

Potential observation of the $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$ transitions at Belle II

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We perform the investigation of two-body hidden-bottom transitions of the $\Upsilon(6S)$, which include $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$ ($J = 1, 2, 3$) decays. For estimating the branching ratios of these processes, we consider contributions from the triangle hadronic loops composed of S -wave $B_{(s)}$ and $B_{(s)}^*$ mesons, which are a bridge to connect the $\Upsilon(6S)$ and final states. Our results show that both of the branching ratios of these decays can reach 10^{-3} . Because of such considerable potential to observe these two-body hidden-bottom transitions of the $\Upsilon(6S)$, we suggest the forthcoming Belle II experiment to explore them.

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I. INTRODUCTION

As an updated accelerator with luminosity $8 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$, SuperKEKB is currently being constructed. The forthcoming Belle II experiment will accumulate 50 times more data than the previous Belle experiment. It is a good time to explore the potential physical issues close to the Belle II.

Since 2007, experimental studies by the Belle Collaboration have focused on the hadronic transitions of the $\Upsilon(10860)$. The Belle measurement shows that the observed hidden-bottom transitions of $\Upsilon(10860)$ have large branching ratios. For example, all the decay widths of the $\Upsilon(10860) \rightarrow \Upsilon(mS)\pi^+\pi^-$ ($m \leq 3$) are around 10^{-1} MeV and that of the $\Upsilon(10860) \rightarrow \Upsilon(1S)\pi^+\pi^-$ is at least 2 orders of magnitude larger than those of $\Upsilon(nS) \rightarrow \Upsilon(1S)\pi^+\pi^-$ ($n \leq 4$) [1]. In addition, the experimental branching ratios of $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ ($J = 0, 1, 2$) transitions can reach up to 10^{-3} [2]. Exploring hidden-bottom dipion decays of the $\Upsilon(10860)$, Belle also discovered two charged bottomonium-like structures $Z_b(10610)$ and $Z_b(10650)$ [3]. As indicated in a serial of theoretical studies [4–10], the hadronic loop mechanism, which is an equivalent description of coupled-channel effects, may play a crucial role to understand

these novel phenomena since the $\Upsilon(10860)$ lies above the $B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}$ thresholds [11].

Very recently the Belle Collaboration observed the $e^+e^- \rightarrow \Upsilon_J(1D)\eta$ process and obtained the branching ratio $B(\Upsilon(5S) \rightarrow \Upsilon_J(1D)\eta) = (4.82 \pm 0.92 \pm 0.67) \times 10^{-3}$ [12], which confirms the predication given in Ref. [13], where the hadronic loop effect was considered in the calculation. This fact again shows that the hadronic loop mechanism has important contributions to the hadronic decays of the $\Upsilon(5S)$.

In the Particle Data Group [11], there is the $\Upsilon(11020)$ above the $\Upsilon(10860)$. Usually, the $\Upsilon(11020)$ is treated as the $n^{2S+1}L_J = 6^3S_1$ state. Thus, in the following discussions, the $\Upsilon(11020)$ is abbreviated as the $\Upsilon(6S)$ for convenience. Here, we want to emphasize that the $\Upsilon(6S)$ has a situation similar to that of the $\Upsilon(10860)$ since the $\Upsilon(6S)$ is also above the $B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}$ thresholds. This fact gives us a reason to believe that the coupled-channel effect cannot be ignored when carrying out the studies around the $\Upsilon(6S)$.

When checking the experimental information of the $\Upsilon(6S)$, only the resonance parameters of the $\Upsilon(6S)$ and partial width of $\Upsilon(6S) \rightarrow e^+e^-$ are listed, which suggests that experimental study of $\Upsilon(6S)$ is necessary, especially with the running of Belle II. Considering the present status of the $\Upsilon(6S)$, in this work we propose that we first investigate two-body hidden-bottom transitions of $\Upsilon(6S)$, i.e., the $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$ ($J = 1, 2, 3$), which is also motivated by the recent observation of $\Upsilon(5S) \rightarrow \Upsilon_J(1D)\eta$ by Belle [12].

In this work, using the hadronic loop mechanism, we calculate the branching ratios of the discussed $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$ decays, by which the potential observation of

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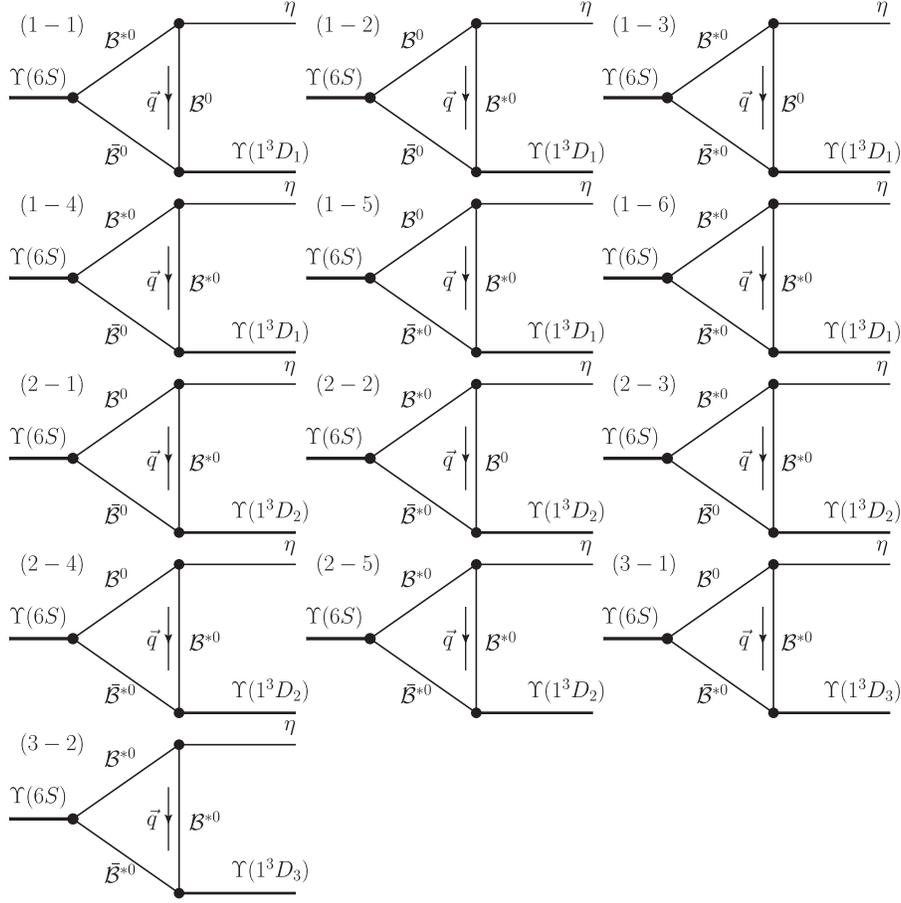


