Revisiting final state interaction in charmless $B_q \to PP$ decays

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Various new measurements in charmless $B_{u,d,s} \to PP$ modes, where P is a low lying pseudoscalar meson, are reported by Belle and LHCb. These include the rates of $B^0 \to \pi^0\pi^0$, $\eta\pi^0$, $B_s \to \eta'\eta'$, $B^0 \to K^+K^-$ and $B_s^0 \to \pi^+\pi^-$ decays. Some of these modes are highly suppressed and are among the rarest B decays. Direct CP asymmetries on various modes are constantly updated. It is well known that direct CP asymmetries and rates of suppressed modes are sensitive to final state interaction (FSI). As new measurements are reported and more data will be collected, it is interesting and timely to revisit the rescattering effects in $B_{u,d,s} \to PP$ states. We perform a χ^2 analysis with all available data on CP-averaged rates and CP asymmetries in $\bar{B}_{u,d,s} \to PP$ decays. Our numerical results are compared to data and those from factorization approach. The quality of the fit is improved significantly from the factorization results in the presence of rescattering. The relations on topological amplitudes and rescattering are explored and they help to provide a better understanding of the effects of FSI. As suggested by U(3) symmetry on topological amplitudes and FSI, a vanishing exchange rescattering scenario is considered. The exchange, annihilation, u-penguin, u-penguin annihilation, and some electroweak penguin amplitudes are enhanced significantly via annihilation and total annihilation rescatterings. In particular, the u-penguin annihilation amplitude is sizably enhanced by the tree amplitude via total annihilation rescattering. These enhancements affect rates and CP asymmetries. Predictions can be checked in the near future.

DOI: 10.1103/PhysRevD.97.093004

I. INTRODUCTION

In recent years there are some experimental progresses on measurements of the charmless $B_{u,d,s} \to PP$ decays. In year 2015, Belle reported a 3.0σ significant measurement on $\bar{B}^0 \to \eta\pi^0$ decay rate with $\mathcal{B}(B^0 \to \eta\pi^0) = (4.1^{+1.7+0.5}_{-1.5-0.7}) \times 10^{-7}$ [1] and $\mathcal{B}(B_s^0 \to K^0\bar{K}^0) = (19.6^{+5.8}_{-5.1} \pm 1.0 \pm 2.0) \times 10^{-6}$ [2] with 5.1σ significance, while LHCb observed $B_s \to \eta'\eta'$ decay at $(3.31 \pm 0.64 \pm 0.28 \pm 0.12) \times 10^{-5}$ at 6.4σ significance [3]. In year 2016, LHCb reported on the observation of annihilation modes with $\mathcal{B}(B^0 \to K^+K^-) = (7.80 \pm 1.27 \pm 0.81 \pm 0.21) \times 10^{-8}$ and $\mathcal{B}(B_s^0 \to \pi^+\pi^-) = (6.91 \pm 0.54 \pm 0.63 \pm 0.19 \pm 0.40) \times 10^{-7}$ [4]. Last year Belle reported the rate of $\bar{B}^0 \to \pi^0\pi^0$ of $\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0) = (1.31 \pm 0.19 \pm 0.18) \times 10^{-6}$ [5]. Some of these modes are highly suppressed and are among the rarest B decays. There were constant updates on other measurements, such as rates and direct CP asymmetries on $B_{(s)} \to K\pi, KK, \pi\pi$ modes [6–8].

It is well known that direct CP asymmetries and rates of suppressed modes are sensitive to final state interaction (FSI) [9,10]. In a study on the effects of FSI on $B_{u,d,s} \to PP$ modes [11], the so called (too large) $\mathcal{B}(\pi^0\pi^0)/\mathcal{B}(\pi^+\pi^-)$ ratio and (nonvanishing) $\Delta\mathcal{A} \equiv \mathcal{A}(K^-\pi^+) - \mathcal{A}(K^-\pi^0)$ direct CP asymmetry puzzles in $\bar{B}_{u,d}$ decays can both be resolved by considering rescattering among PP states.[1] Several rates and CP asymmetries were predicted. The newly observed $B_s^0 \to K^0\bar{K}^0$ rate is consistent with the prediction. However, there are some results that are in tension with the recent measurement. In particular, the predicted $B_s \to \eta'\eta'$ rate is too high compared to data. In fact, its central value is off by a factor of 3. As new measurements are reported and more data will be collected in LHCb, and Belle II will be turned on in the very near future, it is interesting and timely to revisit the subject.

It will be useful to give the physical picture. From the time-invariant property of the Wilson operators in the weak Hamiltonian, one finds that the decay amplitude satisfies [14][2]

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[1]One is referred to [12,13] for some recent analyses on these puzzles.
[2]See Appendix A for a derivation.

$$A_i = \sum_{k=1}^{N} \mathcal{S}_{ik}^{1/2} A_k^0, \qquad (1)$$

where A_i is a $\bar{B}_q \to PP$ decay amplitude with weak as well as strong phases, A_k^0 is a amplitude containing weak phase only, $i = 1, \ldots, n$, denotes all charmless PP states and $k = 1, \ldots, n, n+1, \ldots, N$, denotes *all* possible states that can rescatter into the charmless PP states through the strong interacting S-matrix, $\mathcal{S}$. Strong phases are encoded in the rescattering matrix. This is known as the Watson theorem [15]. There are two points needed to be emphasised. First, the above result is exact. Every $\bar{B}_q \to PP$ decay amplitude should satisfy it. Second, for a typical $\bar{B}_q$ decay, since the B mass is large there is a large number of kinematically allowed states involved in the above equation, i.e., N in the above equation is large. Consequently, the equation is hard to solve.

Although the largeness of the B mass makes it difficult to solve the above equation, it is interesting that on the contrary it is precisely the largeness of m_B that makes the problem somewhat trackable. According to the duality argument, when the contributions from all hadronic states at a large enough energy scale are summed over, one should be able to understand the physics in terms of the quark and gluon degrees of freedom. Indeed, several quantum chromodynamics (QCD)-based factorization approaches, such as pQCD [16], QCD factorization (QCDF) [17,18] and soft collinear effective theory (SCET) [19] make use of the large B mass and give predictions on the factorization amplitudes, A^{fac}. In other words, using the largeness of m_B comparing to Λ_{QCD}, the factorization approaches provide solutions to Eq. (1), i.e., $A_i^{\text{fac}} = \sum_{k=1}^{N} \mathcal{S}_{ik}^{1/2} A_k^0$.

In the infinite m_B limit, the above program may work perfectly. However, in the physical m_B case, power corrections can be important and may not be neglected. In fact, the effects of power corrections are strongly hinted from some unexpected enhancements in rates of several color suppressed modes, such as $\bar{B}^0 \to \pi^0 \pi^0$ decay [6,7], and some unexpected signs of direct CP asymmetries, as in the difference of direct CP asymmetries of $\bar{B}^0 \to \bar{K}^- \pi^+$ and $B^- \to K^- \pi^0$ decays [20]. These anomalies lead to the above mentioned $\pi\pi$ and $K\pi$ puzzles. It is fair to say that the factorization approaches can reasonably produce rates of color allowed modes, but it encounters some difficulties in rates of color-suppressed states and CP asymmeties. It is to plausible to assume that factorization approaches do not give the full solution to Eq. (1), some residual rescattering or residual final state interaction is still allowed and needed in $\bar{B}_q \to PP$ decays. Note that the group of charmless PP states is unique to $\bar{B}_q \to PP$ decays, as P belongs to the same SU(3) multiplet and PP states are well separated from all other states, where the duality argument cannot be applied to these limited number of states [21,22]. Note that residual rescattering among PP modes only slightly affect the rates of color allowed modes, but it can easily change direct CP violation of most modes and the rates of color suppressed modes at the same time. It can be a one stone two birds scenario. It can potentially solve two problems at the same time without affecting the successful results of factorization approach on color allowed rates. In fact, this approach is modest than the factorization approach as it left some rooms for our ignorance on strong dynamics. In the following text, unless indicated otherwise we use rescattering among PP states or rescattering for short to denote this particular type of rescattering, while we assume that FSI contributions from all other states are contained in the factorization amplitudes.

The quark diagram or the so-called topological approach has been extensively used in mesonic modes [9,12,23–27]. It will be useful and interesting to study the FSI effects on topological amplitudes. For some early works in different approach, one is referred to ref. [9]. The relation on topological amplitudes and rescattering will be explored and it can help to provide a better understanding on the effects of residual rescattering.

The layout of the present paper is as follows: In Sec. II we give the formalism. Results and discussions are presented in Sec. IV. Section V contains our conclusions. Some useful formulas and derivations are collected in Appendices A and B.

II. FORMALISM

In this section we will give the rescattering (res.) formulas, topological amplitudes (TA) of $\bar{B}_q \to PP$ decays, and the relations between res. and TA.

A. Rescattering formulas

Most of the following formulas are from [11], but some are new. As noted in the Introduction section, in the rescattering we have (see Appendix A)

$$A_i = \sum_{j=1}^{n} (\mathcal{S}_{\text{res}}^{1/2})_{ij} A_j^{\text{fac}}, \qquad (2)$$

where $i, j = 1, \ldots, n$ denote all charmless PP states. To apply the above formula, we need to specify the factorization amplitudes. In this work, we use the factorization amplitudes obtained in the QCD factorization approach [18].

According to the quantum numbers of the final states, which can be mixed under FSI, $\bar{B}_q \to PP$ decays can be grouped into 4 groups. Explicit formulas are collected in Appendix A. Here we give an example for illustration. The $\bar{B}_d^0 \to K^- \pi^+$ decay can rescatter with three other states, namely $\bar{B}_d^0 \to \bar{K}^0 \pi^0$, $\bar{K}^0 \eta_8$ and $\bar{K}^0 \eta_1$, via charge exchange, singlet exchange and annihilation rescatterings as denoted in Fig. 1(a)–(c). These states are the group-1 modes. The relevant rescattering formula is given by

$$\begin{pmatrix} A_{\bar{B}^0_d \to K^- \pi^+} \\ A_{\bar{B}^0_d \to \bar{K}^0 \pi^0} \\ A_{\bar{B}^0_d \to \bar{K}^0 \eta_8} \\ A_{\bar{B}^0_d \to \bar{K}^0 \eta_1} \end{pmatrix} = \mathcal{S}^{1/2}_{\mathrm{res},1} \begin{pmatrix} A^{\mathrm{fac}}_{\bar{B}^0_d \to K^- \pi^+} \\ A^{\mathrm{fac}}_{\bar{B}^0_d \to \bar{K}^0 \pi^0} \\ A^{\mathrm{fac}}_{\bar{B}^0_d \to \bar{K}^0 \eta_8} \\ A^{\mathrm{fac}}_{\bar{B}^0_d \to \bar{K}^0 \eta_1} \end{pmatrix}, \qquad (3)$$

with $\mathcal{S}^{1/2}_{\mathrm{res},1} = (1 + i\mathcal{T}_1)^{1/2}$ and

$$\mathcal{T}_1 = \begin{pmatrix} r_0 + r_a & \frac{-r_a + r_e}{\sqrt{2}} & \frac{-r_a + r_e}{\sqrt{6}} & \frac{2\bar{r}_a + \bar{r}_e}{\sqrt{3}} \\ \frac{-r_a + r_e}{\sqrt{2}} & r_0 + \frac{r_a + r_e}{2} & \frac{r_a - r_e}{2\sqrt{3}} & -\frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} \\ \frac{-r_a + r_e}{\sqrt{6}} & \frac{r_a - r_e}{2\sqrt{3}} & r_0 + \frac{r_a + 5r_e}{6} & -\frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} \\ \frac{2\bar{r}_a + \bar{r}_e}{\sqrt{3}} & -\frac{2\bar{r}_a + \bar{r}_e}{\sqrt{6}} & -\frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} & \tilde{r}_0 + \frac{4\tilde{r}_a + 2\tilde{r}_e}{3} \end{pmatrix}. \qquad (4)$$

The rescattering parameters $r_{0,a,e,t}$, $\bar{r}_{0,a,e,t}$, $\tilde{r}_{0,a,e,t}$, $\hat{r}_{0,a,e,t}$ and $\check{r}_{0,a,e,t}$ denote[3] rescattering in $\Pi(\mathbf{8})\Pi(\mathbf{8}) \to \Pi(\mathbf{8})\Pi(\mathbf{8})$, $\Pi(\mathbf{8})\Pi(\mathbf{8}) \to \Pi(\mathbf{8})\eta_1$, $\Pi(\mathbf{8})\eta_1 \to \Pi(\mathbf{8})\eta_1$ and $\eta_1\eta_1 \to \eta_1\eta_1$, respectively, with $\Pi(\mathbf{8})$ the SU(3) octet and η_1 the singlet, and the subscripts $0, a, e, t$ represent flavor singlet, annihilation, exchange and total-annihilation rescatterings, respectively (see Fig. 1).

Flavor symmetry requires that $(\mathcal{S}_{\mathrm{res}})^m$ with an arbitrary power of m should also have the same form as $\mathcal{S}_{\mathrm{res}}$. More explicitly, from SU(3) symmetry, we should have

$$(\mathcal{S}_{\mathrm{res}})^m = (1 + i\mathcal{T})^m \equiv 1 + i\mathcal{T}^{(m)}, \qquad (5)$$

where $\mathcal{T}^{(m)}$ is defined through the above equation and its form is given by

$$\mathcal{T}^{(m)} = \mathcal{T} \ \ \text{with} \ \ (r_j, \bar{r}_j, \tilde{r}_j, \check{r}_j) \to (r_j^{(m)}, \bar{r}_j^{(m)}, \tilde{r}_j^{(m)}, \check{r}_j^{(m)}), \qquad (6)$$

for $j = 0, a, e, t$.

It is useful to note that we have $\mathbf{8} \otimes \mathbf{8}$, $\mathbf{8} \otimes \mathbf{1}$, $\mathbf{1} \otimes \mathbf{8}$ and $\mathbf{1} \otimes \mathbf{1}$ SU(3) products for $P_1 P_2$ final states, which has to be symmetric under the exchange of P_1 and P_2 in the $\bar{B} \to PP$ decay as the meson pair is in s-wave configuration and they have to satisfy the Bose-Einstein statistics. The allowed ones are the $\mathbf{27}$, $\mathbf{8}$ and the $\mathbf{1}$ from $\mathbf{8} \otimes \mathbf{8}$, the $\mathbf{8}'$ from the symmetrized $\mathbf{8} \otimes \mathbf{1} + \mathbf{1} \otimes \mathbf{8}$, and $\mathbf{1}'$ from $\mathbf{1} \otimes \mathbf{1}$ (see, e.g., [28], for the decomposition). Hence, from SU(3) symmetry and the Bose-Einstein statistics, we should have

$$(\mathcal{S}_{\mathrm{res}})^m = \sum_{a=1}^{27} |\mathbf{27}; a\rangle e^{2mi\delta_{27}} \langle \mathbf{27}; a|$$
$$+ \sum_{b=1}^{8} \sum_{p,q=8,8'} |p;b\rangle \mathcal{U}^m_{pq} \langle q;b| + \sum_{p,q=1,1'} |p;1\rangle \mathcal{V}^m_{pq} \langle q;1|, \qquad (7)$$

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[3]Note that $\hat{r}$ and $\check{r}$ do not appear in $\mathcal{T}_1$, but they will contribute to some other PP modes.
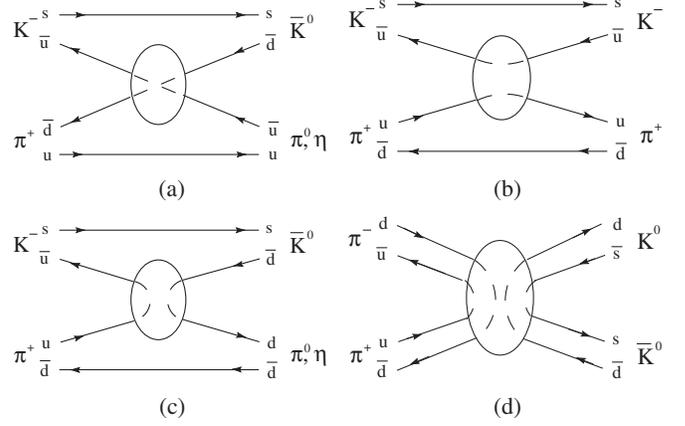
FIG. 1. Pictorial representation of (a) charge exchange r_e, (b) singlet exchange r_0, (c) annihilation r_a and (d) total-annihilation r_t for PP (re)scattering.

where a and b are labels of states within multiplets, and matrices $\mathcal{U}^m$ and $\mathcal{V}^m$ are given by

$$\mathcal{U}^m(\tau, \delta_8, \delta'_8) \equiv \begin{pmatrix} \cos\tau & \sin\tau \\ -\sin\tau & \cos\tau \end{pmatrix} \begin{pmatrix} e^{2mi\delta_8} & 0 \\ 0 & e^{2mi\delta'_8} \end{pmatrix}$$
$$\times \begin{pmatrix} \cos\tau & -\sin\tau \\ \sin\tau & \cos\tau \end{pmatrix},$$

$$\mathcal{V}^m(\nu, \delta_1, \delta'_1) \equiv \begin{pmatrix} \cos\nu & \sin\nu \\ -\sin\nu & \cos\nu \end{pmatrix} \begin{pmatrix} e^{2mi\delta_1} & 0 \\ 0 & e^{2mi\delta'_1} \end{pmatrix}$$
$$\times \begin{pmatrix} \cos\nu & -\sin\nu \\ \sin\nu & \cos\nu \end{pmatrix}, \qquad (8)$$

respectively. Rescattering parameters r_i as the solutions to Eqs. (5) and (6) can be expressed in terms of these angles and phases:

$$1 + i(r_0^{(m)} + r_a^{(m)}) = \frac{2e^{2mi\delta_{27}} + 3\mathcal{U}^m_{11}}{5},$$

$$i(r_e^{(m)} - r_a^{(m)}) = \frac{3e^{2mi\delta_{27}} - 3\mathcal{U}^m_{11}}{5},$$

$$i(r_a^{(m)} + r_t^{(m)}) = \frac{-e^{2mi\delta_{27}} - 4\mathcal{U}^m_{11} + 5\mathcal{V}^m_{11}}{20},$$

$$i(2\bar{r}_a^{(m)} + \bar{r}_e^{(m)}) = \frac{3}{\sqrt{5}}\mathcal{U}^m_{12},$$

$$1 + i\left(\tilde{r}_0^{(m)} + \frac{4\tilde{r}_a^{(m)} + 2\tilde{r}_e^{(m)}}{3}\right) = \mathcal{U}^m_{22},$$

$$i\left(\hat{r}_t^{(m)} + \frac{4\hat{r}_a^{(m)} + 2\hat{r}_e^{(m)}}{3}\right) = \frac{1}{\sqrt{2}}\mathcal{V}^m_{12},$$

$$1 + i\left(\check{r}_0^{(m)} + \frac{4\check{r}_a^{(m)} + 2\check{r}_e^{(m)} + 3\check{r}_t^{(m)}}{6}\right) = \mathcal{V}^m_{22}, \qquad (9)$$

with $\mathcal{U}^m_{ij}$ and $\mathcal{V}^m_{ij}$ given in Eq. (8).

It is interesting to see how the rescattering behaves in a U(3) symmetric case. It is known that the $U_A(1)$ breaking is responsible for the mass difference between η and η' and U(3) symmetry is not a good symmetry for low-lying pseudoscalars. However, U(3) symmetry may still be a reasonable one for a system that rescatters at energies of order m_B. The mass difference between η and η', as an indicator of U(3) symmetry breaking effect, does not lead to sizable energy difference of these particles in charmless B decays. Note that in the literature, some authors also make use of U(3) symmetry in charmless B decays (see, e.g.,[29]). We note that in the U(3) case, we have

$$r_i = \bar{r}_i = \tilde{r}_i = \hat{r}_i = \check{r}_i. \tag{10}$$

Consequently, by requiring

$$\mathcal{T}^{(m)} = \mathcal{T} \quad \text{with} \quad (r_j, \bar{r}_j, \tilde{r}_j, \check{r}_j) \to (r_j^{(m)}, r_j^{(m)}, r_j^{(m)}, r_j^{(m)}), \tag{11}$$

as required by Eq. (10), one must have [11]

$$r_a^{(m)} r_e^{(m)} = 0. \tag{12}$$

There are two solutions, either $r_e^{(m)} = 0$ or $r_a^{(m)} = 0$ [see. Eqs. (A16) and (A17)]. Note that in both solutions, we have

$$\delta_{27} = \delta_8' = \delta_1'. \tag{13}$$

To reduce the number of the rescattering parameters and as a working assumption, the above relations will be used in this work, although we are not imposing the full U(3) symmetry to FSI.

After imposing the above relation and factor out a over phase factor, say δ_{27}, we are left with two mixing angles and two phase differences:

$$\tau, \qquad \nu, \qquad \delta \equiv \delta_8 - \delta_{27}, \qquad \sigma \equiv \delta_1 - \delta_{27}, \tag{14}$$

in the scattering matrices. The rescattering formula Eq. (2) now becomes

$$A = \mathcal{S}_{\text{res}}^{1/2}(\tau, \nu; \delta, \sigma) \cdot A^{\text{fac}}, \tag{15}$$

with the overall phase removed. In summary, 4 additional parameters from Res are introduced to the decay amplitudes.

We find that it is useful to incorporate SU(3) breaking effect in the scattering matrix. The idea is that we try to remove the SU(3) breaking effect in A^{fac} before recattering and put the SU(3) breaking effect back after the rescattering. The underlying reason is as following. In the core of FSI, the rescattering processes are occurring at the m_B energy scale, the SU(3) breaking effect cannot be very important at this stage. Hence the amplitudes to be rescattered are taken in the SU(3) limit, but after the rescattering, as the hadronization process takes place, SU(3) breaking cannot be neglected and their effect needs to be included. In practice we use ratio of decay constants to

model the SU(3) breaking effect. For example, the $B^- \to K^0 K^-$ factorization amplitude is multiplied by $(f_\pi / f_K)^2$ before recattering with other states and is multiplied by $(f_K / f_\pi)^2$ after rescattering. For convenience these two factors are absorbed in $\mathcal{S}_{\text{res}}^{1/2}$. These are new to Ref. [11].

The rescattering matrices needed in this work are collected in Appendix A. As we will see in the next section, including these four rescattering parameters will enhance the agreement of theory and data notably.

B. Rescattering and topological amplitudes in the SU(3) limit

Topological amplitude approach or flavor flow approach is based on SU(3) symmetry. The amplitudes can contain weak and strong phases. FSI will generate additional strong phases and can potentially mixed up different topological amplitudes. It is therefore interesting to investigate the relation of the FSI and topological amplitudes. We will take a closer look of this issue in the presence of the rescattering among PP states. We will consider the topological amplitudes in the SU(3) limit, rescattering of topological amplitudes in the SU(3) limit and, finally, topological amplitudes and rescattering in the U(3) limit. The discussion will be useful to provide a better understanding of the effect of FSI in $\bar{B}_q \to PP$ decays. These are all new to Ref. [11].

1. Topological amplitudes in the SU(3) limit

It is well known that the fields annihilating $B^-, \bar{B}_{d,s}^0$ and creating π, K, η_8 transform respectively as $\bar{3}$ and 8 under SU(3) (see, for example [28]),

$$\bar{B} = (\, B^- \quad \bar{B}^0 \quad \bar{B}_s^0 \,),$$

$$\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix}. \tag{16}$$

For the $b \to u\bar{u}d$ and $b \to q\bar{q}d$ processes, the tree ($\mathcal{O}_T$) and penguin ($\mathcal{O}_P$) operators, respectively, have the following flavor structure,

$$\mathcal{O}_T \sim (\bar{u}b)(\bar{d}u) = H_j^{ik}(\bar{q}_i b)(\bar{q}_k q^j),$$

$$\mathcal{O}_P \sim (\bar{d}b)(\bar{q}_i q^i) = H^k(\bar{q}_k b)(\bar{q}_i q^i),$$

$$\mathcal{O}_{\text{EWP}} \sim Q_j(\bar{d}b)(\bar{q}_j q^j) = H_{\text{EW}}{}_j^{ik}(\bar{q}_i b)(\bar{q}_k q^j), \tag{17}$$

where we define $H^k = \delta_2^k$, $H_j^{ik} = \delta_1^i \delta_j^1 \delta_2^k$ and $(H_{\text{EW}})_j^{ik} = \delta_2^i Q_j \delta_j^k$ (no sum in indices). Note that it is easy to check that we have $H_i^{ik} = H^k$, $H_k^{ik} = 0$, $(H_{\text{EW}})_k^{ik} = 0$, $(H_{\text{EW}})_i^{ik} = Q_2 \delta_2^k = Q_2 H^k$. The flavor structures of $|\Delta S| = 1$ tree and penguin operators can be obtained by replacing d to s, $H^k = \delta_3^k$, $H_j^{ik} = \delta_1^i \delta_j^1 \delta_3^k$ and $(H_{\text{EW}})_j^{ik} = \delta_3^i Q_j \delta_j^k$.

The effective Hamiltonian, in term of the meson degree of freedom, for the $\bar{B} \to PP$ decay should have the same SU(3) transform property of H_W. Consequently, we have

$$H_{\text{eff}} = T\bar{B}_m H_j^{ik}(\Pi^{\text{out}})_k^j(\Pi^{\text{out}})_i^m + C\bar{B}_m H_j^{ik}(\Pi^{\text{out}})_i^j(\Pi^{\text{out}})_k^m + E\bar{B}_k H_j^{ik}(\Pi^{\text{out}})_l^j(\Pi^{\text{out}})_i^l + A\bar{B}_i H_j^{ik}(\Pi^{\text{out}})_l^j(\Pi^{\text{out}})_k^l$$

$$+ P\bar{B}_m H^k(\Pi^{\text{out}})_i^m(\Pi^{\text{out}})_k^i + \frac{1}{2}PA\bar{B}_k H^k(\Pi^{\text{out}})_m^l(\Pi^{\text{out}})_l^m + P_{\text{EW}}\bar{B}_m H_{\text{EW}\,j}^{ik}(\Pi^{\text{out}})_i^m(\Pi^{\text{out}})_k^j + P_{\text{EW}}^C\bar{B}_m H_{\text{EW}\,j}^{ik}(\Pi^{\text{out}})_k^m(\Pi^{\text{out}})_i^j$$

$$+ P_{\text{EW}}^E\bar{B}_k H_{\text{EW}\,j}^{ik}(\Pi^{\text{out}})_l^j(\Pi^{\text{out}})_i^l + P_{\text{EW}}^A\bar{B}_i H_{\text{EW}\,j}^{ik}(\Pi^{\text{out}})_l^j(\Pi^{\text{out}})_k^l + (H_{\text{eff}})_{\text{singlet}}, \tag{18}$$

where the A, P, PA, and P_{EW} terms correspond to annihilation, penguin, penguin annihilation, and electroweak penguin amplitudes, respectively. and $(H_{\text{eff}})_{\text{singlet}}$ is the Hamiltonain involving η_1, given by

$$(H_{\text{eff}})_{\text{singlet}} = \bar{T}\bar{B}_m H_j^{ik}(\Pi^{\text{out}})_k^j(\tilde{\Pi}^{\text{out}})_i^m + \bar{C}_1\bar{B}_m H_j^{ik}(\Pi^{\text{out}})_i^j(\tilde{\Pi}^{\text{out}})_k^m + \bar{C}_2\bar{B}_m H_j^{ik}(\tilde{\Pi}^{\text{out}})_i^j(\Pi^{\text{out}})_k^m + \tilde{C}\bar{B}_m H_j^{ik}(\tilde{\Pi}^{\text{out}})_i^j(\tilde{\Pi}^{\text{out}})_k^m$$

$$+ \bar{E}_1\bar{B}_k H_j^{ik}(\Pi^{\text{out}})_l^j(\tilde{\Pi}^{\text{out}})_i^l + \bar{E}_2\bar{B}_k H_j^{ik}(\Pi^{\text{out}})_l^j(\tilde{\Pi}^{\text{out}})_i^l + \tilde{E}\bar{B}_k H_j^{ik}(\tilde{\Pi}^{\text{out}})_l^j(\tilde{\Pi}^{\text{out}})_i^l + \bar{A}_1\bar{B}_i H_j^{ik}(\Pi^{\text{out}})_l^j(\tilde{\Pi}^{\text{out}})_k^l$$

$$+ \bar{A}_2\bar{B}_i H_j^{ik}(\Pi^{\text{out}})_l^j(\tilde{\Pi}^{\text{out}})_k^l + \bar{P}_1\bar{B}_m H^k(\tilde{\Pi}^{\text{out}})_i^m(\Pi^{\text{out}})_k^i + \bar{P}_2\bar{B}_m H^k(\Pi^{\text{out}})_i^m(\tilde{\Pi}^{\text{out}})_k^i + \tilde{P}\bar{B}_m H^k(\tilde{\Pi}^{\text{out}})_i^m(\tilde{\Pi}^{\text{out}})_k^i$$

$$+ \frac{1}{2}\tilde{P}A\bar{B}_k H^k(\tilde{\Pi}^{\text{out}})_m^l(\tilde{\Pi}^{\text{out}})_l^m + \bar{P}_{\text{EW}}\bar{B}_m H_{\text{EW}\,j}^{ik}(\tilde{\Pi}^{\text{out}})_i^m(\Pi^{\text{out}})_k^j + \bar{P}_{\text{EW},1}^C\bar{B}_m H_{\text{EW}\,j}^{ik}(\tilde{\Pi}^{\text{out}})_k^m(\Pi^{\text{out}})_i^j$$

$$+ \bar{P}_{\text{EW},2}^C\bar{B}_m H_{\text{EW}\,j}^{ik}(\Pi^{\text{out}})_k^m(\tilde{\Pi}^{\text{out}})_i^j + \tilde{P}_{\text{EW}}^C\bar{B}_m H_{\text{EW}\,j}^{ik}(\tilde{\Pi}^{\text{out}})_k^m(\tilde{\Pi}^{\text{out}})_i^j + \bar{P}_{\text{EW},1}^E\bar{B}_k H_{\text{EW}\,j}^{ik}(\Pi^{\text{out}})_l^j(\tilde{\Pi}^{\text{out}})_i^l$$

$$+ \bar{P}_{\text{EW},2}^E\bar{B}_k H_{\text{EW}\,j}^{ik}(\Pi^{\text{out}})_l^j(\tilde{\Pi}^{\text{out}})_i^l + \tilde{P}_{\text{EW}}^E\bar{B}_k H_{\text{EW}\,j}^{ik}(\tilde{\Pi}^{\text{out}})_l^j(\tilde{\Pi}^{\text{out}})_i^l$$

$$+ \bar{P}_{\text{EW},1}^A\bar{B}_i H_{\text{EW}\,j}^{ik}(\tilde{\Pi}^{\text{out}})_l^j(\Pi^{\text{out}})_k^l + \bar{P}_{\text{EW},2}^A\bar{B}_i H_{\text{EW}\,j}^{ik}(\Pi^{\text{out}})_l^j(\tilde{\Pi}^{\text{out}})_k^l + \tilde{P}_{\text{EW}}^A\bar{B}_i H_{\text{EW}\,j}^{ik}(\tilde{\Pi}^{\text{out}})_l^j(\tilde{\Pi}^{\text{out}})_k^l, \tag{19}$$

with $(\tilde{\Pi}^{\text{out}})_j^i = \eta_1^{\text{out}}\delta_j^i/\sqrt{3}$. Note that we introduce P_{EW}^E and P_{EW}^A, namely the electroweak penguin exchange and electroweak penguin annihilation terms for completeness. The above H_{eff} contains all possible SU(3) invariant combinations in first order of H_j^{ik}, H^k and $H_{\text{EW}\,j}^{ik}$. It should be emphasize that the effective Hamiltonian in Eq. (18) is obtained using flavor SU(3) symmetry argument only. The TA amplitude can contain all possible FSI contributions, while the expressions of the decay amplitude in term of these TA will remain the same.

