Large-N_c sum rules for charmed baryons at subleading orders

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Sum rules for the low-energy constants of the chiral SU(3) Lagrangian with charmed baryons of spin $J^P = 1/2^+$ and $J^P = 3/2^+$ baryons are derived from large-N_c QCD. We consider the large-N_c operator expansion at subleading orders for current-current correlation functions in the charmed baryon-ground states for two scalar and two axial-vector currents.

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I. INTRODUCTION

The dependence of the charmed baryon masses on the up, down and strange quark masses encodes useful information on the coupled-channel interaction dynamics of the Goldstone bosons with such baryon states [1–8]. Lattice QCD simulations for the baryon masses at unphysical quark masses are particularly useful [9–14] since they complement the well-known values for the masses of the charmed baryon ground states at physical quark masses [15].

An accurate flavor SU(3) chiral extrapolation of the baryon ground states with zero charm content was established in a series of works [16–20]. Based on the chiral Lagrangian formulated with spin $1/2$ and $3/2$ fields, the available lattice data on the baryon masses were reproduced and accurate predictions for the size of the low-energy parameters relevant at N^{3}LO were made [20]. The success of such analyses relies on two crucial ingredients. First, the chiral expansion is formulated in terms of physical meson and baryon masses rather than bare masses as is requested by traditional chiral perturbation theory (χPT). Second, the flood of low-energy constants that arises at subleading orders is tamed by sum rules for the latter as they arise in the limit of a large number of colors (N_c) in QCD [20,21]. The large-N_c sum rules provide a large parameter reduction that allowed fits at N^{3}LO to the lattice data set that are significant. A corresponding program was started for the charmed baryons [22]. At present, however, the large-N_c sum rules for the charmed baryons are derived at leading order only [22]. It is the purpose of this work to close this gap and establish such sum rules accurate to subleading orders in the $1/N_c$ expansion. This will pave the way to accurate chiral extrapolation studies of the charmed baryon masses.

The desired sum rules can systematically be derived from QCD by a study of current-current correlation functions in the baryon ground states. We study matrix elements of current-current correlation functions in the charmed baryon states [21,23]. The technology developed in [24–26] will be applied. The implications of heavy-quark symmetry on the counterterms was worked out already using a suitable multiplet representation of the charmed baryons [22,27–29].

II. CHIRAL DYNAMICS FOR CHARMED BARYONS

The chiral dynamics for the charmed baryon fields is most economically deduced from an effective chiral Lagrangian that is based on power counting rules. We consider here the flavor antisymmetric antitriplet and the flavor symmetric sextet fields $B_{[\bar 3]}$, $B_{[6]}$ and $B^\mu_{[6]}$ with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ quantum numbers. The chiral Lagrangian consists of all possible interaction terms, formed with the baryon fields and the conventional chiral blocks U_μ and $\chi_\pm$ that include the Goldstone boson fields Φ as well as the classical source functions, s, p and v_μ, a_μ of QCD [30]. Derivatives of the fields must be included in compliance with the local chiral SU(3) symmetry which in turn requests the covariant derivative D_μ to act on the various flavor multiplet fields as follows,

$$
\begin{aligned}
(D_\mu U_\nu)^a_b &= \partial_\mu U^a_{\nu,b} + \Gamma^a_{\mu,l} U^l_{\nu,b} - \Gamma^l_{\mu,b} U^a_{\nu,l}, \\
(D_\mu B_{[6]})^{ab} &= \partial_\mu B^{ab}_{[6]} + \Gamma^a_{\mu,l} B^{lb}_{[6]} + \Gamma^b_{\mu,l} B^{al}_{[6]}, \\
(D_\mu B_{[\bar 3]})^{ab} &= \partial_\mu B^{ab}_{[\bar 3]} + \Gamma^a_{\mu,l} B^{lb}_{[\bar 3]} + \Gamma^b_{\mu,l} B^{al}_{[\bar 3]},
\end{aligned} \tag{1}
$$

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with the chiral connection $\Gamma_\mu = -\Gamma_\mu^\dagger$ given by

$$\Gamma_\mu = \frac{1}{2} e^{-i\frac{\Phi}{2f}} [\partial_\mu - i(v_\mu + a_\mu)] e^{+i\frac{\Phi}{2f}} + \frac{1}{2} e^{+i\frac{\Phi}{2f}} [\partial_\mu - i(v_\mu - a_\mu)] e^{-i\frac{\Phi}{2f}},$$
$$U_\mu = \frac{1}{2} u^\dagger (\partial_\mu e^{i\frac{\Phi}{f}}) u^\dagger - \frac{i}{2} u^\dagger (v_\mu + a_\mu) u + \frac{i}{2} u (v_\mu - a_\mu) u^\dagger, \qquad u = e^{i\frac{\Phi}{2f}}. \tag{2}$$

The various hadron fields can be decomposed into their isospin multiplet components,

$$\begin{aligned}
\Phi &= \tau \cdot \pi(140) + \alpha^\dagger \cdot K(494) + K^\dagger(494) \cdot \alpha + \eta(547)\lambda_8, \\
\sqrt{2} B_{[\bar{3}]} &= \frac{1}{\sqrt{2}} \alpha^\dagger \cdot \Xi_c(2470) - \frac{1}{\sqrt{2}} \Xi_c^T(2470) \cdot \alpha + i\tau_2 \Lambda_c(2284), \\
\sqrt{2} B_{[6]} &= \frac{1}{\sqrt{2}} \alpha^\dagger \cdot \Xi_c(2580) + \frac{1}{\sqrt{2}} \Xi_c^T(2580) \cdot \alpha + \Sigma_c(2455) \cdot \tau i\tau_2 + \frac{\sqrt{2}}{3} (1 - \sqrt{3}\lambda_8) \Omega_c(2704), \\
\sqrt{2} B_{[6]}^\mu &= \frac{1}{\sqrt{2}} \alpha^\dagger \cdot \Xi_c^\mu(2645) + \frac{1}{\sqrt{2}} \Xi_c^{T,\mu}(2645) \cdot \alpha + \Sigma_c^\mu(2520) \cdot \tau i\tau_2 + \frac{\sqrt{2}}{3} (1 - \sqrt{3}\lambda_8) \Omega_c^\mu(2770), \\
\alpha^\dagger &= \frac{1}{\sqrt{2}} (\lambda_4 + i\lambda_5, \lambda_6 + i\lambda_7), \qquad \tau = (\lambda_1, \lambda_2, \lambda_3),
\end{aligned} \tag{3}$$

where the matrices λ_i are the standard Gell-Mann generators of the SU(3) algebra.

The main goal of this work is to derive correlations amongst the low-energy parameters of the chiral Lagrangian as they follow from a $1/N_c$ expansion. For that purpose, we consider QCD's axial-vector and scalar currents,

$$A_\mu^{(a)}(x) = \bar{\Psi}(x) \gamma_\mu \gamma_5 \frac{\lambda_a}{2} \Psi(x),$$
$$S^{(a)}(x) = \bar{\Psi}(x) \frac{\lambda_a}{2} \Psi(x), \tag{4}$$

in baryon matrix elements, where we recall their definitions in terms of the Heisenberg quark-field operators $\Psi(x)$. With λ_a we denote the Gell-Mann flavor matrices supplemented with a singlet matrix $\lambda_0 = \sqrt{2/3}\mathbf{1}$. Given the chiral Lagrangian, it is well defined how to derive the contribution of a given term to such matrix elements. The classical matrices of source functions, a_μ and s, enter the chiral Lagrangian via the building blocks,

$$U_\mu = \frac{i}{2f} \partial_\mu \Phi - i a_\mu + \cdots,$$
$$\chi_+ = \frac{1}{2} (u \chi_0 u + u^\dagger \chi_0 u^\dagger) = 2 B_0 s + \cdots, \tag{5}$$

where, for notational simplicity in the following, we put $B_0 = 1/2$.

