Anomalous current from the covariant Wigner function

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We consider accelerated and rotating media of weakly interacting fermions in local thermodynamic equilibrium on the basis of kinetic approach. Kinetic properties of such media can be described by covariant Wigner function incorporating the relativistic distribution functions of particles with spin. We obtain the formulae for axial current by summation of the terms of all orders of thermal vorticity tensor, chemical potential, both for massive and massless particles. In the massless limit all the terms of fourth and higher orders of vorticity and third order of chemical potential and temperature equal zero. It is shown, that axial current gets a topological component along the 4-acceleration vector. The similarity between different approaches to baryon polarization is established.

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I. INTRODUCTION

Relativistic fermionic liquid is an unusual object for which the laws of quantum field theory become apparent in macroscopic phenomena. The most important effects (see e.g. [1] for reference), chiral magnetic effect (CME) and chiral vortical effect (CVE) [2–11] effectively modify the equations of hydrodynamics [2]. Chiral effects can have different experimental consequences, in particular, resulting in baryon polarisation [7,8,12], approached also in the different context [13,14]. In particular, there are suggestions to investigate P-odd effects in heavy-ion collisions, strongly controlled by CVE [7,12], at the Nuclotron-Based Ion Collider Facility (NICA) which under construction in the Joint Institute for Nuclear Research.

There are currently seemingly different approaches to polarization relying either on relativistic thermodynamics [14] or on anomalous axial current and the respective charge [7,12]. The use of that charge allows to address the baryonic phase performing the calculation of quark anomalies. The other approach to polarization in confined phase was recently suggested by consideration of the vortices cores in pionic superfluid [15]. To compare the approaches and fill the gap between them we explore in detail the appearance of anomalous axial current (used in [7,12]) in the framework of the approach [14].

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To do so we will concentrate on the CVE. The existence of CVE was proven in many different ways, e.g. in hydrodynamic approximation as the result of joint consideration of triangle axial anomaly and second law of thermodynamics [2,16], exploring Kubo formulas and thermal field theory loop calculation [5,17,18], kinetics [9,10], calculations of triangle anomaly in effective quantum field theory and generalization of the form of axial charge in hydrodynamics [3,7,19].

We will consider the medium of weakly interacting fermions in approximation of local thermodynamic equilibrium, using the kinetic approach, developed in the series of papers Refs. [13,14,20–24]. In these works, a simple and natural *ansatz* was proposed and grounded, which simulates local relativistic distribution functions. According to the argument used in the construction of this ansatz, it must describe with a good degree of accuracy the effects associated with acceleration and rotation, at least in the lowest order approximation in vorticity.

Various thermodynamic quantities can be calculated using the covariant Wigner function, the expression for which can be derived on the basis of the ansatz of the distribution functions and using the approximation of weak interaction and small inhomogeneities. Thus, in [14] the effects of the vorticity in the energy-momentum tensor, vector current, spin tensor and recently in the axial current in [24] were investigated.

In our paper we continue to consider the consequences of the anzats of distribution functions from [14]. Using the same approximations for the Wigner function, we show that CVE can be obtained by calculating the average value of the axial current using the distribution functions under consideration, and, therefore, is their direct consequence. This is one of the main results of the present paper. We have reproduced in a new way the well-known formula for axial current, corresponding to CVE, which is linear in vorticity, with the necessary coefficient, depending on μ^2 and T^2 [1–5,9–11,24]. Thus, we confirmed the similar calculation recently made in [24]. Let us notice here, that appearance of CVE in kinetics might be connected to triangle graphs, existing in finite-temperature field theory.[1]

We also go beyond currently formulated region of applicability of the formalism [14] and investigate higher order effects. We managed to go beyond the perturbative approximations used in the previous papers, and derive an exact formula for the axial current. We show analytically that in the massless limit all terms in the axial current, starting from the third order in the chemical potential and temperature, and the fourth order in vorticity and acceleration vectors are cancelled. An intriguing consequence of the expression obtained, in comparison with the usual CVE, is that the axial current obtains a topological component directed along the 4-acceleration vector, which gives a nonzero contribution to the divergence of the axial current. Also, vortical conductivity obtains additional contribution of the second order in vorticity and acceleration.

As soon as we show the existence of CVE and anomalous current in kinetic approach [14], this also directs to the similarity of polarization calculation, based on this kinetic approach [13,14] and approach [7,12], based on the form of anomalous current.