With redefinition of the following amplitudes:

$$2\bar{A} \equiv \bar{A}_1 + \bar{A}_2, \qquad 2\bar{E} \equiv \bar{E}_1 + \bar{E}_2, \qquad 2\bar{P} \equiv \bar{P}_1 + \bar{P}_2,$$
$$2\bar{P}_{\text{EW}}^A \equiv \bar{P}_{\text{EW},1}^A + \bar{P}_{\text{EW},2}^A, \qquad 2\bar{P}_{\text{EW}}^E \equiv \bar{P}_{\text{EW},1}^E + \bar{P}_{\text{EW},2}^E, \tag{20}$$

$(H_{\text{eff}})_{\text{singlet}}$ can be expressed in a more compact form,

$$(H_{\text{eff}})_{\text{singlet}} = (\bar{T} + 2\bar{A})\bar{B}_i H_j^{ik}(\Pi^{\text{out}})_k^j\eta_1^{\text{out}}/\sqrt{3} + (\bar{C}_1 + 2\bar{E})\bar{B}_k H_j^{ik}(\Pi^{\text{out}})_i^j\eta_1^{\text{out}}/\sqrt{3} + \left(\bar{C}_2 + 2\bar{P} - \frac{1}{3}\bar{P}_{\text{EW},2}^C\right)\bar{B}_m H^k(\Pi^{\text{out}})_k^m\eta_1^{\text{out}}/\sqrt{3}$$

$$+ (\bar{P}_{\text{EW}} + 2\bar{P}_{\text{EW}}^A)\bar{B}_i H_{\text{EW}\,j}^{ik}(\Pi^{\text{out}})_k^j\eta_1^{\text{out}}/\sqrt{3} + (\bar{P}_{\text{EW},1}^C + 2\bar{P}_{\text{EW}}^E)\bar{B}_k H_{\text{EW}\,j}^{ik}(\Pi^{\text{out}})_i^j\eta_1^{\text{out}}/\sqrt{3}$$

$$+ \left(\tilde{C} + \tilde{E} + \tilde{P} + \frac{3}{2}\tilde{P}A - \frac{1}{3}\tilde{P}_{\text{EW}}^C - \frac{1}{3}\tilde{P}_{\text{EW}}^E\right)\bar{B}_k H^k\eta_1^{\text{out}}\eta_1^{\text{out}}/3. \tag{21}$$

Using the above approach we can reproduce familiar expressions of decay amplitudes in terms of TA [26,27].[4] Explicitly, we have the following amplitudes:

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[4]See also [30] for a recent discussion.

$$A_{\bar{B}_d^0 \to K^- \pi^+} = T' + P' + \frac{1}{3}(2P_{\text{EW}}^{\prime C} - P_{\text{EW}}^{\prime E}),$$

$$A_{\bar{B}_d^0 \to \bar{K}^0 \pi^0} = \frac{1}{3\sqrt{2}}(3C' - 3P' + 3P_{\text{EW}}' + P_{\text{EW}}^{\prime C} + P_{\text{EW}}^{\prime E}),$$

$$A_{\bar{B}_d^0 \to \bar{K}^0 \eta_8} = \frac{1}{3\sqrt{6}}(3C' - 3P' + 3P_{\text{EW}}' + P_{\text{EW}}^{\prime C} + P_{\text{EW}}^{\prime E}),$$

$$A_{\bar{B}_d^0 \to \bar{K}^0 \eta_1} = \frac{1}{3\sqrt{3}}(3\bar{C}_2' + 6\bar{P}' - \bar{P}_{\text{EW},1}^{\prime C} - \bar{P}_{\text{EW},2}^{\prime C} - 2\bar{P}_{\text{EW}}^{\prime E}), \tag{22}$$

for group-1 modes,

$$A_{B^- \to \bar{K}^0 \pi^-} = A' + P' + \frac{1}{3}(-P_{\text{EW}}^{\prime C} + 2P_{\text{EW}}^{\prime E}),$$

$$A_{B^- \to K^- \pi^0} = \frac{1}{3\sqrt{2}}(3T' + 3C' + 3A' + 3P' + 3P_{\text{EW}}' + 2P_{\text{EW}}^{\prime C} + 2P_{\text{EW}}^{\prime E}),$$

$$A_{B^- \to K^- \eta_8} = \frac{1}{3\sqrt{6}}(3T' + 3C' - 3A' - 3P' + 3P_{\text{EW}}' + 4P_{\text{EW}}^{\prime C} - 2P_{\text{EW}}^{\prime E}),$$

$$A_{B^- \to K^- \eta_1} = \frac{1}{3\sqrt{3}}(3\bar{T}' + 3\bar{C}_2' + 6\bar{A}' + 6\bar{P}' + 2\bar{P}_{\text{EW},1}^{\prime C} - \bar{P}_{\text{EW},2}^{\prime C} + 4\bar{P}_{\text{EW}}^{\prime E}), \tag{23}$$

for group-2 modes,

$$A_{B^- \to \pi^- \pi^0} = \frac{1}{\sqrt{2}}(T + C + P_{\text{EW}} + P_{\text{EW}}^C),$$

$$A_{B^- \to K^0 K^-} = A + P + \frac{1}{3}(-P_{\text{EW}}^C + 2P_{\text{EW}}^E),$$

$$A_{B^- \to \pi^- \eta_8} = \frac{1}{3\sqrt{6}}(3T + 3C + 6A + 6P + 3P_{\text{EW}} + P_{\text{EW}}^C + 4P_{\text{EW}}^E),$$

$$A_{B^- \to \pi^- \eta_1} = \frac{1}{3\sqrt{3}}(3\bar{T} + 3\bar{C}_2 + 6\bar{A} + 6\bar{P} + 2\bar{P}_{\text{EW},1}^C - \bar{P}_{\text{EW},2}^C + 4\bar{P}_{\text{EW}}^E), \tag{24}$$

for group-3 modes,

$$A_{\bar{B}_d^0 \to \pi^+ \pi^-} = T + E + P + PA + \frac{1}{3}(2P_{\text{EW}}^C + P_{\text{EW}}^A - P_{\text{EW}}^E),$$

$$A_{\bar{B}_d^0 \to \pi^0 \pi^0} = \frac{1}{\sqrt{2}}\left(-C + E + P + PA - P_{\text{EW}} - \frac{1}{3}P_{\text{EW}}^C + \frac{1}{3}P_{\text{EW}}^A - \frac{1}{3}P_{\text{EW}}^E\right),$$

$$A_{\bar{B}_d^0 \to \eta_8 \eta_8} = \frac{1}{9\sqrt{2}}(3C + 3E + 3P + 9PA + 3P_{\text{EW}} - P_{\text{EW}}^C - 3P_{\text{EW}}^A - P_{\text{EW}}^E),$$

$$A_{\bar{B}_d^0 \to \eta_8 \eta_1} = \frac{1}{9\sqrt{2}}(3\bar{C}_1 + 3\bar{C}_2 + 6\bar{E} + 6\bar{P} + 3\bar{P}_{\text{EW}} + 6\bar{P}_{\text{EW}}^A - \bar{P}_{\text{EW},1}^C - \bar{P}_{\text{EW},2}^C - 2\bar{P}_{\text{EW}}^E),$$

$$A_{\bar{B}_d^0 \to \eta_1 \eta_1} = \frac{1}{9\sqrt{2}}(6\tilde{C} + 6\tilde{E} + 6\tilde{P} + 9\tilde{P}A - 2\tilde{P}_{\text{EW}}^C - 2\tilde{P}_{\text{EW}}^E),$$

$$A_{\bar{B}_d^0 \to K^+ K^-} = E + PA + \frac{1}{3}P_{\text{EW}}^A,$$

$$A_{\bar{B}_d^0 \to K^0 \bar{K}^0} = P + PA - \frac{1}{3}(P_{\text{EW}}^C + 2P_{\text{EW}}^A + P_{\text{EW}}^E),$$

$$A_{\bar{B}_d^0 \to \pi^0 \eta_8} = \frac{1}{3\sqrt{3}}(3E - 3P + P_{\text{EW}}^C + 3P_{\text{EW}}^A + P_{\text{EW}}^E),$$

$$A_{\bar{B}_d^0 \to \pi^0 \eta_1} = \frac{1}{3\sqrt{6}}(3\bar{C}_1 - 3\bar{C}_2 + 6\bar{E} - 6\bar{P} + 3\bar{P}_{\text{EW}} + 6\bar{P}_{\text{EW}}^A + \bar{P}_{\text{EW},1}^C + \bar{P}_{\text{EW},2}^C + 2\bar{P}_{\text{EW}}^E), \tag{25}$$

for group-4 modes, and the following amplitudes:

$$A_{\bar{B}_s^0 \to K^+\pi^-} = T + P + \frac{1}{3}(2P_{\rm EW}^C - P_{\rm EW}^E),$$

$$A_{\bar{B}_s^0 \to K^0\pi^0} = \frac{1}{3\sqrt{2}}(3C - 3P + 3P_{\rm EW} + P_{\rm EW}^C + P_{\rm EW}^E),$$

$$A_{\bar{B}_s^0 \to K^0\eta_8} = \frac{1}{3\sqrt{6}}(3C - 3P + 3P_{\rm EW} + P_{\rm EW}^C + P_{\rm EW}^E),$$

$$A_{\bar{B}_s^0 \to K^0\eta_1} = \frac{1}{3\sqrt{3}}(3\bar{C}_2 + 6\bar{P} - \bar{P}_{\rm EW,1}^C - \bar{P}_{\rm EW,2}^C - 2\bar{P}_{\rm EW}^E),$$

$$\tag{26}$$

and

$$A_{\bar{B}_s^0 \to \pi^+\pi^-} = E' + PA' + \frac{1}{3}P_{\rm EW}^{\prime A},$$

$$A_{\bar{B}_s^0 \to \pi^0\pi^0} = \frac{1}{\sqrt{2}}\left(E' + PA' + \frac{1}{3}P_{\rm EW}^{\prime A}\right),$$

$$A_{\bar{B}_s^0 \to \eta_8\eta_8} = \frac{1}{9\sqrt{2}}(-6C' + 3E' + 12P' + 9PA'$$
$$\qquad - 6P_{\rm EW}' - 4P_{\rm EW}^{\prime C} - 3P_{\rm EW}^{\prime A} - 4P_{\rm EW}^{\prime E}),$$

$$A_{\bar{B}_s^0 \to \eta_8\eta_1} = \frac{1}{9\sqrt{2}}(3\bar{C}_1' - 6\bar{C}_2' + 6\bar{E}' - 12\bar{P}' + 3\bar{P}_{\rm EW}'$$
$$\qquad + 6\bar{P}_{\rm EW}^{\prime A} + 2\bar{P}_{\rm EW,1}^{\prime C} + 2\bar{P}_{\rm EW,2}^{\prime C} + 4\bar{P}_{\rm EW}^{\prime E}),$$

$$A_{\bar{B}_s^0 \to \eta_1\eta_1} = \frac{1}{9\sqrt{2}}(6\tilde{C}' + 6\tilde{E}' + 6\tilde{P}' + 9\widetilde{PA}' - 2\tilde{P}_{\rm EW}^{\prime C} - 2\tilde{P}_{\rm EW}^{\prime E}),$$

$$A_{\bar{B}_s^0 \to K^+K^-} = T' + E' + P' + PA' + \frac{1}{3}(P_{\rm EW}^{\prime A} + 2P_{\rm EW}^{\prime C} - P_{\rm EW}^{\prime E}),$$

$$A_{\bar{B}_s^0 \to K^0\bar{K}^0} = P' + PA' - \frac{1}{3}(P_{\rm EW}^{\prime C} + 2P_{\rm EW}^{\prime A} + P_{\rm EW}^{\prime E}),$$

$$A_{\bar{B}_s^0 \to \pi^0\eta_8} = \frac{1}{\sqrt{3}}(-C' + E' - P_{\rm EW}' + P_{\rm EW}^{\prime A}),$$

$$A_{\bar{B}_s^0 \to \pi^0\eta_1} = \frac{1}{\sqrt{6}}(\bar{C}_1' + 2\bar{E}' + \bar{P}_{\rm EW}' + 2\bar{P}_{\rm EW}^{\prime A}), \tag{27}$$

for $\bar{B}_s \to PP$ decays, where the T, C, A, P, PA, and $P_{\rm EW}$ terms correspond to color-allowed tree, color-suppressed tree, annihilation, penguin, penguin annihilation, and electroweak penguin amplitudes, respectively. Note that $P_{\rm EW}^E$ and $P_{\rm EW}^A$, namely the electroweak penguin exchange and electroweak penguin annihilation terms, are introduced for completeness. See Appendix B for details. Those with (without) prime are for $\Delta S = -1(0)$ transition.

The one-to-one correspondence of the SU(3) parameters and the topological amplitudes is not a coincidence. It can be understood by using a flavor flow analysis. We take the first term of $H_{\rm eff}$ for illustration. In H_W the decays are governed by the $\mathcal{O}_T \sim (\bar{u}b)(\bar{d}u) = H_j^{ik}(\bar{q}_i b)(\bar{q}_k q^j)$, $b \to q_i\bar{q}^j q_k$ transition with the corresponded H_j^{ik} coupling. The first term of $H_{\rm eff}$ in Eq. (18) is $T\bar{B}_m H_j^{ik}(\Pi^{\rm out})_k^j(\Pi^{\rm out})_i^m$. Note that we use subscript and superscript according to the field convention. For example, we assign a subscript (superscript) to the initial (final) state antiquark $\bar{q}_m$ ($\bar{q}^m$). The $\bar{B}_m(\Pi^{\rm out})_i^m$ part in $T\bar{B}_m H_j^{ik}(\Pi^{\rm out})_k^j(\Pi^{\rm out})_i^m$ can be interpreted as a $\bar{B}_m$ to $(\Pi^{\rm out})_i^m$ transition with the same spectator antiquark $\bar{q}_m$ from $\bar{B}_m$ becoming the final state spectator anti-quark $\bar{q}^m$, which ends up in $(\Pi^{\rm out})_i^m$. The quark q_i from $b \to q_i$ transition also ends up in $(\Pi^{\rm out})_i^m$, while the $(\Pi^{\rm out})_k^j$ part is responsible for the creation of the meson where the W-emitted $\bar{q}^j q_k$ pair ends up with. The above picture clearly corresponds to the external W-emission topology. Similarly, the identification of the other topological amplitudes can be understood similarly.

One can check that all of the above amplitudes can be expressed in terms of the following combinations:

$$T^{(\prime)} + C^{(\prime)}, \qquad C^{(\prime)} - E^{(\prime)}, \qquad A^{(\prime)} + C^{(\prime)},$$

$$P^{(\prime)} - C^{(\prime)} + \frac{1}{3}P_{\rm EW}^{(\prime)C}, \qquad PA^{(\prime)} - \frac{4}{9}C^{(\prime)} + \frac{13}{9}E^{(\prime)} - \frac{1}{3}P_{\rm EW}^{(\prime)C},$$

$$P_{\rm EW}^{(\prime)} + P_{\rm EW}^{(\prime)C}, \qquad P_{\rm EW}^{(\prime)C} - P_{\rm EW}^{(\prime)E}, \qquad P_{\rm EW}^{(\prime)A} + P_{\rm EW}^{(\prime)C},$$

$$\bar{T}^{(\prime)} + 2\bar{A}^{(\prime)}, \qquad \bar{C}_1^{(\prime)} + 2\bar{E}^{(\prime)},$$

$$\bar{C}_2^{(\prime)} + 2\bar{P}^{(\prime)} - \frac{1}{3}\bar{P}_{\rm EW,2}^{(\prime)C}, \bar{P}_{\rm EW}^{(\prime)} + 2\bar{P}_{\rm EW}^{(\prime)A}, \qquad \bar{P}_{\rm EW,1}^{(\prime)C} + 2\bar{P}_{\rm EW}^{(\prime)E},$$

$$\tilde{C}^{(\prime)} + \tilde{E}^{(\prime)} + \tilde{P}^{(\prime)} + \frac{3}{2}\tilde{PA}^{(\prime)} - \frac{1}{3}\tilde{P}_{\rm EW}^{(\prime)C} - \frac{1}{3}\tilde{P}_{\rm EW}^{(\prime)E}. \tag{28}$$

For example, we can express the decay amplitude of $\bar{B}^0 \to K^-\pi^+$ in the following combinations:

$$A(\bar{B}^0 \to K^-\pi^+) = (T' + C') + \left(P' - C' + \frac{1}{3}P_{\rm EW}^{C\prime}\right)$$
$$\qquad\qquad + \frac{1}{3}(P_{\rm EW}^{C\prime} - P_{\rm EW}^{E\prime}). \tag{29}$$

It is interesting to compare the amplitudes expressed in terms of the topological amplitudes with the those in the QCDF calculation. We can obtain the following relations in the SU(3) limit: (using formulas in [18] but taking the SU(3) limit)

$$T^{(\prime)0} = A_{PP}\lambda_p^{(\prime)}\delta_{pu}\alpha_1,$$

$$C^{(\prime)0} = A_{PP}\lambda_p^{(\prime)}\delta_{pu}\alpha_2,$$

$$E^{(\prime)0} = A_{PP}\lambda_p^{(\prime)}\delta_{pu}\beta_1,$$

$$A^{(\prime)0} = A_{PP}\lambda_p^{(\prime)}\delta_{pu}\beta_2,$$

$$P^{(\prime)0} = A_{PP}\lambda_p^{(\prime)}(\alpha_4^p + \beta_3^p),$$

$$PA^{(\prime)0} = 2A_{PP}\lambda_p^{(\prime)}\beta_4^p,$$

$$P_{\rm EW}^{(\prime)0} = \frac{3}{2}A_{PP}\lambda_p^{(\prime)}\alpha_{3,\rm EW}^p,$$

$$P_{\rm EW}^{(\prime)C0} = \frac{3}{2}A_{PP}\lambda_p^{(\prime)}\alpha_{4,\rm EW}^p,$$

$$P_{\rm EW}^{(\prime)E0} = \frac{3}{2}A_{PP}\lambda_p^{(\prime)}\beta_{3,\rm EW}^p,$$

$$P_{\rm EW}^{(\prime)A0} = \frac{3}{2}A_{PP}\lambda_p^{(\prime)}\beta_{4,\rm EW}^p, \tag{30}$$

where we use $\lambda_p^{(\prime)} \equiv V_{pb} V_{pd(s)}^*$, $p = u$, c with $V_{pb,pd(s)}$ the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and summation over p is implied. One can find detail definitions of A_{PP}, α and β in [18]. Note that A_{PP} involves a $\bar{B}_q \to P$ transition and a P decay constant:

$$A_{PP} = \frac{G_F}{\sqrt{2}} F_0^{BP}(m_P^2) f_P m_B^2. \tag{31}$$

It should be note that we have removed an overall i in the definition of A_{PP}. The superscript 0 on TA is denoting the fact that rescattering among PP states has not taken place. In the SU(3) limit, we will use $F_0^{BP}(m_P^2) = F_0^{B\pi}(0)$ and $f_P = f_\pi$ in later discussion.

For B decays to a final state with η_1, things are more complicated. For example, $A_{P\eta_1}$ is in principle different from $A_{\eta_1 P}$. We have in the SU(3) limit: [18]

$$\bar{T}^{(\prime)0} = A_{\eta_1 P} \lambda_p^{(\prime)} \delta_{pu} \alpha_1,$$

$$\bar{C}_1^{(\prime)0} = A_{\eta_1 P} \lambda_p^{(\prime)} \delta_{pu} \alpha_2,$$

$$\bar{C}_2^{(\prime)0} = A_{P\eta_1} \lambda_p^{(\prime)} \delta_{pu} \alpha_2,$$

$$2\bar{E}^{(\prime)0} \equiv \bar{E}_1^{(\prime)0} + \bar{E}_2^{(\prime)0}$$
$$= A_{\eta_1 P} \lambda_p^{(\prime)} \delta_{pu} \beta_1 + A_{P\eta_1} \lambda_p^{(\prime)} \delta_{pu} (\beta_1 + 3\beta_{S1}),$$

$$2\bar{A}^{(\prime)0} \equiv \bar{A}_1^{(\prime)0} + \bar{A}_2^{(\prime)0}$$
$$= A_{\eta_1 P} \lambda_p^{(\prime)} \delta_{pu} \beta_2 + A_{P\eta_1} \lambda_p^{(\prime)} \delta_{pu} (\beta_2 + 3\beta_{S2}),$$

$$2\bar{P}^{(\prime)0} \equiv \bar{P}_1^{(\prime)0} + \bar{P}_2^{(\prime)0}$$
$$= A_{\eta_1 P} \lambda_p^{(\prime)} (\alpha_4^p + \beta_3^p) + A_{P\eta_1} \lambda_p^{(\prime)} (\alpha_4^p + 3\alpha_3^p + \beta_3^p + 3\beta_{S3}^p),$$

$$\bar{P}_{\rm EW}^{(\prime)0} = \frac{3}{2} A_{\eta_1 P} \lambda_p^{(\prime)} \alpha_{3,\rm EW}^p,$$

$$\bar{P}_{\rm EW,1}^{(\prime)C0} = \frac{3}{2} A_{\eta_1 P} \lambda_p^{(\prime)} \alpha_{4,\rm EW}^p,$$

$$\bar{P}_{\rm EW,2}^{(\prime)C0} = \frac{3}{2} A_{P\eta_1} \lambda_p^{(\prime)} \alpha_{4,\rm EW}^p,$$

$$2\bar{P}_{\rm EW}^{(\prime)E0} \equiv \bar{P}_{\rm EW,1}^{(\prime)E0} + \bar{P}_{\rm EW,2}^{(\prime)E0}$$
$$= \frac{3}{2} [A_{\eta_1 P} \lambda_p^{(\prime)} \beta_{3,\rm EW}^p + A_{P\eta_1} \lambda_p^{(\prime)} (\beta_{3,\rm EW}^p + 3\beta_{S3,\rm EW}^p)],$$

$$2\bar{P}_{\rm EW}^{(\prime)A0} \equiv \bar{P}_{\rm EW,1}^{(\prime)A0} + \bar{P}_{\rm EW,2}^{(\prime)A0}$$
$$= \frac{3}{2} [A_{\eta_1 P} \lambda_p^{(\prime)} \beta_{4,\rm EW}^p + A_{P\eta_1} \lambda_p^{(\prime)} (\beta_{4,\rm EW}^p + 3\beta_{4S,\rm EW}^p)]. \tag{32}$$

Note that $A_{P\eta_1}$ involves a $\bar{B} \to P$ transition, while $A_{\eta_1 P}$ involves a $\bar{B} \to \eta_1$ transition:

$$A_{P\eta_1} = \frac{G_F}{\sqrt{2}} F_0^{\bar{B} \to P} f_{\eta_1} m_B^2,$$

$$A_{\eta_1 P} = \frac{G_F}{\sqrt{2}} F_0^{\bar{B} \to \eta_1} f_P m_B^2 \simeq \frac{G_F}{\sqrt{2}} F_0^{\bar{B} \to P} \frac{f_{\eta_1}}{f_P} f_P m_B^2, \tag{33}$$

where in the second equation, we have made use of the approximation from [18]. In fact we have $A_{P\eta_1} \simeq A_{\eta_1 P} \simeq A_{PP}(f_{\eta_1}/f_P)$.

Finally comparing our expressions and those in Ref. [18], we have

$$\tilde{C}^{(\prime)0} = A_{\eta_1 \eta_1} \lambda_p^{(\prime)} \delta_{pu} \alpha_2,$$

$$\tilde{E}^{(\prime)0} = A_{\eta_1 \eta_1} \lambda_p^{(\prime)} \delta_{pu} (\beta_1 + 3\beta_{S1}),$$

$$\tilde{P}^{(\prime)0} = A_{\eta_1 \eta_1} \lambda_p^{(\prime)} (\alpha_4^p + 3\alpha_3^p + \beta_3^p + 3\beta_{S3}^p),$$

$$\widetilde{PA}^{(\prime)} = 2 A_{\eta_1 \eta_1} \lambda_p^{(\prime)} (\beta_4^p + 3\beta_{S4}^p),$$

$$\tilde{P}_{\rm EW}^{(\prime)C0} = \frac{3}{2} A_{\eta_1 \eta_1} \lambda_p^{(\prime)} \alpha_{4,\rm EW}^p,$$

$$\tilde{P}_{\rm EW}^{(\prime)E0} = \frac{3}{2} A_{\eta_1 \eta_1} \lambda_p^{(\prime)} (\beta_{3,\rm EW}^p + 3\beta_{S3,\rm EW}^p), \tag{34}$$

with

$$A_{\eta_1 \eta_1} = \frac{G_F}{\sqrt{2}} F_0^{\bar{B} \to \eta_1} f_{\eta_1} m_B^2 \simeq \frac{G_F}{\sqrt{2}} F_0^{\bar{B} \to P} \frac{f_{\eta_1}}{f_P} f_{\eta_1} m_B^2$$
$$= A_{PP} \left(\frac{f_{\eta_1}}{f_P} \right)^2. \tag{35}$$

In the later discussion, we take $f_{\eta_1} = f_P = f_\pi$.

2. Rescattering of topological amplitudes in the SU(3) limit

We now turn to the rescattering part. The matrices $\mathcal{T}_{1,2,3,4}$ can be obtained through a diagrammatic method by matching the Clebsh-Gordan coefficients of scattering mesons (see Fig. 1) or by using an operator method. We have

$$O_e = {\rm Tr}(\Pi^{\rm in} \Pi^{\rm out} \Pi^{\rm in} \Pi^{\rm out})/2, \quad O_a = {\rm Tr}(\Pi^{\rm in} \Pi^{\rm in} \Pi^{\rm out} \Pi^{\rm out}),$$

$$O_0 = {\rm Tr}(\Pi^{\rm in} \Pi^{\rm out}){\rm Tr}(\Pi^{\rm in} \Pi^{\rm out})/2,$$

$$O_t = {\rm Tr}(\Pi^{\rm in} \Pi^{\rm in}){\rm Tr}(\Pi^{\rm out} \Pi^{\rm out})/4, \tag{36}$$

corresponding to r_e, r_a, r_0, and r_t contributions, in the combination of

$$\mathcal{T}^{(m)} = r_0^{(m)} O_0 + r_e^{(m)} O_e + r_a^{(m)} O_a + r_t^{(m)} O_t + \cdots, \tag{37}$$

where the remaining terms will be specified in below. The above terms exhaust all possible combinations for $\Pi(\mathbf{8})\Pi(\mathbf{8}) \to \Pi(\mathbf{8})\Pi(\mathbf{8})$ scatterings.

To obtain operators involving η_1, we simply replace Π in the above operators to $\Pi + \eta_1 1_{3\times 3}/\sqrt{3}$ and collect terms with different number of η_1 as

$$\sqrt{3}\bar{O}_e = \frac{\sqrt{3}}{2}\bar{O}_a = \mathrm{Tr}(\Pi^{\mathrm{in}}\Pi^{\mathrm{out}}\Pi^{\mathrm{in}})\eta_1^{\mathrm{out}} + \mathrm{Tr}(\Pi^{\mathrm{out}}\Pi^{\mathrm{in}}\Pi^{\mathrm{out}})\eta_1^{\mathrm{in}},$$

$$\tilde{O}_0 = \frac{3}{4}\tilde{O}_a = \frac{3}{2}\tilde{O}_e = \mathrm{Tr}(\Pi^{\mathrm{in}}\Pi^{\mathrm{out}})\eta_1^{\mathrm{in}}\eta_1^{\mathrm{out}},$$

$$4\hat{O}_t = 3\hat{O}_a = 6\hat{O}_e$$
$$= \eta_1^{\mathrm{in}}\eta_1^{\mathrm{in}}\mathrm{Tr}(\Pi^{\mathrm{out}}\Pi^{\mathrm{out}}) + \eta_1^{\mathrm{out}}\eta_1^{\mathrm{out}}\mathrm{Tr}(\Pi^{\mathrm{in}}\Pi^{\mathrm{in}}),$$

$$2\check{O}_0 = 4\check{O}_t = 3\check{O}_a = 6\check{O}_e = \eta_1^{\mathrm{in}}\eta_1^{\mathrm{out}}\eta_1^{\mathrm{in}}\eta_1^{\mathrm{out}}. \qquad (38)$$

Note that it is impossible to obtain a term containing three η_1 as is prohibited from $SU(3)$ symmetry. We now have

$$\mathcal{T}^{(m)} = r_0^{(m)}O_0 + r_e^{(m)}O_e + r_a^{(m)}O_a + r_t^{(m)}O_t + \bar{r}_e^{(m)}\bar{O}_e$$
$$+ \bar{r}_a^{(m)}\bar{O}_a + \tilde{r}_0^{(m)}\tilde{O}_0 + \tilde{r}_a^{(m)}\tilde{O}_a + \tilde{r}_e^{(m)}\tilde{O}_e + \hat{r}_t^{(m)}\hat{O}_t$$
$$+ \hat{r}_a^{(m)}\hat{O}_a + \hat{r}_e^{(m)}\hat{O}_e + \check{r}_0^{(m)}\check{O}_0 + \check{r}_t^{(m)}\check{O}_t$$
$$+ \check{r}_a^{(m)}\check{O}_a + \check{r}_e^{(m)}\check{O}_e. \qquad (39)$$

Using Eq. (38), the above equation can be simplified into

$$\mathcal{T}^{(m)} = r_0^{(m)}O_0 + r_e^{(m)}O_e + r_a^{(m)}O_a + r_t^{(m)}O_t + (\bar{r}_e^{(m)} + 2\bar{r}_a^{(m)})\bar{O}_e + \left(\tilde{r}_0^{(m)} + \frac{4\tilde{r}_a^{(m)} + 2\tilde{r}_e^{(m)}}{3}\right)\tilde{O}_0 + \left(\hat{r}_t^{(m)} + \frac{4\hat{r}_a^{(m)} + 2\hat{r}_e^{(m)}}{3}\right)\hat{O}_t$$

$$+ \left(\check{r}_0^{(m)} + \frac{4\check{r}_a^{(m)} + 2\check{r}_e^{(m)} + 3\check{r}_t^{(m)}}{6}\right)\check{O}_0. \qquad (40)$$

Note that various $\bar{r}_i^{(m)}$, $\tilde{r}_i^{(m)}$, $\hat{r}_i^{(m)}$, and $\check{r}_i^{(m)}$ occur in $\mathcal{T}^{(m)}$ only through some very specific combinations. We still preserve the subscripts ($i = 0, t, a, e$), since these $\bar{r}_i^{(m)}$, $\tilde{r}_i^{(m)}$, $\hat{r}_i^{(m)}$ and $\check{r}_i^{(m)}$ for different i correspond to different flavor flow patterns in rescattering diagrams (see Fig. 1) and they will, in fact, reduce to $r_i^{(m)}$ in the $U(3)$ limit.

It is straightforward to obtain the rescattering effects on topological amplitudes. In analogy to Eq. (15):

$$A = \mathcal{S}_{\mathrm{res}}^{1/2} \cdot A^{\mathrm{fac}} = (1 + i\mathcal{T}^{1/2}) \cdot A^{\mathrm{fac}}, \qquad (41)$$

we have

$$H_{\mathrm{eff}} = (1 + i\mathcal{T}^{1/2}) \cdot H_{\mathrm{eff}}^0 = H_{\mathrm{eff}}^0 + i\mathcal{T}^{1/2} \cdot H_{\mathrm{eff}}^0, \qquad (42)$$

where H_{eff} is given in Eq. (18), $\mathcal{T}^{1/2}$ in Eq. (40) but with $m = 1/2$, H_{eff}^0 is the unscattered effective Hamiltonian with

all TA in H_{eff} replaced by TA^0 and the dot in the above equation implies all possible pairing of the $P^{\mathrm{out}}P^{\mathrm{out}}$ fields in H_{eff}^0 to the $P^{\mathrm{in}}P^{\mathrm{in}}$ fields in $\mathcal{T}^{1/2}$ (the $P^{\mathrm{out}}P^{\mathrm{out}}$ in $\mathcal{T}^{1/2}$ remains unpaired). As noted previously since the effective Hamiltonian in Eq. (18) is obtained using flavor $SU(3)$ symmetry argument only, its flavor structure will not be changed in the presence of rescattering, i.e., Eq. (42) will not modify the flavor structure of H_{eff}. This feature is indeed verified in the explicit computation. Therefore the expressions of the decay amplitude in term of the TA will remain the same, but now these TA contain rescattering contributions.

The effect of rescattering on TA can be obtained using the above equation. The computation is straightforward, but tedious. Here we only give the final results, some derivations using the above equation can be found in Appendix B for illustration. We obtain, in the presence of the rescattering, TA will receive corrections in the following ways:

$$\delta T^{(\prime)} = ir_0'T^{(\prime)0} + ir_e'C^{(\prime)0},$$

$$\delta C^{(\prime)} = ir_0'C^{(\prime)0} + ir_e'T^{(\prime)0},$$

$$\delta E^{(\prime)} = ir_0'E^{(\prime)0} + ir_a'T^{(\prime)0} - \frac{1}{3}i(r_e' + 2r_a')C^{(\prime)0} + \frac{1}{3}i(-2r_e' + 5r_a')E^{(\prime)0} + \frac{1}{3}i(\bar{r}_e' + 2\bar{r}_a')(\bar{C}_1^{(\prime)0} + 2\bar{E}^{(\prime)0}),$$

$$\delta A^{(\prime)} = \frac{1}{3}i(3r_0' - 2r_e' + 5r_a')A^{(\prime)0} - \frac{1}{3}i(r_e' + 2r_a')T^{(\prime)0} + ir_a'C^{(\prime)0} + \frac{1}{3}i(\bar{r}_e' + 2\bar{r}_a')(\bar{T}^{(\prime)0} + 2\bar{A}^{(\prime)0}),$$

$$\delta P^{(\prime)} = ir_0'P^{(\prime)0} + ir_a'T^{(\prime)0} - \frac{1}{3}i(r_e' + 2r_a')C^{(\prime)0} + \frac{1}{3}i(-2r_e' + 5r_a')P^{(\prime)0}$$

$$- \frac{1}{3}ir_a'P_{\mathrm{EW}}^{(\prime)0} + \frac{1}{9}i(r_e' + 2r_a')P_{\mathrm{EW}}^{(\prime)C0} + \frac{1}{3}i(\bar{r}_e' + 2\bar{r}_a')\left(\bar{C}_2^{(\prime)0} + 2\bar{P}^{(\prime)0} - \frac{1}{3}\bar{P}_{\mathrm{EW},2}^{(\prime)C0}\right),$$

$$\delta PA^{(\prime)} = \frac{1}{3}i(3r_0' - r_e' + 16r_a' + 12r_t')PA^{(\prime)0} + ir_t'T^{(\prime)0} + \frac{1}{9}(2ir_e' + 4ir_a' - 3ir_t')C^{(\prime)0}$$
$$+ \frac{2}{9}(ir_e' + 11ir_a' + 12ir_t')E^{(\prime)0} + \frac{2}{9}(ir_e' + 11ir_a' + 12ir_t')P^{(\prime)0} - \frac{1}{3}ir_t'P_{EW}^{(\prime)0} + \frac{1}{27}i(-2r_e' - 4r_a' + 3r_t')P_{EW}^{(\prime)C}$$
$$- \frac{2}{27}i(r_e' + 11r_a' + 12r_t')P_{EW}^{(\prime)E0} - \frac{2}{9}i(\bar{r}_e' + 2\bar{r}_a')\left(\bar{C}_1^{(\prime)0} + 2\bar{E}^{(\prime)0} + \bar{C}_2^{(\prime)0} + 2\bar{P}^{(\prime)0} - \frac{1}{3}\bar{P}_{EW,2}^{(\prime)C0} - \frac{1}{3}\bar{P}_{EW,1}^{(\prime)C0} - \frac{2}{3}\bar{P}_{EW}^{(\prime)E0}\right)$$
$$+ \frac{1}{3}i\left(\hat{r}_t' + \frac{4\hat{r}_a' + 2\hat{r}_e'}{3}\right)\left(\tilde{C}^{(\prime)0} + \tilde{E}^{(\prime)0} + \tilde{P}^{(\prime)0} + \frac{3}{2}\tilde{PA}^{(\prime)0} - \frac{1}{3}\tilde{P}_{EW}^{(\prime)C0} - \frac{1}{3}\tilde{P}_{EW}^{(\prime)E0}\right),$$

$$\delta P_{EW}^{(\prime)} = ir_0'P_{EW}^{(\prime)0} + ir_e'P_{EW}^{(\prime)C0},$$

$$\delta P_{EW}^{(\prime)C} = ir_0'P_{EW}^{(\prime)C0} + ir_e'P_{EW}^{(\prime)0},$$

$$\delta P_{EW}^{(\prime)E} = ir_0'P_{EW}^{(\prime)E0} + ir_a'P_{EW}^{(\prime)0} - \frac{1}{3}i(r_e' + 2r_a')P_{EW}^{(\prime)C0} + \frac{1}{3}i(-2r_e' + 5r_a')P_{EW}^{(\prime)E0} + \frac{1}{3}i(\bar{r}_e' + 2\bar{r}_a')(\bar{P}_{EW,1}^{(\prime)C0} + 2\bar{P}_{EW}^{(\prime)E0}),$$

$$\delta P_{EW}^{(\prime)A} = \frac{1}{3}i(3r_0' - 2r_e' + 5r_a')P_{EW}^{(\prime)A0} - \frac{1}{3}i(r_e' + 2r_a')P_{EW}^{(\prime)0} + ir_a'P_{EW}^{(\prime)C0} + \frac{1}{3}i(\bar{r}_e' + 2\bar{r}_a')(\bar{P}_{EW}^{(\prime)0} + 2\bar{P}_{EW}^{(\prime)A0}), \qquad (43)$$

$$\delta(\bar{T}^{(\prime)} + 2\bar{A}^{(\prime)}) = i(\bar{r}_e' + 2\bar{r}_a')\left(-\frac{2}{3}T^{(\prime)0} + C^{(\prime)0} + \frac{5}{3}A^{(\prime)0}\right) + i\left(\bar{r}_0' + \frac{4\bar{r}_a' + 2\bar{r}_e'}{3}\right)(\bar{T}^{(\prime)0} + 2\bar{A}^{(\prime)0}),$$

$$\delta(\bar{C}_1^{(\prime)} + 2\bar{E}^{(\prime)}) = i(\bar{r}_e' + 2\bar{r}_a')\left(T^{(\prime)0} - \frac{2}{3}C^{(\prime)0} + \frac{5}{3}E^{(\prime)0}\right) + i\left(\bar{r}_0' + \frac{4\bar{r}_a' + 2\bar{r}_e'}{3}\right)(\bar{C}_1^{(\prime)0} + 2\bar{E}^{(\prime)0}),$$

$$\delta\left(\bar{C}_2^{(\prime)} + 2\bar{P}^{(\prime)} - \frac{1}{3}\bar{P}_{EW,2}^{(\prime)C}\right) = i(\bar{r}_e' + 2\bar{r}_a')\left(T^{(\prime)0} - \frac{2}{3}C^{(\prime)0} + \frac{5}{3}P^{(\prime)0} - \frac{1}{3}P_{EW}^{(\prime)0} + \frac{2}{9}P_{EW}^{(\prime)C0}\right)$$
$$+ i\left(\bar{r}_0' + \frac{4\bar{r}_a' + 2\bar{r}_e'}{3}\right)\left(\bar{C}_2^{(\prime)0} + 2\bar{P}^{(\prime)0} - \frac{1}{3}\bar{P}_{EW,2}^{(\prime)C0}\right),$$

$$\delta(\bar{P}_{EW}^{(\prime)} + 2\bar{P}_{EW}^{(\prime)A}) = i(\bar{r}_e' + 2\bar{r}_a')\left(-\frac{2}{3}P_{EW}^{(\prime)0} + P_{EW}^{(\prime)C0} + \frac{5}{3}P_{EW}^{(\prime)A0}\right) + i\left(\bar{r}_0' + \frac{4\bar{r}_a' + 2\bar{r}_e'}{3}\right)(\bar{P}_{EW}^{(\prime)0} + 2\bar{P}_{EW}^{(\prime)A0}),$$

$$\delta(\bar{P}_{EW,1}^{(\prime)C} + 2\bar{P}_{EW}^{(\prime)E}) = i(\bar{r}_e' + 2\bar{r}_a')\left(P_{EW}^{(\prime)0} - \frac{2}{3}P_{EW}^{(\prime)C0} + \frac{5}{3}P_{EW}^{(\prime)E0}\right) + i\left(\bar{r}_0' + \frac{4\bar{r}_a' + 2\bar{r}_e'}{3}\right)(\bar{P}_{EW,1}^{(\prime)C0} + 2\bar{P}_{EW}^{(\prime)E0}), \qquad (44)$$

and

$$\delta\left(\tilde{C}^{(\prime)} + \tilde{E}^{(\prime)} + \tilde{P}^{(\prime)} + \frac{3}{2}\widetilde{PA}^{(\prime)} - \frac{1}{3}\tilde{P}_{EW}^{(\prime)C} - \frac{1}{3}\tilde{P}_{EW}^{(\prime)E}\right)$$
$$= i\left(\hat{r}_t' + \frac{4\hat{r}_a' + 2\hat{r}_e'}{3}\right)\left(\frac{3}{2}T^{(\prime)0} - \frac{1}{2}C^{(\prime)0} + 4E^{(\prime)0} + 4P^{(\prime)0} + 6PA^{(\prime)0} - \frac{1}{2}P_{EW}^0 + \frac{1}{6}(P_{EW}^C)^0 - \frac{4}{3}(P_{EW}^E)^0\right)$$
$$+ i\left(\check{r}_0' + \frac{4\check{r}_a' + 2\check{r}_e' + 3\check{r}_t'}{6}\right)\left(\tilde{C}^{(\prime)0} + \tilde{E}^{(\prime)0} + \tilde{P}^{(\prime)0} + \frac{3}{2}\widetilde{PA}^{(\prime)0} - \frac{1}{3}\tilde{P}_{EW}^{(\prime)C0} - \frac{1}{3}\tilde{P}_{EW}^{(\prime)E0}\right), \qquad (45)$$

where the superscript 0 denote unscattered amplitudes and we define $r_i' \equiv r_i^{(1/2)}, \bar{r}_i' \equiv \bar{r}_i^{(1/2)}, \hat{r}_i' \equiv \hat{r}_i^{(1/2)}, \check{r}_i' \equiv \check{r}_i^{(1/2)}, \tilde{r}_i' \equiv \tilde{r}_i^{(1/2)}$.