We recall all terms in the chiral Lagrangian that are relevant in a chiral extrapolation of the baryon masses at N^{3}LO. Altogether we recalled $34 + 16 = 50$ distinct low-energy constants, which have to be correlated. There are 16 symmetry-breaking counterterms

$$\begin{aligned}
\mathcal{L}_\chi^{(4)} = {} & c_{1,[\bar{3}\bar{3}]} \mathrm{tr}(\bar{B}_{[\bar{3}]} B_{[\bar{3}]}) \mathrm{tr}(\chi_+^2) + c_{2,[\bar{3}\bar{3}]} \mathrm{tr}(\bar{B}_{[\bar{3}]} B_{[\bar{3}]}) (\mathrm{tr}\chi_+)^2 \\
& + c_{3,[\bar{3}\bar{3}]} \mathrm{tr}(\bar{B}_{[\bar{3}]} \chi_+ B_{[\bar{3}]}) \mathrm{tr}(\chi_+) + c_{4,[\bar{3}\bar{3}]} \mathrm{tr}(\bar{B}_{[\bar{3}]} \chi_+^2 B_{[\bar{3}]}) \\
& + c_{1,[66]} \mathrm{tr}(\bar{B}_{[6]} B_{[6]}) \mathrm{tr}(\chi_+^2) + c_{2,[66]} \mathrm{tr}(\bar{B}_{[6]} B_{[6]}) (\mathrm{tr}\chi_+)^2 \\
& + c_{3,[66]} \mathrm{tr}(\bar{B}_{[6]} \chi_+ B_{[6]}) \mathrm{tr}(\chi_+) + c_{4,[66]} \mathrm{tr}(\bar{B}_{[6]} \chi_+^2 B_{[6]}) + c_{5,[66]} \mathrm{tr}(\bar{B}_{[6]} \chi_+ B_{[6]} \chi_+^T) \\
& + c_{1,[\bar{3}6]} \mathrm{tr}(\bar{B}_{[6]} \chi_+ B_{[\bar{3}]} + \mathrm{H.c.}) \mathrm{tr}(\chi_+) + c_{2,[\bar{3}6]} \mathrm{tr}(\bar{B}_{[6]} \chi_+^2 B_{[\bar{3}]} + \mathrm{H.c.}) \\
& - e_{1,[66]} \mathrm{tr}(\bar{B}_{[6]}^\mu g_{\mu\nu} B_{[6]}^\nu) \mathrm{tr}(\chi_+^2) - e_{2,[66]} \mathrm{tr}(\bar{B}_{[6]}^\mu g_{\mu\nu} B_{[6]}^\nu) (\mathrm{tr}\chi_+)^2 \\
& - e_{3,[66]} \mathrm{tr}(\bar{B}_{[6]}^\mu \chi_+ g_{\mu\nu} B_{[6]}^\nu) \mathrm{tr}(\chi_+) - e_{4,[66]} \mathrm{tr}(\bar{B}_{[6]}^\mu g_{\mu\nu} \chi_+^2 B_{[6]}^\nu) \\
& - e_{5,[66]} \mathrm{tr}(\bar{B}_{[6]}^\mu g_{\mu\nu} \chi_+ B_{[6]}^\nu \chi_+^T),
\end{aligned} \tag{6}$$

which contribute to the baryon masses at tree level. Not that as compared to [22] we dropped the flavor redundant term proportional to $c_{3,[\bar{3}6]}$. The symmetry-breaking counterterms contribute to the current-current correlation function of two time-ordered scalar currents

$$S^{ab}(q) = i \int d^4x e^{-iq\cdot x} \mathcal{T} S^{(a)}(x) S^{(b)}(0), \tag{7}$$

in the baryon states. We consider singlet and octet components with $a, b = 0, \ldots, 8$.

In addition, there is a class of 34 symmetry-conserving two-body counterterms that contribute to the baryon masses at the one-loop level. Following [21,22,31] the symmetry-conserving counterterms are classified according to their Dirac structure.

$$\mathcal{L}^{(2)} = \mathcal{L}^{(2)}_\chi + \mathcal{L}^{(S)} + \mathcal{L}^{(V)} + \mathcal{L}^{(A)} + \mathcal{L}^{(T)}. \tag{8}$$

