

Strange baryons with two heavy quarks

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The LHCb Experiment at CERN has observed a doubly-charmed baryon $\Xi_{cc}^{++} = ccu$ with a mass of 3621.40 ± 0.78 MeV, consistent with many predictions. We use the same methods that led us to predict $M(\Xi_{cc}, J^P = 1/2^+) = 3627 \pm 12$ MeV and $M(\Xi_{cc}^*, J^P = 3/2^+) = 3690 \pm 12$ MeV to predict $M(\Omega_{cc}^+, J^P = 1/2^+) = 3692 \pm 16$ MeV and $M(\Omega_{cc}^*, J^P = 3/2^+) = 3756 \pm 16$ MeV. Production and decay are discussed briefly, and predictions for $M(\Omega_{bc})$ and $M(\Omega_{bb})$ are included.

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I. INTRODUCTION

The LHCb Experiment at CERN has observed a doubly-charmed baryon $\Xi_{cc}^{++} = ccu$ with a mass of 3621.40 ± 0.78 MeV [1]. This value is consistent with several predictions, including our value of 3627 ± 12 MeV [2,3]. It is more than 100 MeV above a candidate Ξ_{cc}^+ for an isospin partner claimed by the SELEX Collaboration [4], but not seen by others. Here we use similar methods to those in Ref. [2] and earlier works [5] to predict the mass of the ground-state ccs state with spin-parity $J^P = 1/2^+$ $M(\Omega_{cc}^+) = 3692 \pm 16$ MeV and its hyperfine partner with $J^P = 3/2^+$, $M(\Omega_{cc}^*) = 3756 \pm 16$ MeV. Binding effects lead the difference between the strange and nonstrange doubly charmed baryon masses to be less than half the constituent-quark mass difference between the strange and nonstrange light quarks. These results were obtained using constituent-quark masses appropriate for baryons. Use of quark masses universal for baryons and mesons leads to $M(\Omega_{cc})$ and $M(\Omega_{cc}^*)$ about 40 MeV higher, with similar systematic variations expected for Ω_{bc} and Ω_{bb} , due mostly to uncertainty in how strongly a strange quark binds to a heavy diquark.

In Sec. II we list contributions to $M(\Omega_{cc}^+)$ that are straightforward extrapolations of the calculation of $M(\Xi_{cc})$. The pair of charmed quarks is treated as a (cc) diquark antisymmetric (a 3^*) in color and hence symmetric

in spin ($S = 1$). The difference in binding between a (cc) diquark and a strange quark in comparison with binding between (cc) and a nonstrange quark is discussed in Sec. III. Results using quark masses appropriate for both mesons and baryons are treated in Sec. IV. Production and decay are treated briefly in Sec. V, predictions for $M(\Omega_{bc})$ and $M(\Omega_{bb})$ are presented in Sec. VI, while results and comments on other work are collected in Sec. VII.

II. EXTRAPOLATIONS FROM Ξ_{cc} PREDICTION

We compare the contributions to the Ξ_{cc} mass studied in Ref. [2] to similar contributions to the Ω_{cc} mass in Table I. We take quarks in a baryon to have effective masses $m_q^b = 363$ MeV ($q = u$ or d), $m_s^b = 538$ MeV, $m_c^b = 1710.5$ MeV, and $m_b^b = 5043.5$ MeV. We ignore isospin splitting, treated in [6] and references therein.

The effect of the spin-spin interaction between the q quark and the (cc) diquark is parametrized by a term $-4a/m_q^b m_c^b$, while that between s and (cc) is parametrized by $-4a'/m_s^b m_c^b$ with $a' = am_s^b/m_q^b$ taken so that the two terms

TABLE I. Comparison of contributions to the mass of the lightest doubly charmed baryon Ξ_{cc} [2] with corresponding contributions to the mass of Ω_{cc} .