The full topological amplitudes contain the unscattered and the contribution from the scattering. For example, for the tree amplitude the full amplitude is $T^{(\prime)}$, the unscattered tree amplitude is $T^{(\prime)0}$. After scattering we have

$$T^{(\prime)} = T^{(\prime)0} + \delta T^{(\prime)} = T^{(\prime)0} + ir_0'T^{(\prime)0} + ir_e'C^{(\prime)0}. \quad (46)$$

One can check that the above equations are consistent with the topological amplitude expressions Eqs. (22),

(23), (24), (25), (26), and (27), and the rescattering formulas, Eqs. (A5), (A6), (A7), (A8) and those of B_s decays. It should be pointed out that this is a nontrivial check, as one can see that Eqs. (22), (23), (24), (25), (26), and (27) are rather complicate and a single error in them can easily spoil the consistency check.

Note that decay amplitudes can be expressed in terms of several combinations of topological amplitudes, such as $T + C$, $C - E$ and so on, and FSI affects these combinations only through,

$$1 + i(r'_0 + r'_a), \quad i(r'_e - r'_a), \quad i(r'_a + r'_t), \quad i(2\bar{r}'_a + \bar{r}'_e), \quad 1 + i\left(\tilde{r}'_0 + \frac{4\tilde{r}'_a + 2\tilde{r}'_e}{3}\right), \quad i\left(\hat{r}'_t + \frac{4\hat{r}'_a + 2\hat{r}'_e}{3}\right). \quad (47)$$

We have

$$T^{(\prime)} + C^{(\prime)} = [(1 + ir'_0 + ir'_a) + i(r'_e - r'_a)](T^{(\prime)0} + C^{(\prime)0}) = e^{i\delta_{27}}(T^{(\prime)0} + C^{(\prime)0}),$$

$$C^{(\prime)} - E^{(\prime)} = (1 + ir'_0 + ir'_a)(C^{(\prime)0} - E^{(\prime)0}) + \frac{1}{3}i(r'_e - r'_a)[3(T^{(\prime)0} + C^{(\prime)0}) - 2(C^{(\prime)0} - E^{(\prime)0})]$$
$$- \frac{1}{3}i(\bar{r}'_e + 2\bar{r}'_a)(\bar{C}_1^{(\prime)0} + 2\bar{E}^{(\prime)0}),$$

$$A^{(\prime)} + C^{(\prime)} = (1 + ir'_0 + ir'_a)(A^{(\prime)0} + C^{(\prime)0}) + \frac{2}{3}i(r'_e - r'_a)[(T^{(\prime)0} + C^{(\prime)0}) - (A^{(\prime)0} + C^{(\prime)0})]$$
$$+ \frac{1}{3}i(\bar{r}'_e + 2\bar{r}'_a)(\bar{T}^{(\prime)0} + 2\bar{A}^{(\prime)0}),$$

$$P^{(\prime)} - C^{(\prime)} + \frac{1}{3}P_{\mathrm{EW}}^{(\prime)C} = [(1 + ir'_0 + ir'_a) - \frac{2}{3}i(r'_e - r'_a)]\left(P^{(\prime)0} - C^{(\prime)0} + \frac{1}{3}P_{\mathrm{EW}}^{(\prime)C0}\right)$$
$$+ \frac{1}{3}i(r'_e - r'_a)[-3(T^{(\prime)0} + C^{(\prime)0}) + (P_{\mathrm{EW}}^{(\prime)0} + P_{\mathrm{EW}}^{(\prime)C0})]$$
$$+ \frac{1}{3}i(\bar{r}'_e + 2\bar{r}'_a)\left(\bar{C}_2^{(\prime)0} + 2\bar{P}^{(\prime)0} - \frac{1}{3}\bar{P}_{\mathrm{EW},2}^{(\prime)C0}\right),$$

$$PA^{(\prime)} - \frac{4}{9}C^{(\prime)} + \frac{13}{9}E^{(\prime)} - \frac{1}{3}P_{\mathrm{EW}}^{(\prime)C} = \left(1 + ir'_0 + ir'_a - \frac{1}{3}i(r'_e - r'_a) + 4i(r'_a + r'_t)\right)\left(PA^{(\prime)0} - \frac{4}{9}C^{(\prime)0} + \frac{13}{9}E^{(\prime)0} - \frac{1}{3}P_{\mathrm{EW}}^{(\prime)C}\right)$$
$$+ \left[-\frac{4}{9}i(r'_e - r'_a) + i(r'_a + r'_t)\right](T^{(\prime)0} + C^{(\prime)0}) + \frac{7}{9}\left[\frac{1}{3}i(r'_e - r'_a) + 4i(r'_a + r'_t)\right](C^{(\prime)0} - E^{(\prime)0})$$
$$+ \left[\frac{2}{9}i(r'_e - r'_a) + \frac{8}{3}i(r'_a + r'_t)\right]\left(P^{(\prime)0} - C^{(\prime)0} + \frac{1}{3}P_{\mathrm{EW}}^{(\prime)C0}\right)$$
$$- \frac{1}{3}[i(r'_e - r'_a) + i(r'_a + r'_t)](P_{\mathrm{EW}}^{(\prime)0} + P_{\mathrm{EW}}^{(\prime)C0}) + \left[\frac{2}{27}i(r'_e - r'_a) + \frac{8}{9}i(r'_a + r'_t)\right](P_{\mathrm{EW}}^{(\prime)C0} - P_{\mathrm{EW}}^{(\prime)E0})$$
$$+ \frac{1}{27}i(\bar{r}'_e + 2\bar{r}'_a)\left[7(\bar{C}_1^{(\prime)0} + 2\bar{E}^{(\prime)0}) - 6\left(\bar{C}_2^{(\prime)0} + 2\bar{P}^{(\prime)0} - \frac{1}{3}\bar{P}_{\mathrm{EW},2}^{(\prime)C0}\right) + 2(\bar{P}_{\mathrm{EW},1}^{(\prime)C0} + 2\bar{P}_{\mathrm{EW}}^{(\prime)E0})\right]$$
$$+ \frac{1}{3}i\left(\hat{r}'_t + \frac{4\hat{r}'_a + 2\hat{r}'_e}{3}\right)\left(\tilde{C}^{(\prime)0} + \tilde{E}^{(\prime)0} + \tilde{P}^{(\prime)0} + \frac{3}{2}\tilde{P}A^{(\prime)0} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)C0} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)E0}\right),$$

$$P_{\mathrm{EW}}^{(\prime)} + P_{\mathrm{EW}}^{(\prime)C} = [(1 + ir'_0 + ir'_a) + i(r'_e - r'_a)](P_{\mathrm{EW}}^{(\prime)0} + P_{\mathrm{EW}}^{(\prime)C0}) = e^{i\delta_{27}}(P_{\mathrm{EW}}^{(\prime)0} + P_{\mathrm{EW}}^{(\prime)C0}),$$

$$P_{\mathrm{EW}}^{(\prime)C} - P_{\mathrm{EW}}^{(\prime)E} = (1 + ir'_0 + ir'_a)(P_{\mathrm{EW}}^{(\prime)C0} - P_{\mathrm{EW}}^{(\prime)E0}) + \frac{1}{3}i(r'_e - r'_a)[3(P_{\mathrm{EW}}^{(\prime)0} + P_{\mathrm{EW}}^{(\prime)C0}) - 2(P_{\mathrm{EW}}^{(\prime)C0} - P_{\mathrm{EW}}^{(\prime)E0})]$$
$$- \frac{1}{3}i(\bar{r}'_e + 2\bar{r}'_a)(\bar{P}_{\mathrm{EW},1}^{(\prime)C0} + 2\bar{P}_{\mathrm{EW}}^{(\prime)E0}),$$

$$P_{\mathrm{EW}}^{(\prime)A} + P_{\mathrm{EW}}^{(\prime)C} = (1 + ir'_0 + ir'_a)(P_{\mathrm{EW}}^{(\prime)A0} + P_{\mathrm{EW}}^{(\prime)C0}) + \frac{2}{3}i(r'_e - r'_a)[(P_{\mathrm{EW}}^{(\prime)0} + P_{\mathrm{EW}}^{(\prime)C0}) - (P_{\mathrm{EW}}^{(\prime)A0} + P_{\mathrm{EW}}^{(\prime)C0})]$$
$$+ \frac{1}{3}i(\bar{r}'_e + 2\bar{r}'_a)(\bar{P}_{\mathrm{EW}}^{(\prime)0} + 2\bar{P}_{\mathrm{EW}}^{(\prime)A0}), \quad (48)$$

$$(\bar{T}^{(\prime)} + 2\bar{A}^{(\prime)}) = \left[1 + i\left(\tilde{r}'_0 + \frac{4\tilde{r}'_a + 2\tilde{r}'_e}{3}\right)\right](\bar{T}^{(\prime)0} + 2\bar{A}^{(\prime)0}) + i(\tilde{r}'_e + 2\tilde{r}'_a)\left[-\frac{2}{3}(T^{(\prime)0} + C^{(\prime)0}) + \frac{5}{3}(A^{(\prime)0} + C^{(\prime)0})\right],$$

$$(\bar{C}_1^{(\prime)} + 2\bar{E}^{(\prime)}) = \left[1 + i\left(\tilde{r}'_0 + \frac{4\tilde{r}'_a + 2\tilde{r}'_e}{3}\right)\right](\bar{C}_1^{(\prime)0} + 2\bar{E}^{(\prime)0}) + i(\tilde{r}'_e + 2\tilde{r}'_a)\left[(T^{(\prime)0} + C^{(\prime)0}) - \frac{5}{3}(C^{(\prime)0} - E^{(\prime)0})\right],$$

$$\bar{C}_2^{(\prime)} + 2\bar{P}^{(\prime)} - \frac{1}{3}\bar{P}_{\mathrm{EW},2}^{(\prime)C} = \left[1 + i\left(\tilde{r}'_0 + \frac{4\tilde{r}'_a + 2\tilde{r}'_e}{3}\right)\right]\left(\bar{C}_2^{(\prime)0} + 2\bar{P}^{(\prime)0} - \frac{1}{3}\bar{P}_{\mathrm{EW},2}^{C0}\right) + i(\tilde{r}'_e + 2\tilde{r}'_a)[(T^{(\prime)0} + C^{(\prime)0})$$

$$+ \frac{5}{3}\left(P^{(\prime)0} - C^{(\prime)0} + \frac{1}{3}P_{\mathrm{EW}}^{(\prime)C0}\right) - \frac{1}{3}(P_{\mathrm{EW}}^{(\prime)0} + P_{\mathrm{EW}}^{(\prime)C0})],$$

$$(\bar{P}_{\mathrm{EW}}^{(\prime)} + 2\bar{P}_{\mathrm{EW}}^{(\prime)A}) = \left[1 + i\left(\tilde{r}'_0 + \frac{4\tilde{r}'_a + 2\tilde{r}'_e}{3}\right)\right](\bar{P}_{\mathrm{EW}}^{(\prime)0} + 2\bar{P}_{\mathrm{EW}}^{(\prime)A0}) + i(\tilde{r}'_e + 2\tilde{r}'_a)\left[-\frac{2}{3}(P_{\mathrm{EW}}^{(\prime)0} + P_{\mathrm{EW}}^{(\prime)C0}) + \frac{5}{3}(P_{\mathrm{EW}}^{(\prime)A0} + P_{\mathrm{EW}}^{(\prime)C0})\right],$$

$$(\bar{P}_{\mathrm{EW},1}^{(\prime)C} + 2\bar{P}_{\mathrm{EW}}^{(\prime)E}) = \left[1 + i\left(\tilde{r}'_0 + \frac{4\tilde{r}'_a + 2\tilde{r}'_e}{3}\right)\right](\bar{P}_{\mathrm{EW},1}^{(\prime)C0} + 2\bar{P}_{\mathrm{EW}}^{(\prime)E0}) + i(\tilde{r}'_e + 2\tilde{r}'_a)[(P_{\mathrm{EW}}^{(\prime)0} + P_{\mathrm{EW}}^{(\prime)C0}) - \frac{5}{3}(P_{\mathrm{EW}}^{(\prime)C0} - P_{\mathrm{EW}}^{(\prime)E0})], \quad (49)$$

and

$$\left(\tilde{C}^{(\prime)} + \tilde{E}^{(\prime)} + \tilde{P}^{(\prime)} + \frac{3}{2}\widetilde{PA}^{(\prime)} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)C} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)E}\right)$$

$$= i\left(\hat{r}'_t + \frac{4\hat{r}'_a + 2\hat{r}'_e}{3}\right)\left[\frac{3}{2}(T^{(\prime)0} + C^{(\prime)0}) + \frac{14}{3}(C^{(\prime)0} - E^{(\prime)0}) + 4\left(P^{(\prime)0} - C^{(\prime)0} + \frac{1}{3}P_{\mathrm{EW}}^{(\prime)C0}\right)\right.$$

$$+ 6\left(PA^{(\prime)0} - \frac{4}{9}C^{(\prime)0} + \frac{13}{9}E^{(\prime)0} - \frac{1}{3}P_{\mathrm{EW}}^{(\prime)C0}\right) - \frac{1}{2}(P_{\mathrm{EW}}^{(\prime)0} + P_{\mathrm{EW}}^{(\prime)C0}) + \frac{4}{3}(P_{\mathrm{EW}}^{(\prime)C0} - P_{\mathrm{EW}}^{(\prime)E0})\right]$$

$$+ \left[1 + i\left(\tilde{r}'_0 + \frac{4\tilde{r}'_a + 2\tilde{r}'_e + 3\tilde{r}'_t}{6}\right)\right]\left(\tilde{C}^{(\prime)0} + \tilde{E}^{(\prime)0} + \tilde{P}^{(\prime)0} + \frac{3}{2}\tilde{PA}^{(\prime)0} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)C0} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)E0}\right). \quad (50)$$

With the help of Eq. (9) (with $m = 1/2$) we will be able to study the effect of rescattering to the above combinations and give a clearer picture. Note that the above transformation formulas of the combined topological amplitudes, in Eqs. (48)–(50), are not as powerful compared to transformation formulas of the individual topological amplitudes, Eqs. (43)–(45). They are, however, the ones that can have in terms of the combinations of r'_i [Eq. (47)] and hence the rescattering angles and phases, τ, ν, σ, and δ [see Eqs. (9) and (14)], without introducing additional assumptions.

3. Topological amplitudes and rescattering in the U(3) limit

It is interesting to investigate the above relations in the U(3) limit, where we take Eq. (10) and

$$\bar{T} = \tilde{T} = T, \qquad \bar{C}_1 = \bar{C}_2 = \tilde{C} = C,$$

$$\bar{E} = \tilde{E} = E, \qquad \bar{A} = \tilde{A} = A,$$

$$\bar{P} = \tilde{P} = P, \qquad \bar{P}_{\mathrm{EW}} = \tilde{P}_{\mathrm{EW}} = P_{\mathrm{EW}},$$

$$\bar{P}_{\mathrm{EW},1}^C = \bar{P}_{\mathrm{EW},2}^C = \tilde{P}_{\mathrm{EW}}^C = P_{\mathrm{EW}}^C,$$

$$\bar{P}_{\mathrm{EW}}^E = \tilde{P}_{\mathrm{EW}}^E = P_{\mathrm{EW}}^E, \qquad \bar{P}_{\mathrm{EW}}^A = \tilde{P}_{\mathrm{EW}}^A = P_{\mathrm{EW}}^A,$$

$$\overline{PA} = \widetilde{PA} = PA. \quad (51)$$

Using Eq. (10) and Eqs. (43)–(45), we find that

$$\delta(\bar{T} + 2\bar{A}) - \delta(T + 2A) = 3ir'_e A^0,$$

$$\delta(\bar{C}_1 + 2\bar{E}) - \delta(C + 2E) = 3ir'_e E^0,$$

$$\delta\left(\bar{C}_2 + 2\bar{P} - \frac{1}{3}\bar{P}_{\mathrm{EW},2}^C\right) - \delta\left(C + 2P - \frac{1}{3}P_{\mathrm{EW}}^C\right) = 3ir'_e P^0,$$

$$\delta(\bar{P}_{\mathrm{EW}} + 2\bar{P}_{\mathrm{EW}}^A) - \delta(P_{\mathrm{EW}} + 2P_{\mathrm{EW}}^A) = 3ir'_e P_{\mathrm{EW}}^{A0},$$

$$\delta(\bar{P}_{\mathrm{EW},1}^C + 2\bar{P}_{\mathrm{EW}}^E) - \delta(P_{\mathrm{EW}}^C + 2P_{\mathrm{EW}}^E) = 3ir'_e P_{\mathrm{EW}}^{E0},$$

$$(52)$$

and

$$\delta\left(\tilde{C} + \tilde{E} + \tilde{P} + \frac{3}{2}\widetilde{PA} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^C - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^E\right)$$

$$- \delta\left(C + E + P + \frac{3}{2}PA - \frac{1}{3}P_{\mathrm{EW}}^C - \frac{1}{3}P_{\mathrm{EW}}^E\right)$$

$$= \frac{1}{2}ir'_e(6E^0 + 6P^0 + 9PA^0 - 2P_{\mathrm{EW}}^{E0}). \quad (53)$$

The above relations can be consistent with the relations in the U(3) limit, Eq. (51), only if we take

$$r'_e = 0. \qquad (54)$$

It is useful to recall that by requiring U(3) symmetry to the rescattering matrix $\mathcal{T}$ [Eq. (10)] one only leads to $r'_e r'_a = 0$ [see Eq. (12)], which can either be $r'_e = 0$ or $r'_a = 0$. Now we can select out the $r'_e = 0$ solution. The reason of being more specify here is that we now apply U(3) symmetry to both rescattering matrix $\mathcal{T}$ [Eq. (10)] and to the topological amplitudes [Eq. (51)]. Hence it leads to a more specify relation.

III. NUMERICAL RESULTS

In this section, we will present our numerical results. First, we will give an overview of the results of the fits. We will then discuss the rescattering effects on topological amplitudes. Finally, numerical results for decay rates and CP asymmetries will be shown.

A. Overview of the results of the fits

Before present our numerical results, we specify the inputs used in the following numerical study. Masses of all particles and total widths of $B_{u,d,s}$ mesons are taken from the review of the Particle Data Group (PDG) [7] and the branching ratios of B to charmless meson decays are taken from the latest averages in [6].

For theoretical inputs, we use $f_\pi = 130.2$ MeV, $f_K = 155.6$ MeV and $f_{B_{(s)}} = 187.1$ (227.2) MeV for decay constants and $m_s(2 \text{ GeV}) = 93.5$ MeV for the strange quark mass, which is taken from the central value of the PDG averaged value, 93.5 ± 2 MeV [7].[5] The values of CKM matrix elements, except γ/ϕ_3, are also taken from the central values of the latest PDG's results [7]. We use the QCD factorization calculated amplitudes [18] for the factorization amplitudes in the right-hand side of Eq. (2). We take the renormalization scale as $\mu = 4.2$ GeV and the power correction parameters $X_{A,H} = \ln(m_B/\Lambda_h)(1 + \rho_{A,H} e^{i\phi_{A,H}})$. For meson wave functions, we use the following Gegenbauer moments: $\alpha_1^{\bar{K}} = -\alpha_1^{K} = 0.2$, $\alpha_2^{\bar{K}} = \alpha_2^{K} = 0.1$, $\alpha_1^{\pi} = 0$, $\alpha_2^{\pi} = 0.2$ and $\alpha_{1,2}^{\eta,\eta'} = 0$ [18]. Several hadronic parameters, in additional to the CKM phase γ/ϕ_3, $\rho_{A,H}$ and $\phi_{A,H}$, in factorization amplitudes are fit parameters and are allowed to vary in the following ranges:

$$F_0^{B\pi}(0) = 0.25 \pm 0.05, \quad F_0^{BK}(0) = 0.35 \pm 0.08,$$
$$F^{B_s K}(0) = 0.31 \pm 0.08, \quad \gamma/\phi_3 = (73.2 \pm 10)^\circ,$$
$$\lambda_B = 0.35 \pm 0.25 \text{ GeV}, \quad \lambda_{B_s} = 0.35 \pm 0.25 \text{ GeV}.$$
$$\qquad (55)$$

TABLE I. Numbers of rates $\mathcal{B}$, direct CP asymmetries $\mathcal{A}$ and mixing induced CP asymmetries S of $\bar{B}_q \to PP$ decays involved in this study.

	Number of $\mathcal{B}$	Number of $\mathcal{A}$	Number of S	Number of SL	Total number
All	34	34	24	1	93
Fitted	26	16	5	1	48
Predicted	8	18	19	0	45

These estimations agree with those in [18,31–36], while the ranges of form factors and γ/ϕ_3 are slightly enlarged. For example, the above $F_0^{B\pi}(0)$ can be compared to the following reported values for the quantity: 0.28 ± 0.05 [18], 0.25 [31], 0.29 [32], 0.258 ± 0.031 [33], $0.26^{+0.04}_{-0.03}$ [34], $0.281^{+0.027}_{-0.029}$ [35], and $0.261^{+0.020}_{-0.023}$ [36].[6]

It is known that semileptonic $B \to \pi l\nu$ decays are related to the $B \to \pi$ form factor and the determination of $|V_{ub}|$ [7]. Using data from BABAR [37,38] and Belle [39,40], HFLAG obtain the following result in 2014: [6]

$$F_0^{B\pi}(0)|V_{ub}| = (9.23 \pm 0.24) \times 10^{-4}, \qquad (56)$$

We will use this in our χ^2 analysis.

In summary, 9 hadronic parameters, $\rho_{A,H}$, $\phi_{A,H}$, $F_0^{B\pi}(0)$, $F_0^{BK}(0)$, $F_0^{B_s K}(0)$, λ_B, λ_{B_s}, and one CKM phase, γ/ϕ_3, involved in the QCDF amplitudes will be fitted from data. The residue rescattering part add 4 more parameters, τ, ν, δ, and σ, giving 14 parameters in total. Note that the majority of the fitted parameters are from the factorization part.

In this analysis there are totally 93 measurable quantities, including 34 rates, 34 direct CP asymmetries, 24 mixing induced CP asymmetries and one measurement from semileptonic B decay [Eq. (56)]. Among them we will fit to all available data, including 26 rates, 16 direct CP asymmetries, 5 mixing induced CP asymmetries and 1 semileptonic decay data, giving 48 in total, and will have prediction on 8 rates, 18 direct CP asymmetries and 19 mixing induced CP asymmetries. The explicit list of these 48 items will be shown later. The total numbers of data in fit and in predictions are roughly the same. The summary of these numbers is shown in Table I.

We perform a χ^2 analysis with all available data on CP-averaged rates and CP asymmetries in $\bar{B}_{u,d,s} \to PP$ decays. In the following study we use two different scenarios: Fac and Res. For the formal we use only factorization amplitudes (i.e., $A_i = A_i^{\text{fac}}$), while for the latter we add residue

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[5]Note that in the previous study [11] m_s is taken as a fit parameter in the range of 100 ± 30 MeV, but now as the value becomes more precisely known it is better to use the present central value as an input parameter.

[6]It is preferable to use the form factors as inputs instead of variables in the fit, but in the present situation no definite values for these form factors can be found (see for example the collected $F_0^{B\pi}(0)$ values from [18,31–36]) and we therefore treat them as fitting variables to avoid bias in this work. Hopefully the situation can be improved in future. See also Footnote 5.

TABLE II. Confidence level (C.L.), χ^2_{min}/d.o.f. and various contributions to χ^2_{min} for the best fitted solutions. The p-value of the rescattering (Res) case is 5.5%. Numbers of data used are shown in parentheses.

	χ^2_{min}/d.o.f.	$\chi^2_{\{\mathcal{B}(\bar{B}^0 \to K\pi),...\}}$	$\chi^2_{\{\mathcal{A}(\bar{B}^0 \to K\pi),...\}}$	$\chi^2_{\{\mathcal{B}(B^- \to K\pi),...\}}$	$\chi^2_{\{\mathcal{A}(B^- \to K\pi),...\}}$
Fac	213.4/38 (48)	10.1 (4)	1.8 (3)	24.7 (4)	5.2 (4)
Res	48.1/34 (48)	7.2 (4)	1.1 (3)	6.3 (4)	0.6 (4)
	$\chi^2_{\{\mathcal{B}(B^- \to \pi\pi),...\}}$	$\chi^2_{\{\mathcal{A}(B^- \to \pi\pi),...\}}$	$\chi^2_{\{\mathcal{B}(\bar{B}^0 \to \pi\pi),...\}}$	$\chi^2_{\{\mathcal{A}(\bar{B}^0 \to \pi\pi),...\}}$	$\chi^2_{\{\mathcal{B}(\bar{B}_s),\mathcal{A}(\bar{B}_s)\}}$
Fac	10.6 (4)	6.5 (4)	55.3 (9)	15.7 (3)	64.0 (7)
Res	6.4 (4)	7.5 (4)	7.8 (9)	4.7 (3)	0.6 (7)
	$\chi^2_{\{S(\bar{B}^0)),S(\bar{B}^0_s)\}}$	χ^2_{SL}			
Fac	12.9 (5)	8.0 (1)			
Res	5.2 (5)	0.7 (1)			

FSI effect as well (i.e., $A_i = \sum_{j=1}^n (\mathcal{S}^{1/2}_{res})_{ij} A_j^{fac}$). Both are fitted to data. The confidence levels and χ^2s for the best fitted cases in both senarios are shown in Table II. Contributions to χ^2_{min} from various subsets of data are also given. Modes that are related through the Res are grouped together [see Eq. (A5), and see Eqs. (A6)–(A8) for other groups]. Off course only those with data can contribute to χ^2. Numbers of data used are shown in parentheses. Explicitly, $\chi^2_{\{\mathcal{B}(\bar{B}^0 \to K\pi),...\}}$ and $\chi^2_{\{\mathcal{A}(\bar{B}^0 \to K\pi),...\}}$ in the table denote the χ^2 contribution obtained from 4 CP-average rates and 3 direct CP asymmetries, respectively, of the group-1 modes: consisting of $\bar{B}^0 \to K^-\pi^+, \bar{K}^0\pi^0, \bar{K}^0\eta, \bar{K}^0\eta'$ decays (except $\mathcal{A}(\bar{B}^0 \to \bar{K}^0\eta)$); $\chi^2_{\{\mathcal{B}(B^- \to K\pi),...\}}$ and $\chi^2_{\{\mathcal{A}(B^- \to K\pi),...\}}$ are contributed from the group-2 modes: $B^- \to \bar{K}^0\pi^-, K^-\pi^0, K^-\eta, K^-\eta'$ decays; $\chi^2_{\{\mathcal{B}(B^- \to \pi\pi),...\}}$ and $\chi^2_{\{\mathcal{A}(B^- \to \pi\pi),...\}}$ are contributed from the group-3 modes: $B^- \to \pi^-\pi^0, K^-K^0, \pi^-\eta, \pi^-\eta'$ decays; $\chi^2_{\{\mathcal{B}(\bar{B}^0 \to \pi\pi),...\}}$ is contributed from the group-4 modes: $\bar{B}^0 \to \pi^+\pi^-, \pi^0\pi^0, \eta\eta, \eta\eta', \eta'\eta', K^+K^-, \bar{K}^0K^0, \pi^0\eta, \pi^0\eta'$ decays, while $\chi^2_{\{\mathcal{A}(\bar{B}^0 \to \pi\pi),...\}}$ only contributed from 3 of the above modes, $\bar{B}^0 \to \pi^+\pi^-, \pi^0\pi^0, \bar{K}^0K^0$ decays; $\chi^2_{\{\mathcal{B}(\bar{B}_s),\mathcal{A}(\bar{B}_s)\}}$ is contributed from 5 CP-averaged rates in $\bar{B}^0_s \to K^+\pi^-, \pi^+\pi^-, \eta'\eta', K^+K^-, K^0\bar{K}^0$ decays and from 2 direct CP asymmetries in $\bar{B}^0_s \to K^+\pi^-, K^+K^-$ decays; $\chi^2_{\{S(\bar{B}^0)),S(\bar{B}^0_s))\}}$ is contributed from mixing induced CP asymmetries of $\bar{B}^0 \to \bar{K}^0\pi^0, \bar{K}^0\eta', \pi^+\pi^-, K_SK_S$, and $\bar{B}^0_s \to K^+K^-$ decays. The semileptonic data, Eq. (56) is also included in the fit. The above lists are the 26 rates, 16 direct CP asymmetries, 5 mixing induced asymmetries and 1 semileptonic data [Eq. (56)], 48 in totally, that go into the fit.

Table II shows the overall performances of the fits. We discuss the factorization case first. The χ^2 per degree of freedom of Fac is $213.4/(48-10)$. One can compare the χ^2 values and the numbers of data used in the corresponding groups. When the ratio of χ^2 and the number of data is smaller than one, the fit in the group is reasonably well. By inspecting the table, we see that Fac gives a good fit in the direct CP

asymmetries of group-1 modes ($\bar{B}^0 \to K^-\pi^+, ...$), and produces reasonable fits in the direct CP asymmetries of group-2 modes ($B^- \to \bar{K}^0\pi^-, ...$) and of group-3 modes ($B^- \to \pi^-\pi^0, ...$), but the fits in rates and mixing induced CP asymmetries of all modes (including B_s decay modes) and direct CP asymmetries of group-4 modes are poor. In particular, the ratios of χ^2 per number of data used in rates of the group-2 modes ($B^- \to \bar{K}^0\pi^-, ...$), group-4 modes ($\bar{B}^0 \to \pi^+\pi^-, ...$), in the rates and direct CP asymmetries of B_s modes and in the semileptonic quantity are as large as 24.7/4, 55.3/9, 64.0/7, and 8.0/1, respectively, indicating the badness of the fit in these sectors.

The fit is significant improved when the rescattering is added. In the best fitted case, the χ^2 per degree of freedom of the fit is $48.1/(48-14)$ giving the p-value of 5.5%. It should be noted that with 4 additional parameters the quality of the fit is improved significantly. All χ^2, except the direct CP of group-3 modes ($B^- \to \pi^-\pi^0, ...$), which is slightly enhanced, are reduced. In particular, the χ^2 per number of data of rates of the group-2 modes ($B^- \to \bar{K}^0\pi^-, ...$), group-4 modes ($\bar{B}^0 \to \pi^+\pi^-, ...$), the rates and direct CP asymmetries of B_s modes and in the semileptonic quantity are 6.3/4, 7.8/9, 0.6/7 and 0.7/1, respectively. The performance of the fit in these sector is improved significantly. Detailed results will be shown later.

The fitted parameters are shown in Table III. Uncertainties are obtained by scanning the parameter space with $\chi^2 \le \chi^2_{min} + 1$. The parameters consist of those in factorization amplitude and of Res. The Fac fit gives $F^{B\pi} = 0.239 \pm 0.002$, while the Res fit gives $F^{B\pi} = 0.253 \pm 0.002$. They correspond to $F^{B\pi}|V_{ud}| = (8.55^{+0.08}_{-0.05}) \times 10^{-4}$ and $F^{B\pi}|V_{ud}| = (9.03 \pm 0.09) \times 10^{-4}$ for $|V_{ub}| = 35.76 \times 10^{-4}$ employed in the numerical study, respectively, and they can be compared the HFLAG average, $F_0^{B\pi}(0)|V_{ub}| = (9.23 \pm 0.24) \times 10^{-4}$. The Res result agrees better with the data.

Both fits prefer large λ_{B_s}. Except ρ_A and ϕ_A, most common parameters in Fac and Res have similar fitted values. A closer look reveals that Fac prefers γ/ϕ_3 close to its center value [see Eq. (55)], while Res prefers a lower γ/ϕ_3. Comparing the fitted phases to those in the U(3)

TABLE III. Fitted hadronic and FSI parameters. Upper table contains fitted parameters in factorization amplitudes (Fac), while the lower ones are parameters in the rescattering (Res) case.