A complete list relevant at second order was constructed in [22] with

$$\begin{aligned}
\mathcal{L}^{(S)} = & -g^{(S)}_{0,[\bar{3}\bar{3}]} \mathrm{tr}(\bar{B}_{[\bar{3}]} B_{[\bar{3}]}) \mathrm{tr}(U_\mu U^\mu) - g^{(S)}_{D,[\bar{3}\bar{3}]} \mathrm{tr}(\bar{B}_{[\bar{3}]} \{U_\mu, U^\mu\} B_{[\bar{3}]}) - g^{(S)}_{0,[66]} \mathrm{tr}(\bar{B}_{[6]} B_{[6]}) \mathrm{tr}(U_\mu U^\mu) \\
& - g^{(S)}_{1,[66]} \mathrm{tr}(\bar{B}_{[6]} U^\mu B_{[6]} U^T_\mu) - g^{(S)}_{D,[66]} \mathrm{tr}(\bar{B}_{[6]} \{U_\mu, U^\mu\} B_{[6]}) - g^{(S)}_{D,[\bar{3}6]} \mathrm{tr}(\bar{B}_{[6]} \{U_\mu, U^\mu\} B_{[\bar{3}]} + \mathrm{H.c.}) \\
& + h^{(S)}_{0,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} g_{\mu\nu} B^\nu_{[6]}) \mathrm{tr}(U_\alpha U^\alpha) + h^{(S)}_{1,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} B^\nu_{[6]}) \mathrm{tr}(U_\mu U_\nu) \\
& + h^{(S)}_{2,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} g_{\mu\nu} \{U^\alpha, U_\alpha\} B^\nu_{[6]}) - h^{(S)}_{3,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} \{U_\mu, U_\nu\} B^\nu_{[6]}) \\
& + h^{(S)}_{4,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} g_{\mu\nu} U^\alpha B^\nu_{[6]} U^T_\alpha) + h^{(S)}_{5,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} U_\nu B^\nu_{[6]} U^T_\mu + \bar{B}^\mu_{[6]} U_\mu B^\nu_{[6]} U^T_\nu), \\
\mathcal{L}^{(V)} = & -\frac{1}{2} g^{(V)}_{0,[\bar{3}\bar{3}]} \mathrm{tr}(\bar{B}_{[\bar{3}]} i\gamma^\alpha (D^\beta B_{[\bar{3}]}) \mathrm{tr}(U_\beta U_\alpha) + \mathrm{H.c.}) - \frac{1}{2} g^{(V)}_{1,[\bar{3}\bar{3}]} \mathrm{tr}(\bar{B}_{[\bar{3}]} i\gamma^\alpha U_\beta (D^\beta B_{[\bar{3}]}) U^T_\alpha \\
& + \bar{B}_{[\bar{3}]} i\gamma^\alpha U_\alpha (D^\beta B_{[\bar{3}]}) U^T_\beta + \mathrm{H.c.}) - \frac{1}{2} g^{(V)}_{D,[\bar{3}\bar{3}]} \mathrm{tr}(\bar{B}_{[\bar{3}]} i\gamma^\alpha \{U_\alpha, U_\beta\} (D^\beta B_{[\bar{3}]}) + \mathrm{H.c.}) \\
& - \frac{1}{2} g^{(V)}_{D,[\bar{3}6]} \mathrm{tr}(\bar{B}_{[6]} i\gamma^\alpha \{U_\alpha, U_\beta\} (D^\beta B_{[\bar{3}]}) - (D^\beta \bar{B}_{[6]}) i\gamma^\alpha \{U_\alpha, U_\beta\} B_{[\bar{3}]} + \mathrm{H.c.}) \\
& - \frac{1}{2} g^{(V)}_{0,[66]} (\mathrm{tr}(\bar{B}_{[6]} i\gamma^\alpha (D^\beta B_{[6]})) \mathrm{tr}(U_\beta U_\alpha) + \mathrm{H.c.}) - \frac{1}{4} g^{(V)}_{1,[66]} \mathrm{tr}(\bar{B}_{[6]} i\gamma^\alpha U_\beta (D^\beta B_{[6]}) U^T_\alpha \\
& + \bar{B}_{[6]} i\gamma^\alpha U_\alpha (D^\beta B_{[6]}) U^T_\beta + \mathrm{H.c.}) - \frac{1}{2} g^{(V)}_{D,[66]} \mathrm{tr}(\bar{B}_{[6]} i\gamma^\alpha \{U_\alpha, U_\beta\} (D^\beta B_{[6]}) + \mathrm{H.c.}) \\
& + \frac{1}{2} h^{(V)}_{0,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} g_{\mu\nu} i\gamma^\alpha (D^\beta B^\nu_{[6]}) \mathrm{tr}(U_\alpha U_\beta) + \mathrm{H.c.}) + \frac{1}{4} h^{(V)}_{1,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} g_{\mu\nu} i\gamma^\alpha U_\beta (D^\beta B^\nu_{[6]}) U^T_\alpha \\
& + \bar{B}^\mu_{[6]} g_{\mu\nu} i\gamma^\alpha U_\alpha (D^\beta B^\nu_{[6]}) U^T_\beta + \mathrm{H.c.}) + \frac{1}{2} h^{(V)}_{2,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} g_{\mu\nu} i\gamma^\alpha \{U_\alpha, U_\beta\} (D^\beta B^\nu_{[6]}) + \mathrm{H.c.}), \\
\mathcal{L}^{(A)} = & f^{(A)}_{0,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} \gamma^\nu \gamma_5 B_{[6]} \mathrm{tr}(U_\nu U_\mu) + \mathrm{H.c.}) + f^{(A)}_{1,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} \gamma^\nu \gamma_5 U_\nu B_{[6]} U^T_\mu + \bar{B}^\mu_{[6]} \gamma^\nu \gamma_5 U_\mu B_{[6]} U^T_\nu + \mathrm{H.c.}) \\
& + f^{(A)}_{D,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} \gamma^\nu \gamma_5 \{U_\mu, U_\nu\} B_{[6]} + \mathrm{H.c.}) + f^{(A)}_{F,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} \gamma^\nu \gamma_5 [U_\mu, U_\nu] B_{[6]} + \mathrm{H.c.}) \\
& + f^{(A)}_{1,[\bar{3}6]} \mathrm{tr}(\bar{B}^\mu_{[6]} \gamma^\nu \gamma_5 U_\nu B_{[\bar{3}]} U^T_\mu - \bar{B}^\mu_{[6]} \gamma^\nu \gamma_5 U_\mu B_{[\bar{3}]} U^T_\nu + \mathrm{H.c.}) \\
& + f^{(A)}_{D,[\bar{3}6]} \mathrm{tr}(\bar{B}^\mu_{[6]} \gamma^\nu \gamma_5 \{U_\mu, U_\nu\} B_{[\bar{3}]} + \mathrm{H.c.}) + f^{(A)}_{F,[\bar{3}6]} \mathrm{tr}(\bar{B}^\mu_{[6]} \gamma^\nu \gamma_5 [U_\mu, U_\nu] B_{[\bar{3}]} + \mathrm{H.c.}), \\
\mathcal{L}^{(T)} = & -g^{(T)}_{F,[\bar{3}\bar{3}]} \mathrm{tr}(\bar{B}_{[\bar{3}]} i\sigma^{\alpha\beta} [U_\alpha, U_\beta] B_{[\bar{3}]}) - g^{(T)}_{1,[\bar{3}6]} \mathrm{tr}(\bar{B}_{[6]} i\sigma^{\alpha\beta} U_\alpha B_{[\bar{3}]} U^T_\beta + \mathrm{H.c.}) \\
& - g^{(T)}_{F,[\bar{3}6]} \mathrm{tr}(\bar{B}_{[6]} i\sigma^{\alpha\beta} [U_\alpha, U_\beta] B_{[\bar{3}]} + \mathrm{H.c.}) - g^{(T)}_{F,[66]} \mathrm{tr}(\bar{B}_{[6]} i\sigma^{\alpha\beta} [U_\alpha, U_\beta] B_{[6]}) + h^{(T)}_{F,[66]} \mathrm{tr}(\bar{B}^\mu_{[6]} g_{\mu\nu} i\sigma^{\alpha\beta} [U_\alpha, U_\beta] B^\nu_{[6]}),
\end{aligned} \tag{9}$$

where possible further terms are redundant owing to flavor identities or the on-shell conditions of spin-$\frac{3}{2}$ fields with $\gamma_\mu B^\mu_{[6]} = 0$ and $\partial_\mu B^\mu_{[6]} = 0$. As compared to [22] we further streamlined the notations and dropped the flavor redundant terms proportional to $g^{(S)}_{1,[\bar{3}6]}$ and $g^{(V)}_{1,[\bar{3}6]}$.

The symmetry-conserving parameters contribute to the current-current correlation function of two time-ordered axial-vector currents

$$A^{ab}_{\mu\nu}(q) = i \int d^4x e^{-iq\cdot x} \mathcal{T} A^{(a)}_\mu(x) A^{(b)}_\nu(0), \qquad (10)$$

in the baryon states.

The specific form of the matrix elements of the current-current correlation functions (7) and (10) was already worked out in the previous work [22]. The matrix elements are detailed in the flavor SU(3) limit where the physical baryon states are specified by the momentum p and the flavor indices i, $j = 1, 2, 3$.

III. PRIMER ON LARGE-N_c OPERATOR ANALYSIS

The low-energy constants recalled in (6) and (8) can be analyzed systematically in the $1/N_c$ expansion [21–23,25,26]. Leading-order results have already been worked out in [22]. Here we extend these results to the next accuracy level.

The large-N_c operator expansion is performed in terms of a complete set of static and color-neutral one-body operators that act on effective baryon states rather than the physical states [21,22,24–26]. In our case, the physical and effective baryon states,

$$|p, ij_\pm, S, \chi\rangle, \qquad |ij_\pm, S, \chi), \qquad (11)$$

are specified by the momentum p and the flavor indices i, j, $k = 1, 2, 3$. The spin S and the spin-polarization are $\chi = 1, 2$ for the spin one-half ($S = 1/2$) and $\chi = 1, \ldots, 4$ for the spin three-half states ($S = 3/2$). The flavor sextet and the antitriplet are discriminated by their symmetric (index $+$) and antisymmetric (index $-$) behavior under the exchange of $i \leftrightarrow j$. At leading order in the $1/N_c$ expansion all considered baryon states are mass degenerate. The generic form of the operator expansion takes the form

$$\langle \bar p, mn_\pm, \bar S, \bar\chi | \mathcal{O}_{\rm QCD} | p, kl_\pm, S, \chi\rangle = \sum_{n=0}^{\infty} c_n(\bar p, p)(mn_\pm, \bar S, \bar \chi | \mathcal{O}^{(n)}_{\rm static} | kl_\pm, S, \chi). \qquad (12)$$

It is important to note that unlike the physical baryon states, the effective baryon states do not depend on the momentum p. All dynamical information is moved into appropriate coefficient functions $c_n(\bar p, p)$. The contributions on the right-hand side of (12) can be sorted according to their relevance at large N_c.

