pi} = \left(\frac{3}{5} \right)^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N^2}. \tag{8}$$

The other symbols in the above expressions have their usual meanings.

In QDCSM, the color screening is associated with the color structures of the system under consideration. For the 3-quark baryon and quark-antiquark meson, the unscreened confinement is enough, especially for the low-lying states, and the two-body interaction works well. However, it is different in the multiquark system. The lattice QCD calculations show a stringlike structure [55,56]). The confinement is a genuine multibody interaction, and, in general, one does not expect it to be described by a sum of two-body interactions. To simplify the calculations, the two-body interaction form are still employed to evaluate the matrix elements of hamiltonian. The main physics introduced is the recognition that the confining interaction between two quarks resident in different nucleons might be different from that within one nucleon. So we model the confinement as follows: the interaction takes the normal, unscreened form (quadratic in r_{ij}) when the interacting quark pair always remains in the same cluster orbit, before and after interacting; otherwise the interaction takes the screening form. Although this has not been demonstrated to be correct, it is more sophisticated than the usual, simple two-body confining interaction, and it is expected that it does include some nonlocal, nonperturbative effects of QCD, which is missing in the three-quark baryons and quark-antiquark mesons. In addition, the screened confinement permits the development of quark delocalization in the QDCSM.

Generally, we use the parameters from our previous work on dibaryons [14,57]. However, the model parameters used in the dibaryon calculation can describe the ground baryons well, but cannot fit the masses of the ground mesons, especially the K meson, the obtained mass of which is much higher than the experimental value. This situation will lead to a consequence that some bound states cannot decay to the open channels, because of the much larger mass of K. To solve this problem, we adjust the

TABLE I. Model parameters: $m_\pi = 0.7\,\mathrm{fm}^{-1}$, $m_k = 2.51\,\mathrm{fm}^{-1}$, $m_\eta = 2.77\,\mathrm{fm}^{-1}$, $\Lambda_\pi = 4.2\,\mathrm{fm}^{-1}$, $\Lambda_K = \Lambda_\eta = 5.2\,\mathrm{fm}^{-1}$, $g_{ch}^2/(4\pi) = 0.54$, $\theta_p = -15^0$.

b (fm)	m_u (MeV)	m_s (MeV)	a_c (MeV $\cdot$ fm^{-2})	$V_0^{(qq)}$ (fm^2)	$V_0^{(q\bar{q})}$ (fm^2)
0.518	313	573	58.03	−1.2883	−0.2012
α_s^{uu}	α_s^{us}	α_s^{ss}	$\alpha_s^{u\bar{u}}$	$\alpha_s^{u\bar{s}}$	$\alpha_s^{s\bar{s}}$
0.5652	0.5239	0.4506	1.7930	1.7829	1.5114

TABLE II. The masses (in MeV) of the baryons and mesons obtained from QDCSM. Experimental values are taken from the Particle Data Group (PDG) [35].

	N	Δ	Λ	Σ	Σ^*	Ξ	Ξ^*	Ω
PDG	939	1232	1116	1193	1385	1318	1533	1672
QDCSM	939	1232	1124	1238	1360	1374	1496	1642

	η'	K	K^*	ϕ
PDG	958	495	892	1020
QDCSM	852	495	892	1020

quark-gluon coupling constant α_s of the $q\bar{q}$ pair and keep the other parameters unchanged. By doing this, the parameters can describe the nucleon-nucleon and hyperon-nucleon interaction well, and at the same time, it will lower the mass of K to the experimental value. The model parameters are fixed by fitting the spectrum of baryons and mesons we used in this work. The parameters of the Hamiltonian are given in Table I. Besides, a phenomenological color screening confinement potential is used here, and μ_{ij} is the color screening parameter, which is determined by fitting the deuteron properties, NN scattering phase shifts, and $N\Lambda$ and $N\Sigma$ scattering phase shifts, respectively, with $\mu_{uu} = 0.45$, $\mu_{us} = 0.19$, and $\mu_{ss} = 0.08$, satisfying the relation, $\mu_{us}^2 = \mu_{uu}\mu_{ss}$ [57]. The calculated masses of baryons and mesons in comparison with experimental values are shown in Table II.