We use Minkowskian metric tensor in the form $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ and Levi-Civita symbol $\epsilon^{0123} = 1$. The contraction of the induces is sometimes denoted by dots $\varpi^2 = \varpi_{\mu\nu}\varpi^{\mu\nu} = \varpi{:}\varpi$. For Dirac matrices we take $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. We use the system of units $\hbar = c = k = 1$.

II. THERMAL VORTICITY TENSOR

Thermal vorticity tensor [14,24] contains information about local acceleration, vorticity and temperature gradients in the media in local thermodynamic equilibrium. As it was noticed in [14] the form of this tensor is not strictly defined for local thermodynamic equilibrium and known up to the second order over the gradients $\partial^2\beta$

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu) + \mathcal{O}(\partial^2\beta), \qquad (2.1)$$

where $\beta_\mu = \frac{u_\mu}{T}$ and T is a temperature in the comoving system. In the limit of global thermodynamic equilibrium β_μ satisfies [24]

$$\beta_\mu = b_\mu + \varpi_{\mu\nu}x_\nu, \qquad b_\mu = \mathrm{const},$$
$$\varpi_{\mu\nu} = \mathrm{const}, \qquad \varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu) \qquad (2.2)$$

[1]We are indebted for V. I. Zakharov for pointing out such an effect.

and the ratio of chemical potential to temperature in comoving frame is constant

$$\xi = \frac{\mu}{T} = \mathrm{const}. \qquad (2.3)$$

Taking into account (2.2), (2.3) one may transform the density operator to the form of global thermodynamic equilibrium density operator [24]. Further, in most cases it will not be necessary to know the exact form of expression for ϖ. In fact, it is sufficient to assume that thermal vorticity is an antisymmetric tensor, which follows from the form of the distribution functions given below. But at the final stages, we will consider global thermodynamic equilibrium (2.2) as a particular case to illustrate the content and the consequences of the obtained formulas.

By analogy with electrodynamics it is convenient to expand ϖ tensor into vector and pseudovector. Following to [24] we introduce thermal acceleration vector α_μ and thermal vorticity pseudovector w_μ

$$\alpha_\mu = \varpi_{\mu\nu}u^\nu, \qquad w_\mu = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}u^\nu\varpi^{\alpha\beta}. \qquad (2.4)$$

In accordance with the terminology of electrodynamics, α_μ and w_μ may be called "electric" and "magnetic" components in the comoving frame, respectively. Tensor $\varpi_{\mu\nu}$ can be expressed in terms of these components (2.4) as follows:

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu. \qquad (2.5)$$

For global thermodynamic equilibrium temperature remains constant along the flow direction [24]. Due to this, α_μ and w_μ are proportional in this case to the usual kinematic 4-acceleration and vorticity, respectively

$$\alpha_\mu = \frac{1}{T}u^\nu\partial_\nu u_\mu = \frac{a_\mu}{T},$$
$$w_\mu = \frac{1}{2T}\epsilon_{\mu\nu\alpha\beta}u^\nu\partial^\alpha u^\beta = \frac{\omega_\mu}{T}, \qquad (2.6)$$

which in the local rest frame a and ω are expressed in terms of 3-dimensional vectors

$$a^\mu = (0,\mathbf{a}), \qquad \omega^\mu = (0,\mathbf{w}), \qquad (2.7)$$

where $\mathbf{a}$ and $\mathbf{w}$ are 3-dimensional acceleration and angular velocity. In the limit (2.2) and (2.3) the derivatives of thermal acceleration, vorticity and temperature can be defined [24] as

$$\partial\cdot\alpha = \frac{1}{|\beta|}(2w^2 - \alpha^2), \qquad \partial\cdot w = -\frac{3}{|\beta|}(w\cdot\alpha),$$
$$\partial_\mu T = T^2\alpha_\mu. \qquad (2.8)$$

It is possible to construct tensor $\tilde{\varpi}$, which is dual to ϖ and gives the vorticity vector after projection to 4-velocity

$$\tilde{\varpi}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\varpi^{\alpha\beta} = \epsilon_{\mu\nu\alpha\beta}\alpha^\alpha u^\beta - w_\mu u_\nu + w_\nu u_\mu,$$
$$w_\mu = \tilde{\varpi}_{\nu\mu}u^\nu. \tag{2.9}$$

By analogy with electrodynamics one scalar and one pseudoscalar can be constructed from the thermal vorticity tensor (2.1), $\varpi^2 = \varpi : \varpi$ and $\varpi : \tilde{\varpi}$, respectively

$$\varpi^2 = 2(\alpha^2 - w^2), \qquad \varpi : \tilde{\varpi} = -4(w \cdot \alpha) \tag{2.10}$$

Note that $\varpi^2 = \text{const}$, $\varpi : \tilde{\varpi} = \text{const}$ in the global thermodynamic equilibrium limit as it follows from (2.2). Also, another scalar,

$$\alpha^2 + w^2, \tag{2.11}$$

corresponds to the density of the Hamiltonian in electrodynamics.