The effective baryon states $|ij_\pm, \chi)$ have a mean-field structure that can be generated in terms of effective quark operators q and Q for the light and heavy species respectively. A corresponding complete set of color-neutral one-body operators may be constructed in terms of the very same static quark operators,

$$\mathbb{1} = q^\dagger(\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1})q, \qquad J_i = q^\dagger\left(\frac{\sigma_i}{2}\otimes\mathbf{1}\otimes\mathbf{1}\right)q,$$
$$T^a = q^\dagger\left(\mathbf{1}\otimes\frac{\lambda_a}{2}\otimes\mathbf{1}\right)q, \qquad G^a_i = q^\dagger\left(\frac{\sigma_i}{2}\otimes\frac{\lambda_a}{2}\otimes\mathbf{1}\right)q,$$
$$\mathbb{1}_Q = Q^\dagger(\mathbf{1}\otimes\mathbf{1})Q, \qquad J^i_Q = Q^\dagger\left(\frac{\sigma_i}{2}\otimes\mathbf{1}\right)Q, \qquad (13)$$

with static operators $q = (u, d, s)^T$ and $Q = c$ of the up, down, strange and charm quarks. With λ_a we denote the Gell-Mann matrices supplemented with a singlet matrix $\lambda_0 = \sqrt{2/3}\mathbf{1}$. Here we use a redundant notation with

$$T^0 = \sqrt{\frac{1}{6}}\mathbb{1}, \qquad G^0_i = \sqrt{\frac{1}{6}}J_i, \qquad (14)$$

which will turn useful when analyzing matrix elements of scalar currents.

In the sum of (12) there are infinitely many terms one may write down. The static operators $\mathcal{O}^{(n)}_{\rm static}$ are finite products of the one-body operators J_i, T^a and G^a_i. Terms that break the heavy-quark spin symmetry are exclusively caused by the heavy-spin operator

$$J^i_Q \sim \frac{1}{M_Q}, \qquad (15)$$

with the heavy-quark mass M_Q. In contrast the counting of N_c factors is intricate since there is a subtle balance of suppression and enhancement effects. An r-body operator consisting of the r products of any of the spin and flavor operators receives the suppression factor N_c^{-r}. This is counteracted by enhancement factors for the flavor and spin-flavor operators T^a and G^a_i that are produced by taking baryon matrix elements at $N_c \neq 3$. Altogether, this leads to the effective scaling laws [25,26]:

$$J_i \sim \frac{1}{N_c}, \qquad T^a \sim N_c^0, \qquad G^a_i \sim N_c^0. \qquad (16)$$

According to (16) there is an infinite number of terms contributing at a given order in the $1/N_c$ expansion. Taking higher products of flavor and spin-flavor operators does not reduce the N_c scaling power. A systematic $1/N_c$ expansion is made possible by a set of operator identities [21,25,26], that allows a systematic summation of the infinite number of relevant terms. This can be summarized into two reduction rules:

(i) All operator products in which two flavor indices are contracted using δ_{ab}, f_{abc} or d_{abc} or two spin indices on G's are contracted using δ_{ij} or ϵ_{ijk} can be eliminated.

(ii) All operator products in which two flavor indices are contracted using symmetric or antisymmetric combinations of two different d and/or f symbols can be eliminated. The only exception to this rule is the antisymmetric combination $f_{acg}d_{bch} - f_{bcg}d_{ach}$.

As a consequence the infinite tower of spin-flavor operators truncates at any given order in the $1/N_c$ expansion. We can now turn to the $1/N_c$ expansion of the baryon matrix elements of the QCD's axial-vector and scalar currents. In application of the operator reduction rules, the baryon matrix elements of time-ordered products of the current operators are expanded in powers of the effective one-body operators according to the counting rule (15), (16) supplemented by the reduction rules. In contrast to Jenkins [26], we consider the ratio $N_l/N_c = 1 - 1/N_c$ not as a suppression factor. The strength of the spin-symmetry breaking terms we estimate with $1/M_Q \sim 1/N_c$. In the course of the construction of the various structures, parity and time-reversal transformation properties are taken into account.

All that is needed in any practical application of the $1/N_c$ expansion is the action of any of the one-body operators introduced in (13) on the effective mean-field-type baryon states $|ij_\pm, \chi)$. In fact, it suffices to provide results at the physical value $N_c = 3$, for which a complete list was already generated in [22]. We exemplify such results with

$$
\begin{aligned}
J^k_Q\left|ij_+, \frac{1}{2}, \chi\right) &= -\frac{1}{6}\sigma^{(k)}_{\bar\chi\chi}\left|ij_+, \frac{1}{2}, \bar\chi\right) + \frac{1}{\sqrt{3}}S^{(k)}_{\bar\chi\chi}\left|ij_+, \frac{3}{2}, \bar\chi\right),\\
J^k_Q\left|ij_+, \frac{3}{2}, \chi\right) &= \frac{1}{2}(\vec S\sigma^{(k)}\vec S^\dagger)_{\bar\chi\chi}\left|ij_+, \frac{3}{2}, \bar\chi\right) + \frac{1}{\sqrt{3}}S^{(k)\dagger}_{\bar\chi\chi}\left|ij_+, \frac{1}{2}, \bar\chi\right),\\
J^k_Q\left|ij_-, \frac{1}{2}, \chi\right) &= \frac{1}{2}\sigma^{(k)}_{\bar\chi\chi}\left|ij_-, \frac{1}{2}, \bar\chi\right),\\
J_k\left|ij_+, \frac{1}{2}, \chi\right) &= \frac{2}{3}\sigma^{(k)}_{\bar\chi\chi}\left|ij_+, \frac{1}{2}, \bar\chi\right) - \frac{1}{\sqrt{3}}S^{(k)}_{\bar\chi\chi}\left|ij_+, \frac{3}{2}, \bar\chi\right),\\
J_k\left|ij_+, \frac{3}{2}, \chi\right) &= (\vec S\sigma^{(k)}\vec S^\dagger)_{\bar\chi\chi}\left|ij_+, \frac{3}{2}, \bar\chi\right) - \frac{1}{\sqrt{3}}S^{(k)\dagger}_{\bar\chi\chi}\left|ij_+, \frac{1}{2}, \bar\chi\right),\\
J_k\left|ij_-, \frac{1}{2}, \chi\right) &= 0,
\end{aligned} \tag{17}
$$

where we apply the spin matrices $\sigma^{(k)}$ and $S^{(k)}$ in the convention as used in [22]. Note that an error in the action of the heavy-spin operator J^k_Q on the $|ij_+, \frac{1}{2}, \chi)$ states is corrected here in (17). We affirm that now, with (17), the relations

$$
\begin{aligned}
&[J_i, J_j] = i\epsilon_{ijk}J_k, \quad [J^i_Q, J^j_Q] = i\epsilon_{ijk}J^k_Q, \quad \{J^i_Q, J^i_Q\} = \frac{3}{2}\mathbb{1}_Q,\\
&[J^i_Q, J_j] = 0, \qquad [J^i_Q, G^a_j] = 0, \qquad [J^i_Q, T^a] = 0
\end{aligned} \tag{18}
$$

hold if matrix elements in the charmed baryon states as introduced in (11) are taken. The latter were affected by the error made in [22]. Further corrections for matrix elements of the anticommutator of two one-body operators are considered in the Appendix.