The quark delocalization in QDCSM is realized by specifying the single particle orbital wave function of QDCSM as a linear combination of left and right Gaussians. More details can be seen in Eq. (A15) in the Appendix, in which the mixing parameter ϵ is not an adjusted one but determined variationally by the dynamics of the multiquark system itself. In this way, the multiquark system chooses its favorable configuration in the interacting process. This mechanism has been used to explain the cross-over transition between hadron phase and quark-gluon plasma phase [58].

III. THE RESULTS AND DISCUSSIONS

In this work, we perform a dynamical investigation of the molecular states composed of Σ (or Σ^*) and K (or K^*) in the QDCSM. Our purpose is to understand the interaction properties of the Σ (or Σ^*) and K (or K^*), and to see whether there exist any P_c-like pentaquarks in a hidden strange sector. Moreover, we also attempt to explore if there are any pentaquark states which can be used to explain some nucleon resonances. For the system with isospin $I = \frac{1}{2}$ and $J^P = \frac{1}{2}^-$, we investigate three molecular states ΣK, ΣK^*, and $\Sigma^* K^*$; for the system with isospin $I = \frac{1}{2}$ and $J^P = \frac{3}{2}^-$, we investigate three molecular states ΣK^*, $\Sigma^* K$, and $\Sigma^* K^*$.

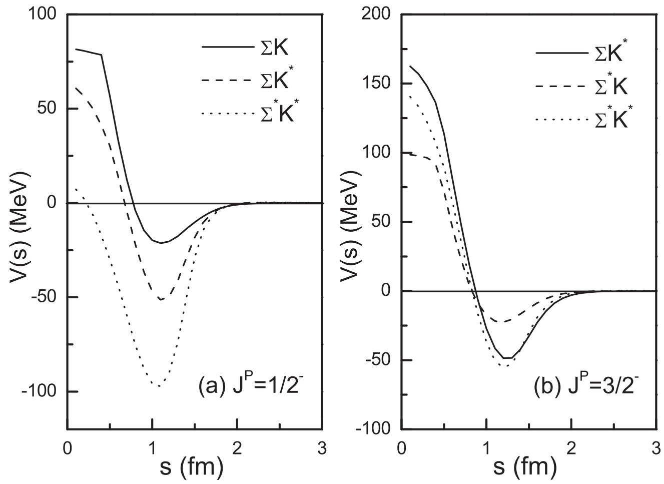

FIG. 1. The potentials of different channels for the $J^P = \frac{1}{2}\frac{1}{2}^-$ and $J^P = \frac{1}{2}\frac{3}{2}^-$ systems.

Since an attractive potential is necessary for forming bound state or resonance, the effective potentials between Σ (or Σ^*) and K (or K^*) are calculated and shown in Figs. 1. The effective potential between two colorless clusters is defined as, $V(s) = E(s) - E(\infty)$, where $E(s)$ is the diagonal matrix element of the Hamiltonian of the system in the generating coordinate. For the $IJ^P = \frac{1}{2}\frac{1}{2}^-$ system (Fig. 1(a)), one sees that the potentials are all attractive for the channels ΣK, ΣK^* and $\Sigma^* K^*$. The attraction between Σ^* and K^* is the largest one, followed by that of the ΣK^* channel, then the ΣK channel. This rule is very similar to the interactions between Σ_c (or Σ_c^*) and D (or D^*) [13]. For the $IJ^P = \frac{1}{2}\frac{3}{2}^-$ system (Fig. 1(b)), the potentials are all attractive for channels ΣK^*, $\Sigma^* K$ and $\Sigma^* K^*$. The attractions of both ΣK^* and $\Sigma^* K^*$ channels are larger than that of the $\Sigma^* K$ channel.

In order to see whether or not there is any bound state, a dynamic calculation is needed. The resonating group method (RGM) [50], a well established method for studying a bound-state problem or a scattering one, is used here. The details of RGM are shown in the Appendix.

For the single channel calculations, the strong attractive interaction between Σ (or Σ^*) and K (or K^*) leads to the total energy below the threshold of the two particles. All the binding energies (labeled as B) and the masses (labeled as M) of molecular pentaquarks are listed in Table III.