Continuing the analogy with electrodynamics, it is convenient to introduce complex vectors, constructed from (2.6)

$$\varphi_\mu = \frac{a_\mu}{2\pi} + \frac{i\omega_\mu}{2\pi}, \qquad \psi_\mu = \varphi_\mu^*, \tag{2.12}$$

With the help of these vectors, the final results can be represented in a compact form reflecting the symmetry between acceleration and rotation.

III. COVARIANT WIGNER FUNCTION AND DISTRIBUTION FUNCTION FOR PARTICLES WITH SPIN

One of the ways of describing the kinetic properties of the medium, allowing one to take into account the quantum effects, is based on the use of the covariant Wigner function. A sequential introduction to the quantum relativistic kinetic theory and formalism using the Wigner function can be found in [25]. For the spin 1/2 particles this Wigner function is a spinorial matrix, expressed in terms of mean value of the operators of Dirac fields

$$W(x,k)_{AB} = -\frac{1}{(2\pi)^4}\int d^4y e^{-ik\cdot y}$$
$$\times \langle :\Psi_A(x-y/2)\bar{\Psi}_B(x+y/2): \rangle. \tag{3.1}$$

Brackets $\langle : : \rangle$ mean ensemble averaging with normal ordering. Wigner function (3.1) can be calculated using the distribution function. In the case when the interaction is weak and the inhomogeneities leading to a gradient of the Wigner function are much smaller than the characteristic length scale (Compton wave length for massive particles and de Broglie wave length for massless case) [14,25], one has

$$W(x,k) = \frac{1}{2}\int \frac{d^3p}{\varepsilon}(\delta^4(k-p)U(p)f(x,p)\bar{U}(p)$$
$$- \delta^4(k+p)V(p)\bar{f}^T(x,p)\bar{V}(p)), \tag{3.2}$$

where $U(p) = (u_+(p), u_-(p))$, $V(p) = (v_+(p), v_-(p))$ are 4×2 matrices, $\bar{U}(p) = U^\dagger(p)\gamma^0$, $\bar{V}(p) = V^\dagger(p)\gamma^0$ are 2×4 matrices, $f(x,p)$ and $\bar{f}(x,p)$ are 2×2 matrices and $u_+(p)$ and $u_-(p)$ are spinors of free Dirac fields with different values of helicity, normalized as usual: $\bar{u}_r u_s = -\bar{v}_r v_s = 2m\delta_{rs}$.

In [14] local equilibrium distribution functions $f(x,p)$ and $\bar{f}(x,p)$ for massive particles and antiparticles with spin in the accelerated and rotating media were introduced. They have the following form

$$f(x,p) = \frac{1}{8\pi^3}\frac{1}{2m}\bar{U}(p)X(x,p)U(p),$$
$$\bar{f}(x,p) = -\frac{1}{8\pi^3}\frac{1}{2m}[\bar{V}(p)\bar{X}(x,p)V(p)]^T \tag{3.3}$$

(note the phase space factor $\frac{1}{8\pi^3}$, cf. [26,24]). Exact form of $X(x,p)$ and $\bar{X}(x,p)$ is unknown. However, it can be stated beforehand that these functions must lead to a Fermi-Dirac distribution in the case of a stationary medium, and to a correct distribution in a rotating medium in the Boltzmann limit obtained in [14,20]. In [20] a simple *ansatz*,[2] complying with these requirements, was suggested to investigate the lowest order corrections for the vorticity

$$X(x,p) = \left(\exp[\beta\cdot p - \xi(x)]\exp\left[-\frac{1}{2}\varpi(x):\Sigma\right] + I\right)^{-1},$$
$$\bar{X}(x,p) = \left(\exp[\beta\cdot p + \xi(x)]\exp\left[\frac{1}{2}\varpi(x):\Sigma\right] + I\right)^{-1}, \tag{3.4}$$

where $\xi(x) = \frac{\mu(x)}{T(x)}$, $\mu(x)$ and $T(x)$ are comoving local chemical potential and temperature and $\Sigma_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]$ are the generators of Lorentz transformation of spinors. One may expect [20] that ansatz (3.4) is a good approximation at least for the lowest order corrections for the vorticity, though we will also apply it to investigate the higher order effects.