IV. TWO SCALAR CURRENTS IN CHARMED BARYON MATRIX ELEMENTS

We turn to a derivation of large-N_c sum rules for the chiral-symmetry breaking low-energy constants introduced in (6). They contribute to the time-ordered product of two scalar currents as evaluated in the baryon states. At N^{2}LO in the $1/N_c$ expansion, we find the relevance of 11 operators

$$
\begin{aligned}
\langle \bar p, mn_\pm, \bar S, \bar\chi|S^{ab}(q)|p, kl_\pm, S, \chi\rangle &= (mn_\pm, \bar S, \bar\chi|\mathcal{O}^{ab}|kl_\pm, S, \chi),\\
\mathcal{O}^{ab} &= \hat c_1\delta_{a0}\delta_{b0}\mathbb{1} + \hat c_2\delta_{ab}\mathbb{1} + \hat c_3(T_a\delta_{b0} + \delta_{a0}T_b) + \hat c_4 d_{abe}T_e + \hat c_5\{T_a, T_b\}\\
&\quad + \hat c_6 d_{abe}\{J^i, G^i_e\} + \hat c_7(\{J^i, G^a_i\}\delta_{b0} + \delta_{a0}\{J^i, G^i_b\})\\
&\quad + \hat c_8(\{J^i, \{T_a, G^i_b\}\} + \{J^i, \{T_b, G^i_a\}\})\\
&\quad + \hat c_9 d_{abe}\{J^i_Q, G^i_e\} + \hat c_{10}(\{J^i_Q, G^a_i\}\delta_{b0} + \delta_{a0}\{J^i_Q, G^i_b\})\\
&\quad + \hat c_{11}(\{J^i_Q, \{T_a, G^i_b\}\} + \{J^i_Q, \{T_b, G^i_a\}\}) + \cdots,
\end{aligned} \tag{19}
$$

where only the first five operators that are required at NLO were considered previously in [22]. We find six additional terms either involving the spin operators J_i or J^i_Q. The three sums in (19) run over $e = 1, \ldots, 8$.

TABLE I. The symmetry-breaking two-body counterterms as introduced in (6) are correlated by the large-N_c operators (19).

$c_{1,[66]}$	$\frac{2}{9}(9\hat{c}_2 - 3\hat{c}_4 - 6\hat{c}_6 - \hat{c}_9)$	$e_{1,[66]}$	$\frac{1}{3}(6\hat{c}_2 - 2\hat{c}_4 - 4\hat{c}_6 - \hat{c}_9)$
$c_{2,[66]}$	$\frac{2}{3}\hat{c}_1$	$e_{2,[66]}$	$\frac{2}{3}\hat{c}_1$
$c_{3,[66]}$	$\frac{2}{3}\sqrt{\frac{2}{3}}(3\hat{c}_3 + 6\hat{c}_7 + \hat{c}_{10})$	$e_{3,[66]}$	$\sqrt{\frac{2}{3}}(2\hat{c}_3 + 4\hat{c}_7 + \hat{c}_{10})$
$c_{4,[66]}$	$\frac{2}{3}(3\hat{c}_4 + 3\hat{c}_5 + 6\hat{c}_6 + 12\hat{c}_8 + \hat{c}_9 + 2\hat{c}_{11})$	$e_{4,[66]}$	$2\hat{c}_4 + 2\hat{c}_5 + 4\hat{c}_6 + 8\hat{c}_8 + \hat{c}_9 + 2\hat{c}_{11}$
$c_{5,[66]}$	$\frac{2}{3}(3\hat{c}_5 + 12\hat{c}_8 + 2\hat{c}_{11})$	$e_{5,[66]}$	$2(\hat{c}_5 + 4\hat{c}_8 + \hat{c}_{11})$
$c_{1,[\bar{3}\bar{3}]}$	$\frac{1}{3}(6\hat{c}_2 - 2\hat{c}_4 + 3\hat{c}_5)$	$c_{4,[\bar{3}\bar{3}]}$	$2(\hat{c}_4 - \hat{c}_5)$
$c_{2,[\bar{3}\bar{3}]}$	$\frac{1}{3}(2\hat{c}_1 - 3\hat{c}_5)$	$c_{1,[\bar{3}6]}$	$-\frac{5\sqrt{2}}{3}\hat{c}_{10}$
$c_{3,[\bar{3}\bar{3}]}$	$2\sqrt{\frac{2}{3}}(\hat{c}_3 + \sqrt{6}\hat{c}_5)$	$c_{2,[\bar{3}6]}$	$-\frac{5}{\sqrt{3}}(\hat{c}_9 + 2\hat{c}_{11})$

The operator truncation (19) can be matched to the tree-level Lagrangian (6). For this the matrix elements of the operators in (19) are derived in the Appendix and [22]. Altogether, we claim the identifications as detailed in Table I.

At NLO with $\hat{c}_{6-10} = 0$ there are $16 - 5 = 11$ sum rules. No spin-symmetry breaking operator J_Q has to be considered at this accuracy level. In turn, we recover the eight heavy-spin symmetry relations in (59) from [22]. Additional relations arise from the large-N_c considerations. We correct two misprints in (60) of [22]. Altogether, the following set of sum rules arises:

$$
\begin{aligned}
&c_{n,[66]} = e_{n,[66]} \quad \text{for } n = 1, \dots, 5, \\
&c_{n,[\bar{3}6]} = 0 \quad \text{for } n = 1, 2, \\
&c_{1,[\bar{3}\bar{3}]} = c_{1,[66]} + \frac{1}{2}c_{5,[66]}, \qquad c_{2,[\bar{3}\bar{3}]} = c_{2,[66]} - \frac{1}{2}c_{5,[66]}, \\
&c_{3,[\bar{3}\bar{3}]} = c_{3,[66]} + 2c_{5,[66]}, \quad c_{4,[\bar{3}\bar{3}]} = c_{4,[66]} - 2c_{5,[66]}.
\end{aligned} \tag{20}
$$

At N^{2}LO, all 11 operators in (19) turn relevant, and we have $16 - 11 = 5$ sum rules:

$$
\begin{aligned}
c_{1,[\bar{3}6]} &= 5\sqrt{3}(c_{3,[66]} - e_{3,[66]}), \\
c_{2,[66]} &= e_{2,[66]}, \\
c_{2,[\bar{3}6]} &= 5\sqrt{3}(c_{4,[66]} - e_{4,[66]}), \\
3c_{1,[\bar{3}\bar{3}]} + c_{2,[\bar{3}\bar{3}]} + c_{4,[\bar{3}\bar{3}]} &= 3c_{1,[66]} + c_{2,[66]} + c_{4,[66]} - c_{5,[66]}, \\
3e_{1,[66]} + e_{4,[66]} - e_{5,[66]} &= 3c_{1,[66]} + c_{4,[66]} - c_{5,[66]}.
\end{aligned} \tag{21}
$$