$\Xi_{cc} = ccq$		$\Omega_{cc} = ccs$	
Contribution	Value (MeV)	Contribution	Value (MeV)
$2m_c^b + m_q^b$	3789.0	$2m_c^b + m_s^b$	3959.0
cc binding	-129.0	cc binding	-129.0
$a_{cc}/(m_c^b)^2$	14.2	$a_{cc}/(m_c^b)^2$	14.2
$-4a/m_q^b m_c^b$	-42.4	$-4a'/m_s^b m_c^b$	-42.4
Total	3626.8 ± 12	Subtotal	3801.8 ± 12

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have the same strength. This is motivated by comparing the spin-spin interaction in the $c\bar{s}$ and $c\bar{q}$ systems: $M(D_s^*) - M(D_s) = 143.8$ MeV is almost the same as $M(D^*) - M(D) = 141.4$ MeV. The smaller magnetic moment of s is compensated by a larger wave function at the origin in the $c\bar{s}$ system. We assume a similar compensation is taking place here. For the mass of the $\Omega_{cc}^*(J^P = 3/2^+)$, we replace the term $-4a'/m_s^b m_c^b = -42.4$ MeV by $+2a'/m_s^b m_c^b = +21.2$ MeV, so $M[\Omega_{cc}^*(J^P = 3/2^+)] - M[\Omega_{cc}(J^P = 1/2^+)] = 63.6$ MeV.

Our calculations of the masses of light hadrons, based on the ideas of Ref. [7], use constituent-quark masses and do not require separate binding energies. However, for systems without q involving heavy quarks one must take into account additional binding. For example, when calculating the mass of the S-wave $c\bar{s}$ system, it was found necessary to include a supplemental binding energy of 69.9 MeV, while a binding energy of 258 MeV was needed to describe S-wave charmonium [2]. Hence the last energy in Table I represents a subtotal; we estimate the binding energy of s with the diquark (cc) in the next section.

III. DIQUARK-LIGHT QUARK BINDING

We shall interpolate between the $\bar{c}s$ and $\bar{c}c$ binding energies to find that between (cc) and s . All three cases involve the interaction of a color antitriplet with a color triplet. We compare the masses m_1, m_2 of the constituents and reduced mass $\mu \equiv m_1 m_2 / (m_1 + m_2)$ of the composite system in Table II. When discussing mesons, we use effective masses $m_q^m = 310$ MeV, $m_s^m = 483$ MeV, $m_c^m = 1663.3$ MeV and $m_b^m = 5003.8$ MeV [2]. The mass of the cc diquark is calculated to be $2m_c^b - B(cc) + a_{cc}/(m_c^b)^2 = 3421.0 - 129.0 + 14.2 = 3306.2$ MeV. For use in subsequent discussion of the masses of $\Omega_{bc} \equiv bcs$ and $\Omega_{bb} \equiv bss$, we include the binding energies between the diquark (bc) and s and between the diquark (bb) and s . The mass of the bc diquark is calculated to be $m_b^b + m_c^b - B(bc) = 5043.5 + 1710.5 - (167.6 \pm 3) = 6586.4 \pm 3$ MeV,

TABLE II. Comparison of constituent masses and reduced masses in MeV for some systems of strange and c or b quarks, in a scheme with separate quark masses for mesons and baryons. Binding energies in MeV are also shown, with two different values averaged and errors reflecting half their difference for the (cc) s , (bc) s , and (bb) s systems (see text).

System	m_1	m_2	μ	B
$\bar{c}s$	1663.3	483	374.3	69.9
$\bar{c}c$	1663.3	1663.3	831.6	258
(cc) s	3306.2	538	462.7	109.4 ± 10.5
(bc) s	6586.4^a	538	497.4	124.1 ± 12.8
(bb) s	9813.4	538	510.0	129.4 ± 13.4

^aMass eigenstates of indefinite bc spin; small hyperfine terms ignored.

where the error reflects uncertainty in the bc binding energy. As the mass eigenstates are of indefinite bc spin (rather, they are approximately states of definite cs spin), we ignore small hyperfine effects. The mass of the bb diquark is calculated to be $2m_b^b - B(bb) + a_{bb}/(m_b^b)^2 = 10087.0 - 281.4 + 7.8 = 9813.4$ MeV.