FIG. 1. Schematic diagrams depicting the $Y(6S) \rightarrow Y(1^3D_J)\eta$ transitions via the hadronic loop mechanism.

these two-body hidden-bottom transitions at Belle II can be suggested. We want to emphasize that the present study must become an important part of the physics around the $Y(6S)$, and further push experimental exploration of these decays.

This paper is organized as follows. After the Introduction, we present the detailed deductions of $Y(6S) \rightarrow Y(1^3D_J)\eta$ via the hadronic loop mechanism in Sec. II. Then, various parameters are determined in Sec. III. In Sec. IV, numerical results are presented. Finally, the paper will end with a short summary.

II. THE $Y(6S) \rightarrow Y(1^3D_J)\eta$ TRANSITIONS VIA THE HADRONIC LOOP MECHANISM

In this work, we adopt the hadronic loop mechanism to study the $Y(6S) \rightarrow Y(1^3D_J)\eta$ transitions. The hadronic loop mechanism has been widely applied to investigate hidden-bottom/hidden-charm decays of the bottomonium/charmonium [13–16]. Under the hadronic loop mechanism, the $Y(6S) \rightarrow Y(1^3D_J)\eta$ decays may occur via the intermediate triangle loops composed of S -wave bottom mesons, where all the diagrams depicting this decay process with triangle loops are listed in Fig. 1.

To calculate the diagrams shown in Fig. 1, we adopt the effective Lagrangian approach. Thus, we first need to introduce the relevant effective Lagrangians describing the involved interactions at the hadron level. Considering the constraints from various symmetries, the involved effective Lagrangians can be constructed. For example, under the heavy quark spin symmetry in the heavy quark limit, interactions between an S -wave (D -wave) bottomonium and a $B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}$ pair can be written as [13,17]

$$\mathcal{L}_S = ig_1 \text{Tr} \left[S^{(Q\bar{Q})} \bar{H}^{(Q\bar{q})} \gamma^\mu \overleftrightarrow{\partial}_\mu \bar{H}^{(Qq)} \right] + \text{H.c.}, \quad (1)$$

$$\mathcal{L}_D = ig_2 \text{Tr} \left[D_{\mu\lambda}^{(Q\bar{Q})} \bar{H}^{(Q\bar{q})} \gamma^\lambda \overleftrightarrow{\partial}^\mu \bar{H}^{(Qq)} \right] + \text{H.c.}, \quad (2)$$

where $S^{(Q\bar{Q})}$ and $D^{(Q\bar{Q})}$ denote the S -wave and D -wave multiplets of the bottomonium, respectively [17], which have the concrete expressions

$$S^{(Q\bar{Q})} = \frac{1 + \not{p}}{2} [\gamma^\mu \gamma_\mu - \eta_b \gamma_5] \frac{1 - \not{p}}{2}, \quad (3)$$

$$D^{(Q\bar{Q})\mu\lambda} = \frac{1 + \not{v}}{2} \left[\Upsilon_3^{\mu\lambda\alpha} \gamma_\alpha + \frac{1}{\sqrt{6}} (\epsilon^{\mu\alpha\beta\rho} v_\alpha \gamma_\beta \Upsilon_{2\rho}^\lambda + \epsilon^{\lambda\alpha\beta\rho} v_\alpha \gamma_\beta \times \Upsilon_{2\rho}^\mu) + \frac{\sqrt{15}}{10} [(\gamma^\mu - v^\mu) \Upsilon_1^\lambda + (\gamma^\lambda - v^\lambda) \Upsilon_1^\mu] \right. \\ \left. - \frac{1}{\sqrt{15}} (g^{\mu\lambda} - v^\mu v^\lambda) \gamma_\alpha \Upsilon_1^\alpha + \eta_{b2}^{\mu\lambda} \gamma_5 \right] \frac{1 - \not{v}}{2}, \quad (4)$$

where v^μ is the 4-velocity. Υ^μ and η_b correspond to S -wave bottomonia with $J^{PC} = 1^{--}$ and 0^{-+} , respectively. Υ_3 , Υ_2 , Υ_1 , and η_{b2} denote to D -wave states in the bottomonium family, which have $J^{PC} = 3^{--}$, 2^{--} , 1^{--} , and 2^{-+} , respectively. For a heavy meson emitting a light Nambu-Goldstone boson, the effective Lagrangian, which is constrained by the heavy quark symmetry and the chiral symmetry, has the form [17–22]

$$\mathcal{L}_P = ig_\pi \text{Tr}[H_b^{(Q\bar{q})} \gamma_\mu \gamma_5 \mathcal{A}_\mu^b \bar{H}_a^{(Q\bar{q})}], \quad (5)$$

where $\mathcal{A}_\mu = 1/2(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$ with $\xi = e^{i\mathcal{M}_P/f_\pi}$, and the pseudoscalar octet \mathcal{M}_P reads as

$$\mathcal{M}_P = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix}. \quad (6)$$