TABLE III. The binding energy and masses (in MeV) of the molecular pentaquarks.

$J^P = \frac{1}{2}^-$		$J^P = \frac{3}{2}^-$	
Channel	B/M	Channel	B/M
ΣK	$-18.8/1669.2$	ΣK^*	$-22.7/2062.3$
ΣK^*	$-7.2/2077.8$	$\Sigma^* K$	$-7.4/1872.6$
$\Sigma^* K^*$	$-21.9/2255.1$	$\Sigma^* K^*$	$-6.8/2270.2$

We need to mention that the mass of the bound state can be generally splitted into three terms: the baryon mass M_{baryon}, the meson mass M_{meson}, and the binding energy B. To minimize the theoretical deviations, the former two terms, M_{baryon} and M_{meson}, are shifted to the experimental values.

To confirm whether or not these bound states can survive as resonance states after coupling to the open channels, the study of the scattering process of the open channels is needed. Resonances are unstable particles usually observed as bell-shaped structures in scattering cross sections of their open channels. For a simple narrow resonance, its fundamental properties correspond to the visible cross-section features: mass is the peak position, and decay width is the half-width of the bell shape. To find the resonance mass and decay width of the bound states in Table III, we can calculate the cross section of the corresponding open channels. The details of the calculation method are shown in the Appendix.

In this work, we study the pentaquarks composed of $udds\bar{s}$, so the open channels composed of $uddu\bar{u}$ are not considered at the present stage. For the $IJ^P = \frac{1}{2}\frac{1}{2}^-$ system, the bound state ΣK can be coupled to one open channel: the S wave ΛK; the bound state ΣK^* can be coupled to eight open channels: the S wave $N\eta'$, $N\phi$, ΛK, ΛK^*, ΣK and the D wave $N\phi$, ΛK^*, $\Sigma^* K$; the bound state $\Sigma^* K^*$ can be coupled to ten open channels: the S wave $N\eta'$, $N\phi$, ΛK, ΛK^*, ΣK, ΣK^* and the D wave $N\phi$, ΛK^*, ΣK^*, $\Sigma^* K$. All these open channels are listed in the first column of Table IV, and the resonance states are listed in the first row of Table IV. We calculate the scattering phase shifts of all these open channels, and then the cross section by using Eq. (A23), finally we can obtain the resonance mass and decay width of the resonance states, which are show in Table IV. For the $IJ^P = \frac{1}{2}\frac{3}{2}^-$ system, we do the same calculation as that of the $IJ^P = \frac{1}{2}\frac{1}{2}^-$ system, and all resonance states and the corresponding open channels, as well as the resonance mass and decay width are shown in Table V. To save space, here we only show the cross section of all open channels for the state ΣK^* with $J^P = \frac{3}{2}^-$ (see Fig. 2). The resonance mass and decay width of this state are obtained from the cross section of those related open channels. There are several features which are discussed below.

First, the bound states in Table III are all resonance states due to the coupling of the corresponding open channels.

TABLE IV. The resonance mass and decay width (in MeV) of the molecular pentaquarks with $J^P = \frac{1}{2}^-$.

	ΣK		ΣK^*		$\Sigma^* K^*$	
S wave	M_r	Γ_i	M_r	Γ_i	M_r	Γ_i
$N\eta'$	⋯	⋯	2079.4	1.1	2246.8	20.0
$N\phi$	⋯	⋯	2080.0	3.6	2237.0	30.0
ΛK	1668.0	1.3	2083.4	1.0	2261.5	20.0
ΛK^*	⋯	⋯	2056.6	0.2	2219.0	58.0
ΣK	⋯	⋯	2071.6	4.6	2252.3	6.0
ΣK^*	⋯	⋯	⋯	⋯	2253.9	16.0
D wave						
$N\phi$	⋯	⋯	2076.3	0.3	2254.4	0.006
ΛK^*	⋯	⋯	2076.3	0.4	2253.6	0.6
ΣK^*	⋯	⋯	⋯	⋯	2254.0	0.06
$\Sigma^* K$	⋯	⋯	2076.8	0.01	2253.3	0.8

TABLE V. The resonance mass and decay width (in MeV) of the molecular pentaquarks with $J^P = \frac{3}{2}^-$.