The reduced mass of the (cc) s system lies between those of $\bar{c}s$ and $\bar{c}c$. Assuming a power-law dependence on μ , $B = A\mu^p$, gives $p = 1.636$ and $B((cc)s) = 98.9$ MeV. An alternate method makes use of the Feynman-Hellmann theorem [8], which relates the derivative of an energy expectation value with respect to a parameter μ to the expectation value of the derivative of the Hamiltonian:

$$\frac{dE_\mu}{d\mu} = \left\langle \frac{dH_\mu}{d\mu} \right\rangle. \quad (1)$$

In the present case, the right-hand side is $-(1/\mu)\langle T \rangle$, where T is the kinetic energy. Let us now assume $\langle T \rangle$ is independent of the reduced mass. This is indeed the case for a logarithmic potential [9,10], which has been shown to suitably interpolate between charmonium and bottomonium. We shall assume T is constant also for our interpolation. Then the shift in binding energy between a system with reduced mass μ_1 and one with μ_2 is

$$\Delta B = \langle T \rangle \int_{\mu_1}^{\mu_2} \frac{d\mu}{\mu} = \langle T \rangle \ln \frac{\mu_2}{\mu_1}. \quad (2)$$

The binding energy increases with increased reduced mass, as expected. One can determine $\langle T \rangle = 235.6$ MeV by comparing $\bar{c}s$ and $\bar{c}c$ binding energies, yielding

$$\begin{aligned} B((cc)s) &= B(\bar{c}s) + \langle T \rangle \ln \frac{462.7}{374.3} \\ &= 69.9 + 50 = 119.9 \text{ MeV}. \end{aligned} \quad (3)$$

The average of the two determinations is 109.4 ± 10.5 MeV, where we take the error to be half of their difference. Similar methods apply to the estimates of $B((bc)s)$ and $B((bb)s)$ quoted in Table II, where the averages are those of the power-law (lesser value) and Feynman-Hellmann (greater value) methods of interpolation. Subtracting this from the subtotal in Table I, whose error was assumed to be the same as in the calculations of $M(\Xi_{cc})$, and adding the error of ± 10.5 MeV in quadrature, we find $M(\Omega_{cc}) = 3692 \pm 16$ MeV, $M(\Omega_{cc}^*) = 3756 \pm 16$ MeV.

IV. UNIVERSAL QUARK MASSES

For many years it has been realized that fits to baryon masses require constituent quarks about 55 MeV heavier than those in fits to low-lying mesons [11,12]. An alternative, secondary, description [13] makes use of quark masses appropriate for both mesons and baryons, adding a term $S = 165.1$ MeV to characterize the extra mass in a

TABLE III. Contributions to $M(\Omega_{cc})$ and $M(\Xi_{cc})$ in a picture with identical quark masses for mesons and baryons. a_{uqm} and a'_{uqm} denote the strengths of cq and cs color hyperfine coupling appropriate for universal quark masses [13].

$M(\Omega_{cc})$		$M(\Xi_{cc})$	
Contribution	Value (MeV)	Contribution	Value (MeV)
$2m_c + m_s$	3793.4	$2m_c + m_q$	3619.7
cc binding	-121.3	cc binding	-121.3
S	165.1	S	165.1
$a_{cc}/(m_c)^2$	14.2	$a_{cc}/(m_c)^2$	14.2
$-4a'_{uqm}/m_s m_c$	-37.6	$-4a_{uqm}/m_q m_c$	-37.6
Subtotal	3813.8 ± 12	Total	3640.1 ± 12

baryon due to a string junction [14]. The contributions to $M(\Omega_{cc})$, before accounting of binding between the (cc) diquark and the strange quark, are shown in Table III. Also shown are contributions to $M(\Xi_{cc})$ in this scheme. Here $m_q = 308.5$ MeV, $m_s = 482.2$ MeV, $m_c = 1655.6$ MeV, and $m_b = 4988.6$ MeV [13].

The (cc) diquark's mass is $M(cc, 3^*) = 2(1655, 6) - 121.3 + 14.2 = 3204.1$ MeV. To account for binding between the s quark and the (cc) diquark, we interpolate as before, with the results shown in Table IV. The binding energy in the $\bar{c}s$ system has been calculated as $B(\bar{c}s) = -[3M(D_s^*) + M(D_s)]/4 + m_s + m_c = -[3(2112.1) + 1968.3]/4 + 482.2 + 1655.6$ MeV = 61.65 MeV.