In Eqs. (1), (2), and (5), $H^{(Q\bar{q})}$ represents the bottom meson spin doublet (\mathcal{B} , \mathcal{B}^*) [17,23–25], i.e.,

$$H^{(Q\bar{q})} = \frac{1 + \not{v}}{2} [\mathcal{B}_\mu^* \gamma^\mu - \mathcal{B} \gamma^5]. \quad (7)$$

$H^{(\bar{Q}q)}$ corresponds to antimeson counterpart of $H^{(Q\bar{q})}$, which can be obtained by performing the charge conjugation transformation.

Further expanding the Lagrangians shown in Eqs. (1) and (5), we get the explicit forms of interaction Lagrangians, i.e.,

$$\mathcal{L}_{\Upsilon\mathcal{B}^{(*)}\mathcal{B}^{(*)}} = -ig_{\Upsilon BB} \Upsilon_\mu (\partial^\mu \mathcal{B} \mathcal{B}^\dagger - \mathcal{B} \partial^\mu \mathcal{B}^\dagger) + g_{\Upsilon B^* B} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \Upsilon_\nu (\mathcal{B}_\alpha^* \overleftrightarrow{\partial}_\beta \mathcal{B}^\dagger - \mathcal{B} \overleftrightarrow{\partial}_\beta \mathcal{B}_\alpha^{*\dagger}) \\ + ig_{\Upsilon B^* B^*} \Upsilon^\mu (\mathcal{B}_\nu^* \partial^\mu \mathcal{B}_\mu^{*\dagger} - \partial^\nu \mathcal{B}_\mu^* \mathcal{B}_\nu^{*\dagger} - \mathcal{B}_\nu^* \overleftrightarrow{\partial}_\mu \mathcal{B}^{*\nu\dagger}), \quad (8)$$

$$\mathcal{L}_{\Upsilon_J \mathcal{B}^{(*)} \mathcal{B}^{(*)}} = g_{\Upsilon_1 B B} \Upsilon_1^\mu (\mathcal{B}^\dagger \partial_\mu \mathcal{B} - \mathcal{B} \partial_\mu \mathcal{B}^\dagger) + ig_{\Upsilon_1 B^* B^*} \epsilon^{\mu\nu\alpha\beta} [\mathcal{B}^\dagger \overleftrightarrow{\partial}_\mu \mathcal{B}_\beta^* - \mathcal{B}_\beta^{*\dagger} \overleftrightarrow{\partial}_\mu \mathcal{B}] \partial_\nu \Upsilon_{1\alpha} \\ + g_{\Upsilon_1 B^* B^*} [-4(\Upsilon_1^\mu \mathcal{B}^{*\nu} \partial_\mu \mathcal{B}_\nu^{*\dagger} - \Upsilon_1^\mu \mathcal{B}_\nu^{*\dagger} \partial_\mu \mathcal{B}^{*\nu}) + \Upsilon_1^\mu \mathcal{B}^{*\nu} \partial_\nu \mathcal{B}_\mu^{*\dagger} - \Upsilon_1^\mu \mathcal{B}^{*\nu\dagger} \partial_\nu \mathcal{B}_\mu^*] \\ + ig_{\Upsilon_2 B B} \Upsilon_2^{\mu\nu} (\mathcal{B}^\dagger \overleftrightarrow{\partial}_\nu \mathcal{B}_\mu^* - \mathcal{B}_\mu^{*\dagger} \overleftrightarrow{\partial}_\nu \mathcal{B}) + g_{\Upsilon_2 B^* B^*} \epsilon_{\alpha\beta\mu\nu} [\mathcal{B}^{*\nu\dagger} \overleftrightarrow{\partial}^\beta \mathcal{B}_\lambda^* + \mathcal{B}^{*\nu} \overleftrightarrow{\partial}^\beta \mathcal{B}_\lambda^{*\dagger}] \partial^\mu \Upsilon_{2\alpha}^\lambda \\ + g_{\Upsilon_3 B^* B^*} \Upsilon_3^{\mu\nu\alpha} [\mathcal{B}_\alpha^{*\dagger} \overleftrightarrow{\partial}_\mu \mathcal{B}_\nu^* + \mathcal{B}_\nu^{*\dagger} \overleftrightarrow{\partial}_\mu \mathcal{B}_\alpha^*], \quad (9)$$

$$\mathcal{L}_{\mathcal{B}^{(*)} \mathcal{B}^{(*)} \eta_8} = ig_{BB^* \eta_8} (\mathcal{B}^\dagger \partial_\mu \eta_8 \mathcal{B}^{*\mu} - \mathcal{B}^{*\dagger \mu} \partial_\mu \eta_8 \mathcal{B}) - g_{B^* B^* \eta_8} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \mathcal{B}_\nu^{*\dagger} \partial_\alpha \mathcal{B}_\beta^* \eta_8, \quad (10)$$

where $\mathcal{B}^{(*)\dagger}$ and $\mathcal{B}^{(*)}$ are defined as $\mathcal{B}^{(*)\dagger} = (B^{(*)+}, B^{(*)0}, B_s^{(*)0})$ and $\mathcal{B}^{(*)} = (B^{(*)-}, \bar{B}^{(*)0}, \bar{B}_s^{(*)0})^T$, respectively.