V. TWO AXIAL CURRENTS IN CHARMED BARYON MATRIX ELEMENTS

We study the time-ordered product of two axial-vector currents. The large-N_c operator expansion was already worked out in [21] at leading order. Matrix elements in charmed baryons were derived in [22]. Here we consider and derive the implication for the chiral two-body interactions introduced (8) at subleading orders in this expansion. At NLO, there are 19 distinct operators to be considered,

$$
\begin{aligned}
\langle \bar{p}, mn_\pm, \bar{S}, \bar{\chi} | A_{ij}^{ab}(q) | p, kl_\pm, S, \chi \rangle &= (mn_\pm, \bar{S}, \bar{\chi} | \mathcal{O}_{ij}^{ab} | kl_\pm, S, \chi), \\
\mathcal{O}_{ij}^{ab} = -\delta_{ij}[&\hat{g}_1 \delta_{ab} \mathbb{1} + \hat{g}_2 d_{abc} T_c + \hat{g}_3 \{T_a, T_b\} + \hat{g}_4 d_{abc} \{J^k, G_c^k\} \\
&+ \hat{g}_5 (\{J^k, \{T_a, G_b^k\}\} + \{J^k, \{G_a^k, T_b\}\}) + \hat{g}_6 d_{abc} \{J_Q^k, G_c^k\} \\
&+ \hat{g}_7 (\{J_Q^k, \{T_a, G_b^k\}\} + \{J_Q^k, \{G_a^k, T_b\}\})] \\
&+ \hat{g}_8 (\{G_a^i, G_b^j\} + \{G_a^j, G_b^i\}) + \hat{g}_9 (\{G_i^a, G_j^b\} - \{G_j^a, G_i^b\}) \\
&+ \epsilon_{ijk} f_{abc} (\hat{g}_{10} G_k^c + \hat{g}_{11} \{J^k, T_c\} + \hat{g}_{12} \{J_Q^k, T_c\}) \\
&+ (\bar{p} + p)_i (\bar{p} + p)_j [\hat{g}_{13} \delta_{ab} \mathbb{1} + \hat{g}_{14} d_{abc} T_c + \hat{g}_{15} \{T_a, T_b\} \\
&+ \hat{g}_{16} d_{abc} \{J^k, G_c^k\} + \hat{g}_{17} (\{J^k, \{T_a, G_b^k\}\} + \{J^k, \{G_a^k, T_b\}\}) \\
&+ \hat{g}_{18} d_{abc} \{J_Q^k, G_c^k\} + \hat{g}_{19} (\{J_Q^k, \{T_a, G_b^k\}\} + \{J_Q^k, \{G_a^k, T_b\}\})] + \cdots,
\end{aligned} \tag{22}
$$

where we focus on the space components of the correlation function. In (22), we have $q = \bar{p} - p$ and $a, b = 1, \dots, 8$. In addition, we consider only terms that arise in the small-momentum expansion and that are required for the desired matching with (8). The dots in (22) represent additional terms that are further suppressed in the $1/N_c$ expansion or for small 3-momenta p and $\bar{p}$.

TABLE II. The symmetry-conserving two-body counterterms as introduced in (8) are correlated by the large-N_c operators (22) with $\hat{g}^{(V)}_{n,[ab]} = 2g^{(V)}_{n,[ab]}/(M^{(1/2)}_{[a]} + M^{(1/2)}_{[b]})$ and $\hat{h}^{(V)}_{n,[aa]} = h^{(V)}_{n,[aa]}/M^{(3/2)}_{[a]}$. In the flavor SU(3) chiral limit we use here three distinct charm baryon masses $M^{(J=1/2)}_{[\bar{3}]}$, $M^{(J=1/2)}_{[6]}$ and $M^{(J=3/2)}_{[6]}$. Only the combinations $g^{(V)}_{D,[\bar{3}\,\bar{3}]} - 2g^{(V)}_{1,[\bar{3}\,\bar{3}]}$ and $g^{(V)}_{0,[\bar{3}\,\bar{3}]} + g^{(V)}_{1,[\bar{3}\,\bar{3}]}$ can be matched at leading order in a nonrelativistic expansion.

$g^{(S)}_{0,[\bar{3}\,\bar{3}]}$	$2\hat{g}_1 - \frac{2}{3}\hat{g}_2 + \hat{g}_3 + \frac{1}{2}\hat{g}_8$	$\hat{g}^{(V)}_{0,[\bar{3}\,\bar{3}]}$	$-\hat{g}^{(V)}_{1,[\bar{3}\,\bar{3}]} + 8\hat{g}_{13} - \frac{8}{3}\hat{g}_{14} + 4\hat{g}_{15}$
$g^{(T)}_{F,[\bar{3}\,\bar{3}]}$	$-\hat{g}_{12}$		
$g^{(S)}_{D,[\bar{3}\,\bar{3}]}$	$\hat{g}_2 - \hat{g}_3 - \frac{3}{2}\hat{g}_8$	$\hat{g}^{(V)}_{D,[\bar{3}\,\bar{3}]}$	$2\hat{g}^{(V)}_{1,[\bar{3}\,\bar{3}]} + 4(\hat{g}_{14} - \hat{g}_{15})$
$g^{(S)}_{0,[66]}$	$2\hat{g}_1 - \frac{2}{3}\hat{g}_2$	$\hat{g}^{(V)}_{0,[66]}$	$\frac{8}{3}(3\hat{g}_{13} - \hat{g}_{14})$
$g^{(S)}_{1,[66]}$	$2\hat{g}_3 + 8\hat{g}_5 - 4\hat{g}_7 - \frac{1}{3}\hat{g}_8$	$\hat{g}^{(V)}_{1,[66]}$	$8(\hat{g}_{15} + 4\hat{g}_{17} - 2\hat{g}_{19})$
$g^{(S)}_{D,[66]}$	$\hat{g}_2 + \hat{g}_3 + 2\hat{g}_4 + 4\hat{g}_5 - \hat{g}_6 - 2\hat{g}_7 - \frac{1}{2}\hat{g}_8$	$\hat{g}^{(V)}_{D,[66]}$	$4(\hat{g}_{14} + \hat{g}_{15} + 2\hat{g}_{16} + 4\hat{g}_{17} - \hat{g}_{18} - 2\hat{g}_{19})$
$g^{(T)}_{F,[\bar{3}6]}$	$\frac{1}{2\sqrt{3}}(-\hat{g}_9 + \hat{g}_{10})$	$g^{(T)}_{F,[66]}$	$\frac{1}{3}(\hat{g}_9 - \hat{g}_{10} - 4\hat{g}_{11} + \hat{g}_{12})$
$g^{(T)}_{1,[\bar{3}6]}$	$\frac{1}{\sqrt{3}}\hat{g}_9$	$f^{(A)}_{1,[\bar{3}6]}$	$-\hat{g}_9$
		$f^{(A)}_{D,[\bar{3}6]}$	0
$\hat{g}^{(V)}_{D,[\bar{3}6]}$	$-2\sqrt{3}(\hat{g}_{18} + 2\hat{g}_{19})$	$f^{(A)}_{F,[\bar{3}6]}$	$-\hat{g}_9 + \hat{g}_{10}$
$g^{(S)}_{D,[\bar{3}6]}$	$-\frac{\sqrt{3}}{2}(\hat{g}_6 + 2\hat{g}_7)$	$\hat{h}^{(V)}_{0,[66]}$	$8\hat{g}_{13} - \frac{8}{3}\hat{g}_{14}$
$h^{(S)}_{0,[66]}$	$2\hat{g}_1 - \frac{2}{3}\hat{g}_2$	$\hat{h}^{(V)}_{1,[66]}$	$8(\hat{g}_{15} + 4\hat{g}_{17} + \hat{g}_{19})$
$h^{(S)}_{1,[66]}$	0	$\hat{h}^{(V)}_{2,[66]}$	$2(2\hat{g}_{14} + 2\hat{g}_{15} + 4\hat{g}_{16} + 8\hat{g}_{17} + \hat{g}_{18} + 2\hat{g}_{19})$
$h^{(S)}_{2,[66]}$	$\hat{g}_2 + \hat{g}_3 + 2\hat{g}_4 + 4\hat{g}_5 + \frac{1}{2}\hat{g}_6 + \hat{g}_7 - \frac{1}{2}\hat{g}_8$	$f^{(A)}_{0,[66]}$	0
$h^{(S)}_{3,[66]}$	0	$f^{(A)}_{1,[66]}$	$\frac{1}{\sqrt{3}}\hat{g}_8$
$h^{(S)}_{4,[66]}$	$2\hat{g}_3 + 8\hat{g}_5 + 2\hat{g}_7 - \hat{g}_8$	$f^{(A)}_{D,[66]}$	0
$h^{(S)}_{5,[66]}$	$\hat{g}_8$	$f^{(A)}_{F,[66]}$	$\frac{1}{\sqrt{3}}(-\hat{g}_9 + \hat{g}_{10} + 4\hat{g}_{11} - 4\hat{g}_{12})$
$h^{(T)}_{F,[66]}$	$\frac{1}{2}(\hat{g}_9 - \hat{g}_{10} - 4\hat{g}_{11} - 2\hat{g}_{12})$		