Interpolating via a power law with $B = A\mu^p$ one finds $p = 1.721$, $B((cc)s) = 75.2$ MeV, while interpolating via the Feynman-Hellmann theorem (2) one finds $\langle T \rangle = 227.4$ MeV and $B((cc)s) = 87.9$ MeV. Hence $B((cc)s) = 81.6 \pm 6.4$ MeV, implying $M(ccs, 1/2^+) = 3732 \pm 14$ MeV. This is 40 MeV above the value we obtained with separate quark masses for meson and baryons. The uncertainty reflects in part the uncertainty in estimating the binding energy between a strange quark and the heavy

TABLE IV. Constituent and reduced masses in MeV for interpolation to find binding energy between s and heavy diquarks (cc) , (bc) , and (bb) , in a scheme with common quark masses for mesons and baryons. For the heavy diquark systems two different values have been averaged; errors reflect half their difference.

System	m_1	m_2	μ	B
$\bar{c}s$	1655.6	482.2	373.4	61.65
$\bar{c}c$	1655.6	1655.6	827.8	242.7 ^a
$(cc)s$	3204.1	482.2	419.1	81.6 ± 6.4
$(bc)s$	6484.9 ^b	482.2	448.8	94.0 ± 9.4
$(bb)s$	9718.9	482.2	459.4	98.4 ± 10.4

^aFrom Ref. [13].

^bMass eigenstates of indefinite bc spin; small hyperfine terms ignored.

diquark. A precise measurement of $M(\Omega_{cc})$ could help distinguish between the two pictures compared here. We also quote the predicted value of $M(\Xi_{cc}) = 3640 \pm 12$ MeV in the scheme with universal quark masses. This is not as close to the experimental value as that in Ref. [2], but still acceptable. For the $\Omega_{cc}^*(J^P = 3/2^+)$ we replace the term $-4a'/m_s m_c = -37.6$ MeV in Table III by $+2a'/m_s m_c = +18.8$ MeV, so we predict $M(\Omega_{cc}^*) - M(\Omega_{cc}) = 56.4$ MeV, or $M(\Omega_{cc}^*) = 3789 \pm 16$ MeV.

In addition to the $(cc)s$ binding energy, Table IV contains also the $(bc)s$ and $(bb)s$ binding energies, obtained in an analogous way. The latter are used in Sec. VI to predict the masses Ω_{bc} and Ω_{bb} .

V. PRODUCTION AND DECAY

We can estimate the rate for production of $\Omega_{cc} = ccs$ by reference to that for $\Xi_{cc}^{++} = ccu$. Imagine that some process gives rise to the (cc) diquark, which then fragments into Ξ_{cc} by picking up a u quark. The corresponding process giving rise to Ω_{cc} then involves (cc) picking up a s quark. What is the ratio of these two processes?

There is information on b quark fragmentation in hadronic collisions from the CDF Collaboration [15], which measures $f_s \simeq 0.3f_u$ in $\bar{p}p$ collisions at $\sqrt{s} = 1.96$ TeV. One could expect a similar ratio for (cc) to pick up a u or s quark. At 13 TeV, in a sample of pp collisions consisting of an integrated luminosity of 1.7 fb^{-1} , the LHCb experiment accumulated 313 ± 33 Ξ_{cc}^{++} events [1]. One might then expect the same sample to contain about $(100 \pm 10)R$ Ω_{cc}^+ identifiable events, where R is the ratio of Ω_{cc} to Ξ_{cc} decays into identifiable branching fractions.

The Ξ_{cc} was seen in the final state $\Lambda_c K^- \pi^+ \pi^+$. One decay process depicted in Fig. 1 of Ref. [1] involves the initial u and one of the initial charmed quarks c in the $\Xi_{cc} = ccu$ ending up in the $\Lambda_c = ucd$. If the initial baryon is $\Omega_{cc} = ccs$, an initial s and one of the initial charmed quarks will end up instead in a $\Xi_c^0 = scd$. The detectability of the Ω_{cc} will then depend on the relative efficiencies for reconstruction of Ξ_c^0 and Λ_c .