	ΣK^*		$\Sigma^* K$		$\Sigma^* K^*$	
S wave	M_r	Γ_i	M_r	Γ_i	M_r	Γ_i
$N\phi$	2060.6	10.4	⋯	⋯	2270.5	0.03
ΛK^*	2046.1	15.0	⋯	⋯	2256.5	2.0
ΣK^*	⋯	⋯	⋯	⋯	2270.6	0.1
$\Sigma^* K$	2054.1	2.3	⋯	⋯	2263.6	3.7
D wave						
$N\eta'$	2061.4	0.001	1875.7	0.0004	2269.2	0.01
$N\phi$	2061.0	0.2	⋯	⋯	2269.3	0.01
ΛK	2060.6	0.9	1871.6	0.08	2269.2	0.02
ΛK^*	2059.1	0.3	⋯	⋯	2269.1	0.05
ΣK	2060.3	0.9	1871.6	0.05	2269.2	0.02
ΣK^*	⋯	⋯	⋯	⋯	2269.2	0.003

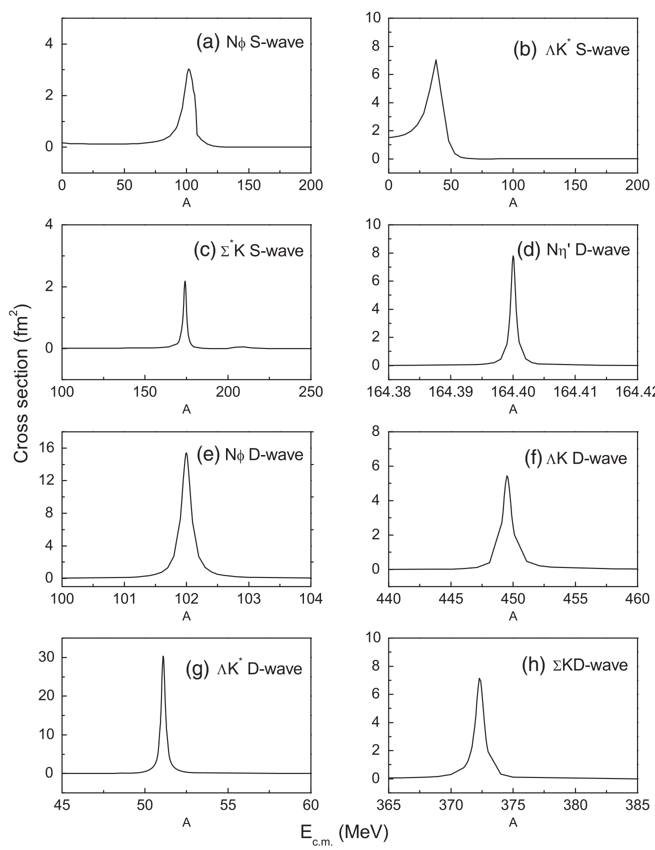

FIG. 2. The cross section of all open channels for the state ΣK^* with $J^P = \frac{3}{2}^-$.

Because only the hidden strange channels are considered here, the total decay width of the states given below is the lower limits. For the $IJ^P = \frac{1}{2}\frac{1}{2}^-$ system, the resonance mass of ΣK is 1668.0 MeV, and the decay width is very small which is only 1.3 MeV; the ΣK^* is also possible a narrow resonance state with the mass range of 2056.6–2083.4 MeV and the decay width is ∼10 MeV; the mass of the resonance $\Sigma^* K^*$ is between 2219.0–2261.5 MeV, while the decay width is much larger, which is about 150 MeV at least. For the $IJ^P = \frac{1}{2}\frac{3}{2}^-$ system, both the $\Sigma^* K$ and $\Sigma^* K^*$ are very narrow resonance states with the mass range of 1871.6–1875.7 MeV and 2256.5–2270.5 MeV, respectively. Besides, the resonance mass range of ΣK^* state is 2046.1–2061.4 MeV and the decay width is about 30 MeV.