Using the Lagrangians above, we can write out the amplitudes of the processes $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$ with $J = 1, 2, 3$, which are depicted in Fig. 1. As for the $\Upsilon(6S) \rightarrow \Upsilon(1^3D_1)\eta$ transition, with $\tilde{g}^{\mu\nu}(p, m_p) \equiv -g^{\mu\nu} + \frac{p^\mu p^\nu}{m_p^2}$, the amplitudes are

$$\mathcal{M}_{(1-1)} = \int \frac{d^4 q}{(2\pi)^4} [g_{\Upsilon B^* \bar{B}} \epsilon^{\mu\nu\alpha\beta} p_{1\mu} \epsilon_{\Upsilon\nu}(k_{1\beta} - k_{2\beta})] [g_{B^* \bar{B} \eta} p_{2\lambda}] [ig_{\bar{B} B \Upsilon_1} \epsilon_{\Upsilon_1 \zeta}^\dagger (k_2^\zeta - q^\zeta)] \frac{\tilde{g}_\alpha^\lambda(k_1, m_{B^*})}{k_1^2 - m_{B^*}^2} \frac{1}{k_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2(q^2), \quad (11)$$

$$\mathcal{M}_{(1-2)} = \int \frac{d^4 q}{(2\pi)^4} [g_{rB\bar{B}} \epsilon_{r\mu} (k_2^\mu - k_1^\mu)] [g_{B\bar{B}^* \eta} p_{2\lambda}] [i g_{\bar{B}B^* \gamma_1} \epsilon^{\zeta\eta\kappa\xi} p_{3\eta} \epsilon_{\gamma_1 \kappa}^* (k_{2\zeta} - q_\zeta)] \frac{1}{k_1^2 - m_B^2} \frac{1}{k_2^2 - m_B^2} \frac{\tilde{g}_\xi^\lambda(q, m_{B^*})}{q^2 - m_B^2} \mathcal{F}^2(q^2), \quad (12)$$

$$\begin{aligned} \mathcal{M}_{(1-3)} &= \int \frac{d^4 q}{(2\pi)^4} [g_{rB^* \bar{B}^*} \epsilon_r^\mu (g_{\mu\alpha} k_{2\beta} - g_{\mu\beta} k_{1\alpha} + g_{\alpha\beta} (k_{1\mu} - k_{2\mu}))] [g_{B^* \bar{B} \eta} p_{2\lambda}] [i g_{\bar{B}^* B \gamma_1} \epsilon^{\zeta\eta\kappa\xi} p_{3\eta} \epsilon_{\gamma_1 \kappa}^* (k_{2\zeta} - q_\zeta)] \\ &\times \frac{\tilde{g}^{\beta\lambda}(k_1, m_{B^*})}{k_1^2 - m_{B^*}^2} \frac{\tilde{g}_\xi^\alpha(k_2, m_{B^*})}{k_2^2 - m_{B^*}^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2(q^2), \end{aligned} \quad (13)$$

$$\begin{aligned} \mathcal{M}_{(1-4)} &= \int \frac{d^4 q}{(2\pi)^4} [g_{rB^* \bar{B}^*} \epsilon^{\mu\nu\alpha\beta} p_{1\mu} \epsilon_{r\nu} (k_{2\beta} - k_{1\beta})] [g_{B^* \bar{B}^* \eta} \epsilon^{\lambda\rho\delta\sigma} k_{1\lambda} q_\delta] [i g_{\bar{B}B^* \gamma_1} \epsilon^{\zeta\eta\kappa\xi} p_{3\eta} \\ &\times \epsilon_{\gamma_1 \kappa}^* (k_{2\zeta} - q_\zeta)] \frac{\tilde{g}_{\alpha\rho}(k_1, m_{B^*})}{k_1^2 - m_{B^*}^2} \frac{1}{k_2^2 - m_B^2} \frac{\tilde{g}_{\sigma\xi}(q, m_{B^*})}{q^2 - m_B^2} \mathcal{F}^2(q^2), \end{aligned} \quad (14)$$

$$\begin{aligned} \mathcal{M}_{(1-5)} &= \int \frac{d^4 q}{(2\pi)^4} [g_{rB\bar{B}^*} \epsilon^{\mu\nu\alpha\beta} p_{1\mu} \epsilon_{r\nu} (k_{1\beta} - k_{2\beta})] [g_{B\bar{B}^* \eta} p_{2\lambda}] [i g_{\bar{B}^* B^* \gamma_1} \epsilon_{\gamma_1 \zeta}^* (4g_{\kappa\xi} (k_2^\zeta - q^\zeta) + g_\xi^\zeta q_\kappa - g_\kappa^\zeta k_{2\xi})] \\ &\times \frac{1}{k_1^2 - m_B^2} \frac{\tilde{g}_\alpha^\kappa(k_2, m_{B^*})}{k_2^2 - m_{B^*}^2} \frac{\tilde{g}^{\lambda\xi}(q, m_{B^*})}{q^2 - m_{B^*}^2} \mathcal{F}^2(q^2), \end{aligned} \quad (15)$$