In the previous work [22], only seven leading-order operators were considered. The ansatz of this work is reproduced with

$$\hat{g}_1 = \frac{1}{3}\hat{g}_2, \qquad \hat{g}_{4-7} = 0, \qquad \hat{g}_{11-12} = 0, \qquad \hat{g}_{13} = \frac{1}{3}\hat{g}_{14}, \qquad \hat{g}_{16-19} = 0. \tag{23}$$

An application of the results of our Appendix leads to the matching result as detailed in Table II.

From the operator analysis (22), we obtain $33 - 7 = 26$ sum rules. We do not reproduce all sum rules as considered first in [22]. In our analysis, we unravel two misprint in (62) of [22]. A complete list of relation reads

$$\begin{aligned}
&g^{(S)}_{D,[\bar{3}\,\bar{3}]} = h^{(S)}_{2,[66]} - h^{(S)}_{4,[66]} - 2h^{(S)}_{5,[66]}, \qquad g^{(S)}_{0,[\bar{3}\,\bar{3}]} = \frac{1}{2}h^{(S)}_{4,[66]} + h^{(S)}_{5,[66]},\\
&g^{(S)}_{0,[66]} = g^{(S)}_{D,[\bar{3}6]} = 0, \qquad g^{(S)}_{1,[66]} = h^{(S)}_{4,[66]} + \frac{2}{3}h^{(S)}_{5,[66]}, \qquad g^{(S)}_{D,[66]} = h^{(S)}_{2,[66]}, \qquad h^{(S)}_{0,[66]} = h^{(S)}_{1,[66]} = h^{(S)}_{3,[66]} = 0,\\
&\hat{g}^{(V)}_{0,[\bar{3}\,\bar{3}]} = -\hat{g}^{(V)}_{1,[\bar{3}\,\bar{3}]} + \frac{1}{2}\hat{h}^{(V)}_{1,[66]}, \qquad \hat{g}^{(V)}_{D,[\bar{3}\,\bar{3}]} = 2\hat{g}^{(V)}_{1,[\bar{3}\,\bar{3}]} - \hat{h}^{(V)}_{1,[66]} + \hat{h}^{(V)}_{2,[66]},\\
&\hat{g}^{(V)}_{0,[66]} = \hat{g}^{(V)}_{D,[\bar{3}6]} = \hat{h}^{(V)}_{0,[66]} = 0, \qquad \hat{g}^{(V)}_{1,[66]} = \hat{h}^{(V)}_{1,[66]}, \qquad \hat{g}^{(V)}_{D,[66]} = \hat{h}^{(V)}_{2,[66]},\\
&f^{(A)}_{1,[\bar{3}6]} = -\sqrt{3}g^{(T)}_{1,[\bar{3}6]}, \qquad f^{(A)}_{F,[\bar{3}6]} = -2h^{(T)}_{F,[66]}, \qquad f^{(A)}_{D,[\bar{3}6]} = f^{(A)}_{0,[66]} = f^{(A)}_{D,[66]} = 0,\\
&f^{(A)}_{1,[66]} = \frac{1}{\sqrt{3}}h^{(S)}_{5,[66]}, \qquad f^{(A)}_{F,[66]} = -\frac{2}{\sqrt{3}}h^{(T)}_{F,[66]}, \qquad g^{(T)}_{F,[\bar{3}\,\bar{3}]} = 0, \qquad g^{(T)}_{F,[\bar{3}6]} = -\frac{1}{\sqrt{3}}h^{(T)}_{F,[66]}, \qquad g^{(T)}_{F,[66]} = \frac{2}{3}h^{(T)}_{F,[66]},
\end{aligned} \tag{24}$$

where the two identities in the fourth line of (24) were not presented correctly in [22]. We confirm the result of [22] that the combination of the heavy spin-symmetry sum rules as summarized in (41) of [22] with the large-N_c sum rules (24) does lead to one extra relation:

$$f^{(A)}_{1,[66]} = 0. \tag{25}$$

As argued already in [22], this does not contradict the systematics of the large-N_c operator expansion. Though the operator analysis is not predicting such a feature, it can not be excluded.

We turn to the central result of our work. At NLO, we have derived $33 - 19 = 14$ novel sum rules:

$$\begin{aligned}
&g^{(S)}_{D,[\bar{3}6]} = \frac{1}{\sqrt{3}}(g^{(S)}_{D,[66]} - h^{(S)}_{2,[66]}), \qquad g^{(S)}_{0,[66]} = h^{(S)}_{0,[66]}, \qquad h^{(S)}_{1,[66]} = h^{(S)}_{3,[66]} = 0,\\
&\hat{g}^{(V)}_{D,[\bar{3}6]} = \frac{1}{\sqrt{3}}(\hat{g}^{(V)}_{D,[66]} - \hat{h}^{(V)}_{2,[66]}), \qquad \hat{g}^{(V)}_{0,[66]} = \hat{h}^{(V)}_{0,[66]},\\
&f^{(A)}_{1,[\bar{3}6]} = -\sqrt{3}g^{(T)}_{1,[\bar{3}6]}, \qquad f^{(A)}_{1,[66]} = \frac{1}{\sqrt{3}}h^{(S)}_{5,[66]}, \qquad f^{(A)}_{D,[\bar{3}6]} = f^{(A)}_{0,[66]} = f^{(A)}_{D,[66]} = 0,\\
&f^{(A)}_{F,[\bar{3}6]} = 2\sqrt{3}g^{(T)}_{F,[\bar{3}6]}, \qquad f^{(A)}_{F,[66]} = -2\sqrt{3}g^{(T)}_{F,[66]} + \frac{2}{\sqrt{3}}h^{(T)}_{F,[66]},\\
&g^{(T)}_{F,[\bar{3}\,\bar{3}]} = -g^{(T)}_{F,[66]} + \frac{2}{3}h^{(T)}_{F,[66]}.
\end{aligned} \tag{26}$$

VI. SUMMARY

In this work, we considered the chiral Lagrangian for charmed baryons based on the flavor SU(3) symmetry. Current-current correlation functions in the baryon states were evaluated at tree level and analyzed in an expansion of $1/N_c$ at subleading orders. Sum rules for the symmetry-breaking and symmetry-conserving low-energy constants are systematically derived and presented. We correct various misprints found in previous works and establish novel sum rules that are valid at subleading orders in the $1/N_c$ expansion. Such parameter correlations are of crucial importance in chiral extrapolation studies of the charmed baryon masses at N^{3}LO, but also they determine, to a large extent, the coupled-channel interaction of the charmed baryons with the set of Goldstone bosons.