	ρ_A	ρ_H	$\phi_A(°)$	$\phi_H(°)$	$F_0^{B\pi}(0)$	$F_0^{BK}(0)$	$F_0^{B_sK}(0)$
Fac	$0.97^{+0.01}_{-0.02}$	$2.82^{+0.20}_{-0.61}$	$-28.4^{+0.3}_{-0.1}$	$-111.5^{+4.4}_{-13.6}$	0.239 ± 0.002	$0.27^{+0.00}_{-0.00}$	$0.23^{+0.00}_{-0.00}$
Res	$2.87^{+0.02}_{-0.03}$	2.33 ± 0.63	165.1 ± 0.9	-111.7 ± 20.6	0.253 ± 0.002	0.28 ± 0.01	0.24 ± 0.01

	λ_B (GeV)	λ_{B_s} (GeV)	$\gamma/\phi_3(°)$	$\tau(°)$	$\nu(°)$	$\delta(°)$	$\sigma(°)$
Fac	$0.19^{+0.02}_{-0.05}$	$0.60^{+0.00}_{-0.04}$	$75.4^{+1.7}_{-1.6}$	–	–	–	–
Res	0.22 ± 0.06	$0.45^{+0.15}_{-0.34}$	68.9 ± 1.8	22.2 ± 2.2	78.1 ± 2.9	23.3 ± 4.0	120.7 ± 22.3

exchange-type solution [see Eq. (A17)] $\tau = 24.1°$, $\nu = 35.3°$ and $\sigma - \delta = 0$ and in the U(3) annihilation-type solution [see Eq. (A16)] $\tau = -41.8°$, $\nu = -19.5°$ and $\sigma - \delta \neq 0$, we see that the fitted $\tau \simeq 22°$ and $\nu \simeq 78°$ seem to prefer the exchange-type solution, while the fitted $\sigma - \delta \simeq 97.4°$ supports the annihilation-type solution.

B. Rescattering effects on topological amplitudes

In this part, we will show the rescattering effects on topological amplitudes in certain combinations and on

some individual topological amplitudes of interest. Note that the discussion in the first part is generic, while we need to impose further assumption in the second part.

1. Rescattering effects on the combinations of topological amplitudes

It is useful to show the fitted results on residual rescattering parameters r_i' (or $r_i^{(1/2)}$):

$$1 + i(r_0' + r_a') = (0.979^{+0.007}_{-0.008}) \exp[i(11.98^{+1.66}_{-1.81})° + i\delta_{27}],$$

$$i(r_e' - r_a') = (0.208^{+0.028}_{-0.031}) \exp[i(-78.36 \pm 2.02)° + i\delta_{27}],$$

$$i(r_a' + r_t') = (0.059 \pm 0.009) \exp[i(-92.06^{+9.09}_{-13.21})° + i\delta_{27}],$$

$$i(2\bar{r}_a' + \bar{r}_e') = (0.189^{+0.048}_{-0.044}) \exp[i(-78.36 \pm 2.02)° + i\delta_{27}],$$

$$1 + i\left(\tilde{r}_0' + \frac{4\tilde{r}_a' + 2\tilde{r}_e'}{3}\right) = (0.990^{+0.004}_{-0.006}) \exp[i(3.27^{+1.24}_{-1.01})° + i\delta_{27}],$$

$$i\hat{r}_t' + i\frac{4\hat{r}_a' + 2\hat{r}_e'}{3} = (0.248^{+0.067}_{-0.068}) \exp[i(-29.66^{+11.13}_{-11.13})° + i\delta_{27}],$$

$$1 + i\left(\check{r}_0' + \frac{4\check{r}_a' + 2\check{r}_e' + 3\check{r}_t'}{6}\right) = (0.936^{+0.031}_{-0.041}) \exp[i(118.43^{+22.24}_{-21.73})° + i\delta_{27}]. \tag{57}$$

From the above equation, we see that most of these parameters have large phases (with respect to δ_{27}). Note that $i\hat{r}_t' + i(4\hat{r}_a' + 2\hat{r}_e')/3$, $i(r_e' - r_a')$ and $i(2\bar{r}_a' + \bar{r}_e')$ are three most sizable combinations and they are close to λ, $-i\lambda$ and $-i\lambda$ (taking the overall phase $\delta_{27} = 0$), respectively, where λ is the Wolfenstein parameter.

In Tables IV and V we show the topological amplitudes of $\bar{B}_q \to PP$ and $B_q \to PP$ decays before

rescattering (A^0) and after rescattering (A_{FSI}) in the unit of 10^{-8} GeV.[7] These amplitudes are expressed in certain combinations as noted in Eq. (28). Note that without lost of generality the overall phase (δ_{27}) is set to 0 from now on for simplicity. The ratios A_{FSI}/A^0 are also shown. These results are obtained using the best fitted solution and Eqs. (30), (33), (34), (48)–(50). Both $\Delta S = 0$ and $\Delta S = -1$ amplitudes are shown. Note that we do not use them directly in the fitting. In fact, they can be obtained only after the best fit result is available. Nevertheless they will provide useful information.

From Table IV, we see that before rescattering, we have the following order for $\bar{B}_q \to PP$ amplitudes:

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[7]The A^0 are obtained by using the rescattering parameters as shown in Table III, but with τ, ν, δ, and σ set to zero. Do not confuse it with the annihilation amplitude, where they may share the same notation occasionally.

TABLE IV. Combinations of topological amplitudes of $\Delta S = 0$, $\bar{B}_q \to PP$, and $B_q \to PP$ decays before rescattering (A^0) and after rescattering (A_{FSI}) in the unit of 10^{-8} GeV. These results are obtained using the best fitted solution and Eqs. (30), (33), (34), (48)–(50). Without loss of generality the overall phase (δ_{27}) for A_{FSI} is set to 0 for simplicity.

	$A^0(\bar{B})$	$A_{FSI}(\bar{B})$	$A_{FSI}/A^0(\bar{B})$	$A^0(B)$	$A_{FSI}(B)$	$A_{FSI}/A^0(B)$
$T + C$	$3.23e^{-i79.8°}$	$3.23e^{-i79.8°}$	1	$3.23e^{i57.9°}$	$3.23e^{i57.9°}$	1
$C - E$	$1.13e^{-i119.5°}$	$1.58e^{-i118.8°}$	$1.40e^{i0.7°}$	$1.18e^{i18.2°}$	$1.58e^{i19.0°}$	$1.40e^{i0.7°}$
$A + C$	$1.07e^{-i122.4°}$	$1.52e^{-i120.7°}$	$1.42e^{i1.7°}$	$1.07e^{i15.3°}$	$1.52e^{i17.0°}$	$1.42e^{i1.7°}$
$P - C + \frac{1}{3}P_{EW}^C$	$1.77e^{i34.1°}$	$2.23e^{i44.1°}$	$1.26e^{i10.0°}$	$0.80e^{-i102.1°}$	$0.94e^{-i128.3°}$	$1.17e^{-i26.2°}$
$PA - \frac{4}{9}C + \frac{13}{9}E - \frac{1}{3}P_{EW}^C$	$0.56e^{i75.4°}$	$0.45e^{i69.7°}$	$0.81e^{-i5.7°}$	$0.64e^{-i160.6°}$	$0.80e^{-i141.5°}$	$1.25e^{i19.1°}$
$P_{EW} + P_{EW}^C$	$0.10e^{i11.9°}$	$0.10e^{i11.9°}$	1	$0.10e^{-i32.1°}$	$0.10e^{-i32.1°}$	1
$P_{EW}^C - P_{EW}^E$	$0.04e^{-i36.7°}$	$0.05e^{-i31.0°}$	$1.40e^{i5.7°}$	$0.04e^{-i79.5°}$	$0.05e^{-i74.9°}$	$1.37e^{i4.6°}$
$P_{EW}^A + P_{EW}^C$	$0.03e^{-i44.2°}$	$0.05e^{-i38.5°}$	$1.53e^{i5.7°}$	$0.03e^{-i89.8°}$	$0.05e^{-i83.6°}$	$1.55e^{i6.1°}$
$\bar{T} + 2\bar{A}$	$2.66e^{-i63.9°}$	$2.43e^{-i56.2°}$	$0.92e^{i7.7°}$	$2.66e^{i73.8°}$	$2.43e^{i81.5°}$	$0.92e^{i7.7°}$
$\bar{C}_1 + 2\bar{E}$	$0.90e^{-i134.9°}$	$1.29e^{-i129.3°}$	$1.44e^{i5.6°}$	$0.96e^{i2.8°}$	$1.29e^{i8.4°}$	$1.44e^{i5.6°}$
$\bar{C}_2 + 2\bar{P} - \frac{1}{3}\bar{P}_{EW,2}^C$	$1.97e^{-i36.6}$	$2.22e^{-i49.1}$	$1.13e^{-i12.5°}$	$2.99e^{-i29.7}$	$3.34e^{-i27.1}$	$1.12e^{i2.6°}$
$\bar{P}_{EW} + 2\bar{P}_{EW}^A$	$0.09e^{i28.4°}$	$0.08e^{i37.4°}$	$0.92e^{i8.9°}$	$0.09e^{-i15.8°}$	$0.08e^{-i6.7°}$	$0.92e^{i9.2°}$
$\bar{P}_{EW,1}^C + 2\bar{P}_{EW}^E$	$0.04e^{-i58.8°}$	$0.05e^{-i46.9°}$	$1.34e^{i11.9°}$	$0.03e^{-i72.1°}$	$0.04e^{-i68.8°}$	$1.53e^{i3.3°}$
$\tilde{C} + \tilde{E} + \tilde{P} + \frac{3}{2}\tilde{P}A - \frac{1}{3}\tilde{P}_{EW}^C - \frac{1}{3}\tilde{P}_{EW}^E$	$1.34e^{-i65.0°}$	$1.56e^{-i2.6°}$	$1.16e^{i62.4°}$	$1.89e^{-i32.2°}$	$1.92e^{i59.1°}$	$1.02e^{i91.3°}$

TABLE V. Same as Table IV, but for $\Delta S = -1$ transition decay amplitudes.

	$A^0(\bar{B})$	$A_{FSI}(\bar{B})$	$A_{FSI}/A^0(\bar{B})$	$A^0(B)$	$A_{FSI}(B)$	$A_{FSI}/A^0(B)$
$T' + C'$	$0.75e^{-i79.8°}$	$0.75e^{-i79.8°}$	1	$0.75e^{i57.9°}$	$0.75e^{i57.9°}$	1
$C' - E'$	$0.26e^{-i119.5°}$	$0.36e^{-i118.8°}$	$1.40e^{i0.7°}$	$0.26e^{i18.2°}$	$0.36e^{i19.0°}$	$1.40e^{i0.7°}$
$A' + C'$	$0.25e^{-i122.4°}$	$0.35e^{-i120.7°}$	$1.42e^{i1.7°}$	$0.25e^{i15.3°}$	$0.35e^{i17.0°}$	$1.42e^{i1.7°}$
$P' - C' + \frac{1}{3}P_{EW}'^C$	$4.36e^{i164.2°}$	$4.00e^{i174.0°}$	$0.92e^{i9.8°}$	$4.64e^{i170.3°}$	$4.48e^{-i178.6°}$	$0.97e^{i11.1°}$
$PA' - \frac{4}{9}C' + \frac{13}{9}E' - \frac{1}{3}P_{EW}'^C$	$0.42e^{i3.5°}$	$0.99e^{i73.2°}$	$2.37e^{i69.7°}$	$0.29e^{-i31.2°}$	$0.74e^{i81.8°}$	$2.58e^{i113.0°}$
$P_{EW}' + P_{EW}'^C$	$0.46e^{i168.9°}$	$0.46e^{i168.9°}$	1	$0.46e^{i171.0°}$	$0.46e^{i171.0°}$	1
$P_{EW}'^C - P_{EW}'^E$	$0.18e^{i120.8°}$	$0.26e^{i124.8°}$	$1.40e^{i3.9°}$	$0.18e^{i122.9°}$	$0.26e^{i126.8°}$	$1.40e^{i3.9°}$
$P_{EW}'^A + P_{EW}'^C$	$0.14e^{i113.6°}$	$0.22e^{i118.9°}$	$1.53e^{i5.3°}$	$0.14e^{i115.6°}$	$0.22e^{i121.0°}$	$1.53e^{i5.4°}$
$\bar{T}' + 2\bar{A}'$	$0.61e^{-i63.9°}$	$0.56e^{-i56.2°}$	$0.92e^{i7.7°}$	$0.61e^{i73.8°}$	$0.56e^{i81.5°}$	$0.92e^{i7.7°}$
$\bar{C}_1' + 2\bar{E}'$	$0.21e^{-i134.9°}$	$0.30e^{-i129.3°}$	$1.44e^{i5.6°}$	$0.21e^{i2.8°}$	$0.30e^{i8.4°}$	$1.44e^{i5.6°}$
$\bar{C}_2' + 2\bar{P}' - \frac{1}{3}\bar{P}_{EW,2}'^C$	$10.62e^{i152.6}$	$11.13e^{i149.9}$	$1.05e^{-i2.7°}$	$10.38e^{i152.4}$	$10.85e^{i148.8}$	$1.05e^{-i3.6°}$
$\bar{P}_{EW}' + 2\bar{P}_{EW}'^A$	$0.42e^{-i174.8°}$	$0.38e^{-i165.9°}$	$0.92e^{i8.8°}$	$0.42e^{-i172.6°}$	$0.38e^{-i163.8°}$	$0.92e^{i8.9°}$
$\bar{P}_{EW,1}'^C + 2\bar{P}_{EW}'^E$	$0.08e^{i61.0°}$	$0.12e^{i93.9°}$	$1.48e^{i33.0°}$	$0.08e^{i63.0°}$	$0.12e^{i93.0°}$	$1.48e^{i33.0°}$
$\tilde{C}' + \tilde{E}' + \tilde{P}' + \frac{3}{2}\tilde{P}A' - \frac{1}{3}\tilde{P}_{EW}'^C - \frac{1}{3}\tilde{P}_{EW}'^E$	$6.04e^{i140.0°}$	$5.01e^{-i141.5°}$	$0.83e^{i78.5°}$	$5.89e^{i138.1°}$	$4.88e^{-i146.1°}$	$0.83e^{i75.8°}$

$$|T^0 + C^0| > |\bar{T}^0 + 2\bar{A}^0| > \left|\bar{C}_2^0 + 2\bar{P}^0 - \frac{1}{3}\bar{P}_{EW,2}^{C0}\right| > \left|P^0 - C^0 + \frac{1}{3}P_{EW}^{C0}\right| > \left|\tilde{C}^0 + \tilde{E}^0 + \tilde{P}^0 + \frac{3}{2}\tilde{P}A^0 - \frac{1}{3}\tilde{P}_{EW}^{C0} - \frac{1}{3}\tilde{P}_{EW}^{E0}\right| > |C^0 - E^0|$$

$$\gtrsim |A^0 + C^0| \gtrsim |\bar{C}_1^0 + 2\bar{E}^0| > \left|PA^0 - \frac{4}{9}C^0 + \frac{13}{9}E^0 - \frac{1}{3}P_{EW}^{C0}\right|,$$

while the rest are rather small. After rescattering, we have:

$$|T + C| > |\bar{T} + 2\bar{A}| > \left|P - C + \frac{1}{3}P_{EW}^C\right| \gtrsim \left|\bar{C}_2 + 2\bar{P} - \frac{1}{3}\bar{P}_{EW,2}^C\right| > \left|\tilde{C} + \tilde{E} + \tilde{P} + \frac{3}{2}\tilde{P}A - \frac{1}{3}\tilde{P}_{EW}^C - \frac{1}{3}\tilde{P}_{EW}^E\right| \gtrsim |C - E|$$

$$\gtrsim |A + C| > |\bar{C}_1 + 2\bar{E}| > \left|PA - \frac{4}{9}C + \frac{13}{9}E - \frac{1}{3}P_{EW}^C\right|,$$

where $|C - E|$, $|A + C|$ and $|\bar{C}_1 + 2\bar{E}|$ are enhanced by $40 \sim 44\%$, $|P - C + \frac{1}{3}P_{EW}^C|$ by 26% and $|\tilde{C} + \tilde{E} + \tilde{P} + \frac{3}{2}\widetilde{PA} - \frac{1}{3}\tilde{P}_{EW}^C - \frac{1}{3}\tilde{P}_{EW}^E|$ by 16%. Note that the orders of $|\bar{C}_2 + 2\bar{P} - \frac{1}{3}\bar{P}_{EW,2}^C|$ and $|P - C + \frac{1}{3}P_{EW}^C|$ are switched after turning on Res. Subleading tree amplitudes and penguin amplitudes are enhanced. We will return to this shortly. Note that except in $\tilde{C} + \tilde{E} + \tilde{P} + \frac{3}{2}\widetilde{PA} - \frac{1}{3}\tilde{P}_{EW}^C - \frac{1}{3}\tilde{P}_{EW}^E$ Res does not introduce sizable phases to these topological amplitude combinations.

Similarly, from Table IV, we see that before rescattering, we have the following order for the conjugated $B_q \to PP$ decay amplitudes:

$$|T^0 + C^0| > \left|\bar{C}_2^0 + 2\bar{P}^0 - \frac{1}{3}\bar{P}_{EW,2}^{C0}\right| > |\bar{T}^0 + 2\bar{A}^0| > \left|\tilde{C}^0 + \tilde{E}^0 + \tilde{P}^0 + \frac{3}{2}\tilde{P}A^0 - \frac{1}{3}\tilde{P}_{EW}^{C0} - \frac{1}{3}\tilde{P}_{EW}^{E0}\right| > |C^0 - E^0| \gtrsim |A^0 + C^0|$$

$$\gtrsim |\bar{C}_1^0 + 2\bar{E}^0| > \left|P^0 - C^0 + \frac{1}{3}P_{EW}^{C0}\right| > \left|PA^0 - \frac{4}{9}C^0 + \frac{13}{9}E^0 - \frac{1}{3}P_{EW}^{C0}\right|,$$

while the rest are rather small. Note that the above order is different form the one in $\bar{B}_q \to PP$ decays. After rescattering, only the first two terms switch order, where $|\bar{C}_2 + 2\bar{P} - \frac{1}{3}\bar{P}_{EW,2}^C|$ is enhanced by 12%, $|P - C + \frac{1}{3}P_{EW}^C|$ by 17% and $|C - E|$, $|A + C|$ and $|\bar{C}_1 + 2\bar{E}|$ by $40 \sim 44\%$. Note that Res introduces sizable phases to some of these topological amplitude combinations and $|\bar{C}_2 + 2\bar{P} - \frac{1}{3}\bar{P}_{EW,2}^C|$, $|\tilde{C} + \tilde{E} + \tilde{P} + \frac{3}{2}\widetilde{PA} - \frac{1}{3}\tilde{P}_{EW}^C - \frac{1}{3}\tilde{P}_{EW}^E|$, and $|PA - \frac{4}{9}C + \frac{13}{9}E - \frac{1}{3}P_{EW}^C|$ are quite different to those in $\bar{B}_q \to PP$ decays.

Some comments will be useful. (i) A large number of combinations of topological amplitudes are sizable. (ii) After rescattering one sees that the phases introduced to $\bar{B} \to PP$ and $B \to PP$ amplitudes are quite different. (iii) The above facts imply that the effect of Res on direct CP violations can be complicate and rich. (iv) The enhancement of rescattering on some of the $\Delta S = 0$ topological amplitudes can be up to 55%, such as on $P_{EW}^A + P_{EW}^C$, but their sizes are still small even after the enhancement. Nevertheless this may have impact on some suppressed modes.

It is useful to see the above enhancements in details. It is clear from Eq. (48) that the effects of Res on $T + C$ and $P_{EW} + P_{EW}^C$ are just adding the common phase δ_{27} to them. The effects on other combinations of topological amplitudes are more interesting. In $\bar{B}_q \to PP$ decays, considering only the dominant contributions in Eq. (48), we have

$$C - E \simeq (1 + ir_0' + ir_a')(C^0 - E^0) + i(r_e' - r_a')(T^0 + C^0),$$

$$A + C \simeq (1 + ir_0' + ir_a')(A^0 + C^0) + \frac{2}{3}i(r_e' - r_a')(T^0 + C^0)$$

$$+ \frac{1}{3}i(\bar{r}_e' + 2\bar{r}_a')(\bar{T}^0 + 2\bar{A}^0). \quad (58)$$

We can estimation the above values by taking the central values of $(1 + ir_0' + ir_a')$, $i(r_e' - r_a')$ and $i(\bar{r}_e' + 2\bar{r}_a')$ from Eq. (57) and the central values of $C^0 - E^0$, $A^0 + C^0$, $T^0 + C^0$ and $\bar{T}^0 + 2\bar{A}^0$ from Table IV, obtaining

$$\frac{C - E}{C^0 - E^0} \simeq 1.4e^{-i7°}, \qquad \frac{A + C}{A^0 + C^0} \simeq 1.4e^{-i4°}, \quad (59)$$

which are close the values of $1.40e^{i0.7°}$ and $1.42e^{i1.7°}$ shown in Table IV. Even using a crude estimation by taking $(1 + ir_0' + ir_a') \simeq 1$ and $i(r_e' - r_a') \simeq i(\bar{r}_e' + 2\bar{r}_a') \simeq -i\lambda$, one still get $1.5e^{-i19°}$ and $1.5e^{-i16°}$, which are not too far off. It is clear that the effect of Res in $C - E$ mainly comes from the exchange and annihilation rescatterings fed from the $T^0 + C^0$ amplitude, while those in $A + C$ comes from the exchange and annihilation rescatterings fed from both $T^0 + C^0$ and $\bar{T}^0 + 2\bar{A}^0$ amplitudes.

Similarly from Eq. (50), we have

$$\left(\tilde{C} + \tilde{E} + \tilde{P} + \frac{3}{2}\widetilde{PA} - \frac{1}{3}\tilde{P}_{EW}^C - \frac{1}{3}\tilde{P}_{EW}^E\right)$$

$$\simeq \left[1 + i\left(\check{r}_0' + \frac{4\check{r}_a' + 2\check{r}_e' + 3\check{r}_t'}{6}\right)\right]$$

$$\times \left(\tilde{C}^0 + \tilde{E}^0 + \tilde{P}^0 + \frac{3}{2}\tilde{P}A^0 - \frac{1}{3}\tilde{P}_{EW}^{C0} - \frac{1}{3}\tilde{P}_{EW}^{E0}\right)$$

$$+ i\left(\hat{r}_t' + \frac{4\hat{r}_a' + 2\hat{r}_e'}{3}\right)\left[\frac{3}{2}(T^0 + C^0) + \frac{14}{3}(C^0 - E^0)\right.$$

$$\left. + 4\left(P^0 - C^0 + \frac{1}{3}P_{EW}^{C0}\right) + 6\left(PA^0 - \frac{4}{9}C^0 + \frac{13}{9}E^0 - \frac{1}{3}P_{EW}^{C0}\right)\right],$$

$$(60)$$

and we find that the $T^0 + C^0$ and $C^0 - E^0$ terms give (sizable) destructive contributions, while $P^0 - C^0 + \frac{1}{3}P_{EW}^{C0}$ and $PA^0 - \frac{4}{9}C^0 + \frac{13}{9}E^0 - \frac{1}{3}P_{EW}^{C0}$ terms give (sizable) constructive contributions via the same Res parameter $i\hat{r}_t' + i(4\hat{r}_a' + 2\hat{r}_e')/3$. The final result of the 16% enhancement in $|\tilde{C} + \tilde{E} + \tilde{P} + \frac{3}{2}\tilde{P}A - \frac{1}{3}\tilde{P}_{EW}^C - \frac{1}{3}\tilde{P}_{EW}^E|$ is the complicate interplay of these contributions.

We now turn to the Res effect on the penguin amplitudes. From [see Eq. (48)]

$$P - C + \frac{1}{3} P_{EW}^C \simeq \left[(1 + ir_0' + ir_a') - \frac{2}{3} i(r_e' - r_a') \right]$$
$$\times \left(P^0 - C^0 + \frac{1}{3} P_{EW}^{C0} \right) - i(r_e' - r_a')(T^0 + C^0)$$
$$+ \frac{1}{3} i(\bar{r}_e' + 2\bar{r}_a') \left(\bar{C}_2^0 + 2\bar{P}^0 - \frac{1}{3} \bar{P}_{EW,2}^{C0} \right), \tag{61}$$

we obtain for $\bar{B} \to PP$ decay:

$$\frac{P - C + \frac{1}{3} P_{EW}^C}{P^0 - C^0 + \frac{1}{3} P_{EW}^{C0}} \simeq 1.3 e^{i10°}, \tag{62}$$

which is close to the value $1.26 e^{i10.0°}$ shown in Table IV. where the main contribution is from the $r_e' - r_a'$ rescattering term fed from $T^0 + C^0$.

We now turn to $\Delta S = -1$ processes. The results are shown in Table V. We see from the table that before rescattering, we have the following order for $\bar{B}_q \to PP$ amplitudes:

$$\left| \bar{C}_2'^0 + 2\bar{P}'^0 - \frac{1}{3} \bar{P}_{EW,2}'^{C0} \right|$$
$$> \left| \tilde{C}'^0 + \tilde{E}'^0 + \tilde{P}'^0 + \frac{3}{2} \tilde{P} \tilde{A}'^0 - \frac{1}{3} \tilde{P}_{EW}'^{C0} - \frac{1}{3} \tilde{P}_{EW}'^{E0} \right|$$
$$> \left| P'^0 - C^0 + \frac{1}{3} P_{/EW}^{C0} \right| \gg |T'^0 + C'^0|$$
$$> |\bar{T}'^0 + 2\bar{A}'^0| > |P_{EW}'^0 + P_{EW}'^{C0}|$$
$$\gtrsim \left| PA'^0 - \frac{4}{9} C'^0 + \frac{13}{9} E'^0 - \frac{1}{3} P_{EW}'^{C0} \right|$$
$$\gtrsim |\bar{P}_{EW}'^0 + 2\bar{P}_{EW}'^{A0}| > |C'^0 - E'^0| \gtrsim |A'^0 + C'^0|,$$

while the rest are rather small. Note that as expected penguin amplitudes dominate over trees. In fact, even the electroweak penguin amplitudes, which were neglected in the $\Delta S = 0$ case, cannot be neglected now. After rescattering, the above orders are rearranged into:

$$\left| \bar{C}_2' + 2\bar{P}' - \frac{1}{3} \bar{P}_{EW,2}'^C \right| > \left| \tilde{C}' + \tilde{E}' + \tilde{P}' + \frac{3}{2} \tilde{P} \tilde{A}' - \frac{1}{3} \tilde{P}_{EW}'^C - \frac{1}{3} \tilde{P}_{EW}'^E \right|$$
$$> \left| P' - C' + \frac{1}{3} P_{EW}'^C \right|$$
$$\gg \left| PA' - \frac{4}{9} C' + \frac{13}{9} E' - \frac{1}{3} P_{EW}'^C \right|$$
$$> |T' + C'| > |\bar{T}' + 2\bar{A}'|$$
$$> |P_{EW}' + P_{EW}'^C| > |\bar{P}_{EW}' + 2\bar{P}_{EW}'^A|$$
$$\gtrsim |C' - E'| \gtrsim |A' + C'|.$$

We see that the combinations with sub-leading tree amplitudes, $C' - E'$ and $A' + C'$, are enhanced, while the one with the penguin term, $P' - C' + P_{EW}'^C/3$, is slightly reduced. Note that $|PA' - \frac{4}{9} C' + \frac{13}{9} E' - \frac{1}{3} P_{EW}'^C|$ is enhanced by a factor of 2, but $|\tilde{C}' + \tilde{E}' + \tilde{P}' + \frac{3}{2} \tilde{P} \tilde{A}' - \frac{1}{3} \tilde{P}_{EW}'^C - \frac{1}{3} \tilde{P}_{EW}'^E|$ is reduced by about 20%. Similar pattern occurs in the conjugated $B_q \to PP$ decays.

The effect of rescattering on $A' + C'$ is similar to the one in $A + C$. It is enhanced from the exchange and annihilation rescatterings fed from both $T'^0 + C'^0$ and $\bar{T}'^0 + 2\bar{A}'^0$ amplitudes. We also note that the effect of rescattering on $P_{EW}'^C - P_{EW}'^E$ is similar to the one in $C' - E'$, but with tree amplitudes replaced by electroweak penguins. Hence $P_{EW}'^C - P_{EW}'^E$ is affected most from $P_{EW}'^0 + P_{EW}'^{C0}$ and the effect is an enhancement in size.

It is useful to see the enhancement and reduction in $|PA' - \frac{4}{9} C' + \frac{13}{9} E' - \frac{1}{3} P_{EW}'^C|$ and $|\tilde{C}' + \tilde{E}' + \tilde{P}' + \frac{3}{2} \tilde{P} \tilde{A}' - \frac{1}{3} \tilde{P}_{EW}'^C - \frac{1}{3} \tilde{P}_{EW}'^E|$, respectively, in more detail. In $\bar{B}_q \to PP$ decays, keeping only the $(PA'^0 - \frac{4}{9} C'^0 + \frac{13}{9} E'^0 - \frac{1}{3} P_{EW}'^C)$ and the $(P'^0 - C'^0 + \frac{1}{3} P_{EW}'^{C0})$ terms in the corresponding formula shown in Eq. (48), we obtain

$$\frac{PA' - \frac{4}{9} C' + \frac{13}{9} E' - \frac{1}{3} P_{EW}'^C}{PA'^0 - \frac{4}{9} C'^0 + \frac{13}{9} E'^0 - \frac{1}{3} P_{EW}'^{C0}} \simeq 2.6 e^{i52°}, \tag{63}$$

which is close to the value $2.37 e^{i69.7°}$ shown in Table V. Similarly using the corresponding formula in Eq. (48) and keep only the $(\tilde{C}'^0 + \tilde{E}'^0 + \tilde{P}'^0 + \frac{3}{2} \tilde{P} \tilde{A}^{(\prime)0} - \frac{1}{3} \tilde{P}_{EW}'^{C0} - \frac{1}{3} \tilde{P}_{EW}'^{E0})$ and the $(P'^0 - C'^0 + \frac{1}{3} P_{EW}'^{C0})$ terms we obtain

$$\frac{\tilde{C}' + \tilde{E}' + \tilde{P}' + \frac{3}{2} \tilde{P} \tilde{A}' - \frac{1}{3} \tilde{P}_{EW}'^C - \frac{1}{3} \tilde{P}_{EW}'^E}{\tilde{C}'^0 + \tilde{E}'^0 + \tilde{P}'^0 + \frac{3}{2} \tilde{P} \tilde{A}'^0 - \frac{1}{3} \tilde{P}_{EW}'^{C0} - \frac{1}{3} \tilde{P}_{EW}'^{E0}} \simeq 0.8 e^{i76°}, \tag{64}$$

which is close to the value $0.83 e^{i78.5°}$ shown in Table V. In both cases the most important contributions are from the $(P'^0 - C'^0 + \frac{1}{3} P_{EW}'^{C0})$ term.

2. Rescattering effects on some individual topological amplitudes

The results in Tables IV and V are all we can have, if no further assumption is made. It is, however, desirable to reveal the effect of Res on some individual topological amplitudes instead of their combinations. To explore the effect one needs the information of various r_i' instead of their combinations shown in Eq. (57). For example, the Res effect on exchange amplitude is given by [see Eq. (43)]

TABLE VI. Some topological amplitudes of $\Delta S = 0, -1$, $\bar{B}_q \to PP$ and $B_q \to PP$ decays before rescattering (A^0) and after rescattering (A_{FSI}) in the unit of 10^{-9} GeV. These results are obtained using the best fitted solution and Eqs. (30), (33), (34) and (43). We use an additional assumption, $r'_e = 0$ as suggested from U(3) symmetry on TA [see Eq. (54)]. Without lost of generality the overall phase (δ_{27}) for A_{FSI} is set to 0. Results in combinations of $\bar{A}$ and $\tilde{A}$ can be found in Tables IV and V.

	$A^0(\bar{B})$	$A_{\text{FSI}}(\bar{B})$	$A_{\text{FSI}}/A^0(\bar{B})$	$A^0(B)$	$A_{\text{FSI}}(B)$	$A_{\text{FSI}}/A^0(B)$
T	$25.84e^{-i63.5°}$	$25.84e^{-i63.5°}$	1	$25.84e^{i74.2°}$	$25.84e^{i74.2°}$	1
C	$10.45e^{-i123.9°}$	$10.45e^{-i123.9°}$	1	$10.45e^{i13.8°}$	$10.45e^{i13.8°}$	1
E	$1.19e^{i102.6°}$	$5.46e^{i71.0°}$	$4.61e^{-i31.5°}$	$1.19e^{-i119.7°}$	$5.46e^{-i151.2°}$	$4.61e^{-i31.5°}$
A	$0.38e^{-i77.4°}$	$4.78e^{-i113.8°}$	$12.67e^{-i36.4°}$	$0.38e^{i60.3°}$	$4.78e^{i23.9°}$	$12.67e^{-i36.4°}$
P	$8.89e^{i8.6°}$	$12.26e^{i34.3°}$	$1.38e^{i25.7°}$	$9.94e^{-i31.6°}$	$6.43e^{-i47.6°}$	$0.65e^{-i16.0°}$
PA	$0.76e^{-i166.4°}$	$7.95e^{-i116.2°}$	$10.50e^{i50.2°}$	$0.76e^{i149.3°}$	$5.19e^{-i1.6°}$	$6.86e^{-i150.9°}$
P_{EW}	$0.86e^{i29.0°}$	$0.86e^{i29.0°}$	1	$0.86e^{-i15.3°}$	$0.86e^{-i15.3°}$	1
P_{EW}^C	$0.29e^{-i46.8°}$	$0.29e^{-i46.8°}$	1	$0.29e^{-i89.1°}$	$0.29e^{-i89.1°}$	1
P_{EW}^E	$0.11e^{i170.5°}$	$0.27e^{i166.0°}$	$2.42e^{-i4.4°}$	$0.11e^{i126.2°}$	$0.26e^{i121.2°}$	$2.31e^{-i5.0°}$
P_{EW}^A	$0.02e^{i13.6°}$	$0.18e^{-i24.7°}$	$11.21e^{-i38.3°}$	$0.02e^{-i30.7°}$	$0.18e^{-i68.8°}$	$11.08e^{-i.38.1°}$
T'	$5.98e^{-i63.5°}$	$5.98e^{-i63.5°}$	1	$5.98e^{i74.2°}$	$5.98e^{i74.2°}$	1
C'	$2.41e^{-i123.9°}$	$2.41e^{-i123.9°}$	1	$2.41e^{i13.8°}$	$2.41e^{i13.8°}$	1
E'	$0.27e^{i102.6°}$	$1.26e^{i71.0°}$	$4.61e^{-i31.5°}$	$0.27e^{-i119.7°}$	$1.26e^{-i151.2°}$	$4.61e^{-i31.5°}$
A'	$0.09e^{-i77.4°}$	$1.10e^{-i113.8°}$	$12.67e^{-i36.4°}$	$0.09e^{i60.3°}$	$1.10e^{i23.9°}$	$12.67e^{-i36.4°}$
P'	$44.12e^{i167.6°}$	$40.99e^{i177.5°}$	$0.93e^{i9.9°}$	$43.90e^{i169.6°}$	$42.29e^{-i178.7°}$	$0.96e^{i11.7°}$
PA'	$3.54e^{-i9.6°}$	$7.43e^{i78.3°}$	$2.10e^{i87.9°}$	$3.54e^{-i7.5°}$	$9.56e^{i68.4°}$	$2.70e^{i75.9°}$
P'_{EW}	$4.01e^{-i174.2°}$	$4.01e^{-i174.2°}$	1	$4.01e^{-i172.1°}$	$4.01e^{-i172.1°}$	1
$P_{\text{EW}}'^C$	$1.38e^{i111.0°}$	$1.38e^{i111.0°}$	1	$1.38e^{i113.0°}$	$1.38e^{i113.0°}$	1
$P_{\text{EW}}'^E$	$0.53e^{-i32.7°}$	$1.27e^{-i40.3°}$	$2.40e^{-i7.6°}$	$0.53e^{-i30.6°}$	$1.27e^{-i38.2°}$	$2.41e^{-i7.6°}$
$P_{\text{EW}}'^A$	$0.07e^{i170.4°}$	$0.83e^{i132.2°}$	$11.14e^{-i38.2°}$	$0.07e^{i172.5°}$	$0.83e^{i134.3°}$	$11.15e^{-i38.2°}$

$$\delta E^{(\prime)} = ir'_0 E^{(\prime)0} + ir'_a T^{(\prime)0} - \frac{1}{3}i(r'_e + 2r'_a)C^{(\prime)0}$$
$$+ \frac{1}{3}i(-2r'_e + 5r'_a)E^{(\prime)0}$$
$$+ \frac{1}{3}i(\bar{r}'_e + 2\tilde{r}'_a)(\bar{C}_1^{(\prime)0} + 2\bar{E}^{(\prime)0}). \tag{65}$$

It is clear that we need the information of r'_0, r'_a, r'_e and so on to obtain $\delta E^{(\prime)}$. From the fit we only have information on some combinations of these rescattering parameters, such as $1 + i(r'_0 + r'_a)$, $i(r'_e - r'_a)$ and so on [see Eq. (57)], but not on individual ones. To study the effect of Res on individual topological amplitudes, we make an additional assumption:

$$r'_e = 0, \tag{66}$$

which is suggested by the U(3) symmetry on TA [see Eq. (54)]. Note that we only assume $r'_e = 0$ and do not impose any condition on $\bar{r}'_e$, $\hat{r}'_e$ and $\check{r}'_e$. Hence we are not using the full U(3) symmetry, but rather consider the case of a suppressed r'_e. Using the above assumption and the results in Eq. (57) one can now extract the effect of Res on some individual TAs of interest. The results are shown in Table VI. One should keep in mind of the assumption made. Note that the above assumption will affect our interpretation of the effect of Res on individual topological

amplitudes, but not on the interpretation of the effect of Res on the combinations of topological amplitudes as discussed previously. In other words, the above assumption will affect the results stated in Table VI, but not on those in Tables IV and V.