Another potentially useful decay mode of Ξ_{cc}^{++} is into $\pi^+ \Xi_c^+$. Its visibility at LHCb will depend on relative

TABLE V. Summary of predictions of Ω_{QQ} masses, in MeV. ‘‘Separate’’ denotes separate quark masses for mesons and baryons; ‘‘universal’’ denotes universal quark masses for mesons and baryons.

	Separate	Universal
$M(\Omega_{cc})$	3692 ± 16	3732 ± 14
$M(\Omega_{bc})$	6968 ± 19	7013 ± 16
$M(\Omega'_{bc})$	6984 ± 19	7025 ± 16
$M(\Omega_{bb})$	10208 ± 18	10255 ± 16

TABLE VI. Comparison of nonlattice predictions for $M(\Omega_{cc})$.

Reference	Value (MeV)	Method
Present work	3692 ± 16	Separate baryonic quark masses
Present work	3732 ± 14	Universal quark masses
[7]	3730–3940	QCD-motivated quark model
[17]	3690	Bag model
[18]	3664	Bag model
[19]	3819 ± 57	QCD-motivated quark model
[20]	3811	QCD-motivated quark model
[21]	3703	Potential models
[21]	3657	Bag models
[22]	3760.7 ± 2.4^a	Potential approach
[23]	3720	Potential model
[24]	3710	Heavy quark effective theory
[25]	3737	Potential model
[26]	3740 ± 80	Feynman-Hellmann+semiempirical
[27]	3787	Mass sum rules
[28]	3760	Relativistic quasipotential quark model
[29]	3710	Three-body Faddeev equations
[30]	3804 ± 8	Quadratic mass relations
[31]	3598	Bootstrap quark model+Faddeev eqs.
[32]	3650 ± 50	Nonrelativistic QCD sum rules
[33]	3749 ± 10	Quark model
[34]	3590 ± 50	Potential approach+QCD sum rules
[35]	3594	Potential model
[36]	3860	Nonperturbative string
[37]	3778	Relativistic quark-diquark
[38]	3619	Bag model
[39]	3637 ± 23	Lattice; exact chiral symmetry
[40]	3732	Relativistic quark model+Bethe-Salpeter
[41]	3702^{+41}	Variational
[42]	3815	Quark model
[43]	3719	Relativistic quark model
[44]	3650.4 ± 6.3^b	Quadratic mass relations
[45]	3697	Quark model+QCD
[46]	3710 ± 140	QCD sum rules
[47]	3635 ± 15	Instantaneous approx+Bethe-Salpeter
[48]	$3566 \div 3687$	Potential model
[49]	4250 ± 200	QCD sum rules
[50]	3710	Modified bag model
[51]	3648	Anti-de Sitter/QCD inspired potl.
[52]	3630^b	QCD sum rules
[53]	3667	Preferred potential model
[54]	3650 ± 40^b	Quadratic mass relations

^aSpin-weighted average of $M(\Omega_{cc})$ and $M(\Omega_{cc}^*)$.^bSELEX [4] $M(ccd, 1/2^+) = 3519$ MeV candidate as input.

efficiencies for reconstruction of Ξ_c^+ and Λ_c . The corresponding decay mode of Ω_{cc} is into $\pi^+\Omega_c$. The LHCb experiment has detected not only the Ω_c but several excited states of it [16] in the final state $\Xi_c^+ K^-$, providing a test of ability to reconstruct Ξ_c^+ .

VI. MASSES OF $\Omega_{bc} = bcs$ AND $\Omega_{bb} = bbs$

We have seen that much of the uncertainty in prediction of $M(\Omega_{cc})$ lies in uncertainty of the binding energy between the (cc) diquark and the strange quark. The same is true when predicting the masses of Ω_{bc} and Ω_{bb} . Extrapolating our results for nonstrange states [2] to ones in which $q = u, d$ is replaced with s , we take account of (1) the $s - q$ mass difference, (2) differences in $((QQ')q)$ and $((QQ')s)$ binding, and (3) small differences in hyperfine splittings, to obtain the results in Table V. The use of universal quark masses raises the prediction of all $\Omega_{Q_1 Q_2}$ masses by about 40 MeV.