$$\begin{aligned} \mathcal{M}_{(1-6)} &= \int \frac{d^4 q}{(2\pi)^4} [g_{rB^* \bar{B}^*} \epsilon_r^\mu (g_{\mu\alpha} k_{2\beta} - g_{\mu\beta} k_{1\alpha} + g_{\alpha\beta} (k_{1\mu} - k_{2\mu}))] [g_{B^* \bar{B}^* \eta} \epsilon^{\lambda\rho\delta\sigma} k_{1\lambda} q_\delta] \\ &\times [i g_{\bar{B}^* B^* \gamma_1} \epsilon_{\gamma_1 \zeta}^* (4g_{\kappa\xi} (k_{2\zeta} - q_\zeta) + g_{\zeta\xi} q_\kappa - g_{\zeta\kappa} k_{2\xi})] \frac{\tilde{g}_\rho^\beta(k_1, m_{B^*})}{k_1^2 - m_{B^*}^2} \frac{\tilde{g}^{\alpha\kappa}(k_2, m_{B^*})}{k_2^2 - m_{B^*}^2} \times \frac{\tilde{g}_\sigma^\xi(q, m_{B^*})}{q^2 - m_{B^*}^2} \mathcal{F}^2(q^2). \end{aligned} \quad (16)$$

As for the $\Upsilon(6S) \rightarrow \Upsilon(1^3D_2)\eta$ transition, the amplitudes read as

$$\mathcal{M}_{(2-1)} = \int \frac{d^4 q}{(2\pi)^4} [g_{rB\bar{B}} \epsilon_{r\mu} (k_2^\mu - k_1^\mu)] [g_{B\bar{B}^* \eta} p_{2\lambda}] [g_{\bar{B}B^* \gamma_2} \epsilon_{\gamma_2 \zeta \eta}^* (k_2^\eta - q^\eta)] \frac{1}{k_1^2 - m_B^2} \frac{1}{k_2^2 - m_B^2} \frac{\tilde{g}^{\zeta\lambda}(q, m_{B^*})}{q^2 - m_B^2} \mathcal{F}^2(q^2), \quad (17)$$

$$\begin{aligned} \mathcal{M}_{(2-2)} &= \int \frac{d^4 q}{(2\pi)^4} [g_{rB^* \bar{B}^*} \epsilon_r^\mu (g_{\mu\alpha} k_{2\beta} - g_{\mu\beta} k_{1\alpha} + g_{\alpha\beta} (k_{1\mu} - k_{2\mu}))] [g_{B^* \bar{B} \eta} p_{2\lambda}] [g_{\bar{B}^* B \gamma_2} \epsilon_{\gamma_2 \zeta \eta}^* (k_2^\eta - q^\eta)] \frac{\tilde{g}^{\beta\lambda}(k_1, m_{B^*})}{k_1^2 - m_{B^*}^2} \\ &\times \frac{\tilde{g}^{\zeta\alpha}(k_2, m_{B^*})}{k_2^2 - m_{B^*}^2} \frac{1}{q^2 - m_B^2} \mathcal{F}^2(q^2), \end{aligned} \quad (18)$$

$$\begin{aligned} \mathcal{M}_{(2-3)} &= \int \frac{d^4 q}{(2\pi)^4} [g_{rB^* \bar{B}^*} \epsilon^{\mu\nu\alpha\beta} p_{1\mu} \epsilon_{r\nu} (k_{1\beta} - k_{2\beta})] [g_{B^* \bar{B}^* \eta} \epsilon^{\lambda\rho\delta\sigma} k_{1\lambda} q_\delta] [g_{\bar{B}B^* \gamma_2} \epsilon_{\gamma_2 \zeta \eta}^* (q^\eta - k_2^\eta)] \\ &\times \frac{\tilde{g}_{\alpha\rho}(k_1, m_{B^*})}{k_1^2 - m_{B^*}^2} \frac{1}{k_2^2 - m_B^2} \frac{\tilde{g}_\sigma^\zeta(k_1, m_{B^*})}{q^2 - m_{B^*}^2} \mathcal{F}^2(q^2), \end{aligned} \quad (19)$$

$$\begin{aligned} \mathcal{M}_{(2-4)} &= \int \frac{d^4 q}{(2\pi)^4} [g_{rB\bar{B}^*} \epsilon^{\mu\nu\alpha\beta} p_{1\mu} \epsilon_{r\nu} (k_{1\beta} - k_{2\beta})] [g_{B\bar{B}^* \eta} p_{2\lambda}] [g_{\bar{B}^* B^* \gamma_2} \epsilon^{\kappa\xi\zeta\eta} p_{3\zeta} \epsilon_{\gamma_2 \kappa \omega}^* (g_\tau^\omega g_{\chi\eta} - g_{\tau\eta} g_\chi^\omega) (q_\xi - k_{2\xi})] \frac{1}{k_1^2 - m_B^2} \\ &\times \frac{\tilde{g}_\alpha^\tau(k_2, m_{B^*})}{k_2^2 - m_{B^*}^2} \frac{\tilde{g}^{\lambda\xi}(q, m_{B^*})}{q^2 - m_{B^*}^2} \mathcal{F}^2(q^2), \end{aligned} \quad (20)$$

$$\begin{aligned} \mathcal{M}_{(2-5)} &= \int \frac{d^4 q}{(2\pi)^4} [g_{rB^* \bar{B}^*} \epsilon_r^\mu (g_{\mu\alpha} k_{2\beta} - g_{\mu\beta} k_{1\alpha} + g_{\alpha\beta} (k_{1\mu} - k_{2\mu}))] [g_{B^* \bar{B}^* \eta} \epsilon^{\lambda\rho\delta\sigma} k_{1\lambda} q_\delta] \\ &\times [g_{\bar{B}^* B^* \gamma_2} \epsilon^{\kappa\xi\zeta\eta} p_{3\zeta} \epsilon_{\gamma_2 \kappa \omega}^* (g_\tau^\omega g_{\chi\eta} - g_{\tau\eta} g_\chi^\omega) (q_\xi - k_{2\xi})] \frac{\tilde{g}_\rho^\beta(k_1, m_{B^*})}{k_1^2 - m_{B^*}^2} \frac{\tilde{g}^{\alpha\tau}(k_2, m_{B^*})}{k_2^2 - m_{B^*}^2} \frac{\tilde{g}_\sigma^\xi(q, m_{B^*})}{q^2 - m_{B^*}^2} \mathcal{F}^2(q^2). \end{aligned} \quad (21)$$