From Table VI we see that, before Res, for $\bar{B}_q \to PP$ and $B_q \to PP$ decays, we have

$$|T^0| > |C^0| > |P^0| \gg |E^0| > |P_{\text{EW}}^0| > |PA^0| > |A^0|$$
$$\gtrsim |P_{\text{EW}}^{0C}| > |P_{\text{EW}}^{0E}| \gg |P_{\text{EW}}^{0A}|,$$
$$|P'^0| \gg |T'^0| > |P_{\text{EW}}'^0| > |PA'^0|$$
$$\gtrsim |C'^0| > |P_{\text{EW}}'^{C0}| > |P_{\text{EW}}'^{E0}| > |E'^0| \gg |A'^0| \gtrsim |P_{\text{EW}}'^{A0}|, \tag{67}$$

while after Res, we have

$$|T| > |P| > |C| > |PA| > |E| > |A|$$
$$\gg |P_{\text{EW}}| > |P_{\text{EW}}^C| > |P_{\text{EW}}^E| > |P_{\text{EW}}^A|,$$
$$|P'| \gg |PA'| > |T'| > |P_{\text{EW}}'| > |C'| > |P_{\text{EW}}'^C| > |P_{\text{EW}}'^E|$$
$$\gtrsim |E'| > |A'| > |P_{\text{EW}}'^A|, \tag{68}$$

for $\bar{B}_q \to PP$ decays, and

$$|T| > |C| > |P| > |E| > |PA| > |A|$$
$$\gg |P_{\text{EW}}| > |P_{\text{EW}}^C| > |P_{\text{EW}}^E| > |P_{\text{EW}}^A|,$$
$$|P'| \gg |PA'| > |T'| > |P_{\text{EW}}'| > |C'| > |P_{\text{EW}}'^C| > |P_{\text{EW}}'^E|$$
$$\gtrsim |E'| > |A'| > |P_{\text{EW}}'^A|, \tag{69}$$

for $B_q \to PP$ decays. Note that the positions of $|P|$ and $|PA|$ in the above orders are different in $\bar{B}_q \to PP$ and $B_q \to PP$ decays. We will come to that later.

We see from Table VI that $|E|$, $|E'|$, $|A|$, $|A'|$, $|PA|$, $|PA'|$, $|P_{\text{EW}}^{A,E}|$ and $|P_{\text{EW}}'^{A,E}|$ are enhanced significantly with factors ranging from $2 \sim 11$, while $|P|$ is enhanced by 35% in $\bar{B}_q \to PP$ decay, but is suppressed by 35% in $B_q \to PP$ decay and $|P'|$ are suppressed by 6% and 3% in $\bar{B}_q \to PP$ and $B_q \to PP$ decays, respectively. Note that in particular $|A|$ and $|A'|$ are enhanced by a factor of 11.5. It is useful to look into the enhancement. From Eq. (43), we have

$$A^{(\prime)} = \left(1 + ir_0' - \frac{2}{3}ir_e' + \frac{5}{3}ir_a'\right)A^{(\prime)0} - \frac{1}{3}i(r_e' + 2r_a')T^{(\prime)0}$$
$$+ ir_a'C^{(\prime)0} + \frac{1}{3}i(\bar{r}_e' + 2\bar{r}_a')(\bar{T}^{(\prime)0} + 2\bar{A}^{(\prime)0}). \tag{70}$$

Now make use of $r_e' = 0$ and Eq. (57), we obtain

$$\frac{A^{(\prime)}}{A^{(\prime)0}} = 0.99e^{i20.0°} + 9.48e^{-i64.4°} + 5.75e^{i55.2°} + 4.45^{-i64.8°}$$
$$= 12.67e^{-i36.4}, \tag{71}$$

where the terms in the right-hand side of the first equality are from $A^{(\prime)0}$, $T^{(\prime)0}$, $C^{(\prime)0}$, $\bar{T}^{(\prime)0} + 2\bar{A}^{(\prime)0}$ contributions, respectively. We see that the $T^{(\prime)0}$, $C^{(\prime)0}$, $\bar{T}^{(\prime)0} + 2\bar{A}^{(\prime)0}$ terms give sizable contributions to $A^{(\prime)}$, via r_a', r_a', and $\bar{r}_e + 2\bar{r}_a'$ rescatterings, respectively, and enhance its size significantly. Similarly we have

$$\frac{E^{(\prime)}}{E^{(\prime)0}} = 0.99e^{i20.0°} + 4.53e^{-i64.4°} + 1.22e^{i55.2°} + 0.48^{i44.2°}$$
$$= 4.61e^{-i31.5}, \tag{72}$$

where the terms in the right-hand side of the first equality are from $A^{(\prime)0}$, $T^{(\prime)0}$, $C^{(\prime)0}$, $\bar{C}_1^{(\prime)0} + 2\bar{E}^{(\prime)0}$ contributions, respectively. The dominate contribution is from $T^{(\prime)0}$ via annihilation rescattering r_a'.

As noted previously $P^{(\prime)}$ and $PA^{(\prime)}$ receive different Res contributions in $\bar{B}_q \to PP$ and $B_q \to PP$ decays. It is interesting to investigate the effects of Res on these penguin amplitudes in details. First, we decompose $P^{(\prime)}$ into the so-called u-penguin ($P^{(\prime)u}$) and c-penguin ($P^{(\prime)c}$) as $P^{(\prime)} = P^{(\prime)u} + P^{(\prime)c}$ according to the different CKM factors. Now from Eq. (43), we have

$$P^{(\prime)u} = \left[1 + ir_0' + \frac{1}{3}i(-2r_e' + 5r_a')\right]P^{(\prime)u0} + ir_a'T^{(\prime)0}$$
$$- \frac{1}{3}i(r_e' + 2r_a')C^{(\prime)0} - \frac{1}{3}ir_a'P_{\text{EW}}^{(\prime)u0} + \frac{1}{9}i(r_e' + 2r_a')P_{\text{EW}}^{(\prime)uC0}$$
$$+ \frac{1}{3}i(\bar{r}_e' + 2\bar{r}_a')\left(\bar{C}_2^{(\prime)0} + 2\bar{P}^{(\prime)u0} - \frac{1}{3}\bar{P}_{\text{EW},2}^{(\prime)uC0}\right),$$

$$P^{(\prime)c} = \left[1 + ir_0' + \frac{1}{3}i(-2r_e' + 5r_a')\right]P^{(\prime)c0}$$
$$- \frac{1}{3}ir_a'P_{\text{EW}}^{(\prime)c0} + \frac{1}{9}i(r_e' + 2r_a')P_{\text{EW}}^{(\prime)cC0}$$
$$+ \frac{1}{3}i(\bar{r}_e' + 2\bar{r}_a')\left(2\bar{P}^{(\prime)c0} - \frac{1}{3}\bar{P}_{\text{EW},2}^{(\prime)cC0}\right). \tag{73}$$

Using these formulas and the best fit parameters, we obtain

$$\frac{P^{(\prime)u}}{P^{(\prime)u0}} = 0.99e^{i20.0°} + 1.53e^{-i70.2°} + 0.41e^{i49.4°}$$
$$+ 0.01^{-i56.1°} + 0.10^{-i2.3°} = 1.99e^{-i23.6°}, \tag{74}$$

where the terms in the right-hand side of the first equality are from $P^{(\prime)u0}$, $T^{(\prime)0}$, $C^{(\prime)0}$, $P_{\text{EW}}^{(\prime)u0,(\prime)uC0}$, and $\bar{C}_2^{(\prime)0} + 2\bar{P}^{(\prime)u0} - \frac{1}{3}\bar{P}_{\text{EW},2}^{(\prime)uC0}$, respectively, and

$$\frac{P^{(\prime)c}}{P^{(\prime)c0}} = 0.99e^{i20.0°} + 0.01^{-i46.8°} + 0.15^{-i94.6°} = 0.94e^{i11.2°},$$
$$\tag{75}$$

where the terms in the right-hand side of the first equality are from $P^{(\prime)c0}$, $P_{\text{EW}}^{c(\prime)0,c(\prime)C0}$, and $2\bar{P}^{(\prime)c0} - \frac{1}{3}\bar{P}_{\text{EW},2}^{(\prime)cC0}$, respectively. It is clear that $T^{(\prime)0}$ and $C^{(\prime)0}$ only contribute to $P^{(\prime)u}$ (via the annihilation rescattering r_a') and $|P^{(\prime)u}|$ is enhanced by about a factor of 2. On the other hand $P^{(\prime)c}$ is only slightly affected by rescattering and is still close to the original $P^{(\prime)c0}$. The results are shown in Table VII.

It is useful to note that the ratio of u-penguin and c-penguin in $\Delta S = 0$ process before rescattering is expected to proportional to the CKM factors giving

$$\left|\frac{P^u}{P^c}\right| \simeq \left|\frac{V_{ub}V_{ud}^*}{V_{cb}V_{cd}^*}\right| \simeq 0.38. \tag{76}$$

The estimation is close to the ratio $|P^{u0}/P^{c0}| = 3.51/10.09 \simeq 0.35$ using P^{u0} and P^{c0} shown in Table VII. The CKM ratio implies that u-penguin and the c-penguin are not as hierarchical as in the $\Delta S = -1$ case. Furthermore, when rescattering is turned on, the u-penguin and c-penguin receive different contributions as only P^u can receive contribution fed from T^0, see Eq. (74), and, consequently, the above ratio is enhanced to $6.98/9.49 \simeq 0.74$ (see Table VII). These will affect the CP asymmetries of $\Delta S = 0$ modes to be discussed later.

We now turn to $PA^{(\prime)}$. Similarly we decompose $PA^{(\prime)}$ into $PA^{(\prime)u} + PA^{(\prime)c}$ and from Eq. (43) we have

TABLE VII. Same as Table VI, but for u-penguins and c-penguins.

	$A^0(\bar{B})$	$A_{\mathrm{FSI}}(\bar{B})$	$A_{\mathrm{FSI}}/A^0(\bar{B})$	$A^0(B)$	$A_{\mathrm{FSI}}(B)$	$A_{\mathrm{FSI}}/A^0(B)$
P^u	$3.51e^{i108.3°}$	$6.98e^{i84.7°}$	$1.99e^{-i23.6°}$	$3.51e^{-i113.9°}$	$6.98e^{-i137.6°}$	$1.99e^{-i23.6°}$
P^c	$10.09e^{-i11.4°}$	$9.49e^{-i0.2°}$	$0.94e^{i11.2°}$	$10.09e^{-i11.4°}$	$9.49e^{-i0.2°}$	$0.94e^{i11.2°}$
PA^u	$0.31e^{-i77.4°}$	$5.98e^{-i120.1°}$	$19.55e^{-i42.7°}$	$0.31e^{i60.3°}$	$5.98e^{i17.6°}$	$19.55e^{-i42.7°}$
PA^c	$0.81e^{i171.4°}$	$2.02e^{-i104.6°}$	$2.49e^{i84.0°}$	$0.81e^{i171.4°}$	$2.02e^{-i104.6°}$	$2.49e^{i84.0°}$
$P^{u\prime}$	$0.81e^{i108.3°}$	$1.61e^{i84.7°}$	$1.99e^{-i23.6°}$	$0.81e^{-i113.9°}$	$1.61e^{-i137.6°}$	$1.99e^{-i23.6°}$
$P^{c\prime}$	$43.71e^{i168.6°}$	$41.10e^{i179.8°}$	$0.94e^{i11.2°}$	$43.71e^{i168.6°}$	$41.10e^{i179.8°}$	$0.94e^{i11.2°}$
$PA^{u\prime}$	$0.07e^{-i77.4°}$	$1.38e^{-i120.1°}$	$19.55e^{-i42.7°}$	$0.07e^{i60.3°}$	$1.38e^{i17.6°}$	$19.55e^{-i42.7°}$
$PA^{c\prime}$	$3.51e^{-i8.6°}$	$8.75e^{i75.4°}$	$2.49e^{i84.0°}$	$3.51e^{-i8.6°}$	$8.75e^{i75.4°}$	$2.49e^{i84.0°}$

$$PA^{(\prime)u} = \frac{1}{3}(3 + 3ir_0' - ir_e' + 16ir_a' + 12ir_t')PA^{(\prime)u0} + ir_t'T^{(\prime)0} + \frac{1}{9}(2ir_e' + 4ir_a' - 3ir_t')C^{(\prime)0}$$

$$+ \frac{2}{9}(ir_e' + 11ir_a' + 12ir_t')E^{(\prime)0} + \frac{2}{9}(ir_e' + 11ir_a' + 12ir_t')P^{(\prime)u0}$$

$$+ \left(-\frac{1}{3}ir_t'P_{\mathrm{EW}}^{(\prime)u0} + \frac{1}{27}i(-2r_e' - 4r_a' + 3r_t')P_{\mathrm{EW}}^{(\prime)uC0} - \frac{2}{27}i(r_e' + 11r_a' + 12r_t')P_{\mathrm{EW}}^{(\prime)uE0}\right)$$

$$- \frac{2}{9}i(\bar{r}_e' + 2\bar{r}_a')\left(\bar{C}_1^{(\prime)0} + 2\bar{E}^{(\prime)0} + \bar{C}_2^{(\prime)0} + 2\bar{P}^{(\prime)u0} - \frac{1}{3}\bar{P}_{\mathrm{EW,2}}^{(\prime)uC0} - \frac{1}{3}\bar{P}_{\mathrm{EW,1}}^{(\prime)uC0} - \frac{2}{3}\bar{P}_{\mathrm{EW}}^{(\prime)uE0}\right)$$

$$+ \frac{1}{3}i\left(\hat{r}_t' + \frac{4\hat{r}_a' + 2\hat{r}_e'}{3}\right)\left(\tilde{C}^{(\prime)0} + \tilde{E}^{(\prime)0} + \tilde{P}^{(\prime)u0} + \frac{3}{2}\widetilde{PA}^{(\prime)u0} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)uC0} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)uE0}\right),$$

$$PA^{(\prime)c} = \frac{1}{3}(3 + 3ir_0' - ir_e' + 16ir_a' + 12ir_t')PA^{(\prime)c0} + \frac{2}{9}(ir_e' + 11ir_a' + 12ir_t')P^{(\prime)c0}$$

$$+ \left(-\frac{1}{3}ir_t'P_{\mathrm{EW}}^{(\prime)c0} + \frac{1}{27}i(-2r_e' - 4r_a' + 3r_t')P_{\mathrm{EW}}^{(\prime)cC0} - \frac{2}{27}i(r_e' + 11r_a' + 12r_t')P_{\mathrm{EW}}^{(\prime)cE0}\right)$$

$$- \frac{2}{9}i(\bar{r}_e' + 2\bar{r}_a')\left(2\bar{P}^{(\prime)c0} - \frac{1}{3}\bar{P}_{\mathrm{EW,2}}^{(\prime)cC0} - \frac{1}{3}\bar{P}_{\mathrm{EW,1}}^{(\prime)cC0} - \frac{2}{3}\bar{P}_{\mathrm{EW}}^{(\prime)cE0}\right)$$

$$+ \frac{1}{3}i\left(\hat{r}_t' + \frac{4\hat{r}_a' + 2\hat{r}_e'}{3}\right)\left(\tilde{P}^{(\prime)c0} + \frac{3}{2}\widetilde{PA}^{(\prime)c0} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)cC0} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)cE0}\right), \tag{77}$$

Using these formulas and the best fit parameters, we obtain

$$\frac{PA^{(\prime)u}}{PA^{(\prime)u0}} = 0.94e^{i2.2°} + 22.39e^{-i67.4°} + 6.17e^{i53.7°} + 0.78^{i91.1°}$$

$$+ 2.31^{i96.8°} + 0.09^{-i47.6°} + 1.91^{i28.5°} + 1.52^{-i99.7°}$$

$$= 19.55e^{-i42.7°}, \tag{78}$$

where the terms in the right-hand side of the first equality are from $PA^{(\prime)u0}$, $T^{(\prime)0}$, $C^{(\prime)0}$, $E^{(\prime)0}$, $P^{(\prime)u0}$, $P_{\mathrm{EW}}^{(\prime)u0,(\prime)uC0,(\prime)uE0}$, $\bar{C}_1^{(\prime)0} + 2\bar{E}^{(\prime)0} + \bar{C}_2^{(\prime)0} + 2\bar{P}^{(\prime)u0} - \frac{1}{3}\bar{P}_{\mathrm{EW}}^{(\prime)uC0} - \frac{1}{3}\bar{P}_{\mathrm{EW}}^{(\prime)uC0} - \frac{2}{3}\bar{P}_{\mathrm{EW}}^{(\prime)uE0}$, and $\tilde{C}^{(\prime)0} + \tilde{E}^{(\prime)0} + \tilde{P}^{(\prime)u0} + \frac{3}{2}\widetilde{PA}^{(\prime)u0} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)uC0} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)uE0}$ contributions, respectively. Note that $|PA^{(\prime)u}|$ is enhanced by a factor of 18, and the main contributions are from $T^{(\prime)0}$, $C^{(\prime)0}$ and $P^{(\prime)u0}$ terms via the total annihilation rescattering r_t', the annihilation r_a' and total annihilation r_t' rescatterings, respectively. In particular, the enhancement from $T^{(\prime)0}$ via r_t' is the most prominent one.

Similarly we have

$$\frac{PA^{(\prime)c}}{PA^{(\prime)c0}} = 0.94e^{i2.2°} + 2.51e^{i88.2°} + 0.09^{-i47.0°}$$

$$+ 1.26^{-i97.3°} + 1.41e^{i117.7°} = 2.49e^{i84.0°}, \tag{79}$$

where the terms in the right-hand side of the first equality are from $PA^{(\prime)c0}$, $P^{(\prime)c0}$, $P_{\mathrm{EW}}^{(\prime)c0,(\prime)cC0,(\prime)cE0}$, $2\bar{P}^{(\prime)c0} - \frac{1}{3}\bar{P}_{\mathrm{EW,2}}^{(\prime)cC0} - \frac{1}{3}\bar{P}_{\mathrm{EW,1}}^{(\prime)cC0} - \frac{2}{3}\bar{P}_{\mathrm{EW}}^{(\prime)cE0}$, and $\tilde{P}^{(\prime)c0} + \frac{3}{2}\widetilde{PA}^{(\prime)c0} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)cC0} - \frac{1}{3}\tilde{P}_{\mathrm{EW}}^{(\prime)cE0}$ contributions, respectively. Note that $|PA^{(\prime)c}|$ is enhanced by a factor of 2.5, while the main contribution is from the $P^{(\prime)c0}$ term via the annihilation r_a' and total annihilation r_t' rescatterings. The effect of rescattering in $PA^{(\prime)c}$ is not as prominent as in the $PA^{(\prime)u}$ case.

We see that in the presence of rescattering, the resulting $|PA^u|$ is even greater than $|PA^c|$, while $PA^{(\prime)u}$ can no longer

TABLE VIII. Branching ratios of various $\bar{B}_{u,d} \to PP$ modes in units of 10^{-6}. Fac and Res denote factorization and rescattering results, respectively. Experimental results are taken from [6,7]. Contributions to $\chi^2_{\min}$ from the best fitted solutions are also shown. The values in parenthesis are the results from the rescattering solution, but with all rescattering phases turn off.

Mode	Exp	Fac	Res	$\chi^{2(\mathrm{Fac})}_{\min}$	$\chi^{2(\mathrm{Res})}_{\min}$
$\bar{B}^0 \to K^-\pi^+$	$19.57^{+0.53}_{-0.52}$	$19.3^{+0.3}_{-0.3}$	$20.7^{+0.3}_{-0.3}$ (23.1)	0.2	4.7 (44.7)
$\bar{B}^0 \to \bar{K}^0\pi^0$	9.93 ± 0.49	8.5 ± 0.1	$9.6^{+0.2}_{-0.1}$ (10.7)	8.2	0.4 (2.4)
$\bar{B}^0 \to \bar{K}^0\eta$	$1.23^{+0.27}_{-0.24}$	1.3 ± 0.1	1.6 ± 0.1 (1.6)	0.0	1.6 (1.8)
$\bar{B}^0 \to \bar{K}^0\eta'$	66.1 ± 3.1	70.0 ± 1.2	$68.3^{+2.6}_{-1.4}$ (64.6)	1.6	0.5 (0.2)
$B^- \to \bar{K}^0\pi^-$	23.79 ± 0.75	21.1 ± 0.3	22.5 ± 0.3 (25.4)	11.8	3.1 (4.5)
$B^- \to K^-\pi^0$	$12.94^{+0.52}_{-0.51}$	12.1 ± 0.1	12.3 ± 0.2 (13.8)	2.7	1.7 (2.7)
$B^- \to K^-\eta$	$2.36^{+0.22}_{-0.21}$	1.7 ± 0.1	$2.1^{+0.1}_{-0.2}$ (2.1)	8.2	1.5 (1.1)
$B^- \to K^-\eta'$	71.1 ± 2.6	74.7 ± 1.3	$71.4^{+2.9}_{-1.5}$ (66.5)	1.9	0.0 (3.1)
$B^- \to \pi^-\pi^0$	$5.48^{+0.35}_{-0.34}$	4.7 ± 0.1	$4.9^{+0.2}_{-0.1}$ (4.9)	5.7	2.9 (2.9)
$B^- \to K^0K^-$	1.32 ± 0.14	1.43 ± 0.03	1.31 ± 0.03 (1.5)	0.6	0.0 (1.8)
$B^- \to \pi^-\eta$	4.02 ± 0.27	3.4 ± 0.1	4.2 ± 0.1 (4.3)	4.1	0.3 (0.8)
$B^- \to \pi^-\eta'$	$2.7^{+0.5}_{-0.4}$	2.9 ± 0.1	3.5 ± 0.1 (3.3)	0.1	3.3 (1.5)
$\bar{B}^0 \to \pi^+\pi^-$	5.10 ± 0.19	6.2 ± 0.1	5.3 ± 0.1 (6.0)	36.1	0.7 (23.7)
$\bar{B}^0 \to \pi^0\pi^0$	$1.59 \pm 0.26^{\mathrm{a}}$	$0.98^{+0.05}_{-0.03}$	$1.09^{+0.06}_{-0.05}$ (0.82)	5.6	3.7 (9.7)
$\bar{B}^0 \to \eta\eta$	0.76 ± 0.29	0.28 ± 0.01	$0.41^{+0.04}_{-0.06}$ (0.11)	2.8	1.5 (5.1)
$\bar{B}^0 \to \eta\eta'$	$0.5 \pm 0.4 (<1.2)$	$0.32^{+0.02}_{-0.01}$	$0.30^{+0.05}_{-0.04}$ (0.26)	0.2	0.2 (0.4)
$\bar{B}^0 \to \eta'\eta'$	$0.6 \pm 0.6 (<1.7)$	0.24 ± 0.01	$0.40^{+0.15}_{-0.12}$ (0.08)	0.4	0.1 (0.7)
$\bar{B}^0 \to K^+K^-$	0.084 ± 0.024	0.065 ± 0.002	$0.100^{+0.012}_{-0.007}$ (0.03)	0.6	0.5 (4.3)
$\bar{B}^0 \to K^0\bar{K}^0$	1.21 ± 0.16	1.67 ± 0.03	1.19 ± 0.03 (1.21)	8.4	0.0 (0.0)
$\bar{B}^0 \to \pi^0\eta$	0.41 ± 0.17	0.37 ± 0.01	$0.36^{+0.02}_{-0.00}$ (0.41)	0.1	0.1 (0.0)
$\bar{B}^0 \to \pi^0\eta'$	$1.2 \pm 0.6^{\mathrm{b}}$	0.52 ± 0.02	0.60 ± 0.02 (0.47)	1.3	1.0 (1.5)

$^{\mathrm{a}}$An S factor of 1.4 is included in the uncertainty.
$^{\mathrm{b}}$Taken from PDG with an S factor of 1.7 included in the uncertainty.

be neglected (see Table VII). The above observations can shed light on the results in the following discussions.

C. Numerical results for decay rates and CP asymmetries

In this part we will present the numerical results on rates in $\bar{B}^0$ and B^- decays, direct CP violations in $\bar{B}^0$ and B^- Decays, rates and direct CP asymmetries in $\bar{B}^0_s$ decays, and time-dependent CP violations in $\bar{B}^0$ and $\bar{B}^0_s$ decays.

1. Rates in $\bar{B}^0$ and B^- decays

In Table VIII, we show the CP-average rates of $\bar{B}^0, B^- \to PP$ decays. In the table, Fac and Res denote the factorization (without rescattering) and the rescattering results, respectively. To see the effect of rescattering, we also show the results from the rescattering solution, but with all rescattering phases turn off, i.e., with rescattering turn off, in the parentheses. In the table the contributions from various modes to $\chi^2_{\min}$ in the best fitted solutions are also shown.

From the table, we see that, except for rates in $\bar{B}^0 \to K^-\pi^+$, $\bar{K}^0\eta$, and $B^- \to \pi^-\eta'$ decays, the χ^2 in Res for the other modes are lower than the Fac ones. In particular, the χ^2 in the $\bar{B}^0 \to \bar{K}^0\pi^0$, $\pi^+\pi^-$, $K^0\bar{K}^0$, and $B^- \to \bar{K}^0\pi^-$, $K^-\eta$,

$\pi^-\pi^0$, $\pi^-\eta$ rates are improved significantly, as Fac encounters difficulties to fit some of these rates well. In fact, in Fac the χ^2 in $\bar{B}^0 \to \pi^+\pi^-$ is as large as 36.1, while it is reduced to 0.7 in Res. We see that in each group the χ^2 is improved in the presence of Res. The total χ^2 from these $21(=4+4+4+9)$ modes reduced from $100.7(=10.1+24.7+10.6+55.3)$ to $27.7(=7.2+6.3+6.4+7.8)$ (the breakdown can be found in Table II as well). Overall speaking rescattering significantly improves the fit in this sector, especially in the last group, and can reproduce all the measured $\bar{B}_{u,d} \to PP$ rates reasonably well.

Note that both Fac and Res can successfully reproduce the newly measured $\bar{B}^0 \to \pi^0\eta$ and K^+K^- rates [1,4]. On the other hand, both Fac and Res results on the $\bar{B}^0 \to \pi^0\pi^0$ rate have tension with the data, while Res is somewhat better as its $\chi^2(=3.7)$ is smaller than the one (5.6) in Fac. It should be note that the uncertainty in the present data is still large and it will be interesting to see the updated measurement. Both Fac and Res fits on the $B^- \to \pi^-\pi^0$ rates are smaller than the experimental result. The χ^2 from Fac on this mode is 5.7, while the Res fit improves it to 2.9 with a slightly large rate, but both results are in tension with data.

We will investigate how rescattering improves the fit in $\bar{B}^0_d \to \pi^+\pi^-, \pi^0\pi^0, K^+K^-$ and $B^- \to \bar{K}^-\pi^0$ rates. For

simplicity we will concentrate on the dominant contributions to the decay amplitudes in the following discussion. By neglecting the electroweak penguin contributions, the $\bar{B}^0 \to \pi^+ \pi^-$ amplitude in Eq. (25) can be expressed as

$$A_{\bar{B}_d^0 \to \pi^+ \pi^-} \simeq T + P + E + PA. \tag{80}$$

Using the results in Sec. III B, we see that before rescattering and after rescattering, we have (in unit of 10^{-8} GeV)

$$(A_{\bar{B}_d^0 \to \pi^+ \pi^-})^0 \simeq 2.58e^{-i63.5°} + 0.89e^{i8.6°} + 0.19e^{i102.6°} + 0.08e^{-i166.4°}$$

$$\simeq 2.98e^{-i47.0°} + 0.14e^{i135.4°} \simeq 2.84e^{-i47.1°},$$

$$(A_{B_d^0 \to \pi^+ \pi^-})^0 \simeq 2.58e^{i74.2°} + 0.99e^{-i31.6°} + 0.12e^{-i119.7°} + 0.08e^{i149.3°}$$

$$\simeq 2.50e^{i51.8°} + 0.14e^{-i152.6°} \simeq 2.38e^{i53.2°},$$

$$(A_{\bar{B}_d^0 \to \pi^+ \pi^-})_{\text{FSI}} \simeq 2.58e^{-i63.5°} + 1.23e^{i34.3°} + 0.55e^{i71.0°} + 0.79e^{-i116.2°}$$

$$\simeq 2.71e^{-i136.8°} + 0.26e^{-i131.4°} \simeq 2.70e^{-i42.4°},$$

$$(A_{B_d^0 \to \pi^+ \pi^-})_{\text{FSI}} \simeq 2.58e^{i74.2°} + 0.64e^{-i47.6°} + 0.55e^{-i152.2°} + 0.52e^{-i1.6°}$$

$$\simeq 2.31e^{i60.6°} + 0.28e^{-i81.7°} \simeq 2.10e^{i55.9°}, \tag{81}$$

respectively, where expressions with four terms are given in the order of T, P, E, and PA and those in two terms are with the first two terms ($T + P$) and the last two terms ($E + PA$) summed separately. Before we proceed we may compare the above estimation to our full numerical results, where we have $(A_{\bar{B}_d^0 \to \pi^+ \pi^-})^0$, $(A_{B_d^0 \to \pi^+ \pi^-})^0$, $(A_{\bar{B}_d^0 \to \pi^+ \pi^-})_{\text{FSI}}$, and $(A_{B_d^0 \to \pi^+ \pi^-})_{\text{FSI}}$ given by $2.86e^{-i47.1°}$, $2.36e^{i52.8°}$, $2.71e^{-i40.8°}$, and $2.16e^{i57.2°}$ (in unit of 10^{-8} GeV), respectively, which are close to the above estimation.

Note that $T + P$ are dominant contributions, while $E + PA$ are subleading contributions, and these two groups interfere destructively. In the presence of rescattering, the sizes of the dominant parts, $T + P$, are reduced, while the sizes of the destructive and subleading parts, $E + PA$, are

enhanced, resulting more effective destructive interferences. From the estimation we see that the $\bar{B}_d^0 \to \pi^+ \pi^-$ rate is reduced by about 15% bringing $\mathcal{B}(\bar{B}^0 \to \pi^+ \pi^-) \simeq 6 \times 10^{-6}$ down to $\sim 5 \times 10^{-6}$, which agrees well with the data $[(5.1 \pm 0.19) \times 10^{-6}]$ shown in Table VIII and, consequently, the quality of the fit is improved significantly.

Similarly for $\bar{B}^0 \to \pi^0 \pi^0$ decays, we have

$$\sqrt{2} A_{\bar{B}_d^0 \to \pi^0 \pi^0} \simeq -C + P + E + PA, \tag{82}$$

which is close to the above $\bar{B}^0 \to \pi^+ \pi^-$ amplitudes, but with T replaced by $-C$. Before rescattering and after rescattering, we have (in unit of 10^{-8} GeV)

$$\sqrt{2}(A_{\bar{B}_d^0 \to \pi^0 \pi^0})^0 \simeq 1.05e^{i56.1°} + 0.89e^{i8.6°} + 0.19e^{i102.6°} + 0.08e^{-i166.4°}$$

$$\simeq 1.82e^{i37.9°} + 0.08e^{-i166.4°} \simeq 1.75e^{i38.9°},$$

$$\sqrt{2}(A_{B_d^0 \to \pi^0 \pi^0})^0 \simeq 1.05e^{-i166.2°} + 0.99e^{-i31.6°} + 0.12e^{-i119.7°} + 0.08e^{i149.3°}$$

$$\simeq 0.90e^{-i104.5°} + 0.08e^{i149.3°} \simeq 0.88e^{-i109.2°},$$

$$\sqrt{2}(A_{\bar{B}_d^0 \to \pi^0 \pi^0})_{\text{FSI}} \simeq 1.05e^{i56.1°} + 1.23e^{i34.3°} + 0.55e^{i71.0°} + 0.79e^{-i116.2°}$$

$$\simeq 2.73e^{i49.5°} + 0.79e^{-i116.2°} \simeq 1.97e^{i43.8°},$$

$$\sqrt{2}(A_{B_d^0 \to \pi^0 \pi^0})_{\text{FSI}} \simeq 1.05e^{-i166.2°} + 0.64e^{-i47.6°} + 0.55e^{-i152.2°} + 0.52e^{-i1.6°}$$

$$\simeq 1.45e^{-i137.0°} + 0.52e^{-i1.6°} \simeq 1.14e^{-i118.3°}, \tag{83}$$

respectively, where terms are given in the order of $-C$, P, E, and PA and the expressions with the first three terms $(-C + P + E)$ combined are also shown. The above estimation is close to the values in the full numerical results

with $\sqrt{2}(A_{\bar{B}_d^0 \to \pi^0 \pi^0})^0$, $\sqrt{2}(A_{B_d^0 \to \pi^0 \pi^0})^0$, $\sqrt{2}(A_{\bar{B}_d^0 \to \pi^0 \pi^0})_{\text{FSI}}$ and $\sqrt{2}(A_{B_d^0 \to \pi^0 \pi^0})_{\text{FSI}}$ given by $1.67e^{i39.6°}, 0.89e^{-i114.8°}, 1.96e^{i45.0°}$ and $1.08e^{-i126.0°}$ in the unit of 10^{-8} GeV, respectively.

In the above estimation the first three terms and the last term interfere destructively. With Res P and E are enhanced giving a larger $-C + P + E$, while the enhanced PA cannot be neglected anymore, producing a slightly larger decay amplitude and resulting a 35% enhancement in rate, which brings the rate up from $\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0) \simeq 0.8 \times 10^{-6}$ to $\sim 1.1 \times 10^{-6}$ as shown in Table VIII. As noted previously the rate is still smaller than the central value of the data, which however accompanies with large uncertainty.