VII. RESULTS

Using the same methods used to obtain an accurate prediction of the mass of the recently discovered doubly-charmed baryons Ξ_{cc}^{++} , we predict the mass of its strange partner: $M(\Omega_{cc}) = 3692 \pm 16$ MeV. The hyperfine partner of this state, with $J^P = 3/2^+$, is predicted to have a mass $M(\Omega_{cc}^*) = 3756 \pm 16$ MeV. Predictions for the ground state masses of the bcs baryons Ω_{bc} and Ω'_{bc} and the bbs baryon Ω_{bb} are also presented. The use of universal quark masses with an added “string-junction” contribution for baryons raises these predictions by about 40 MeV.

Our predictions for $M(\Omega_{cc})$ are compared with a number of others in Tables VI (non-lattice) and VII (lattice). The predictions based on lattice gauge theory are shown separately as they have less of a spread. The corresponding values are plotted in Figs. 1 and 2.

In the picture with separate quark masses for mesons and baryons, the prediction of a rather large value of $B((cc)s)$ distinguishes our approach from a number of others [3] in

TABLE VII. Comparison of lattice predictions for $M(\Omega_{cc})$ with our result.

Reference	Value (MeV)	Method
Present work	3692 ± 16	Separate baryonic quark masses
Present work	3732 ± 14	Universal quark masses
[55]	$3747(9)_{(47)}^{(11)} \div 3727(9)_{(40)}^{(16)}$	Quenched lattice (LGT)
[56]	$3663(11)(17)(95)$	Quenched lattice
[57]	$37636 \pm 19 \pm 2_{-79}^{+13}$	Lattice, domain-wall + KS fermions
[58]	$3704(5)(16)$	Lattice, $N_f = 2 + 1$
[59]	$3679(40)(17)(5)$	LGT, $N_f = 2 + 1$, $m_\pi = 200$ MeV
[60]	$3658(11)(16)(50)$	LGT, $N_f = 2 + 1$, $m_\pi = 210$ MeV
[61]	$3738(20)(20)$	Lattice
[62]	$(3640 \pm 173) \div (3663 \pm 230)$	Lattice; on-shell renormalization
[63]	$3711(5)(30)$	LGT, clover-improved, physical m_π