As for the $\Upsilon(6S) \rightarrow \Upsilon(1^3D_3)\eta$ transition, the amplitudes can be expressed as

$$\begin{aligned} \mathcal{M}_{(3-1)} = & \int \frac{d^4q}{(2\pi)^4} [g_{\Upsilon B \bar{B}^*} \epsilon^{\mu\nu\alpha\beta} p_{1\mu} \epsilon_{\Upsilon\nu}(k_{1\beta} - k_{2\beta})] [g_{B \bar{B}^* \eta} p_{2\lambda}] [i g_{B^* \bar{B}^* \Upsilon_3} \epsilon_{\Upsilon_3 \zeta \eta \kappa}^* (g_\tau^\eta g_\omega^\kappa + g_\tau^\kappa g_\omega^\eta) (k_2^\zeta - q^\zeta)] \frac{1}{k_1^2 - m_B^2} \frac{\tilde{g}_\alpha^\tau(k_2, m_{B^*})}{k_2^2 - m_{B^*}^2} \\ & \times \frac{\tilde{g}^{\lambda\omega}(q, m_{B^*})}{q^2 - m_{B^*}^2} \mathcal{F}^2(q^2), \end{aligned} \quad (22)$$

$$\begin{aligned} \mathcal{M}_{(3-2)} = & \int \frac{d^4q}{(2\pi)^4} [g_{\Upsilon B^* \bar{B}^*} \epsilon_\Upsilon^\mu (g_{\mu\alpha} k_{2\beta} - g_{\mu\beta} k_{1\alpha} + g_{\alpha\beta} (k_{1\mu} - k_{2\mu}))] [g_{B^* \bar{B}^* \eta} \epsilon^{\lambda\rho\delta\sigma} k_{1\lambda} q_\delta] [i g_{B^* \bar{B}^* \Upsilon_3} \epsilon_{\Upsilon_3 \zeta \eta \kappa}^* (g_\tau^\eta g_\omega^\kappa + g_\tau^\kappa g_\omega^\eta) (k_2^\zeta - q^\zeta)] \\ & \times \frac{\tilde{g}_\rho^\beta(k_1, m_{B^*})}{k_1^2 - m_{B^*}^2} \frac{\tilde{g}^{\alpha\tau}(k_2, m_{B^*})}{k_2^2 - m_{B^*}^2} \frac{\tilde{g}_\sigma^\rho(q, m_{B^*})}{q^2 - m_{B^*}^2} \mathcal{F}^2(q^2). \end{aligned} \quad (23)$$

In the above amplitudes, the monopole form factor $\mathcal{F}(q^2) = (m_E^2 - \Lambda^2)/(q^2 - \Lambda^2)$ is introduced in our calculation since the structure effect of the interaction vertex cannot be ignored and off-shell effect from the exchanged bottom mesons in triangle loops should be compensated by this way. This type of form factor is supported by the QCD sum rule study [26]. Here, m_E is the mass of the exchanged bottom meson B^* , and the cutoff Λ is parametrized as $\Lambda = m_E + \alpha_\Lambda \Lambda_{\text{QCD}}$ with $\Lambda_{\text{QCD}} = 0.22$ GeV (see Refs. [14,27,28] for more details).

Finally, the total sum of amplitudes reads as

$$\mathcal{M}_J^{\text{Total}} = 4 \sum_j \mathcal{M}_{(J-j)}^q + 2 \sum_j \mathcal{M}_{(J-j)}^s \quad (24)$$

with $J = 1, 2, 3$, which correspond to the $\Upsilon(6S) \rightarrow \Upsilon(1^3D_1)\eta$, $\Upsilon(6S) \rightarrow \Upsilon(1^3D_2)\eta$, and $\Upsilon(6S) \rightarrow \Upsilon(1^3D_3)\eta$ transitions, respectively. The triangle loops can be composed of either bottom mesons or bottom-strange mesons. To distinguish them, we adopt the superscripts in amplitudes $\mathcal{M}_{(J-j)}^q$ and $\mathcal{M}_{(J-j)}^s$. The factor 4 in the first term of the right-hand side of Eq. (24) comes from the charge conjugation transformation $B^{(*)} \leftrightarrow \bar{B}^{(*)}$ and the isospin transformations $B^{(*)0} \leftrightarrow B^{(*)+}$ and $\bar{B}^{(*)0} \leftrightarrow B^{(*)-}$, while the factor 2 in the second term of the right-hand side of Eq. (24) is due to the charge conjugation transformation $B_s^{(*)} \leftrightarrow \bar{B}_s^{(*)}$.

With the total sum of amplitudes for each process, the general expression of the partial decay widths is

$$\Gamma_J = \frac{1}{3} \frac{1}{8\pi} \frac{|\vec{p}_\eta|}{m_{\Upsilon(6S)}^2} |\overline{\mathcal{M}_J^{\text{Total}}}|^2, \quad (25)$$

which averages over the polarization of initial $\Upsilon(6S)$ and sums over the polarizations of the $\Upsilon(1^3D_J)$.