Second, it is obvious that the decay width of decaying to D-wave channels is much smaller than that of decaying to the S-wave channels. This is reasonable. In our quark model calculation, the coupling between S-wave channels is through the central force, while the coupling between S- and D-wave channels is dominated by the tensor force, and the effect of the tensor force is much smaller than that of the central force. This conclusion is consistent with our previous calculation of the dibaryon systems [59,60]. Besides, we only consider the two-body decay channels in this work. The calculation of more decay channels will change the total decay width of the resonance states.

Third, our results in the hidden strange sector are similar to our previous study of the hidden charm molecular pentaquarks [13]. In Ref. [13], we found that three states with $J^P = \frac{1}{2}^-$: $\Sigma_c D$, $\Sigma_c D^*$, and $\Sigma_c^* D^*$, and the other three states with $J^P = \frac{3}{2}^-$: $\Sigma_c D^*$, $\Sigma_c^* D$, and $\Sigma_c^* D^*$ were all quasistable states. Analogously, in this work, we find that three states with $J^P = \frac{1}{2}^-$: ΣK, ΣK^*, and $\Sigma^* K^*$ and the other three states with $J^P = \frac{3}{2}^-$: ΣK^*, $\Sigma^* K$, and $\Sigma^* K^*$ are all resonance states. Besides, in Ref. [13], the molecular pentaquark $\Sigma_c^* D$ with quantum numbers $IJ^P = \frac{1}{2}\frac{3}{2}^-$ can be used to explain the LHCb $P_c(4380)$ state. So, here, the molecular pentaquark $\Sigma^* K$ with quantum numbers $IJ^P = \frac{1}{2}\frac{3}{2}^-$ can be seen as a strange partner of the LHCb $P_c(4380)$ state. This conclusion is consistent with the work on the hadron level [49].

Finally, to identify some resonances with the states observed experimentally is possible. On the hadron level, the previous work suggested that the nucleon resonance $N^*(1875)$ can be explained as the molecular state $\Sigma^* K$ with quantum numbers $IJ^P = \frac{1}{2}\frac{3}{2}^-$ [49]. In the present calculation, we find the quantum numbers and the mass of $\Sigma^* K(IJ^P = \frac{1}{2}\frac{3}{2}^-)$ is consistent with those of $N^*(1875)$ [35]. Moreover, Ref. [49] also found that the ΣK^* interaction produced a bound state with quantum numbers $IJ^P = \frac{1}{2}\frac{3}{2}^-$, which was related to the experimentally observed $N^*(2100)$ in the ϕ photo-production. Our results of the ΣK^* state in the quark level are also consistent with that of Ref. [49], so some nucleon resonances can be identified as baryon-meson molecular states.

IV. SUMMARY

In summary, we perform a dynamical investigation of the molecular states composed of Σ (or Σ^*) and K (or K^*) within the QDCSM. We calculate the effective potential, mass, and decay widths of these molecular states. Our results show the following: (1) The interactions between Σ (or Σ^*) and K (or K^*) are strong enough to form the bound states, which are ΣK, ΣK^*, and $\Sigma^* K^*$ with $IJ^P = \frac{1}{2}\frac{1}{2}^-$ and ΣK^*, $\Sigma^* K$, and $\Sigma^* K^*$ with $IJ^P = \frac{1}{2}\frac{3}{2}^-$. And all these states are transferred to the resonance states by coupling the open channels. (2) Our results in the hidden strange sector are similar to our previous study of the hidden charm molecular pentaquarks [13], and the molecular pentaquark $\Sigma^* K$ with quantum numbers $IJ^P = \frac{1}{2}\frac{3}{2}^-$ can be seen as a strange partner of the LHCb $P_c(4380)$ state. (3) The quantum numbers and masses of $\Sigma^* K$ are consistent with the nucleon resonances $N^*(1875)$ and $N^*(2100)$, to identify the molecular states as nucleon resonances is possible.

In this work, we only study the pentaquarks composed of $udds\bar{s}$, so the open channels composed of $uddu\bar{u}$ are not considered at the present stage. Besides, we only consider the two-body decay channels. The calculation of more decay channels will change the total decay width of the resonance states. We will do this work in the future.