For the newly observed $\bar{B}^0_d \to K^+K^-$ mode, we note that as shown in Table VIII rescattering enhances the rate by $0.100/0.03 = 3.33$ times. It will be useful to see the enhancement in details. From Tables IV and VII and Eq. (25),

$$A_{\bar{B}^0_d \to K^+K^-} = E + PA^u + PA^c + \frac{1}{3}P^A_{\text{EW}}, \qquad (84)$$

we have (in unit of 10^{-8} GeV)

$$(A_{\bar{B}^0_d \to K^+K^-})^0 \simeq 0.119e^{i102.6°} + 0.031e^{-i77.4°} + 0.081e^{i171.4°} + 0.001e^{i13.6°} \simeq 0.139e^{i135.2°},$$

$$(A_{B^0_d \to K^+K^-})^0 \simeq 0.119e^{-i119.7°} + 0.031e^{i60.3°} + 0.081e^{i171.4°} + 0.001e^{-i30.7°} \simeq 0.139e^{-i152.4°},$$

$$(A_{\bar{B}^0_d \to K^+K^-})_{\text{FSI}} \simeq 0.546e^{71.0°} + 0.598e^{-i120.1°} + 0.202e^{-i104.6°} + 0.006e^{-i24.7°} \simeq 0.261e^{-i130.1°}$$

$$(A_{B^0_d \to K^+K^-})_{\text{FSI}} \simeq 0.546e^{-i151.2°} + 0.552e^{i17.1°} + 0.202e^{-i105.4°} + 0.006e^{-i68.8°} \simeq 0.286e^{-i81.4°}, \qquad (85)$$

for the decay amplitudes before and after rescattering, where terms are given in the order of E, PA^u, PA^c, and $P^A_{\text{EW}}/3$. Compare the above estimation to the values in our full numerical result, which have $0.200e^{i135.0°}$, $0.200e^{-i152.6°}$, $0.332e^{-i139.8°}$, and $0.350e^{-i87.2°}$ for $(A_{\bar{B}^0_d \to K^+K^-})^0$, $(A_{B^0_d \to K^+K^-})^0$, $(A_{\bar{B}^0_d \to K^+K^-})_{\text{FSI}}$, and $(A_{B^0_d \to K^+K^-})_{\text{FSI}}$ in unit of 10^{-8} GeV, respectively. The discrepancy is mainly from SU(3) breaking effects, which are not included in the above equation. In fact, by scaling the numbers in Eq. (85) by $(f_K/f_\pi)^2$, the sizes become 0.199, 0.199, 0.373 and 0.409, which agree better to the above values now.

From the above equation, we see that E, PA^u, PA^c and P^A_{EW} are all enhanced. Note that E interferes destructively with PA^u and PA^c in $A_{\bar{B}^0_d \to K^+K^-}$, while PA^u interferes destructively with E and PA^c in $A_{B^0_d \to K^+K^-}$. The result is an enhancement of 3.8 in the averaged rate, which is close to our numerical result $(0.100/0.03 = 3.33)$ as shown in Table VIII. We will return to this mode again in the discussion of direct CP asymmetry.

Finally we turn to the $B^- \to \bar{K}^0\pi^-$ decay. From Eq. (23) we have

$$A_{B^- \to \bar{K}^0\pi^-} = A' + P'^u + P'^c + \frac{1}{3}(-P'^C_{\text{EW}} + 2P'^E_{\text{EW}}), \qquad (86)$$

which gives before and after rescattering (in unit of 10^{-8} GeV)

$$(A_{B^- \to \bar{K}^0\pi^-})^0 \simeq 0.01e^{-i77.4°} + 0.08e^{i108.3°} + 4.37e^{i168.6°}$$
$$+ 0.08e^{-i53.3°} \simeq 4.35e^{i168.4°},$$

$$(A_{B^+ \to K^0\pi^+})^0 \simeq 0.01e^{i60.3°} + 0.08e^{-i113.9°} + 4.37e^{i168.6°}$$
$$+ 0.08e^{-i51.2°} \simeq 4.33e^{i170.2°},$$

$$(A_{B^- \to \bar{K}^0\pi^-})_{\text{FSI}} \simeq 0.11e^{-i113.8°} + 0.16e^{i84.7°} + 4.11e^{i179.8°}$$
$$+ 0.13e^{-i50.3°} \simeq 4.06e^{-i179.7°},$$

$$(A_{B^+ \to K^0\pi^+})_{\text{FSI}} \simeq 0.11e^{i23.9°} + 0.16e^{-i137.6°} + 4.11e^{i179.8°}$$
$$+ 0.13e^{-i48.2°} \simeq 4.05e^{-i178.0°}, \qquad (87)$$

respectively, where terms are given in the order of A', P'^u, P'^c, and $(-P'^C_{\text{EW}} + 2P'^E_{\text{EW}})/3$. Note that in our numerical result, we have $5.17e^{i167.2°}$, $5.29e^{i171.2°}$, $4.86e^{i179.6°}$, and $4.98e^{i175.7°}$, for $(A_{B^- \to \bar{K}^0\pi^-})^0$, $(A_{B^+ \to K^0\pi^+})^0$, $(A_{B^- \to \bar{K}^0\pi^-})_{\text{FSI}}$ and $(A_{B^+ \to K^0\pi^+})_{\text{FSI}}$ in unit of 10^{-8} GeV, respectively. By scaling the values in the Eq. (87) by f_K/f_π, the sizes become 5.20, 5.17, 4.85 and 4.83, respectively, which are close to the numerical results. In the full numerical result either in the presence of rescattering or without it, the sizes of $A_{B^+ \to K^0\pi^+}$ is slightly greater than $A_{B^- \to \bar{K}^0\pi^-}$, but it is the other way around in the estimation. In fact, in the numerical result, we have $P'^u = 0.10e^{i107.8°}$ and $P'^c = 5.19e^{i167.5°}$ in $(A_{B^- \to \bar{K}^0\pi^-})^0$ and $P'^u = 0.10e^{-i114.4°}$ and $P'^c = 5.33e^{i169.6°}$ in $(A_{B^+ \to K^0\pi^+})^0$. The latter $|P'^c|$ in $(A_{B^+ \to K^0\pi^+})^0$ is greater than the one in $(A_{B^- \to \bar{K}^0\pi^-})^0$. The difference can be traced to the nonvanishing first Gegenbauer moment of the kaon wave function $(\alpha^{\bar{K}}_1 = -\alpha^K_1 = 0.2)$, which will change sign in changing from $\bar{K}$ to K. This will affect the direct CP asymmetry and such a feature is absent in the above estimation.

From Eq. (87) we see that $A' + \frac{1}{3}(-P'^C_{\text{EW}} + 2P'^E_{\text{EW}})$ interferes destructively to the dominating P'^c term. Since the sizes of A' and $\frac{1}{3}(-P'^C_{\text{EW}} + 2P'^E_{\text{EW}})$ are enhanced, while the size of P'^c is slightly reduced, the size of the total amplitude is reduced under the rescattering resulting a reduction of 13% in the averaged rate, which brings the rate from $\mathcal{B}(B^- \to \bar{K}^0\pi^+) \simeq 25 \times 10^{-6}$ to $\sim 22 \times 10^{-6}$, which is closer to the data $[(23.79 \pm 0.75) \times 10^{-6}]$ as shown in Table VIII.

2. Direct CP violations in $\bar{B}^0$ and B^- decays

Results for direct CP asymmetries ($\mathcal{A}$) in $\bar{B}_{u,d} \to PP$ decays are summarized in Table IX. The Fac and Res fits give similar results in the first group of data, namely the direct CP asymmetries in $\bar{B}^0 \to K^-\pi^+$, $\bar{K}^0\pi^0$ and $\bar{K}^0\eta'$ decays. Both can explain the so-call $K\pi$ CP puzzle by

TABLE IX. Same as Table VIII, except for the direct CP asymmetries $\mathcal{A}$ (in units of percent) in various $\bar{B}_{u,d} \to PP$ modes.

Mode	Exp	Fac	Res	$\chi^{2(Fac)}_{min}$	$\chi^{2(Res)}_{min}$
$\bar{B}^0 \to K^-\pi^+$	-8.2 ± 0.6	-8.0 ± 0.1	$-8.2 \pm 0.3 \ (-9.5)$	0.1	0.0 (4.8)
$\bar{B}^0 \to \bar{K}^0\pi^0$	-1 ± 13^a	-15.2 ± 0.6	$-14.3 \pm 1.0 \ (-8.5)$	1.2	1.0 (0.3)
$\bar{B}^0 \to \bar{K}^0\eta$	$-$	$-29.3^{+1.3}_{-1.9}$	$-27.7^{+1.4}_{-2.2} \ (-17.5)$	$-$	$-$
$\bar{B}^0 \to \bar{K}^0\eta'$	5 ± 4	7.8 ± 0.2	$6.1^{+1.3}_{-0.9} \ (6.3)$	0.5	0.1 (0.1)
$B^- \to \bar{K}^0\pi^-$	-1.7 ± 1.6	-3.5 ± 0.1	$-2.4^{+0.6}_{-0.4} \ (-2.3)$	1.2	0.2 (0.1)
$B^- \to K^-\pi^0$	4.0 ± 2.1	4.0 ± 0.4	$4.9^{+0.8}_{-1.1} \ (-1.9)$	0.0	0.2 (7.8)
$B^- \to K^-\eta$	-37 ± 8	$-42.0^{+2.5}_{-3.7}$	$-33.9 \pm 2.6 \ (-10.8)$	0.4	0.1 (10.7)
$B^- \to K^-\eta'$	1.3 ± 1.7	$4.5^{+0.2}_{-0.1}$	$1.8^{+1.6}_{-0.7} \ (2.7)$	3.6	0.1 (0.7)
$B^- \to \pi^-\pi^0$	2.6 ± 3.9	-0.11 ± 0.00	$-0.09 \pm 0.01 \ (-0.09)$	0.5	0.5 (0.5)
$B^- \to K^0 K^-$	-8.7 ± 10.0	-5.7 ± 0.1	$-4.8^{+3.8}_{-5.3} \ (-8.8)$	0.1	0.2 (0.0)
$B^- \to \pi^-\eta$	-14 ± 5	$-11.9^{+0.8}_{-0.7}$	$-10.3^{+1.7}_{-1.6} \ (0.8)$	0.2	0.5 (8.7)
$B^- \to \pi^-\eta'$	6 ± 15	$37.8^{+0.8}_{-1.3}$	$43.6^{+2.0}_{-2.4} \ (34.6)$	4.5	6.3 (3.6)
$\bar{B}^0 \to \pi^+\pi^-$	31 ± 5	14.0 ± 0.4	$22.5^{+0.9}_{-1.0} \ (19.1)$	11.5	2.9 (5.7)
$\bar{B}^0 \to \pi^0\pi^0$	34 ± 22	$79.1^{+1.2}_{-1.5}$	$53.7^{+3.3}_{-7.1} \ (55.9)$	4.2	0.8 (1.0)
$\bar{B}^0 \to \eta\eta$	$-$	$-64.5^{+1.5}_{-1.4}$	$-31.1^{+7.2}_{-5.5} \ (-73.5)$	$-$	$-$
$\bar{B}^0 \to \eta\eta'$	$-$	-35.6 ± 1.1	$-29.8^{+9.4}_{-8.0} \ (-52.1)$	$-$	$-$
$\bar{B}^0 \to \eta'\eta'$	$-$	-20.0 ± 0.4	$-7.6^{+19.2}_{-19.8} \ (-12.9)$	$-$	$-$
$\bar{B}^0 \to K^+ K^-$	$-$	0	$-5.2^{+5.2}_{-5.0} \ (0)$	$-$	$-$
$\bar{B}^0 \to K^0\bar{K}^0$	-6 ± 36^b	-8.4 ± 0.1	$-41.8^{+2.6}_{-3.9} \ (-10.0)$	0.0	1.0 (0.0)
$\bar{B}^0 \to \pi^0\eta$	$-$	$-45.6^{+1.8}_{-1.7}$	$-40.9^{+4.6}_{-3.6} \ (-36.3)$	$-$	$-$
$\bar{B}^0 \to \pi^0\eta'$	$-$	$-30.4^{+0.9}_{-0.5}$	$-8.8 \pm 1.4 \ (-8.8)$	$-$	$-$

[a]An S factor of 1.4 is included in the uncertainty.
[b]An S factor of 1.4 is included in the uncertainty.

producing positive $\mathcal{A}(B^- \to K^-\pi^0)$ and negative $\mathcal{A}(\bar{B}^0 \to K^-\pi^+)$, but the Res give a slightly larger $\mathcal{A}(B^- \to K^-\pi^0)$. Fac fits better than Res in the $B^- \to \pi^-\eta'$ and $\bar{B}^0 \to K^0\bar{K}^0$ modes, while Res fits better than Fac in the $B^- \to \bar{K}^0\pi^-$, $K^-\eta'$, $\bar{B}^0 \to \pi^+\pi^-$, and $\pi^0\pi^0$ modes. In particular, the χ^2 in $\mathcal{A}$ of $\bar{B}^0 \to \pi^+\pi^-$ is reduced significantly from 11.5 (Fac) to 2.9 (Res). Overall speaking the fit in Res in this sector (see also Table II) is better than Fac, as the corresponding χ^2 are $13.9(= 1.1 + 0.6 + 7.5 + 4.7)$ and $29.2(= 1.8 + 5.2 + 6.5 + 15.7)$, respectively.

It is interesting to see how rescattering solve the so-called $K\pi CP$ puzzle, where experimental data gives $\Delta\mathcal{A} \equiv \mathcal{A}(K^-\pi^+) - \mathcal{A}(K^-\pi^0) = (12.2 \pm 2.2)\%$, in details. The $\bar{B}^0 \to K^-\pi^+$ and $B^- \to K^-\pi^0$ decay amplitudes can be expressed as

$$A_{\bar{B}^0_d \to K^-\pi^+} = T' + P' + \frac{1}{3}(2P'^C_{EW} - P'^E_{EW}),$$

$$\sqrt{2}A_{B^- \to K^-\pi^0} = T' + C' + A' + P' + P'_{EW} + \frac{2}{3}P'^C_{EW} + \frac{2}{3}P'^E_{EW}. \tag{88}$$

It is useful to note that these two amplitudes are related by the following relation:

$$\sqrt{2}A_{B^- \to K^-\pi^0} = A_{\bar{B}^0_d \to K^-\pi^+} + C' + A' + P'_{EW} + P'^E_{EW}. \tag{89}$$

Using the values in Table V and the above equation, we have (in unit of 10^{-8} GeV and in the corresponding order of the above equation) before and after Res

$$\sqrt{2}(A_{B^- \to K^-\pi^0})^0 \simeq 4.12e^{i173.0°} + 0.24e^{-i123.9°} + 0.01e^{-i77.4°} + 0.40e^{-i174.2°} + 0.05e^{-i32.7°} \simeq 4.58e^{i177.2°},$$

$$\sqrt{2}(A_{B^+ \to K^+\pi^0})^0 \simeq 4.45e^{i160.8°} + 0.24e^{i13.8°} + 0.01e^{i60.3°} + 0.40e^{-i172.1°} + 0.05e^{-i30.6°} \simeq 4.55e^{i161.5°},$$

$$\sqrt{2}(A_{B^- \to K^-\pi^0})_{FSI} \simeq 3.90e^{-i176.4°} + 0.24e^{-i123.9°} + 0.11e^{-i113.8°} + 0.40e^{-i174.2°} + 0.13e^{-i40.3°} \simeq 4.43e^{-i171.3°},$$

$$\sqrt{2}(A_{B^+ \to K^+\pi^0})_{FSI} \simeq 4.18e^{-i171.9°} + 0.24e^{i13.8°} + 0.11e^{i23.9°} + 0.40e^{-i172.1°} + 0.13e^{-i38.2°} \simeq 4.14e^{i172.2°}, \tag{90}$$

respectively. In our full numerical results, for $\bar{B}^0 \to K^- \pi^+$ decay, we have $4.91e^{i172.0°}$, $5.40e^{i161.8°}$, $4.68e^{-i176.8°}$ and $5.08e^{i174.0°}$ for $(A_{\bar{B}^0_d \to K^- \pi^+})^0$, $(A_{B^0_d \to K^+ \pi^-})^0$, $(A_{\bar{B}^0_d \to K^- \pi^+})_{FSI}$ and $(A_{B^0_d \to K^+ \pi^-})_{FSI}$ in unit of 10^{-8} GeV, respectively, which are close to the scaled (by f_K/f_π) estimations, $4.93e^{i177.2°}$, $5.32e^{i161.5°}$, $4.66e^{-i171.3°}$, and $4.99e^{i172.2°}$, from Eq. (90). For $B^- \to K^- \pi^0$ decays, we have $5.40e^{i176.4°}$, $5.50e^{i162.7°}$, $5.26e^{-i171.6°}$, and $5.01e^{i174.8°}$ for $\sqrt{2}(A_{B^- \to K^- \pi^0})^0$, $\sqrt{2}(A_{B^- \to K^- \pi^0})^0$, $\sqrt{2}(A_{B^- \to K^- \pi^0})_{FSI}$, and $\sqrt{2}(A_{B^- \to K^- \pi^0})_{FSI}$ in unit of 10^{-8} GeV, respectively, which are close to the scaled (by f_K/f_π) estimations, $5.48e^{i177.2°}$, $5.44e^{i161.5°}$, $5.29e^{-i171.3°}$, and $4.94e^{i172.2°}$, from Eq. (90).

From Eq. (90) we see that the asymmetries are $\mathcal{A}(\bar{B}^0_d \to K^- \pi^+) \simeq -7.7\%$, $\mathcal{A}(B^- \to K^- \pi^0) \simeq 0.6\%$ and $\Delta\mathcal{A} \simeq 8.3\%$ before Res, which are not too far from the values -9.5%, -1.9% and 7.6% shown in Table IX, and $\mathcal{A}(\bar{B}^0_d \to K^- \pi^+) \simeq -6.8\%$, $\mathcal{A}(B^- \to K^- \pi^0) \simeq 6.8\%$ and $\Delta\mathcal{A} \simeq 13.6\%$ after Res, which are close to the values 4.9, $-8.2\%\%$, and 13.0% shown in Table IX. As noted in the discussion of the $B^- \to \bar{K}^0 \pi^-$ rate in the last subsection, the first Gegenbauer moment of the kaon wave function is the main source of the discrepancies between the estimations and the full numerical results.

As shown in Eq. (90), it is interesting that before rescattering the C' and P'_{EW} terms are the sources of deviation of $\mathcal{A}(B^- \to K^- \pi^0)$ from $\mathcal{A}(\bar{B}^0_d \to K^- \pi^+)$, while with the presence of Res, the sizes of A' and P'^E_{EW} are enhanced and hence further enlarges the deviation of $\mathcal{A}(\bar{B}^0_d \to K^- \pi^+)$ and $\mathcal{A}(B^- \to K^- \pi^0)$ producing a larger $\Delta\mathcal{A}$. Note that comparing to the discussion in $\bar{B}^0 \to \pi^+ \pi^-$ and $\pi^0 \pi^0$ decay rates [see discussion after Eq. (81)], we see that the correlation of the effects of Res on these two sectors is not prominent. Indeed, in the $\pi^0 \pi^0$ mode the most

affected TAs under rescattering are P, E, and PA, while at here A' and P'^E_{EW} are the most affected and relevant ones.

We now turn to $\mathcal{A}(\bar{B}^0 \to \pi^+ \pi^-)$. From previous discussion [see Eq. (81)], we find that before Res $A(\bar{B}^0 \to \pi^+ \pi^-) \simeq 2.84e^{-i47.1°} \times 10^{-8}$ GeV and $A(B^0 \to \pi^+ \pi^-) \simeq 2.38e^{i53.2°} \times 10^{-8}$ GeV, giving $\mathcal{A} \simeq 18\%$, while in the presence of Res, the sizes of the dominant parts, $T+P$, are reduced, but the sizes of the destructive but the subleading parts, $E+PA$, are enhanced, resulting richer interferences, giving $A(\bar{B}^0 \to \pi^+ \pi^-) \simeq 2.70e^{-i42.4°} \times 10^{-8}$ GeV and $A(B^0 \to \pi^+ \pi^-) \simeq 2.10e^{i55.9°} \times 10^{-8}$ GeV, and, consequently, producing an enhanced $\mathcal{A} \simeq 24.7\%$, which is closer to the data, $(31 \pm 5)\%$.

Note that the results of Fac and Res in $\mathcal{A}(\bar{B}^0 \to K^0 \bar{K}^0)$ are different, while with large uncertainty the present data, $\mathcal{A}(\bar{B}^0 \to K^0 \bar{K}^0) = (-6 \pm 36)\%$, allows both. Note that the uncertainty in the data is enlarged by an S factor of 1.4, as Belle and BABAR give very different results in $\mathcal{A}(\bar{B}^0 \to K_s K_s)$, namely, Belle gives $\mathcal{A}(\bar{B}^0 \to K_s K_s) = -0.38 \pm 0.38 \pm 0.5$ [41], while BABAR gives $0.40 \pm 0.41 \pm 0.06$ [42]. The result of Res, $\mathcal{A}(\bar{B}^0 \to K^0 \bar{K}^0) = -0.418^{+0.026}_{-0.039}$, prefers the Belle result. One should be reminded that Res can reproduce the $\bar{B}^0 \to K^0 \bar{K}^0$ CP-averaged rate much better than Fac (see Table VIII). We need more data to clarify the situation and to verify these predictions.

It will be useful to see the effect of Res on the $\bar{B}^0_d \to K^0 \bar{K}^0$ direct CP asymmetry. From Eq. (25), we can approximate the $\bar{B}^0_d \to K^0 \bar{K}^0$ amplitude as

$$A_{\bar{B}^0_d \to K^0 \bar{K}^0} \simeq P + PA \simeq P^u + PA^u + P^c + PA^c. \quad (91)$$

From Table VII, before Res and after FSI, we have (in unit of 10^{-8} GeV)

$$(A_{\bar{B}^0_d \to K^0 \bar{K}^0})^0 \simeq 0.35e^{i108.3°} + 0.03e^{-i77.4°} + 1.01e^{-i11.4°} + 0.08e^{i171.4°} \simeq 0.81e^{i8.1°},$$

$$(A_{B^0_d \to K^0 \bar{K}^0})^0 \simeq 0.35e^{-i113.9°} + 0.03e^{i60.3°} + 1.01e^{-i11.4°} + 0.08e^{i171.4°} \simeq 0.92e^{-i31.6°},$$

$$(A_{\bar{B}^0_d \to K^0 \bar{K}^0})_{FSI} \simeq 0.70e^{i84.7°} + 0.60e^{-i120.1°} + 0.95e^{-i0.2°} + 0.20e^{-i104.6°} \simeq 0.66e^{-i1.9°},$$

$$(A_{B^0_d \to K^0 \bar{K}^0})_{FSI} \simeq 0.70e^{-i137.6°} + 0.60e^{i17.6°} + 0.95e^{-i0.2°} + 0.20e^{-i104.6°} \simeq 1.07e^{-i27.2°}, \quad (92)$$

respectively, where the values of P^u, PA^u, P^c, and PA^c are shown in the corresponding order. In our full numerical result, we have $1.12e^{i8.6°}$, $1.24e^{-i33.3°}$, $0.90e^{i1.6°}$, and $1.40e^{-i27.8°}$ for $(A_{\bar{B}^0_d \to K^0 \bar{K}^0})^0$, $(A_{B^0_d \to K^0 \bar{K}^0})^0$, $(A_{\bar{B}^0_d \to K^0 \bar{K}^0})_{FSI}$, and $(A_{B^0_d \to K^0 \bar{K}^0})_{FSI}$ in units of 10^{-8} GeV, respectively, which are close to the scaled [by $(f_K/f_\pi)^2$] estimations, $1.16e^{i8.1°}$, $1.31e^{-i31.6°}$, $0.95e^{-i1.9°}$, and $1.53e^{-i27.2°}$, from Eq. (92).

In Eq. (92), we see that both P^u and the PA^u terms are enhanced under Res (mainly through rescattering from T^0)

and produce richer inference pattern contributing to the direct CP asymmetry. The $\bar{B}^0_d \to K^0 \bar{K}^0$ amplitude is reduced, while the amplitude of the conjugated decay mode, $B^0_d \to K^0 \bar{K}^0$, is enhanced under Res, producing an enlarged direct CP asymmetry, which is changed from -12% to -45% and hence close to the Belle result.

As shown in Table IX, we see that before Res the direct CP asymmetry of $\bar{B}^0 \to K^+ K^-$ is vanishing. Indeed, as one can infer from Eq. (85) that the rates of $\bar{B}^0 \to K^+ K^-$ and $B^0 \to K^+ K^-$ are the same before Res. This can be understood in the

TABLE X. Same as Table VIII, except for the branching ratios (upper table) in the unit of 10^{-6} and direct CP asymmetries (lower table) in the unit of percent for various $\bar{B}_s \to PP$ modes.

Mode	Exp	Fac	Res	$\chi^{2(\text{Fac})}_{\min}$	$\chi^{2(\text{Res})}_{\min}$
$\mathcal{B}(\bar{B}_s^0 \to K^+\pi^-)$	5.5 ± 0.5	5.5 ± 0.1	$5.5^{+0.4}_{-1.8}$ (6.3)	0.0	0.0 (2.6)
$\mathcal{B}(\bar{B}_s^0 \to K^0\pi^0)$	–	0.59 ± 0.01	$1.02^{+3.64}_{-0.13}$ (0.68)	–	–
$\mathcal{B}(\bar{B}_s^0 \to K^0\eta)$	–	$0.18^{+0.01}_{-0.00}$	$0.48^{+1.87}_{-0.06}$ (0.22)	–	–
$\mathcal{B}(\bar{B}_s^0 \to K^0\eta')$	–	1.76 ± 0.03	$2.02^{+4.30}_{-0.19}$ (1.75)	–	–
$\mathcal{B}(\bar{B}_s^0 \to \pi^+\pi^-)$	0.671 ± 0.083	0.30 ± 0.01	$0.67^{+0.49}_{-0.06}$ (0.14)	20.2	0.0 (41.1)
$\mathcal{B}(\bar{B}_s^0 \to \pi^0\pi^0)$	–	0.15 ± 0.00	$0.33^{+0.25}_{-0.03}$ (0.07)	–	–
$\mathcal{B}(\bar{B}_s^0 \to \eta\eta)$	–	$24.7^{+0.3}_{-0.4}$	$19.6^{+0.6}_{-6.5}$ (20.4)	–	–
$\mathcal{B}(\bar{B}_s^0 \to \eta\eta')$	–	$67.2^{+0.9}_{-1.4}$	$75.1^{+67.4}_{-3.5}$ (68.7)	–	–
$\mathcal{B}(\bar{B}_s^0 \to \eta'\eta')$	33.1 ± 7.1	$60.5^{+0.8}_{-1.1}$	$34.9^{+16.0}_{-4.7}$ (46.6)	16.0	0.0 (3.6)
$\mathcal{B}(\bar{B}_s^0 \to K^+K^-)$	24.8 ± 1.7	$32.7^{+0.5}_{-0.6}$	$24.6^{+2.7}_{-0.6}$ (24.5)	21.3	0.0 (0.0)
$\mathcal{B}(\bar{B}_s^0 \to K^0\bar{K}^0)$	19.6 ± 9.5	$34.3^{+0.5}_{-0.6}$	$24.6^{+0.7}_{-1.0}$ (25.6)	2.4	0.3 (0.4)
$\mathcal{B}(\bar{B}_s^0 \to \pi^0\eta)$	–	0.07 ± 0.00	$0.07^{+0.09}_{-0.00}$ (0.06)	–	–
$\mathcal{B}(\bar{B}_s^0 \to \pi^0\eta')$	–	$0.09^{+0.00}_{-0.00}$	$0.11^{+0.10}_{-0.01}$ (0.10)	–	–
$\mathcal{A}(\bar{B}_s^0 \to K^+\pi^-)$	26 ± 4	$17.4^{+0.4}_{-0.5}$	$24.8^{+22.1}_{-1.0}$ (28.2)	4.6	0.1 (0.3)
$\mathcal{A}(\bar{B}_s^0 \to K^0\pi^0)$	–	$66.8^{+1.5}_{-1.6}$	$74.9^{+4.8}_{-50.8}$ (53.7)	–	–
$\mathcal{A}(\bar{B}_s^0 \to K^0\eta)$	–	$88.1^{+0.9}_{-1.2}$	$81.2^{+6.9}_{-54.8}$ (78.2)	–	–
$\mathcal{A}(\bar{B}_s^0 \to K^0\eta')$	–	$-38.7^{+0.9}_{-0.5}$	$-38.6^{+13.0}_{-2.2}$ (−34.4)	–	–
$\mathcal{A}(\bar{B}_s^0 \to \pi^+\pi^-)$	–	0	$1.7^{+0.5}_{-2.5}$ (0)	–	–
$\mathcal{A}(\bar{B}_s^0 \to \pi^0\pi^0)$	–	0	$1.7^{+0.5}_{-2.5}$ (0)	–	–
$\mathcal{A}(\bar{B}_s^0 \to \eta\eta)$	–	-2.4 ± 0.1	$-3.7^{+0.6}_{-8.2}$ (−2.8)	–	–
$\mathcal{A}(\bar{B}_s^0 \to \eta\eta')$	–	-0.01 ± 0.01	$0.95^{+0.39}_{-0.19}$ (−0.01)	–	–
$\mathcal{A}(\bar{B}_s^0 \to \eta'\eta')$	–	2.0 ± 0.0	$-1.2^{+1.0}_{-4.7}$ (1.9)	–	–
$\mathcal{A}(\bar{B}_s^0 \to K^+K^-)$	-14 ± 11	-5.8 ± 0.0	$-10.5^{+1.1}_{-0.4}$ (−9.9)	0.6	0.1 (0.1)
$\mathcal{A}(\bar{B}_s^0 \to K^0\bar{K}^0)$	–	-0.9 ± 0.0	$0.9^{+2.2}_{-0.3}$ (−0.6)	–	–
$\mathcal{A}(\bar{B}_s^0 \to \pi^0\eta)$	–	$46.0^{+1.5}_{-1.2}$	$92.9^{+2.9}_{-15.4}$ (69.9)	–	–
$\mathcal{A}(\bar{B}_s^0 \to \pi^0\eta')$	–	$64.3^{+1.4}_{-1.1}$	$77.7^{+8.5}_{-6.9}$ (54.0)	–	–

following. In QCDF, E, PA and P^A_{EW} can be expressed in terms of the so-called A^i_1 and A^i_2 terms, and these A^i_1 and A^i_2 terms are identical when the asymptotic distribution amplitudes are used (as in the present case) [18]. Since we have $A_{\bar{B}^0 \to K^+K^-} = E + PA + P^A_{\text{EW}}/3$ and these three topological amplitudes all have a common strong phase resulting a vanishing direct CP asymmetry. Note that in the presence of Res, E, and PA^u are enhanced mostly from T^0 [see Eqs. (72) and (78)], while PA^c from P^c [see Eq. (79)], consequently, the strong phases of these terms are no longer degenerate. In fact, from Eq. (85) one can infer that the direct CP asymmetry is estimated to be -18%, which can be compared to the value of $(-7.7^{+6.0}_{-6.2})\%$ obtained in the full numerical result as shown in Table IX.

For prediction, we see that except $\bar{B}^0 \to K^+K^-$, the sizes of the predicted direct CP asymmetries from Res are smaller than those in Fac.

3. Rates and direct CP asymmetries in $\bar{B}_s^0$ Decays

We show the CP-averaged rates and direct CP violations of $\bar{B}_s^0 \to PP$ decays in Table X. There are five measured $\bar{B}_s$ decay rates, namely $K^+\pi^-$, $\pi^+\pi^-$, $\eta'\eta'$, K^+K^- and $K^0\bar{K}^0$

decay rates. Among them $\bar{B}_s \to \pi^+\pi^-$ and $\eta'\eta'$ decays are newly observed by LHCb [3,4]. From the table we see that both Fac and Res can fit the $\bar{B}_s \to K^+\pi^-$ rate well, but Fac is having difficulties in fitting all other four modes: in particular the χ^2 of $\pi^+\pi^-$, $\eta'\eta'$ and K^+K^- are as large as 20.2, 16.0 and 21.3, respectively, while Res can fit all B_s decay modes very well and brings down these χ^2 efficiently, giving 0.0, 0.0, and 0.0, respectively. Note that the rates of the two newly measured modes ($\pi^+\pi^-$ and $\eta'\eta'$) can be easily reproduced in the Res fit, but not in the Fac fit. For other modes, we see from the table that Res predicts larger rates in $\bar{B}_s^0 \to K^0\pi^0$, $K^0\eta$, $\pi^0\pi^0$ decays, but gives similar predictions on $K^0\eta'$, $\eta\eta$, $\eta\eta'$, $\pi^0\eta$, and $\pi^0\eta'$ rates.

The $\bar{B}_s^0 \to \pi^+\pi^-$ rate in the factorization calculation is too small compared to data. As shown in Table X, through Res the rate can be enhanced significantly. It is useful to see the enhancement of the $\pi^+\pi^-$ rate more closely. From Eq. (27),

$$A_{\bar{B}_s^0 \to \pi^+\pi^-} = E' + PA'^u + PA'^c + \frac{1}{3}P'^A_{\text{EW}}, \quad (93)$$

and the values in Tables V and VII, before and after Res, we have (in unit of 10^{-9} GeV)

$$(A_{\bar{B}_s^0 \to \pi^+\pi^-})^0 \simeq 0.27e^{i102.6°} + 0.07e^{-i77.4°} + 3.51e^{-i8.6°} + 0.02e^{i170.4} \simeq 3.42e^{-i5.4°},$$

$$(A_{\bar{B}_s^0 \to \pi^+\pi^-})^0 \simeq 0.27e^{-i120.1°} + 0.07e^{i60.3°} + 3.51e^{-i8.6°} + 0.02e^{i172.5} \simeq 3.42e^{-i11.7°},$$

$$(A_{\bar{B}_s^0 \to \pi^+\pi^-})_{\mathrm{FSI}} \simeq 1.26e^{i71.0°} + 1.38e^{-i120.1°} + 8.75e^{i75.4°} + 0.28e^{i132.2°} \simeq 8.88e^{i78.7°},$$

$$(A_{\bar{B}_s^0 \to \pi^+\pi^-})_{\mathrm{FSI}} \simeq 1.26e^{-i151.2°} + 1.38e^{i17.6°} + 8.75e^{i75.4°} + 0.28e^{i134.3°} \simeq 8.76e^{i75.3°}, \quad (94)$$

respectively, where terms are given in the order of E', PA'^u, PA'^c, and $P'^A_{\mathrm{EW}}/3$. In our full numerical result, we have $4.17e^{-i5.3°}$, $4.17e^{-i11.7°}$, $9.19e^{i66.7°}$, and $9.04e^{i64.8°}$ for $(A_{\bar{B}_s^0 \to \pi^+\pi^-})^0$, $(A_{\bar{B}_s^0 \to \pi^+\pi^-})^0$, $(A_{\bar{B}_s^0 \to \pi^+\pi^-})_{\mathrm{FSI}}$, and $(A_{\bar{B}_s^0 \to \pi^+\pi^-})_{\mathrm{FSI}}$ in unit of 10^{-9} GeV, respectively, which are close to the scaled (by $f_{B_s}/f_B \simeq f_K/f_\pi$) estimations, $4.15e^{-i5.4°}$, $4.15e^{-i11.8°}$, $10.74e^{i78.7°}$ and $10.64e^{i75.3°}$, from Eq. (94).