III. INPUT PARAMETERS

Before displaying our results, we have to illustrate various parameters, including masses and coupling constants used in this work. First, we need the input of the masses. For the masses of the bottomonia $\Upsilon(1^3D_1)$ and

$\Upsilon(1^3D_3)$, 10.153 GeV [13,29] and 10.174 GeV [13,29] are adopted, respectively, whereas Particle Data Group values [11] are used for other bottomonia involved in this work.

Utilizing the partial decay widths of $\Upsilon(6S) \rightarrow B_{(s)}^{(*)} \bar{B}_{(s)}^{(*)}$ estimated in Ref. [30], we can obtain the coupling constants $g_{\Upsilon(6S)B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}}$ in Eq. (8). In Table I we list the calculated partial decay widths in Ref. [30] as well as the extracted coupling constants [31].

The coupling constants $g_{\Upsilon(1^3D_J)B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}}$ are related to one coupling g_2 of Eq. (1) under the heavy quark symmetry:

$$\begin{aligned} g_{\Upsilon_1 B \bar{B}} &= -2g_2 \frac{\sqrt{15}}{3} \sqrt{m_{\Upsilon_1} m_B m_{\bar{B}}}, \\ g_{\Upsilon_1 B \bar{B}^*} &= g_2 \frac{\sqrt{15}}{3} \sqrt{m_B m_{B^*} / m_{\Upsilon_1}}, \\ g_{\Upsilon_1 B^* \bar{B}^*} &= g_2 \frac{\sqrt{15}}{15} \sqrt{m_{\Upsilon_1} m_{B^*} m_{\bar{B}^*}}, \\ g_{\Upsilon_2 B \bar{B}^*} &= 2g_2 \sqrt{\frac{3}{2}} \sqrt{m_{\Upsilon_2} m_B m_{B^*}}, \\ g_{\Upsilon_2 B^* \bar{B}^*} &= -2g_2 \sqrt{\frac{1}{6}} \sqrt{m_{B^*} m_{B^*} / m_{\Upsilon_2}}, \\ g_{\Upsilon_3 B^* \bar{B}^*} &= 2g_2 \sqrt{m_{\Upsilon_3} m_{B^*} m_{\bar{B}^*}}. \end{aligned}$$

We now have to determine the last remaining coupling constant g_2 . For g_2 , we fix it as follows [13]: we first define the decay constant of the vector meson $\Upsilon(1^3D_1)$ [32]

TABLE I. The partial decay widths given in Ref. [30] and the extracted coupling constants $g_{\Upsilon(6S)B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}}$ [31].

| Final State | Decay Width (MeV) | Coupling Constant |
|--------------------|-----------------------|-------------------------|
| $B\bar{B}$ | 1.32 | 0.654 |
| $B\bar{B}^*$ | 7.59 | 0.077 GeV ⁻¹ |
| $B^*\bar{B}^*$ | 5.89 | 0.611 |
| $B_s\bar{B}_s$ | 1.31×10^{-3} | 0.043 |
| $B_s\bar{B}_s^*$ | 0.136 | 0.023 GeV ⁻¹ |
| $B_s^*\bar{B}_s^*$ | 0.310 | 0.354 |

$$\langle 0|Q\gamma^\mu\bar{Q}|V\rangle = f_V M_V \epsilon_V^\mu, \quad (26)$$

with M_V and ϵ_V^μ being the mass and the polarization vector of $\Upsilon(1^3D_1)$, respectively. Then, using the relation $g_{VBB} \simeq M_V/f_V$ under the vector meson dominance ansatz [33–35], we can get $g_{\Upsilon(1^3D_1)BB}$, and g_2 eventually.

The decay constant f_V can be obtained by fitting to the leptonic decay width $\Gamma[\Upsilon(1^3D_1) \rightarrow e^+e^-] = 1.38$ eV [30]. Using the relation [32]

$$\Gamma_{V \rightarrow e^+e^-} = \frac{4\pi}{3} \frac{\alpha^2}{M_V} f_V^2 C_V, \quad (27)$$

where α is the fine-structure constant and $C_V = 1/9$ for the $\Upsilon(1^3D_1)$ meson, we get $f_V = 23.8$ MeV. Therefore, we get $g_2 = 9.83$ GeV^{-3/2}.

We now turn to the coupling constants between η and $B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}$. Relations can be extracted from Eq. (5):

$$\frac{g_{BB^*\eta_8}}{\sqrt{m_B m_{B^*}}} = g_{B^*B^*\eta_8} = \frac{2}{\sqrt{6}} \frac{g_\pi}{f_\pi},$$

$$\frac{g_{B_s B_s^* \eta_8}}{\sqrt{m_{B_s} m_{B_s^*}}} = g_{B_s^* B_s^* \eta_8} = -2\sqrt{\frac{2}{3}} \frac{g_\pi}{f_\pi},$$

with $f_\pi = 131$ MeV and $g_\pi = 0.569$. Furthermore, η is a mixture between octet η_8 and singlet η_1 :

$$|\eta\rangle = \cos\theta|\eta_8\rangle - \sin\theta|\eta_1\rangle, \quad |\eta'\rangle = \sin\theta|\eta_8\rangle + \cos\theta|\eta_1\rangle, \quad (28)$$

where $\theta = -19.1^\circ$ is fixed by experimental data [36,37].

IV. ESTIMATE OF BRANCHING RATIOS

With above preparation, we now evaluate the branching ratios of the $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$ transitions. In Ref. [13], we have predicted the branching ratios of the $\Upsilon(5S) \rightarrow \Upsilon(1^3D_J)\eta$ by considering the hadronic loop mechanism, which is consistent with the experimental measurement given by Belle later [12]. In this theoretical calculation [13], $\alpha_\Lambda = 0.5 \sim 1.0$ was taken. Because of the similarity between $\Upsilon(5S) \rightarrow \Upsilon(1^3D_J)\eta$ and $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$, in this work we set the same α_Λ range to predict the decay behaviors of $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$. In Fig. 2, we illustrate the α_Λ dependence of the branching ratios, while in Fig. 3, the α_Λ dependence of the relative ratios among these branching fractions is presented.