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APPENDIX: RESONATING GROUP METHOD FOR BOUND-STATE AND SCATTERING PROBLEMS

We use the resonating group method (RGM) to carry out a dynamical calculation. For a bound-state problem, we write the wave function of the baryon-meson system as

$$\Psi_{5q} = \mathcal{A}\sum_L [[\hat{\phi}_A(\boldsymbol{\rho}_A, \boldsymbol{\lambda}_A)\hat{\phi}_B(\boldsymbol{\rho}_B)]^{[\sigma]IS} \otimes \chi_L(\boldsymbol{R})]^J, \quad \text{(A1)}$$

where $[\sigma] = [222]$ gives the total color symmetry and all other symbols have their usual meanings. The symbol $\mathcal{A}$ is the antisymmetrization operator defined as

$$\mathcal{A} = 1 - P_{14} - P_{24} - P_{34}. \quad \text{(A2)}$$

where 1, 2, and 3 stand for the quarks in the baryon cluster, and 4 stands for the quark in the meson cluster. $\hat{\phi}_A$ and $\hat{\phi}_B$ are the antisymmetrized internal cluster wave functions of the baryon A and meson B:

$$\hat{\phi}_A(\boldsymbol{\rho}_A, \boldsymbol{\lambda}_A) = \left(\frac{2}{3\pi b^2}\right)^{3/4}\left(\frac{1}{2\pi b^2}\right)^{3/4} e^{-\left(\frac{\lambda_A^2}{3b^2}+\frac{\rho_A^2}{4b^2}\right)}\eta_{I_A S_A}\chi_c(A), \tag{A3}$$

$$\hat{\phi}_B(\boldsymbol{\rho}_B) = \left(\frac{1}{2\pi b^2}\right)^{3/4} e^{-\frac{\rho_B^2}{4b^2}}\eta_{I_B S_B}\chi_c(B). \tag{A4}$$

where $\eta_{I_A S_A}$ and $\chi_c(A)$ are the internal flavor-spin and color wave functions of the baryon cluster A. The Jacobi coordinates are defined as follows:

$$\begin{aligned}
&\boldsymbol{\rho}_A = \boldsymbol{r}_1 - \boldsymbol{r}_2, \qquad \boldsymbol{\rho}_B = \boldsymbol{r}_4 - \boldsymbol{r}_5,\\
&\boldsymbol{\lambda}_A = \boldsymbol{r}_3 - \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2),\\
&\boldsymbol{R}_A = \frac{1}{3}(\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3), \qquad \boldsymbol{R}_B = \frac{1}{2}(\boldsymbol{r}_4 + \boldsymbol{r}_5),\\
&\boldsymbol{R} = \boldsymbol{R}_A - \boldsymbol{R}_B, \qquad \boldsymbol{R}_G = \frac{3}{5}\boldsymbol{R}_A + \frac{2}{5}\boldsymbol{R}_B.
\end{aligned} \tag{A5}$$

From the variational principle, after variation with respect to the relative motion wave function $\chi(\boldsymbol{R}) = \sum_L \chi_L(\boldsymbol{R})$, one obtains the RGM equation

$$\int H(\boldsymbol{R}'', \boldsymbol{R}')\chi(\boldsymbol{R}')d\boldsymbol{R}' = E\int N(\boldsymbol{R}'', \boldsymbol{R}')\chi(\boldsymbol{R}')d\boldsymbol{R}', \tag{A6}$$

where Hamiltonian kernel $H(\boldsymbol{R}'', \boldsymbol{R}')$ and normalization kernel $N(\boldsymbol{R}'', \boldsymbol{R}')$ can, respectively, be calculated by

$$\left\{\begin{matrix} H(\boldsymbol{R}'', R') \\ N(\boldsymbol{R}'', R') \end{matrix}\right\} = \langle \mathcal{A}[\hat{\phi}_A(\boldsymbol{\rho}_A, \boldsymbol{\lambda}_A)\hat{\phi}_B(\boldsymbol{\rho}_B)\delta(\boldsymbol{R} - \boldsymbol{R}'')] \times \left|\left\{\begin{matrix} H \\ 1 \end{matrix}\right\}\right| \mathcal{A}[\hat{\phi}_A(\boldsymbol{\rho}_A, \boldsymbol{\lambda}_A)\hat{\phi}_B(\boldsymbol{\rho}_B)\delta(\boldsymbol{R} - \boldsymbol{R}')]\rangle. \tag{A7}$$