From Eq. (94), we see that the sizes of the amplitudes of the $\bar{B}_s^0$ and the conjugated B_s^0 decays are enhanced by factors of 2.58 and 2.56, respectively, where the enhancements are mainly from the enhancement in PA'^c. Consequently, the CP-averaged rate is enhanced by a factor of 6.6, while $\mathcal{A}$ is changed from 0 to 0.9% as E' and PA'^u are also enhanced. Note that the above estimation of rate enhancement is somewhat larger than the one in our full numerical result $(0.67/0.14 = 4.79)$ in Table X, but the direct CP asymmetry is close the value (1.9%) shown in the table. The reason of the vanishing $\mathcal{A}$ before Res is similar to those in the $\bar{B}^0 \to K^+K^-$ decay as discussed previously. Hence, in the presence of Res, E' and PA'^u are enhanced mostly from T'^0 [see Eqs. (72) and (78)], while PA'^c from P'^c [see Eq. (79)], which help to enhance the $\bar{B}_s^0 \to \pi^+\pi^-$ rate and bring in nonvanishing direct CP asymmetry.

We now compare our results to the data in direct CP asymmetries. There are two reported measurements in direct CP asymmetries of $\bar{B}_s$ modes: $\mathcal{A}(\bar{B}_s^0 \to K^+\pi^-)$ and $\mathcal{A}(\bar{B}_s \to K^+K^-)$. A better measurement is reported in the $K^+\pi^-$ mode with a much reduced uncertainty. From the table we see that Res gives a better fit to this data than Fac with $\chi^{2(\mathrm{Fac})} = 4.6$ and $\chi^{2(\mathrm{Res})} = 0.1$. On the other hand both Fac and Res can fit $\mathcal{A}(\bar{B}_s \to K^+K^-)$ well, as the uncertainty in data is still large to accommodate both results, but Res has a smaller χ^2.

For predictions on direct CP asymmetries, we note that the signs of $\mathcal{A}(\bar{B}_s \to \eta'\eta')$ and $\mathcal{A}(\bar{B}_s \to K^0\bar{K}^0)$ are opposite in Fac and Res; Res predicts nonvanishing $\mathcal{A}(\bar{B}_s \to \pi^+\pi^-, \pi^0\pi^0)$ and larger $\mathcal{A}(\bar{B}_s \to \pi^0\eta)$, while predictions of Fac and Res on other modes are similar. These predictions can be checked in near future.

4. Time-dependent CP violations in $\bar{B}^0$ and $\bar{B}_s^0$ decays

Results on time-dependent CP-asymmetries S are given in Table XI. We fit to data on mixing induced CP asymmetries. There are reported experimental results of mixing induced CP asymmetries in the following 5 modes:

$\bar{B}^0 \to \bar{K}^0\pi^0$, $\bar{B}^0 \to \bar{K}^0\eta'$, $\bar{B}^0 \to \pi^+\pi^-$, $\bar{B}^0 \to K_S\bar{K}_S$ and $\bar{B}_s^0 \to K^+K^-$. Since the measurements are subtle, the experimental progress in this sector is slower than those in rates and direct CP asymmetries. Currently, the $\bar{B}^0 \to \bar{K}^0\pi^0$ mode was updated up to 2010; the $\bar{B}^0 \to \bar{K}^0\eta'$ mode was updated up to 2014; the $\bar{B}^0 \to \pi^+\pi^-$ mode was updated up to 2013, the $\bar{B}^0 \to K_S\bar{K}_S$ mode was updated up to 2007 and the $\bar{B}_s \to K^+K^-$ mode was included in these measurement in 2013 [41–44]. New data are eagerly awaited. Note that for the $\bar{B}^0 \to K^0\bar{K}^0$ mode, the mixing induced CP asymmetry obtained by Belle ($-0.38^{+0.69}_{-0.77} \pm 0.09$ [41]) and BABAR ($-1.28^{+0.80+0.11}_{-0.73-0.16}$ [42]) are different. As the central value of the latter exceeds the physical range, we only include the former one in our fit.

From Table XI we see that fit in Res for the $\bar{B}^0 \to \pi^+\pi^-$ mode is much better than the one in Fac, where the χ^2 are 1.1 and 9.3 for the former and the latter, respectively. On the contrary, the fit in Fac is better than Res in the $\bar{B}_s \to K^+K^-$ mode, where the χ^2 are 0.6 and 1.4 for the former and the latter, respectively. Note that the uncertainty in the data of the $\bar{B}_s \to K^+K^-$ mode is much larger than the one in the $\bar{B}^0 \to \pi^+\pi^-$ mode. It will be interesting to see the updated data on the $\bar{B}_s \to K^+K^-$ mode. Overall speaking the quality of fit to mixing induced CP asymmetries is improved (χ^2 reduced from 12.9 to 5.2, see also Table II) in the presence of Res.

It is useful to look into the mixing induced asymmetry in the $\bar{B}_d^0 \to K^0\bar{K}^0$ mode. Recall in Eq. (92) that, before and after Res, we have (in unit of 10^{-8} GeV, without SU(3) breaking correction)

$$(A_{\bar{B}_d^0 \to K^0\bar{K}^0})^0 \simeq 0.81e^{i8.1°}, \quad (A_{B_d^0 \to K^0\bar{K}^0})^0 \simeq 0.92e^{-i31.6°},$$

$$(A_{\bar{B}_d^0 \to K^0\bar{K}^0})_{\mathrm{FSI}} \simeq 0.66e^{-i1.9°}, \quad (A_{B_d^0 \to K^0\bar{K}^0})_{\mathrm{FSI}} \simeq 1.07e^{-i27.2°}, \quad (95)$$

respectively. Using the well known formula:

$$S = \frac{2\mathrm{Im}\lambda_A}{1 + |\lambda_A|^2} \quad (96)$$

with

$$\lambda_A \equiv \frac{q}{p}\frac{A_{\bar{B}_d^0 \to K^0\bar{K}^0}}{A_{B_d^0 \to K^0\bar{K}^0}} = e^{-i2\beta}\frac{A_{\bar{B}_d^0 \to K^0\bar{K}^0}}{A_{B_d^0 \to K^0\bar{K}^0}}, \quad (97)$$

we obtain $S \simeq -0.08$ and -0.29 without and with Res, respectively, which are close to the values reported in Table XI. As explained previously, although $\bar{B}_d^0 \to K^0\bar{K}^0$ is a pure penguin mode, its S is not necessary close to

TABLE XI. Results on the time-dependent CP asymmetry $\sin 2\beta_{\mathrm{eff}}$ (for the first three modes) and S of various $\bar{B}_{d,s} \to PP$ modes.

Mode	Exp	Fac	Res	$\chi^{2(\mathrm{Fac})}_{\min}$	$\chi^{2(\mathrm{Res})}_{\min}$
$\bar{B}^0 \to \bar{K}^0 \pi^0$	0.57 ± 0.17	0.798 ± 0.002	$0.806^{+0.010}_{-0.003}$ (0.793)	1.8	1.9 (1.7)
$\bar{B}^0 \to \bar{K}^0 \eta$	$-$	$0.672^{+0.009}_{-0.015}$	$0.728^{+0.030}_{-0.018}$ (0.757)		
$\bar{B}^0 \to \bar{K}^0 \eta'$	0.63 ± 0.06	$0.689^{+0.001}_{-0.002}$	$0.683^{+0.006}_{-0.008}$ (0.693)	1.0	0.8 (1.1)
$\bar{B}^0 \to \pi^+ \pi^-$	-0.66 ± 0.06	$-0.477^{+0.039}_{-0.041}$	-0.598 ± 0.040 (-0.578)	9.3	1.1 (1.9)
$\bar{B}^0 \to \pi^0 \pi^0$	$-$	0.602 ± 0.023	$0.675^{+0.055}_{-0.049}$ (0.778)		
$\bar{B}^0 \to \eta\eta$	$-$	$-0.741^{+0.014}_{-0.015}$	$-0.663^{+0.031}_{-0.033}$ (-0.669)		
$\bar{B}^0 \to \eta\eta'$	$-$	$-0.847^{+0.013}_{-0.014}$	$-0.953^{+0.028}_{-0.021}$ (-0.795)		
$\bar{B}^0 \to \eta'\eta'$	$-$	$-0.922^{+0.003}_{-0.004}$	$-0.753^{+0.067}_{-0.089}$ (-0.962)		
$\bar{B}^0 \to K^+ K^-$	$-$	$-0.835^{+0.016}_{-0.017}$	$-0.992^{+0.017}_{-0.007}$ (-0.895)		
$\bar{B}^0 \to K_S \bar{K}_S$	$-0.38^{+0.69}_{-0.77} \pm 0.09$ $-1.28^{+0.80+0.11}_{-0.73-0.16}$	-0.016 ± 0.002	$-0.231^{+0.048}_{-0.042}$ (-0.037)	0.2	0.0 (0.2)
$\bar{B}^0 \to \pi^0 \eta$	$-$	$0.215^{+0.005}_{-0.006}$	$-0.473^{+0.043}_{-0.068}$ (-0.494)		
$\bar{B}^0 \to \pi^0 \eta'$	$-$	$-0.002^{+0.010}_{-0.012}$	$-0.414^{+0.035}_{-0.025}$ (-0.440)		
$\bar{B}^0_s \to \pi^+ \pi^-$	$-$	0.152 ± 0.001	$0.071^{+0.011}_{-0.009}$ (0.149)		
$\bar{B}^0_s \to \pi^0 \pi^0$	$-$	0.152 ± 0.001	$0.071^{+0.011}_{-0.009}$ (0.149)		
$\bar{B}^0_s \to \eta\eta$	$-$	-0.005 ± 0.000	$-0.035^{+0.004}_{-0.067}$ (-0.027)		
$\bar{B}^0_s \to \eta\eta'$	$-$	-0.004 ± 0.000	$0.005^{+0.007}_{-0.001}$ (0.006)		
$\bar{B}^0_s \to \eta'\eta'$	$-$	0.021 ± 0.000	$0.046^{+0.006}_{-0.003}$ (0.025)		
$\bar{B}^0_s \to K^+ K^-$	0.30 ± 0.13	0.200 ± 0.002	$0.149^{+0.005}_{-0.066}$ (0.176)	0.6	1.4 (1.0)
$\bar{B}^0_s \to K^0 \bar{K}^0$	$-$	$-0.022^{+0.001}_{-0.000}$	$-0.019^{+0.004}_{-0.017}$ (-0.027)		
$\bar{B}^0_s \to \pi^0 \eta$	$-$	$-0.059^{+0.009}_{-0.004}$	$0.100^{+0.050}_{-0.475}$ (0.308)		
$\bar{B}^0_s \to \pi^0 \eta'$	$-$	$0.232^{+0.013}_{-0.008}$	$-0.016^{+0.065}_{-0.319}$ (0.053)		
$\bar{B}^0_s \to K_S \pi^0$	$-$	$-0.738^{+0.017}_{-0.020}$	$-0.311^{+0.541}_{-0.092}$ (-0.784)		
$\bar{B}^0_s \to K_S \eta$	$-$	$-0.296^{+0.041}_{-0.037}$	$0.274^{+0.369}_{-0.076}$ (-0.273)		
$\bar{B}^0_s \to K_S \eta'$	$-$	$-0.395^{+0.011}_{-0.004}$	$-0.049^{+0.367}_{-0.052}$ (-0.276)		

$-\sin 2\beta$, as the u-penguin contribution is not negligible ($|P^{0u}/P^{0c}| \simeq 0.35$, see Table VII). When Res is turned on, the u-penguin and c-penguin receive different contributions, where it is clear that trees can only contribute to the former giving $|P^u/P^c| \simeq 0.74$ (see Table VII), and, consequently, the value of S can be changed drastically.

We now compare the predictions of Fac and Res on mixing induced CP asymmetries. We note that they have different predictions on the mixing induced CP asymmetries of $\bar{B}^0 \to \eta\eta, \eta\eta', \pi^0\eta, \pi^0\eta', B_s \to \pi^0\eta, \pi^0\eta', K_s\pi^0, K_s\eta$, and $K_s\eta'$ modes. In particular, the signs of central values of the asymmetries of $\bar{B}^0 \to \pi^0\eta, \pi^0\eta', \bar{B}_s \to \pi^0\eta, \pi^0\eta'$, and $K_s\eta$ are opposite.

IV. CONCLUSION

Various new measurements in charmless $B_{u,d,s} \to PP$ modes are reported by Belle and LHCb. These include the rates of $B^0 \to \pi^0\pi^0, \eta\pi^0, B_s \to \eta'\eta', B^0 \to K^+K^-$ and $B^0_s \to \pi^+\pi^-$ decays. Some of these modes are highly suppressed and are among the rarest B decays. Direct CP asymmetries on various modes are constantly updated. It is well known that direct CP asymmetries and rates of suppressed modes are sensitive to final state interaction. As new measurements are reported and more data will be collected, it is interesting and timely to studied the rescattering on $B_{u,d,s} \to PP$ decays. We perform a χ^2 analysis with all available data on CP-averaged rates and CP asymmetries in $\bar{B}_{u,d,s} \to PP$ decays. Our numerical results are compared to data and those from factorization approach. The quality of the fit is improved significantly in the presence of Res, especially in the decay rates in the $\bar{B}^0$ $\Delta S = 0$ sector and in rates and direct CP asymmetries in the $\bar{B}^0_s$ decay modes. Indeed, the χ^2 in the $\bar{B}^0 \to \bar{K}^0\pi^0, \pi^+\pi^-, K^0\bar{K}^0, B^- \to \bar{K}^0\pi^-, K^-\eta, \pi^-\pi^0, \pi^-\eta$, and $\bar{B}^0_s \to \pi^+\pi^-, \eta'\eta'$, and K^+K^- rates, and in $\bar{B}^0 \to \pi^+\pi^-$ and $\bar{B}^0_s \to K^+K^-$ direct CP asymmetries are improved significantly. Res also fit better to the semi-leptonic data on $|V_{ub}|F^{B\pi}(0)$ [see Eq. (56)].

The relations on topological amplitudes and rescattering are explored and they help to provide a better understanding of the effects of rescattering. As suggested by U(3) symmetry on topological amplitudes and FSI, a vanishing exchange rescattering scenario is considered. The exchange, annihilation, u-penguin, u-penguin annihilation,

and some electroweak penguin amplitudes are enhanced significantly via annihilation and total annihilation rescatterings. In particular, the u-penguin annihilation amplitude is sizably enhanced by the tree amplitude via total annihilation rescattering. These enhancements affect rates and CP asymmetries. For example, the enhanced PA^u changes the $\bar{B}_d^0 \to K^0 \bar{K}^0$ direct CP asymmetry significantly; the enhanced P, E, and PA produce (through complicated interference) a slightly larger $\bar{B}^0 \to \pi^0 \pi^0$ decay amplitude and resulting a 35% enhancement in rate; A' and $P'^E_{\rm EW}$ are enhanced and enlarges the deviation of $\mathcal{A}(\bar{B}_d^0 \to K^- \pi^+)$ and $\mathcal{A}(B^- \to K^- \pi^0)$ producing a larger $\Delta\mathcal{A}$; the $\bar{B}_s^0 \to \pi^+ \pi^-$ rate is sizably enhanced through the enhancement in PA'^c; the $|P^u/P^c|$ ratio is enhanced from 0.35 to 0.74 and can change mixing induced CP asymmetries drastically.

For the comparison of the predictions of Fac and Res, we observed the following points. (i) Belle and *BABAR* give very different results in $\mathcal{A}(\bar{B}^0 \to K_s K_s)$ mode, namely Belle gives $\mathcal{A}(\bar{B}^0 \to K_s K_s) = -0.38 \pm 0.38 \pm 0.5$ [41], while *BABAR* gives $0.40 \pm 0.41 \pm 0.06$ [42]. The result of Res prefers the Belle result, while Fac prefers a negative but less sizable direct CP asymmetry. (ii) Except $\bar{B}^0 \to K^+ K^-$, the sizes of the predicted direct CP asymmetries of B^-, $\bar{B}^0 \to PP$ modes from Res are smaller than those in Fac. (iii) For B_s decay rates, Res predicts larger rates in $\bar{B}_s^0 \to K^0 \pi^0$, $K^0 \eta$, $\pi^0 \pi^0$ decays, but gives similar predictions on $K^0 \eta'$, $\eta\eta$, $\eta\eta'$, $\pi^0 \eta$, and $\pi^0 \eta'$ rates. (iv) For predictions on direct CP asymmetries, we note that the signs of $\mathcal{A}(\bar{B}_s \to \eta\eta')$ and $\mathcal{A}(\bar{B}_s \to K^0 \bar{K}^0)$ are opposite in Fac and Res; Res predicts nonvanishing $\mathcal{A}(\bar{B}_s \to \pi^+ \pi^-, \pi^0 \pi^0)$ and larger $\mathcal{A}(\bar{B}_s \to \pi^0 \eta)$, while predictions of Fac and Res on other modes are similar. (v) Finally, Fac and Res have different predictions on the mixing induced CP asymmetries of $\bar{B}^0 \to \eta\eta$, $\eta\eta'$, $\pi^0 \eta$, $\pi^0 \eta'$, $\bar{B}_s \to \pi^0 \eta$, $\pi^0 \eta'$, $K_s \pi^0$, $K_s \eta$, and $K_S \eta'$ modes. In particular, the signs of central values of the asymmetries of $\bar{B}^0 \to \pi^0 \eta$, $\pi^0 \eta'$, $\bar{B}_s \to \pi^0 \eta$, $\pi^0 \eta'$, and $K_s \eta$ modes are opposite. These predictions can be checked in the future.

ACKNOWLEDGMENTS

APPENDIX A: FSI FORMULAS

The weak Hamiltonian for charmless $\bar{B}_q$ decays can be written as $H_{\rm W} = (G_f/\sqrt{2})\sum_{r,l} V_{qb} V_{rq}^* c_l^r \mathcal{O}_l^r$, where c_l^r are Wilson coefficients, $\mathcal{O}_l^r$ are Wilson operators and $V_{qb,rq}$ are the relevant CKM matrix elements, see for example [45]. From the time invariant of the Wilson operator $\mathcal{O}_l^r$, we obtain $(\langle i; {\rm out}|\mathcal{O}_l^r|\bar{B}\rangle)^* = (\langle i; {\rm out}|)^* U_T^\dagger U_T (\mathcal{O}_l^r)^* U_T^\dagger U_T|B\rangle^*$, where U_T is the time-reversal transformation operator.

Using the time-reversal invariant of the operators, $\mathcal{O}_l^r = U_T(\mathcal{O}_l^r)^* U_T^\dagger$ and the appropriate phase convention of states, $U_T|{\rm out(in)}\rangle^* = |{\rm in(out)}\rangle$, we have

$$
(\langle i; {\rm out}|\mathcal{O}_l^r|\bar{B}\rangle)^* = \langle i; {\rm in}|\mathcal{O}_l^r|B\rangle
$$
$$
= \sum_j \langle i; {\rm in}|j; {\rm out}\rangle\langle j; {\rm out}|\mathcal{O}_i|\bar{B}\rangle,
$$

where we have inserted a complete set in the last step. Therefore using the time-reversal invariant property of the operator $\mathcal{O}_l^r$ in a $\bar{B} \to PP$ decay, one obtains

$$
(\langle i; {\rm out}|\mathcal{O}_l^r|\bar{B}\rangle)^* = \sum_j \mathcal{S}_{ji}^* \langle j; {\rm out}|\mathcal{O}_l^r|\bar{B}\rangle, \qquad ({\rm A1})
$$

where $\mathcal{S}_{ij} \equiv \langle i; {\rm out}|j; {\rm in}\rangle$ is the strong interaction S-matrix element, j denotes all possible states. Equation (A1) is the master formula of FSI.

One can easily verify that the solution of the above equation is given by

$$
A_i(\mathcal{O}_l^r) = \sum_{k=1}^N \mathcal{S}_{ik}^{1/2} A_k^0(\mathcal{O}_l^r), \qquad ({\rm A2})
$$

where we have $A_i(\mathcal{O}_l^r) \equiv \langle i; {\rm out}|\mathcal{O}_l^r|\bar{B}\rangle$ and $A^0(\mathcal{O}_l^r)$ are real amplitudes. Putting back the coefficients, we obtain the master formula Eq. (1), and we can now state clearly $A_i \equiv \langle i; {\rm out}|H_{\rm W}|\bar{B}\rangle = (G_f/\sqrt{2})\sum_{r,l} V_{qb} V_{rq}^* c_l^r A_i(\mathcal{O}_l^r)$ and $A_i^0 \equiv (G_f/\sqrt{2})\sum_{r,l} V_{qb} V_{rq}^* c_l^r A_i^0(\mathcal{O}_l^r)$.

Without loss of generality, we can reexpress the S-matrix in Eq. (1) as

$$
\mathcal{S}_{ik} = \sum_{j=1}^n (\mathcal{S}_1)_{ij} (\mathcal{S}_2)_{jk}, \qquad ({\rm A3})
$$

where $\mathcal{S}_1$ is a nonsingular $n \times n$ matrix with n the total number of charmless PP states and $\mathcal{S}_2$ is defined through the above equation, i.e., $\mathcal{S}_2 \equiv \mathcal{S}_1^{-1}\mathcal{S}$. As mentioned previously (in the introduction) the factorization amplitudes contain a large portion of rescattering effects as encoded in $\mathcal{S}_2$, while some residual rescattering among a small group of states is still allowed and needs to be explored:

$$
\mathcal{S}_1 = \mathcal{S}_{\rm res}, \quad A_j^{\rm fac} = \sum_{k=1}^N (\mathcal{S}_2^{1/2})_{jk} A_k^0, \qquad ({\rm A4})
$$

with N the total number of states entering Eq. (1), $A_j^{\rm fac}$ the factorization amplitude and $\mathcal{S}_{\rm res}$ the rescattering matrix to govern rescattering among PP states.

We collect the rescattering formulas used in this work. We have

$$
\begin{pmatrix}
A_{\bar{B}_d^0 \to K^- \pi^+} \\
A_{\bar{B}_d^0 \to \bar{K}^0 \pi^0} \\
A_{\bar{B}_d^0 \to \bar{K}^0 \eta_8} \\
A_{\bar{B}_d^0 \to \bar{K}^0 \eta_1}
\end{pmatrix}
= \mathcal{S}_{\mathrm{res},1}^{1/2}
\begin{pmatrix}
A_{\bar{B}_d^0 \to K^- \pi^+}^{\mathrm{fac}} \\
A_{\bar{B}_d^0 \to \bar{K}^0 \pi^0}^{\mathrm{fac}} \\
A_{\bar{B}_d^0 \to \bar{K}^0 \eta_8}^{\mathrm{fac}} \\
A_{\bar{B}_d^0 \to \bar{K}^0 \eta_1}^{\mathrm{fac}}
\end{pmatrix},
\tag{A5}
$$

for group-1 modes,

$$
\begin{pmatrix}
A_{B^- \to \bar{K}^0 \pi^-} \\
A_{B^- \to K^- \pi^0} \\
A_{B^- \to K^- \eta_8} \\
A_{B^- \to K^- \eta_1}
\end{pmatrix}
= \mathcal{S}_{\mathrm{res},2}^{1/2}
\begin{pmatrix}
A_{B^- \to \bar{K}^0 \pi^-}^{\mathrm{fac}} \\
A_{B^- \to K^- \pi^0}^{\mathrm{fac}} \\
A_{B^- \to K^- \eta_8}^{\mathrm{fac}} \\
A_{B^- \to K^- \eta_1}^{\mathrm{fac}}
\end{pmatrix},
\tag{A6}
$$

for group-2 modes,

$$
\begin{pmatrix}
A_{B^- \to \pi^- \pi^0} \\
A_{B^- \to K^0 K^-} \\
A_{B^- \to \pi^- \eta_8} \\
A_{B^- \to \pi^- \eta_1}
\end{pmatrix}
= \mathcal{S}_{\mathrm{res},3}^{1/2}
\begin{pmatrix}
A_{B^- \to \pi^- \pi^0}^{\mathrm{fac}} \\
A_{B^- \to K^0 K^-}^{\mathrm{fac}} \\
A_{B^- \to \pi^- \eta_8}^{\mathrm{fac}} \\
A_{B^- \to \pi^- \eta_1}^{\mathrm{fac}}
\end{pmatrix},
\tag{A7}
$$

for group-3 modes and

$$
\begin{pmatrix}
A_{\bar{B}_d^0 \to \pi^+ \pi^-} \\
A_{\bar{B}_d^0 \to \pi^0 \pi^0} \\
A_{\bar{B}_d^0 \to \eta_8 \eta_8} \\
A_{\bar{B}_d^0 \to \eta_8 \eta_1} \\
A_{\bar{B}_d^0 \to \eta_1 \eta_1} \\
A_{\bar{B}_d^0 \to K^+ K^-} \\
A_{\bar{B}_d^0 \to K^0 \bar{K}^0} \\
A_{\bar{B}_d^0 \to \pi^0 \eta_8} \\
A_{\bar{B}_d^0 \to \pi^0 \eta_1}
\end{pmatrix}
= \mathcal{S}_{\mathrm{res},4}^{1/2}
\begin{pmatrix}
A_{\bar{B}_d^0 \to \pi^+ \pi^-}^{\mathrm{fac}} \\
A_{\bar{B}_d^0 \to \pi^0 \pi^0}^{\mathrm{fac}} \\
A_{\bar{B}_d^0 \to \eta_8 \eta_8}^{\mathrm{fac}} \\
A_{\bar{B}_d^0 \to \eta_8 \eta_1}^{\mathrm{fac}} \\
A_{\bar{B}_d^0 \to \eta_1 \eta_1}^{\mathrm{fac}} \\
A_{\bar{B}_d^0 \to K^+ K^-}^{\mathrm{fac}} \\
A_{\bar{B}_d^0 \to K^0 \bar{K}^0}^{\mathrm{fac}} \\
A_{\bar{B}_d^0 \to \pi^0 \eta_8}^{\mathrm{fac}} \\
A_{\bar{B}_d^0 \to \pi^0 \eta_1}^{\mathrm{fac}}
\end{pmatrix},
\tag{A8}
$$

for group-4 modes, where we define $\mathcal{S}_{\mathrm{res},i}^{1/2} = (1 + i\mathcal{T}_i)^{1/2} \equiv 1 + i\mathcal{T}_i^{(1/2)}$, before incorporating SU(3) breaking effect, with

$$
\mathcal{T}_1 =
\begin{pmatrix}
r_0 + r_a & \frac{-r_a + r_e}{\sqrt{2}} & \frac{-r_a + r_e}{\sqrt{6}} & \frac{2\bar{r}_a + \bar{r}_e}{\sqrt{3}} \\
\frac{-r_a + r_e}{\sqrt{2}} & r_0 + \frac{r_a + r_e}{2} & \frac{r_a - r_e}{2\sqrt{3}} & -\frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} \\
\frac{-r_a + r_e}{\sqrt{6}} & \frac{r_a - r_e}{2\sqrt{3}} & r_0 + \frac{r_a + 5r_e}{6} & -\frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} \\
\frac{2\bar{r}_a + \bar{r}_e}{\sqrt{3}} & -\frac{2\bar{r}_a + \bar{r}_e}{\sqrt{6}} & -\frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} & \tilde{r}_0 + \frac{4\tilde{r}_a + 2\tilde{r}_e}{3}
\end{pmatrix},
\quad
\mathcal{T}_2 =
\begin{pmatrix}
r_0 + r_a & \frac{r_a - r_e}{\sqrt{2}} & \frac{-r_a + r_e}{\sqrt{6}} & \frac{2\bar{r}_a + \bar{r}_e}{\sqrt{3}} \\
\frac{r_a - r_e}{\sqrt{2}} & r_0 + \frac{r_a + r_e}{2} & \frac{-r_a + r_e}{2\sqrt{3}} & \frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} \\
\frac{-r_a + r_e}{\sqrt{6}} & \frac{-r_a + r_e}{2\sqrt{3}} & r_0 + \frac{r_a + 5r_e}{6} & -\frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} \\
\frac{2\bar{r}_a + \bar{r}_e}{\sqrt{3}} & \frac{2\bar{r}_a + \bar{r}_e}{\sqrt{6}} & -\frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} & \tilde{r}_0 + \frac{4\tilde{r}_a + 2\tilde{r}_e}{3}
\end{pmatrix},
$$

$$
\mathcal{T}_3 =
\begin{pmatrix}
r_0 + r_a & 0 & 0 & 0 \\
0 & r_0 + r_a & \sqrt{\frac{2}{3}}(r_a - r_e) & \frac{2\bar{r}_a + \bar{r}_e}{\sqrt{3}} \\
0 & \sqrt{\frac{2}{3}}(r_a - r_e) & r_0 + \frac{2r_a + r_e}{3} & \frac{\sqrt{2}}{3}(2\bar{r}_a + \bar{r}_e) \\
0 & \frac{2\bar{r}_a + \bar{r}_e}{\sqrt{3}} & \frac{\sqrt{2}}{3}(2\bar{r}_a + \bar{r}_e) & \tilde{r}_0 + \frac{4\tilde{r}_a + 2\tilde{r}_e}{3}
\end{pmatrix},
\tag{A9}
$$

and

$$\mathcal{T}_4 = \mathrm{diag}(r_0, r_0, r_0, \tilde{r}_0, \check{r}_0, r_0, r_0, r_0, \tilde{r}_0)$$

$$
+ \begin{pmatrix}
2r_a + r_t & \frac{2r_a - r_e + r_t}{\sqrt{2}} & \frac{2r_a + r_e + 3r_t}{3\sqrt{2}} & \frac{\sqrt{2}(2\bar{r}_a + \bar{r}_e)}{3} & \frac{4\hat{r}_a + 2\hat{r}_e + 3\hat{r}_t}{3\sqrt{2}} & r_a + r_t & r_a + r_t & 0 & 0 \\[6pt]
\frac{2r_a - r_e + r_t}{\sqrt{2}} & \frac{2r_a + r_e + r_t}{2} & \frac{2r_a + r_e + 3r_t}{6} & \frac{2\bar{r}_a + \bar{r}_e}{3} & \frac{4\hat{r}_a + 2\hat{r}_e + 3\hat{r}_t}{6} & \frac{r_a + r_t}{\sqrt{2}} & \frac{r_a + r_t}{\sqrt{2}} & 0 & 0 \\[6pt]
\frac{2r_a + r_e + 3r_t}{3\sqrt{2}} & \frac{2r_a + r_e + 3r_t}{6} & \frac{2r_a + r_e + r_t}{2} & -\frac{2\bar{r}_a + \bar{r}_e}{3} & \frac{4\hat{r}_a + 2\hat{r}_e + 3\hat{r}_t}{6} & \frac{5r_a - 2r_e + 3r_t}{3\sqrt{2}} & \frac{5r_a - 2r_e + 3r_t}{3\sqrt{2}} & 0 & 0 \\[6pt]
\frac{\sqrt{2}(2\bar{r}_a + \bar{r}_e)}{3} & \frac{2\bar{r}_a + \bar{r}_e}{3} & -\frac{2\bar{r}_a + \bar{r}_e}{3} & \frac{4\bar{r}_a + 2\bar{r}_e}{3} & 0 & -\frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} & -\frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} & 0 & 0 \\[6pt]
\frac{4\hat{r}_a + 2\hat{r}_e + 3\hat{r}_t}{3\sqrt{2}} & \frac{4\hat{r}_a + 2\hat{r}_e + 3\hat{r}_t}{6} & \frac{4\hat{r}_a + 2\hat{r}_e + 3\hat{r}_t}{6} & 0 & \frac{4\check{r}_a + 2\check{r}_e + 3\check{r}_t}{6} & \frac{4\hat{r}_a + 2\hat{r}_e + 3\hat{r}_t}{3\sqrt{2}} & \frac{4\hat{r}_a + 2\hat{r}_e + 3\hat{r}_t}{3\sqrt{2}} & 0 & 0 \\[6pt]
r_a + r_t & \frac{r_a + r_t}{\sqrt{2}} & \frac{5r_a - 2r_e + 3r_t}{3\sqrt{2}} & -\frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} & \frac{4\hat{r}_a + 2\hat{r}_e + 3\hat{r}_t}{3\sqrt{2}} & 2r_a + r_t & r_a + r_t & \frac{r_a - r_e}{\sqrt{3}} & \frac{2\bar{r}_a + \bar{r}_e}{\sqrt{6}} \\[6pt]
r_a + r_t & \frac{r_a + r_t}{\sqrt{2}} & \frac{5r_a - 2r_e + 3r_t}{3\sqrt{2}} & -\frac{2\bar{r}_a + \bar{r}_e}{3\sqrt{2}} & \frac{4\hat{r}_a + 2\hat{r}_e + 3\hat{r}_t}{3\sqrt{2}} & r_a + r_t & 2r_a + r_t & \frac{-r_a + r_e}{\sqrt{3}} & -\frac{2\bar{r}_a + \bar{r}_e}{\sqrt{6}} \\[6pt]
0 & 0 & 0 & 0 & 0 & \frac{r_a - r_e}{\sqrt{3}} & \frac{-r_a + r_e}{\sqrt{3}} & \frac{2r_a + r_e}{3} & \frac{\sqrt{2}(2\bar{r}_a + \bar{r}_e)}{3} \\[6pt]
0 & 0 & 0 & 0 & 0 & \frac{2\bar{r}_a + \bar{r}_e}{\sqrt{6}} & -\frac{2\bar{r}_a + \bar{r}_e}{\sqrt{6}} & \frac{\sqrt{2}(2\bar{r}_a + \bar{r}_e)}{3} & \frac{4\check{r}_a + 2\check{r}_e}{3}
\end{pmatrix}.
$$

$$\text{(A10)}$$

Note that for identical particle final states, such as $\pi^0\pi^0$, factors of $1/\sqrt{2}$ are included in the amplitudes and the corresponding $\mathcal{S}_{\mathrm{res}}$ matrix elements. The rescattering formulas for $\bar{B}_s^0 \to \bar{P}\bar{P}$ decays are similar to the $\bar{B}_d^0 \to PP$ ones, since strong interaction respect charge conjugation. For example, the rescattering formula for $\bar{B}_s^0 \to \bar{K}^+\bar{\pi}^-$ is similar to those of $\bar{B}_d^0 \to K^-\pi^+$ with trivial replacement on amplitudes.

To include the SU(3) breaking effect, we proceed as outlined in the main text. First we remove the SU(3) breaking effect in A^{fac} before recattering and put it back after the rescattering. For the reasoning one is referred to the main text. For convenient we absorb these two action into the rescattering matrices. We use ratios of decay constants to model the SU(3) breaking effect. For example, in the group-3 modes, in the $\pi^-\pi^0$–K^0K^-–$\pi^-\eta_q$–$\pi^-\eta_s$ basis, we have

$$
\mathcal{S}_{\mathrm{res},3}^{1/2} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \left(\frac{f_K}{f_\pi}\right)^2 & 0 & 0 \\
0 & 0 & \frac{f_{\eta_q}}{f_\pi} & 0 \\
0 & 0 & 0 & \frac{f_{\eta_s}}{f_\pi}
\end{pmatrix}
\left(1 + i\mathcal{T}_3^{(1/2)}\right)
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \left(\frac{f_\pi}{f_K}\right)^2 & 0 & 0 \\
0 & 0 & \frac{f_\pi}{f_{\eta_q}} & 0 \\
0 & 0 & 0 & \frac{f_\pi}{f_{\eta_s}}
\end{pmatrix}.
\tag{A11}
$$

In the numerical study we follow [46] to use $f_{\eta_q}/f_\pi = 1.07$ and $f_{\eta_s}/f_\pi = 1.34$. It is clear that when FSI is turned off the above $\mathcal{S}_{\mathrm{res},3}^{1/2}$ is just an identity matrix. The SU(3) breaking effects are incorporated in other $\mathcal{S}_{\mathrm{res},i}^{1/2}$ in a similar fashion. Note that $\mathcal{T}_i$ in Eqs. (A9) and (A10) are given in the η_8–η_1 basis and to incorporate the SU(3) effect, one needs to transform $\mathcal{T}_i$ to the η_q–η_s basis (see below).