Our calculation shows that the branching ratios of $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$ shown in Fig. 2 can reach up to 10^{-3} , i.e.,

$$\begin{aligned} \mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_1)\eta] &= (0.174\text{--}1.720) \times 10^{-3}, \\ \mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_2)\eta] &= (0.149\text{--}1.440) \times 10^{-3}, \\ \mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_3)\eta] &= (0.228\text{--}2.210) \times 10^{-3}. \end{aligned}$$

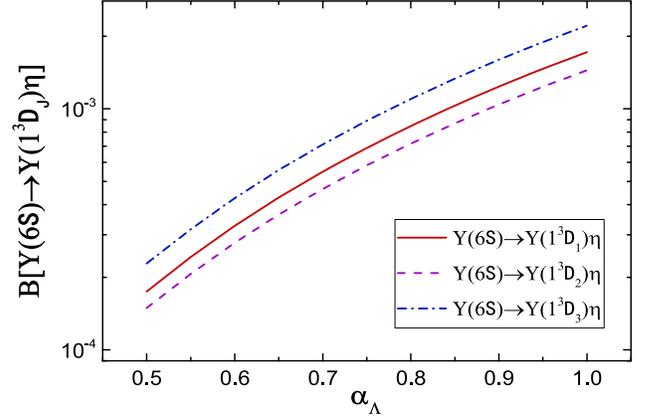


FIG. 2. The α_Λ dependence of the branching ratios for the processes $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$ ($J = 1, 2, 3$).

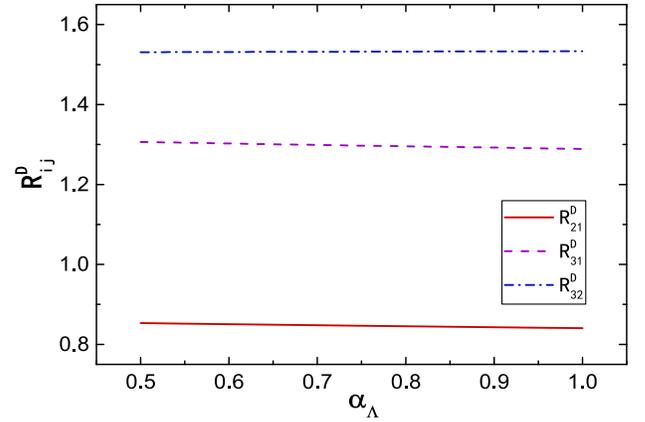


FIG. 3. The α_Λ dependence of the ratios $\mathcal{R}_{21}^D = \mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_2)\eta]/\mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_1)\eta]$, $\mathcal{R}_{31}^D = \mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_3)\eta]/\mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_1)\eta]$, and $\mathcal{R}_{32}^D = \mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_3)\eta]/\mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_2)\eta]$.

These significant values of the branching ratios show that the $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$ decays can be accessible at Belle II.

We also obtain three ratios as shown in Fig. 3, where these ratios weakly depend on α_Λ . The predicted ratios include

$$\begin{aligned} \mathcal{R}_{21}^D &= \frac{\mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_2)\eta]}{\mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_1)\eta]} \approx 0.841\text{--}0.853, \\ \mathcal{R}_{31}^D &= \frac{\mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_3)\eta]}{\mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_1)\eta]} \approx 1.289\text{--}1.306, \\ \mathcal{R}_{32}^D &= \frac{\mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_3)\eta]}{\mathcal{B}[\Upsilon(6S) \rightarrow \Upsilon(1^3D_2)\eta]} \approx 1.531\text{--}1.533, \end{aligned}$$

which can be tested further in future experiments like Belle and Belle II.

V. SUMMARY

Since 2007, Belle has paid more attention to the hidden-bottom hadronic transitions of the $\Upsilon(5S)$, and has found anomalous hadronic transitions like $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) [1] and $\Upsilon(5S) \rightarrow \chi_{bJ}\omega$ ($J = 0, 1, 2$) [2]. Targeted on these anomalies, theorists have made great efforts to explore the underlying mechanisms [4–10,13,15], and have revealed that the hadronic loop mechanism, which is an equivalent description of the coupled-channel effect, plays an important role for understanding these anomalous transitions of $\Upsilon(5S)$.

Besides these observations, Belle discovered the $\Upsilon(5S)$ decays into $\Upsilon(1^3D_J)\eta$ very recently, which has large branching ratios [12]. In fact, this Belle measurement confirmed the prediction made in Ref. [13], which again shows the important role of the hadronic loop effect to the $\Upsilon(5S)$ hidden-bottom decays. What is more important is that this fact also inspires our ambition to further explore the hidden-bottom decays of the $\Upsilon(6S)$.

In this work, we have selected the $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$, and have studied the potential discovery of these decays in a future experiment. Our calculation has shown that the branching ratios of the $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$ can reach up to 10^{-3} when the hadronic loop effect is introduced. It is evident that these significant branching ratios could arouse experimentalist's interest in finding them.

With the running of Belle II, we expect the observation of these anomalous $\Upsilon(6S) \rightarrow \Upsilon(1^3D_J)\eta$ transitions, which will make our understanding of higher bottomonia more in-depth and thorough. Notably, the present study should become a part of the whole research around the $\Upsilon(6S)$.

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