For a bound-state problem, the energies and the wave functions $\chi(\boldsymbol{R})$ are obtained by solving the RGM equation. In practice, it is not convenient to work with the RGM expressions. We introduce generator coordinates $\boldsymbol{S}_i$ to expand the Lth relative motion wave function $\chi_L(\boldsymbol{R})$:

$$\begin{aligned}
\chi_L(\boldsymbol{R}) &= \frac{1}{\sqrt{4\pi}}\left(\frac{6}{5\pi b^2}\right)^{3/4} \times \sum_{i=1}^{n} C_i \int \exp\left[-\frac{3}{5b^2}(\boldsymbol{R} - \boldsymbol{S}_i)^2\right] Y^L(\hat{\boldsymbol{S}}_i)d\hat{\boldsymbol{S}}_i\\
&= \sum_{i=1}^{n} C_i \frac{u_L(R, S_i)}{R} Y^L(\hat{\boldsymbol{R}}),
\end{aligned} \tag{A8}$$

with

$$u_L(R, S_i) = \sqrt{4\pi}\left(\frac{6}{5\pi b^2}\right)^{3/4} R\exp\left[-\frac{3}{5b^2}(R^2 - S_i^2)\right] \times i^L j_L\left(-i\frac{6}{5b^2}RS_i\right). \tag{A9}$$

where C_i is expansion coefficients, n is the number of the Gaussian bases, which is determined by the stability of the results, and j_L is the Lth spherical Bessel function. Then the relative motion wave function $\chi(\boldsymbol{R})$ is

$$\chi(\boldsymbol{R}) = \frac{1}{\sqrt{4\pi}}\sum_L\left(\frac{6}{5\pi b^2}\right)^{3/4} \times \sum_{i=1}^{n} C_{i,L}\int e^{-\frac{3}{5b^2}(\boldsymbol{R}-\boldsymbol{S}_i)^2} Y^L(\hat{\boldsymbol{S}}_i)d\Omega_{\boldsymbol{S}_i}. \tag{A10}$$

After the inclusion of the center of mass motion,

$$\Phi_G(\boldsymbol{R}_G) = \left(\frac{5}{\pi b^2}\right)^{3/4} e^{-\frac{5}{2b^2}\boldsymbol{R}_G^2}, \tag{A11}$$

the total wave function Eq. (A1) can be rewritten as

$$\Psi_{5q} = \mathcal{A}\sum_{i,L} C_{i,L}\int \frac{d\Omega_{\boldsymbol{S}_i}}{\sqrt{4\pi}}\prod_{\alpha=1}^{3}\phi_\alpha(\boldsymbol{S}_i)\prod_{\beta=4}^{5}\phi_\beta(-\boldsymbol{S}_i) \times [[\eta_{I_A S_A}\eta_{I_B S_B}]^{IS} Y^L(\hat{\boldsymbol{S}}_i)]^J [\chi_c(A)\chi_c(B)]^{[\sigma]}. \tag{A12}$$

where $\phi_\alpha(\boldsymbol{S}_i)$ and $\phi_\beta(-\boldsymbol{S}_i)$ are the single-particle orbital wave functions with different reference centers:

$$\begin{aligned}
\phi_\alpha(\boldsymbol{S}_i) &= \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2}(\boldsymbol{r}_\alpha - \frac{2}{5}\boldsymbol{S}_i)^2},\\
\phi_\beta(-\boldsymbol{S}_i) &= \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2}(\boldsymbol{r}_\beta + \frac{3}{5}\boldsymbol{S}_i)^2}.
\end{aligned} \tag{A13}$$

With the reformulated Eq. (A12), the RGM equation (A6) becomes an algebraic eigenvalue equation,

$$\sum_{j,L} C_{j,L} H_{i,j}^{L,L'} = E\sum_j C_{j,L'} N_{i,j}^{L'}, \tag{A14}$$

where $N_{i,j}^{L'}$ and $H_{i,j}^{L,L'}$ are the wave function (A12) overlaps and Hamiltonian matrix elements (without the summation over L'), respectively. By solving the generalized eigenproblem, we obtain the energies of the five-quark systems and corresponding wave functions. In our calculation, the distribution of Gaussians is fixed by the stability of the results. The results are stable when the largest distance between the baryon-meson clusters is around 6 fm. To keep the dimensions of matrix manageably small, the baryon-meson separation is taken to be less than 6 fm.