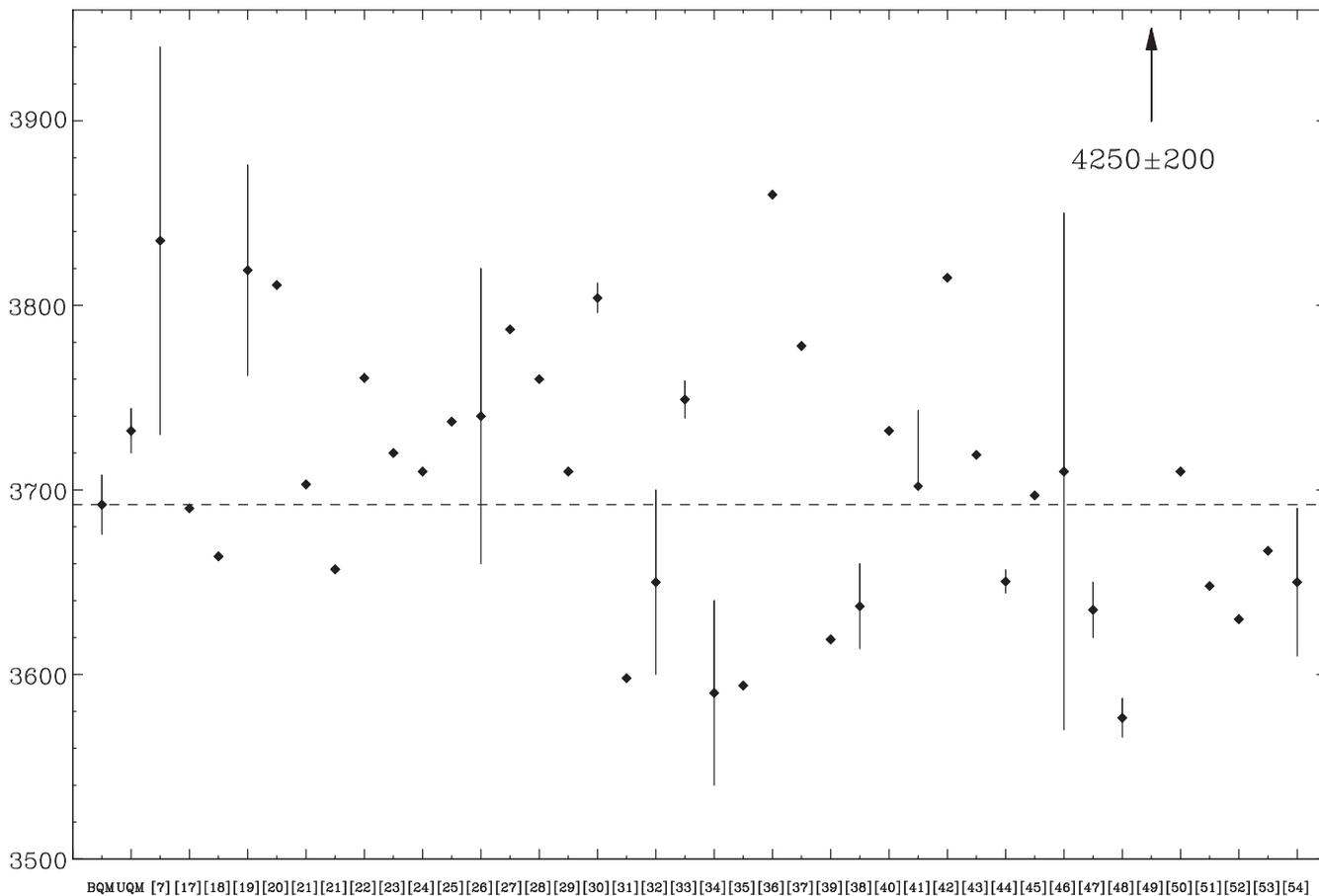


FIG. 1. Comparison of non-lattice predictions for $M(\Omega_{cc})$. The first two points are our predictions for baryonic quark mass (BQM; dashed line) and universal quark masses (UQM).

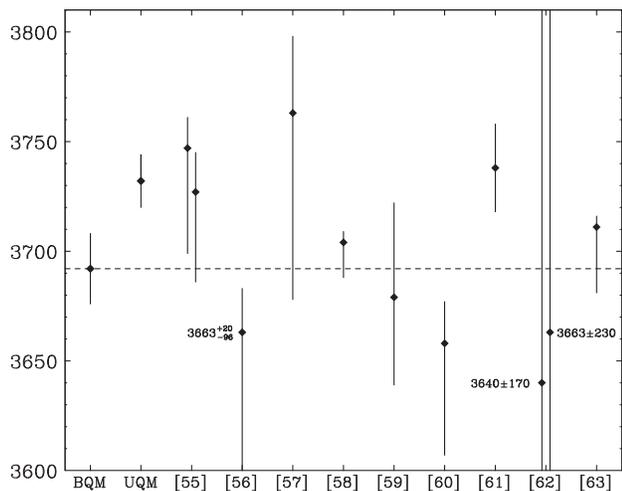


FIG. 2. Comparison of lattice predictions for $M(\Omega_{cc})$. The first two points are our (nonlattice) predictions for baryonic quark mass (BQM; dashed line) and universal quark masses (UQM).

which the difference $M(\Omega_{cc}) - M(\Xi_{cc})$ is larger than our central value of 65 MeV. In our calculation more than half of the mass quark difference $m_s^b - m_q^b = 175$ MeV is canceled by increased binding. For comparison, a lattice gauge theory calculation [61] finds $M(\Xi_{cc}) = 3610(23) \times (22)$ MeV, $M(\Xi_{cc}^*) = 3692(28)(21)$ MeV, $M(\Omega_{cc}) = 3738(20)(20)$ MeV, $M(\Omega_{cc}^*) = 3820(20)(22)$ MeV, implying a difference between the strange and nonstrange states of 128 MeV. This is closer to the value of 105 MeV we find in the picture with universal quark masses.

The production cross section for Ω_{cc} was estimated to be about 0.3 times that for Ξ_{cc}^{++} . Its detectability then depends on the relative efficiency for reconstructing Ξ_c^0 and Λ_c .

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