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APPENDIX

We consider matrix elements of the symmetric product of two one-body operators $\mathcal{O}$ in the charmed baryon ground state at $N_c = 3$. The generic notation,

$$\langle \mathcal{O} \rangle^{ab}_{\bar{S}S} \equiv (mn_a, \bar{S}, \bar{\chi}|\mathcal{O}|kl_b, S, \chi), \tag{A1}$$

with $a, b = \pm$ will be applied. The results are expressed in terms of the flavor structures $\Lambda^{(a),(rs)_\pm}_{(kl)_+}$ and $\Lambda^{(a),(rs)_\pm}_{(kl)_-}$ and the spin structures σ_i and S_i as introduced in [22]. We correct and supplement the matrix elements that involve the spin operator J_Q with

$$\begin{aligned}
&\langle\{J^i_Q, J_j\}\rangle^{--}_{\frac{1}{2}\frac{1}{2}} = 0, \qquad \langle\{J^i_Q, J^j_Q\}\rangle^{--}_{\frac{1}{2}\frac{1}{2}} = \frac{1}{2}\delta_{ij}\delta_{\bar{\chi}\chi}\delta^{(mn)_-}_{(kl)_-},\\
&\langle\{J^i_Q, T^a\}\rangle^{--}_{\frac{1}{2}\frac{1}{2}} = \sigma^{(i)}_{\bar{\chi}\chi}\Lambda^{(a),(mn)_-}_{(kl)_-}, \qquad \langle\{J^i_Q, G^a_j\}\rangle^{--}_{\frac{1}{2}\frac{1}{2}} = 0,\\
&\langle\{J^i_Q, \{T^a, G^b_i\}\}\rangle^{--}_{\frac{1}{2}\frac{1}{2}} = 0,\\
&\langle\{J^i_Q, J_j\}\rangle^{+-}_{\frac{1}{2}\frac{1}{2}} = 0, \qquad \langle\{J^i_Q, J^j_Q\}\rangle^{+-}_{\frac{1}{2}\frac{1}{2}} = 0,\\
&\langle\{J^i_Q, T^a\}\rangle^{+-}_{\frac{1}{2}\frac{1}{2}} = 0, \qquad \langle\{J^i_Q, G^a_j\}\rangle^{+-}_{\frac{1}{2}\frac{1}{2}} = -\frac{1}{2\sqrt{3}}(\delta_{ij} - i\epsilon_{ijk}\sigma_k)_{\bar{\chi}\chi}\Lambda^{(a),(mn)_+}_{(kl)_-},
\end{aligned}$$

$$
\begin{aligned}
\langle\{J^i_Q,\{T^a,G^b_i\}\}\rangle^{+-}_{\frac{11}{22}} &= -\frac{\sqrt{3}}{2}\delta_{\bar\chi\chi}(\Lambda^{(a),(mn)_+}_{(rs)_+}\Lambda^{(b),(rs)_+}_{(kl)_-}+\Lambda^{(b),(mn)_+}_{(rs)_-}\Lambda^{(a),(rs)_-}_{(kl)_-}),\\
\langle\{J^i_Q,J_j\}\rangle^{++}_{\frac{11}{22}} &= -\frac{2}{3}\delta_{ij}\delta_{\bar\chi\chi}\delta^{(mn)_+}_{(kl)_+}, \qquad \langle\{J^i_Q,J^j_Q\}\rangle^{++}_{\frac{11}{22}} = \frac{1}{2}\delta_{ij}\delta_{\bar\chi\chi}\delta^{(mn)_+}_{(kl)_+},\\
\langle\{J^i_Q,T^a\}\rangle^{++}_{\frac{11}{22}} &= -\frac{1}{3}\sigma^{(i)}_{\bar\chi\chi}\Lambda^{(a),(mn)_+}_{(kl)_+}, \qquad \langle\{J^i_Q,G^a_j\}\rangle^{++}_{\frac{11}{22}} = -\frac{1}{3}\delta_{ij}\delta_{\bar\chi\chi}\Lambda^{(a),(mn)_+}_{(kl)_+},\\
\langle\{J^i_Q,\{T^a,G^b_i\}\}\rangle^{++}_{\frac{11}{22}} &= -\delta_{\bar\chi\chi}\left(\Lambda^{(a),(mn)_+}_{(rs)_+}\Lambda^{(b),(rs)_+}_{(kl)_+}+\Lambda^{(b),(mn)_+}_{(rs)_+}\Lambda^{(a),(rs)_+}_{(kl)_+}\right),\\
\langle\{J^i_Q,J_j\}\rangle^{+-}_{\frac{31}{22}} &= 0, \qquad \langle\{J^i_Q,J^j_Q\}\rangle^{+-}_{\frac{31}{22}} = 0,\\
\langle\{J^i_Q,T^a\}\rangle^{+-}_{\frac{31}{22}} &= 0, \qquad \langle\{J^i_Q,G^a_j\}\rangle^{+-}_{\frac{31}{22}} = -\frac{1}{2}(S_j\sigma_i)_{\bar\chi\chi}\Lambda^{(a),(mn)_+}_{(kl)_-},\\
\langle\{J^i_Q,\{T^a,G^b_i\}\}\rangle^{+-}_{\frac{31}{22}} &= 0,\\
\langle\{J^i_Q,J_j\}\rangle^{++}_{\frac{31}{22}} &= \frac{1}{\sqrt{3}}(2S_i\sigma_j-S_j\sigma_i)_{\bar\chi\chi}\delta^{(mn)_+}_{(kl)_+}, \qquad \langle\{J^i_Q,J^j_Q\}\rangle^{++}_{\frac{31}{22}} = 0,\\
\langle\{J^i_Q,T^a\}\rangle^{++}_{\frac{31}{22}} &= \frac{2}{\sqrt{3}}S^{(i)}_{\bar\chi\chi}\Lambda^{(a),(mn)_+}_{(kl)_+},\\
\langle\{J^i_Q,G^a_j\}\rangle^{++}_{\frac{31}{22}} &= \frac{1}{2\sqrt{3}}(2S_i\sigma_j-S_j\sigma_i)_{\bar\chi\chi}\Lambda^{(a),(mn)_+}_{(kl)_+},\\
\langle\{J^i_Q,\{T^a,G^b_i\}\}\rangle^{++}_{\frac{31}{22}} &= 0,\\
\langle\{J^i_Q,J_j\}\rangle^{++}_{\frac{33}{22}} &= (\delta_{ij}-S_iS^\dagger_j-S_jS^\dagger_i)_{\bar\chi\chi}\delta^{(mn)_+}_{(kl)_+}, \qquad \langle\{J^i_Q,J^j_Q\}\rangle^{++}_{\frac{33}{22}} = \frac{1}{2}\delta_{ij}\delta_{\bar\chi\chi}\delta^{(mn)_+}_{(kl)_+},\\
\langle\{J^i_Q,T^a\}\rangle^{++}_{\frac{33}{22}} &= (\vec{S}\sigma_i\vec{S}^\dagger)_{\bar\chi\chi}\Lambda^{(a),(mn)_+}_{(kl)_+},\\
\langle\{J^i_Q,G^a_j\}\rangle^{++}_{\frac{33}{22}} &= \frac{1}{2}(\delta_{ij}-S_iS^\dagger_j-S_jS^\dagger_i)_{\bar\chi\chi}\Lambda^{(a),(mn)_+}_{(kl)_+},\\
\langle\{J^i_Q,\{T^a,G^b_i\}\}\rangle^{++}_{\frac{33}{22}} &= \frac{1}{2}\delta_{\bar\chi\chi}(\Lambda^{(a),(mn)_+}_{(rs)_+}\Lambda^{(b),(rs)_+}_{(kl)_+}+\Lambda^{(b),(mn)_+}_{(rs)_+}\Lambda^{(a),(rs)_+}_{(kl)_+}),
\end{aligned}
\tag{A2}
$$

where

$$
2S_i\sigma_j - S_j\sigma_i = \frac{1}{2}(S_i\sigma_j+S_j\sigma_i)-\frac{3}{2}i\epsilon_{ijk}S_k. \tag{A3}
$$

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