The physical η, η' mesons are defined through

$$
\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix},
\tag{A12}
$$

with the mixing angle $\vartheta \simeq -15.4°$ [46]. For the $\eta^{(\prime)}\eta^{(\prime)}$ states, we have

$$
\begin{pmatrix} \eta\eta \\ \eta\eta' \\ \eta'\eta' \end{pmatrix} = \begin{pmatrix}
\cos^2\vartheta & -\sqrt{2}\cos\vartheta\sin\vartheta & \sin^2\vartheta \\
\sqrt{2}\cos\vartheta\sin\vartheta & \cos^2\vartheta - \sin^2\vartheta & -\sqrt{2}\cos\vartheta\sin\vartheta \\
\sin^2\vartheta & \sqrt{2}\cos\vartheta\sin\vartheta & \cos^2\vartheta
\end{pmatrix} \begin{pmatrix} \eta_8\eta_8 \\ \eta_8\eta_1 \\ \eta_1\eta_1 \end{pmatrix},
\tag{A13}
$$

where the identical particle factor of $1/\sqrt{2}$ is properly included in the mixing matrix. Note that the above formulas can be easily used to transform the η_q–η_s basis into the η_8–η_1 basis by replacing the above ϑ by $\tan^{-1}\sqrt{2}$.

Rescattering parameters enter $\mathcal{S}_{\text{res}}$ only through 7 independent combinations: $1 + i(r_0 + r_a)$, $i(r_e - r_a)$, $i(r_a + r_t)$, $i(2\bar{r}_a + \bar{r}_e)$, $1 + i[\tilde{r}_0 + (4\tilde{r}_a + 2\tilde{r}_e)/3]$, $i(4\hat{r}_a + 2\hat{r}_e + 3\hat{r}_t)$ and $1 + i[\check{r}_0 + (4\check{r}_a + 2\check{r}_e + 3\check{r}_t)/6]$. The solutions to Eqs. (5) and (6) are given in Eq. (9).

If the full U(3) symmetry is a good symmetry, it requires:

$$r_i = \bar{r}_i = \tilde{r}_i = \hat{r}_i = \check{r}_i, \tag{A14}$$

for each $i = 0, a, e, t$. We are constrained to have

$$r_e^{(m)} r_a^{(m)} = 0. \tag{A15}$$

Consequently, there are two different solutions: (a) the annihilation type ($r_a^{(m)} \neq 0$, $r_e^{(m)} = 0$) with

$$\delta_{27} = \delta_8' = \delta_1', \qquad \delta_8, \delta_1,$$
$$\tau = -\frac{1}{2}\sin^{-1}\frac{4\sqrt{5}}{9}, \qquad \nu = -\frac{1}{2}\sin^{-1}\frac{4\sqrt{2}}{9}, \tag{A16}$$

and (b) the exchange type ($r_e^{(m)} \neq 0$, $r_a^{(m)} = r_t^{(m)} = 0$) with

$$\delta_{27} = \delta_8' = \delta_1', \qquad \delta_8 = \delta_1,$$
$$\tau = \frac{1}{2}\sin^{-1}\frac{\sqrt{5}}{3}, \qquad \nu = \frac{1}{2}\sin^{-1}\frac{2\sqrt{2}}{3}. \tag{A17}$$

It is interesting to note that in both solutions of the U(3) case, a common constraint

$$\delta_{27} = \delta_8' = \delta_1', \tag{A18}$$

has to be satisfied.

APPENDIX B: DERIVATION OF THE RESCATTERING EFFECTS ON TOPOLOGICAL AMPLITUDES

It is straightforward to obtain the rescattering effects on topological amplitudes. In analogy to Eq. (15): $A = \mathcal{S}_{\text{res}}^{1/2} \cdot A^{\text{fac}} = (1 + i\mathcal{T}^{1/2}) \cdot A^{\text{fac}}$, we have $H_{\text{eff}} = (1 + i\mathcal{T}^{1/2}) \cdot H_{\text{eff}}^0 = H_{\text{eff}}^0 + i\mathcal{T}^{1/2} \cdot H_{\text{eff}}^0$, where H_{eff} is given in Eq. (18), $\mathcal{T}^{1/2}$ in Eq. (40), H_{eff}^0 is the unscattered effective Hamiltonian with all TA in H_{eff} replaced by TA^0 and the dot in the above equation implies all possible pairing of the $P^{\text{out}}P^{\text{out}}$ fields in H_{eff}^0 to the $P^{\text{in}}P^{\text{in}}$ fields in $\mathcal{T}^{1/2}$. It is useful to use $H_i^{ik} = H^k$, $H_k^{ik} = 0$, $(H_{\text{EW}})_k^{ik} = 0$, $(H_{\text{EW}})_i^{ik} = -\frac{1}{3}H^k$, $(\Pi^{\text{in}})_a^a = (\Pi^{\text{out}})_a^a = 0$ and the fact that the paring of creation and annihilation fields gives the following flavor structure: $\langle(\Pi^{\text{out}})_k^j(\Pi^{\text{in}})_b^a\rangle \rightarrow \delta_b^j\delta_k^a - \frac{1}{3}\delta_k^j\delta_b^a$.

Below we work out the contribution from T^0 via the rescattering among PP states for illustration. We shall concentrate on the flavor structures after the pairings in $(i\mathcal{T}^{1/2} \cdot H_{\text{eff}}^0)$ and compare them to the operators in H_{eff}.[8]

1. Pairing $T^0 \bar{B}_m H_j^{ik}(\Pi^{\text{out}})_k^j(\Pi^{\text{out}})_i^m$ and $(ir_0'/2)\text{Tr}(\Pi^{\text{in}}\Pi^{\text{out}})\text{Tr}(\Pi^{\text{in}}\Pi^{\text{out}})$

Pairing the T^0 term in H_{eff}^0 and the ir_0' term from $\mathcal{T}^{1/2}$ gives:

$$T^0 \bar{B}_m H_j^{ik}(\Pi^{\text{out}})_k^j(\Pi^{\text{out}})_i^m \cdot \left[\frac{ir_0'}{2}(\Pi^{\text{in}})_b^a(\Pi^{\text{out}})_a^b(\Pi^{\text{in}})_d^c(\Pi^{\text{out}})_c^d\right]$$

$$= \frac{ir_0'}{2}T^0\bar{B}_m H_j^{ik}[\langle(\Pi^{\text{out}})_k^j(\Pi^{\text{in}})_b^a\rangle\langle(\Pi^{\text{out}})_i^m(\Pi^{\text{in}})_d^c\rangle + \langle(\Pi^{\text{out}})_k^j(\Pi^{\text{in}})_d^c\rangle\langle(\Pi^{\text{out}})_i^m(\Pi^{\text{in}})_b^a\rangle](\Pi^{\text{out}})_a^b(\Pi^{\text{out}})_c^d$$

$$= \frac{ir_0'}{2}T^0\bar{B}_m H_j^{ik}\left[\left(\delta_b^j\delta_k^a - \frac{1}{3}\delta_k^j\delta_b^a\right)\left(\delta_d^m\delta_i^c - \frac{1}{3}\delta_i^m\delta_d^c\right) + \left(\delta_d^j\delta_k^c - \frac{1}{3}\delta_k^j\delta_d^c\right)\left(\delta_b^m\delta_i^a - \frac{1}{3}\delta_i^m\delta_b^a\right)\right](\Pi^{\text{out}})_a^b(\Pi^{\text{out}})_c^d$$

$$= \frac{ir_0'}{2}T^0\bar{B}_m H_j^{ik}[(\Pi^{\text{out}})_k^j(\Pi^{\text{out}})_i^m + (\Pi^{\text{out}})_i^m(\Pi^{\text{out}})_k^j]$$

$$= ir_0'T^0\bar{B}_m H_j^{ik}(\Pi^{\text{out}})_k^j(\Pi^{\text{out}})_i^m. \tag{B1}$$

We note that the last term has the same form of the T operator in H_{eff} and we denote it as $\delta T(T^0)\bar{B}_m H_j^{ik}(\Pi^{\text{out}})_k^j(\Pi^{\text{out}})_i^m$. From the above equation we obtain,

---

[8]There are integrations of momentum and so on, which will not shown explicitly in the following derivation and are absorbed in the definition of r_i'. See Ref. [47] for the treatment on this issue.

$$\delta T(T^0) = ir_0' T^0. \tag{B2}$$

2. Pairing $T^0 \bar{B}_m H_j^{ik}(\Pi^{\text{out}})_k^j(\Pi^{\text{out}})_i^m$ and $ir_e' \text{Tr}(\Pi^{\text{in}}\Pi^{\text{out}}\Pi^{\text{in}}\Pi^{\text{out}})/2$

Pairing the T^0 term in H_{eff}^0 and the ir_e' term from $\mathcal{T}^{1/2}$ gives:

$$
T^0 \bar{B}_m H_j^{ik}(\Pi^{\text{out}})_k^j(\Pi^{\text{out}})_i^m \cdot \left[\frac{ir_e'}{2}(\Pi^{\text{in}})_b^a(\Pi^{\text{out}})_c^b(\Pi^{\text{in}})_d^c(\Pi^{\text{out}})_a^d\right]
$$

$$
= ir_e' \frac{1}{2} T^0 \bar{B}_m H_j^{ik}[\langle(\Pi^{\text{out}})_k^j(\Pi^{\text{in}})_b^a\rangle\langle(\Pi^{\text{out}})_i^m(\Pi^{\text{in}})_d^c\rangle + \langle(\Pi^{\text{out}})_k^j(\Pi^{\text{in}})_d^c\rangle\langle(\Pi^{\text{out}})_i^m(\Pi^{\text{in}})_b^a\rangle](\Pi^{\text{out}})_c^b(\Pi^{\text{out}})_a^d
$$

$$
= ir_e' \frac{1}{2} T \bar{B}_m H_j^{ik}[(\Pi^{\text{out}})_i^j(\Pi^{\text{out}})_k^m - \frac{1}{3}\delta_i^m(\Pi^{\text{out}})_c^j(\Pi^{\text{out}})_k^c + (\Pi^{\text{out}})_k^m(\Pi^{\text{out}})_i^j - \frac{1}{3}\delta_i^m(\Pi^{\text{out}})_k^a(\Pi^{\text{out}})_a^j]
$$

$$
= ir_e' T^0 \bar{B}_m H_j^{ik}(\Pi^{\text{out}})_i^j(\Pi^{\text{out}})_k^m - \frac{1}{3}ir_e' T^0 \bar{B}_i H_j^{ik}(\Pi^{\text{out}})_l^j(\Pi^{\text{out}})_k^l
$$

$$
= \delta C(T^0) \bar{B}_m H_j^{ik}(\Pi^{\text{out}})_i^j(\Pi^{\text{out}})_k^m + \delta A(T^0) \bar{B}_i H_j^{ik}(\Pi^{\text{out}})_l^j(\Pi^{\text{out}})_k^l, \tag{B3}
$$

which leads to

$$\delta C(T^0) = ir_e' T^0, \qquad \delta A(T^0) = -\frac{1}{3}ir_e' T^0. \tag{B4}$$

3. Pairing $T^0 \bar{B}_m H_j^{ik}(\Pi^{\text{out}})_k^j(\Pi^{\text{out}})_i^m$ and $ir_a' \text{Tr}(\Pi^{\text{in}}\Pi^{\text{in}}\Pi^{\text{out}}\Pi^{\text{out}})$

Pairing the T^0 term in H_{eff}^0 and the ir_a' term from $\mathcal{T}^{1/2}$ gives:

$$
ir_a' T^0 \bar{B}_m H_j^{ik}(\Pi^{\text{out}})_k^j(\Pi^{\text{out}})_i^m[(\Pi^{\text{in}})_b^a(\Pi^{\text{in}})_c^b(\Pi^{\text{out}})_d^c(\Pi^{\text{out}})_a^d]
$$

$$
= ir_a' T^0 \bar{B}_m H_j^{ik}[\langle(\Pi^{\text{out}})_k^j(\Pi^{\text{in}})_b^a\rangle\langle(\Pi^{\text{out}})_i^m(\Pi^{\text{in}})_c^b\rangle + \langle(\Pi^{\text{out}})_k^j(\Pi^{\text{in}})_c^b\rangle\langle(\Pi^{\text{out}})_i^m(\Pi^{\text{in}})_b^a\rangle](\Pi^{\text{out}})_d^c(\Pi^{\text{out}})_a^d
$$

$$
= ir_a' T^0 \bar{B}_m H^k(\Pi^{\text{out}})_i^m(\Pi^{\text{out}})_k^i + ir_a' T^0 \bar{B}_k H_j^{ik}(\Pi^{\text{out}})_l^j(\Pi^{\text{out}})_i^l - \frac{2}{3}ir_a' T^0 \bar{B}_i H_j^{ik}(\Pi^{\text{out}})_l^j(\Pi^{\text{out}})_k^l
$$

$$
= \delta P(T^0) \bar{B}_m H^k(\Pi^{\text{out}})_i^m(\Pi^{\text{out}})_k^i + \delta E(T^0) \bar{B}_k H_j^{ik}(\Pi^{\text{out}})_l^j(\Pi^{\text{out}})_i^l + \delta A(T^0) \bar{B}_i H_j^{ik}(\Pi^{\text{out}})_l^j(\Pi^{\text{out}})_k^l, \tag{B5}
$$

which leads to

$$\delta P(T^0) = ir_a' T^0, \qquad \delta E(T^0) = ir_a' T^0, \qquad \delta A(T^0) = -\frac{2}{3}ir_a' T^0. \tag{B6}$$

4. Pairing $T^0 \bar{B}_m H_j^{ik}(\Pi^{\text{out}})_k^j(\Pi^{\text{out}})_i^m$ and $ir_t' \text{Tr}(\Pi^{\text{in}}\Pi^{\text{in}})\text{Tr}(\Pi^{\text{out}}\Pi^{\text{out}})/4$

Pairing the T^0 term in H_{eff}^0 and the ir_t' term from $\mathcal{T}^{1/2}$ gives:

$$
ir_t' \frac{1}{4} T^0 \bar{B}_m H_j^{ik}(\Pi^{\text{out}})_k^j(\Pi^{\text{out}})_i^m[(\Pi^{\text{in}})_b^a(\Pi^{\text{in}})_a^b(\Pi^{\text{out}})_d^c(\Pi^{\text{out}})_c^d]
$$

$$
= ir_t' \frac{1}{4} T^0 \bar{B}_m H_j^{ik}[\langle(\Pi^{\text{out}})_k^j(\Pi^{\text{in}})_b^a\rangle\langle(\Pi^{\text{out}})_i^m(\Pi^{\text{in}})_a^b\rangle + \langle(\Pi^{\text{out}})_k^j(\Pi^{\text{in}})_a^b\rangle\langle(\Pi^{\text{out}})_i^m(\Pi^{\text{in}})_b^a\rangle](\Pi^{\text{out}})_d^c(\Pi^{\text{out}})_c^d
$$

$$
= \frac{1}{2}ir_t' T^0 \bar{B}_k H^k(\Pi^{\text{out}})_m^l(\Pi^{\text{out}})_l^m
$$

$$
= \frac{1}{2}\delta PA(T^0) \bar{B}_k H^k(\Pi^{\text{out}})_m^l(\Pi^{\text{out}})_l^m, \tag{B7}
$$

which leads to

$$\delta PA(T^0) = ir_t' T^0. \tag{B8}$$

5. Pairing $T^0 \bar{B}_m H_j^{ik} (\Pi^{\mathrm{out}})_k^j (\Pi^{\mathrm{out}})_i^m$ and $i(\bar{r}_e' + 2\bar{r}_a')$ $\mathrm{Tr}(\Pi^{\mathrm{in}} \Pi^{\mathrm{out}} \Pi^{\mathrm{in}}) \eta_1^{\mathrm{out}} / \sqrt{3}$

Pairing the T^0 term in H_{eff}^0 and the $i(\bar{r}_e' + 2\bar{r}_a')$ term from $\mathcal{T}^{1/2}$ gives:

$$i(\bar{r}_e' + 2\bar{r}_a') T^0 \bar{B}_m H_j^{ik} (\Pi^{\mathrm{out}})_k^j (\Pi^{\mathrm{out}})_i^m [(\Pi^{\mathrm{in}})_b^a (\Pi^{\mathrm{in}})_c^b (\Pi^{\mathrm{out}})_a^c \eta_1^{\mathrm{out}}]$$

$$= i(\bar{r}_e' + 2\bar{r}_a') T^0 \bar{B}_m H_j^{ik} [\langle (\Pi^{\mathrm{out}})_k^j (\Pi^{\mathrm{in}})_b^a \rangle \langle (\Pi^{\mathrm{out}})_i^m (\Pi^{\mathrm{in}})_c^b \rangle + \langle (\Pi^{\mathrm{out}})_k^j (\Pi^{\mathrm{in}})_c^b \rangle \langle (\Pi^{\mathrm{out}})_i^m (\Pi^{\mathrm{in}})_b^a \rangle] (\Pi^{\mathrm{out}})_a^c \eta_1^{\mathrm{out}} / \sqrt{3}$$

$$= i(\bar{r}_e' + 2\bar{r}_a') T \bar{B}_m H_j^{ik} \left[\delta_i^j (\Pi^{\mathrm{out}})_k^m \eta_1^{\mathrm{out}} / \sqrt{3} + \delta_k^m (\Pi^{\mathrm{out}})_i^j \eta_1^{\mathrm{out}} / \sqrt{3} - \frac{1}{3} \delta_i^m (\Pi^{\mathrm{out}})_k^j \eta_1^{\mathrm{out}} / \sqrt{3} - \frac{1}{3} \delta_i^m (\Pi^{\mathrm{out}})_k^j \eta_1^{\mathrm{out}} / \sqrt{3}) \right]$$

$$= i(\bar{r}_e' + 2\bar{r}_a') T^0 \bar{B}_m H^k (\Pi^{\mathrm{out}})_k^m \eta_1^{\mathrm{out}} / \sqrt{3} + i(\bar{r}_e' + 2\bar{r}_a') T^0 \bar{B}_k H_j^{ik} (\Pi^{\mathrm{out}})_i^j \eta_1^{\mathrm{out}} / \sqrt{3} - \frac{2}{3} i(\bar{r}_e' + 2\bar{r}_a') T^0 \bar{B}_i H_j^{ik} (\Pi^{\mathrm{out}})_k^j \eta_1^{\mathrm{out}} / \sqrt{3}$$

$$= \delta \left(\bar{C}_2 + \bar{P}_1 + \bar{P}_2 - \frac{1}{3} \bar{P}_{\mathrm{EW},2}^C \right) (T^0) \bar{B}_m H^k (\Pi^{\mathrm{out}})_k^m \eta_1^{\mathrm{out}} / \sqrt{3} + \delta(\bar{C}_1 + \bar{E}_1 + \bar{E}_2)(T^0) \bar{B}_k H_j^{ik} (\Pi^{\mathrm{out}})_i^j \eta_1^{\mathrm{out}} / \sqrt{3}$$

$$+ \delta(\bar{T} + \bar{A}_1 + \bar{A}_2)(T^0) \bar{B}_i H_j^{ik} (\Pi^{\mathrm{out}})_k^j \eta_1^{\mathrm{out}} / \sqrt{3}, \tag{B9}$$

which is similar to the pairing of $T^0 \, ir_a'$ and leads to

$$\delta \left(\bar{C}_2 + \bar{P}_1 + \bar{P}_2 - \frac{1}{3} \bar{P}_{\mathrm{EW},2}^C \right)(T^0) = i(\bar{r}_e' + 2\bar{r}_a') T^0$$

$$\delta(\bar{C}_1 + \bar{E}_1 + \bar{E}_2)(T^0) = i(\bar{r}_e' + 2\bar{r}_a') T^0$$

$$\delta(\bar{T} + \bar{A}_1 + \bar{A}_2)(T^0) = -\frac{2}{3} i(\bar{r}_e' + 2\bar{r}_a') T^0. \tag{B10}$$

6. Pairing $T^0 \bar{B}_m H_j^{ik} (\Pi^{\mathrm{out}})_k^j (\Pi^{\mathrm{out}})_i^m$ and $i(\tilde{r}_0' + \frac{4\tilde{r}_a' + 2\tilde{r}_e'}{3})$ $\mathrm{Tr}(\Pi^{\mathrm{in}} \Pi^{\mathrm{out}}) \eta_1^{\mathrm{in}} \eta_1^{\mathrm{out}}$

Pairing the T^0 term in H_{eff}^0 and the $i(\tilde{r}_0' + \frac{4\tilde{r}_a' + 2\tilde{r}_e'}{3})$ term from $\mathcal{T}^{1/2}$ gives vanishing result.

7. Pairing $T^0 \bar{B}_m H_j^{ik} (\Pi^{\mathrm{out}})_k^j (\Pi^{\mathrm{out}})_i^m$ and $i(\hat{r}_t' + \frac{4\hat{r}_a' + 2\hat{r}_e'}{3}) \eta_1^{\mathrm{out}} \eta_1^{\mathrm{out}} \mathrm{Tr}(\Pi^{\mathrm{in}} \Pi^{\mathrm{in}}) / 4$

Pairing the T^0 term in H_{eff}^0 and the $i(\tilde{r}_0' + \frac{4\tilde{r}_a' + 2\tilde{r}_e'}{3})$ term from $\mathcal{T}^{1/2}$ gives:

$$i \left(\hat{r}_t' + \frac{4\hat{r}_a' + 2\hat{r}_e'}{3} \right) \frac{1}{4} T^0 \bar{B}_m H_j^{ik} (\Pi^{\mathrm{out}})_k^j (\Pi^{\mathrm{out}})_i^m [(\Pi^{\mathrm{in}})_b^a (\Pi^{\mathrm{in}})_a^b \eta_1^{\mathrm{out}} \eta_1^{\mathrm{out}}]$$

$$= i \left(\hat{r}_t' + \frac{4\hat{r}_a' + 2\hat{r}_e'}{3} \right) \frac{1}{4} T^0 \bar{B}_m H_j^{ik} [\langle (\Pi^{\mathrm{out}})_k^j (\Pi^{\mathrm{in}})_b^a \rangle \langle (\Pi^{\mathrm{out}})_i^m (\Pi^{\mathrm{in}})_a^b \rangle + \langle (\Pi^{\mathrm{out}})_k^j (\Pi^{\mathrm{in}})_a^b \rangle \langle (\Pi^{\mathrm{out}})_i^m (\Pi^{\mathrm{in}})_b^a \rangle] \eta_1^{\mathrm{out}} \eta_1^{\mathrm{out}}$$

$$= \frac{1}{2} i \left(\hat{r}_t' + \frac{4\hat{r}_a' + 2\hat{r}_e'}{3} \right) T^0 \bar{B}_k H^k \eta_1^{\mathrm{out}} \eta_1^{\mathrm{out}}$$

$$= \delta \left(\tilde{C} + \tilde{E} + \tilde{P} + \frac{3}{2} \widetilde{PA} - \frac{1}{3} \tilde{P}_{\mathrm{EW}}^C - \frac{1}{3} \tilde{P}_{\mathrm{EW}}^E \right) (T^0) \bar{B}_k H^k \eta_1^{\mathrm{out}} \eta_1^{\mathrm{out}} / 3, \tag{B11}$$

which is similar to the pairing of $T \, ir_t'$ and leads to

$$\delta \left(\tilde{C} + \tilde{E} + \tilde{P} + \frac{3}{2} \widetilde{PA} - \frac{1}{3} \tilde{P}_{\mathrm{EW}}^C - \frac{1}{3} \tilde{P}_{\mathrm{EW}}^E \right)(T^0) = \frac{3}{2} i \left(\hat{r}_t' + \frac{4\hat{r}_a' + 2\hat{r}_e'}{3} \right) T^0. \tag{B12}$$

8. Pairing $T^0 \bar{B}_m H_j^{ik} (\Pi^{\mathrm{out}})_k^j (\Pi^{\mathrm{out}})_i^m$ and $i(\check{r}_0^{(m)} + \frac{4\hat{r}_a^{(m)} + 2\hat{r}_e^{(m)} + 3\hat{r}_t^{(m)}}{6}) \frac{1}{2} \eta_1^{\mathrm{in}} \eta_1^{\mathrm{out}} \eta_1^{\mathrm{in}} \eta_1^{\mathrm{out}}$

Pairing the T^0 term in H_{eff}^0 and the $i(\check{r}_0^{(m)} + \frac{4\hat{r}_a^{(m)} + 2\hat{r}_e^{(m)} + 3\hat{r}_t^{(m)}}{6})$ term from $\mathcal{T}^{1/2}$ gives vanishing result.

The results of rescattering effects from T^0 are collected in Eqs. (43), (44) and (43). Rescattering effects from other TA are obtained and collected similarly.

[1] B. Pal *et al.* (Belle Collaboration), Evidence for the decay $B^0 \to \eta\pi^0$, Phys. Rev. D **92**, 011101 (2015).

[2] B. Pal *et al.* (Belle Collaboration), Observation of the Decay $B_s^0 \to K^0 \bar{K}^0$, Phys. Rev. Lett. **116**, 161801 (2016).

[3] R. Aaij *et al.* (LHCb Collaboration), Observation of the $B_s^0 \to \eta'\eta'$ Decay, Phys. Rev. Lett. **115**, 051801 (2015).

[4] R. Aaij *et al.* (LHCb Collaboration), Observation of the Annihilation Decay Mode $B^0 \to K^+ K^-$, Phys. Rev. Lett. **118**, 081801 (2017).

[5] T. Julius *et al.* (Belle Collaboration), Measurement of the branching fraction and *CP* asymmetry in $B^0 \to \pi^0\pi^0$ decays, and an improved constraint on ϕ_2, Phys. Rev. D **96**, 032007 (2017).

[6] Y. Amhis *et al.* (Heavy Flavor Averaging Group (HFAG) Collaboration), Averages of *b*-hadron, *c*-hadron, and τ-lepton properties as of summer 2014, arXiv:1412.7515; online update at http://www.slac.stanford.edu/xorg/hfag.

[7] C. Patrignani *et al.* (Particle Data Group), Review of particle physics, Chin. Phys. C **40**, 100001 (2016) and 2017 update.

[8] T. Gershon and V. V. Gligorov, *CP* violation in the *B* system, Rep. Prog. Phys. **80**, 046201 (2017).

[9] B. Blok, M. Gronau, and J. L. Rosner, Annihilation, Rescattering, and *CP* Asymmetries in *B* Meson Decays, Phys. Rev. Lett. **78**, 3999 (1997); M. Gronau and J. L. Rosner, Rescattering information from $B \to K\bar{K}$ decays, Phys. Rev. D **58**, 113005 (1998); M. Gronau, D. London, and J. L. Rosner, Rescattering contributions to rare *B*-meson decays, Phys. Rev. D **87**, 036008 (2013).

[10] H. Y. Cheng, C. K. Chua, and A. Soni, Final state interactions in hadronic *B* decays, Phys. Rev. D **71**, 014030 (2005).

[11] C. K. Chua, Rescattering effects in charmless $\bar{B}_{u,d,s} \to PP$ decays, Phys. Rev. D **78**, 076002 (2008).

[12] H. Y. Cheng, C. W. Chiang, and A. L. Kuo, Updating $B \to PP, VP$ decays in the framework of flavor symmetry, Phys. Rev. D **91**, 014011 (2015).

[13] H.-n. Li and S. Mishima, Possible resolution of the $B \to \pi\pi, \pi K$ puzzles, Phys. Rev. D **83**, 034023 (2011); Glauber gluons in spectator amplitudes for $B \to \pi M$ decays, Phys. Rev. D **90**, 074018 (2014); H. Y. Cheng and C. K. Chua, Resolving *B*-CP Puzzles in QCD factorization, Phys. Rev. D **80**, 074031 (2009); S. H. Zhou, Q. A. Zhang, W. R. Lyu, and C. D. Lü, Analysis of charmless two-body B decays in factorization assisted topological amplitude approach, Eur. Phys. J. C **77**, 125 (2017).

[14] M. Suzuki and L. Wolfenstein, Final state interaction phase in B decays, Phys. Rev. D **60**, 074019 (1999).

[15] K. M. Watson, The effect of final state interactions on reaction cross-sections, Phys. Rev. **88**, 1163 (1952).

[16] Y. Y. Keum, H.-n. Li, and A. I. Sanda, Fat penguins and imaginary penguins in perturbative QCD, Phys. Lett. B **504**, 6 (2001); Penguin enhancement and $B \to K\pi$ decays in perturbative QCD, Phys. Rev. D **63**, 054008 (2001).

[17] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, QCD factorization in $B \to \pi K, \pi\pi$ decays and extraction of Wolfenstein parameters, Nucl. Phys. **B606**, 245 (2001).

[18] M. Beneke and M. Neubert, QCD factorization for B → PP and B → PV decays, Nucl. Phys. **B675**, 333 (2003); Flavor-singlet B decay amplitudes in QCD factorization, Nucl. Phys. **B651**, 225 (2003).

[19] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, An effective field theory for collinear and soft gluons: Heavy to light decays, Phys. Rev. D **63**, 114020 (2001); C. W. Bauer, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, B → M(1) M(2): Factorization, charming penguins, strong phases, and polarization, Phys. Rev. D **70**, 054015 (2004).

[20] S. W. Lin *et al.* (Belle Collaboration), Difference in direct charge-parity violation between charged and neutral *B* meson decays, Nature (London) **452**, 332 (2008).

[21] W. S. Hou (private communication).

[22] C. K. Chua, W. S. Hou, and K. C. Yang, Indication for large rescatterings in charmless rare B decays, Mod. Phys. Lett. A **18**, 1763 (2003).

[23] D. Zeppenfeld, SU(3) relations for *B* meson decays, Z. Phys. C **8**, 77 (1981).

[24] L. L. Chau and H. Y. Cheng, Analysis of exclusive two-body decays of charm mesons using the quark diagram scheme, Phys. Rev. D **36**, 137 (1987).

[25] L. L. Chau, H. Y. Cheng, W. K. Sze, H. Yao, and B. Tseng, Charmless nonleptonic rare decays of *B* mesons, Phys. Rev. D **43**, 2176 (1991); Erratum, **58**, 019902 (1998).

[26] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Decays of *B* mesons to two light pseudoscalars, Phys. Rev. D **50**, 4529 (1994).

[27] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Electroweak penguins and two-body *B* decays, Phys. Rev. D **52**, 6374 (1995).

[28] T. D. Lee, Particle physics and introduction to field theory, Contemp. Concepts Phys. **1**, 1 (1981).

[29] T. N. Pham, Nonet symmetry in η, η' and $B \to K\eta, K\eta'$ decays, Phys. Rev. D **77**, 014024 (2008); Erratum, **77**, 019905(E) (2008).

[30] X. G. He and W. Wang, Flavor SU(3) Topological Diagram and Irreducible Representation Amplitudes for Heavy Meson Charmless Hadronic Decays: Mismatch and Equivalence, arXiv:1803.04227 [Phys. Rev. Lett. (to be published)].

[31] H. Y. Cheng, C. K. Chua, and C. W. Hwang, Covariant light-front approach for *s*-wave and *p*-wave mesons: Its application to decay constants and form factors, Phys. Rev. D **69**, 074025 (2004).

[32] D. Melikhov and B. Stech, Weak form factors for heavy meson decays: An update, Phys. Rev. D **62**, 014006 (2000).

[33] P. Ball and R. Zwicky, New results on $B \to \pi, K, \eta$ decay formfactors from light-cone sum rules, Phys. Rev. D **71**, 014015 (2005).

[34] G. Duplancic, A. Khodjamirian, T. Mannel, B. Melic, and N. Offen, Light-cone sum rules for $B \to \pi$ form factors revisited, J. High Energy Phys. 04 (2008) 014.

[35] A. Khodjamirian, T. Mannel, N. Offen, and Y.-M. Wang, $B \to \pi\ell\nu_l$ width and $|V_{ub}|$ from QCD light-cone sum rules, Phys. Rev. D **83**, 094031 (2011).

[36] A. Bharucha, Two-loop corrections to the $B to\pi$ form factor from QCD sum rules on the light-cone and $|V_{ub}|$, J. High Energy Phys. 05 (2012) 092.

[37] P. del Amo Sanchez *et al.* (*BABAR* Collaboration), Study of $B \to \pi\ell\nu$ and $B \to \rho\ell\nu$ decays and determination of $|V_{ub}|$, Phys. Rev. D **83**, 032007 (2011).

[38] J. P. Lees *et al.* (*BABAR* Collaboration), Branching fraction and form-factor shape measurements of exclusive charmless

semileptonic B decays, and determination of $|V_{ub}|$, Phys. Rev. D **86,** 092004 (2012).

[39] H. Ha *et al.* (Belle Collaboration), Measurement of the decay $B^0 \to \pi^- \ell^+ \nu$ and determination of $|V_{ub}|$, Phys. Rev. D **83,** 071101 (2011).

[40] A. Sibidanov *et al.* (Belle Collaboration), Study of exclusive $B^0 \to X_u \ell_\nu$ decays and extraction of $|V_{ub}|$ using full reconstruction tagging at the Belle experiment, Phys. Rev. D **88,** 032005 (2013).

[41] Y. Nakahama *et al.* (Belle Collaboration), Measurement of Time-Dependent CP-Violating Parameters in $B^0 \to K_S^0 K_S^0$ decays, Phys. Rev. Lett. **100,** 121601 (2008).

[42] B. Aubert *et al.* (*BABAR* Collaboration), Observation of $B^+ \to \bar{K}^0 K^+$ and $B^0 \to K^0 \bar{K}^0$, Phys. Rev. Lett. **97,** 171805 (2006).

[43] B. Aubert *et al.* (*BABAR* Collaboration), Measurement of time dependent CP asymmetry parameters in B^0 meson decays to omega K_S^0, $\eta' K^0$, and $\pi^0 K_S^0$, Phys. Rev. D **79,** 052003 (2009); M. Fujikawa *et al.* (Belle Collaboration), Measurement of CP asymmetries in $B^0 \to K^0 \pi^0$ decays, Phys. Rev. D **81,** 011101 (2010); L. antelj *et al.* (Belle

Collaboration), Measurement of time-dependent CP violation in $B^0 \to \eta' K^0$ decays, J. High Energy Phys. 10 (2014) 165; J. P. Lees *et al.* (*BABAR* Collaboration), Measurement of CP asymmetries and branching fractions in charmless two-body B-meson decays to pions and kaons, Phys. Rev. D **87,** 052009 (2013); I. Adachi *et al.* (Belle Collaboration), Measurement of the CP violation parameters in $B^0 \to \pi^+ \pi^-$ decays, Phys. Rev. D **88,** 092003 (2013).

[44] R. Aaij *et al.* (LHCb Collaboration), First measurement of time-dependent CP violation in $B_s^0 \to K^+ K^-$ decays, J. High Energy Phys. 10 (2013) 183.

[45] A. J. Buras, Weak Hamiltonian, CP violation and rare decays, arXiv:hep-ph/9806471.

[46] T. Feldmann, P. Kroll, and B. Stech, Mixing and decay constants of pseudoscalar mesons, Phys. Rev. D **58,** 114006 (1998); Mixing and decay constants of pseudoscalar mesons: The sequel, Phys. Lett. B **449,** 339 (1999).

[47] C. K. Chua, W. S. Hou, and K. C. Yang, Final state rescattering and color suppressed $\bar{B}^0 \to D^{0(*)} h^0$ decays, Phys. Rev. D **65,** 096007 (2002).