In the QDCSM, the single-particle orbital wave functions are delocalized. To implement this here, we modify Eqs. (A13) as follows:

$$\phi_\alpha(\boldsymbol{S}_i) \to \psi_\alpha(\boldsymbol{S}_i,\epsilon) = (\phi_\alpha(\boldsymbol{S}_i) + \epsilon\phi_\alpha(-\boldsymbol{S}_i))/N(\epsilon),$$
$$\phi_\beta(\boldsymbol{S}_i) \to \psi_\beta(\boldsymbol{S}_i,\epsilon) = (\phi_\beta(\boldsymbol{S}_i) + \epsilon\phi_\beta(-\boldsymbol{S}_i))/N(\epsilon),$$
$$N(\epsilon) = \sqrt{1+\epsilon^2+2\epsilon e^{-S_i^2/4b^2}}. \tag{A15}$$

For a scattering problem, the relative wave function is expanded as

$$\chi_L(\boldsymbol{R}) = \sum_{i=1}^{n} C_i \frac{\tilde{u}_L(R,S_i)}{R} Y^L(\hat{\boldsymbol{R}}). \tag{A16}$$

with

$$\tilde{u}_L(R,S_i) = \begin{cases} \alpha_i u_L(R,S_i), & R \le R_C \\ [h_L^-(k,R) - s_i h_L^+(k,R)]R, & R \ge R_C \end{cases} \tag{A17}$$

where u_L is from Eq. (A9), $h_L^\pm$ is the Lth spherical Hankel functions, k is the momentum of relative motion with $k=\sqrt{2\mu E_{cm}}$, μ is the reduced mass of two hadrons (A and B) of the open channel, E_{cm} is the incident energy, and R_C is a cutoff radius beyond which all the strong interaction can be disregarded. Besides, α_i and s_i are complex parameters which are determined by the smoothness condition at $R=R_C$ and C_i satisfy $\sum_{i=1}^n C_i = 1$. After performing variational procedure, a Lth partial-wave equation for the scattering problem can be deduced as

$$\sum_{j=1}^{n} \mathcal{L}_{ij}^L C_j = \mathcal{M}_i^L (i=0,1,\ldots,n-1), \tag{A18}$$

with

$$\mathcal{L}_{ij}^L = \mathcal{K}_{ij}^L - \mathcal{K}_{i0}^L - \mathcal{K}_{0j}^L + \mathcal{K}_{00}^L, \tag{A19}$$

$$\mathcal{M}_i^L = \mathcal{K}_{00}^L - \mathcal{K}_{i0}^L, \tag{A20}$$

and

$$\mathcal{K}_{ij}^L = \left\langle \hat{\phi}_A\hat{\phi}_B \frac{\tilde{u}_L(R',S_i)}{R'} Y^L(\hat{\boldsymbol{R}}')|H-E| \right.$$
$$\left. \times \mathcal{A}\left[\hat{\phi}_A\hat{\phi}_B \frac{\tilde{u}_L(R,S_j)}{R} Y^L(\hat{\boldsymbol{R}})\right]\right\rangle. \tag{A21}$$

By solving Eq. (A18), we can obtain the expansion coefficients C_i. Then the S–matrix element S_L and the phase shifts δ_L are given by

$$S_L \equiv e^{2i\delta_L} = \sum_{i=1}^{n} C_i s_i. \tag{A22}$$

Finally, the cross section can be obtained from the scattering phase shifts by the formula:

$$\sigma_L = \frac{4\pi}{k^2}\cdot(2L+1)\cdot \sin^2\delta_L. \tag{A23}$$

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