One can also consider distribution functions (3.4) as generalization of formulas (introduced in [14] for global equilibrium) to the case local equilibrium. The generalization corresponds to change of quantities, which are constant in global equilibrium, such as temperature,

[2] We are indebted for F. Becattini for pointing out the approximate nature of the distribution functions.

thermal vorticity tensor and chemical potential, to space-time dependent quantities.

Mean values can be now calculated with a help of Wigner functions $W(x,p)$, and we will now concentrate on the axial current.

IV. AXIAL CURRENT

A. Massive particles

In general, the mean value $\langle :\bar{\Psi}A\Psi: \rangle$ of an operator A containing 4×4 matrix, has the form [14]:

$$\begin{aligned}\langle :\bar{\Psi}(x)A\Psi(x): \rangle &= \int d^4k \mathrm{tr}(AW(x,k)) \\ &= \int \frac{d^3p}{2\varepsilon}\mathrm{tr}_2(f(x,p)\bar{U}(p)AU(p)) \\ &\quad - \mathrm{tr}_2(\bar{f}^T(x,p)\bar{V}(p)AV(p)),\end{aligned} \tag{4.1}$$

where tr_2 means the trace of 2×2 matrix, corresponding to sum over polarizations. For axial current $j^5_\mu = \bar{\Psi}\gamma_\mu\gamma^5\Psi$ (4.1) leads to

$$\langle : j^5_\mu : \rangle = -\frac{1}{16\pi^3}\epsilon_{\mu\alpha\nu\beta}\int \frac{d^3p}{\varepsilon}p^\alpha\{\mathrm{tr}(X\Sigma^{\nu\beta}) - \mathrm{tr}(\bar{X}\Sigma^{\nu\beta})\}. \tag{4.2}$$

The traces $\mathrm{tr}(X\Sigma^{\nu\beta})$ $\mathrm{tr}(\bar{X}\Sigma^{\nu\beta})$ in (4.2) can be calculated in the lowest order in vorticity [14], for which (3.4) and (3.2) is expected to be a good approximation. However, one can go beyond perturbation theory and obtain exact expressions for these traces:

$$\begin{aligned}\mathrm{tr}(X\Sigma^{\nu\beta}) = &\{(\exp[(\beta\cdot p - \xi - g_1 + ig_2)] + 1)^{-1} - (\exp[(\beta\cdot p - \xi + g_1 - ig_2)] + 1)^{-1}\} \\ &\times \frac{1}{4(g_1 - ig_2)}[\varpi^{\nu\beta} - i\,\mathrm{sgn}(\varpi:\tilde{\varpi})\tilde{\varpi}^{\nu\beta}] \\ &+ \{(\exp[(\beta\cdot p - \xi - g_1 - ig_2)] + 1)^{-1} - (\exp[(\beta\cdot p - \xi + g_1 + ig_2)] + 1)^{-1}\} \\ &\times \frac{1}{4(g_1 + ig_2)}[\varpi^{\nu\beta} + i\,\mathrm{sgn}(\varpi:\tilde{\varpi})\tilde{\varpi}^{\nu\beta}],\end{aligned} \tag{4.3}$$

where *sgn* is a sign-function and g_1 and g_2 are scalars, depending on vorticity

$$\begin{aligned}g_1 &= \frac{1}{4}(\sqrt{(\varpi:\varpi)^2 + (\varpi:\tilde{\varpi})^2} + \varpi:\varpi)^{1/2}, \\ g_2 &= \frac{1}{4}(\sqrt{(\varpi:\varpi)^2 + (\varpi:\tilde{\varpi})^2} - \varpi:\varpi)^{1/2}.\end{aligned} \tag{4.4}$$

Derivation of (4.3) is given in Appendix. The trace $\mathrm{tr}(\bar{X}\Sigma^{\nu\beta})$ can be formally obtained from (4.3) by replacement $\xi \to -\xi$ and change of overall sign.

The integral in (4.2) has a form $\int \frac{d^3p}{\varepsilon}p^\alpha f(\beta\cdot p)$, where $f(\beta\cdot p)$ is a scalar function of $(\beta\cdot p)$. From Lorentz-covariance one obtains

$$\int \frac{d^3p}{\varepsilon}p^\alpha f(\beta\cdot p) = u^\alpha \int d^3p f\left(\frac{\varepsilon}{T}\right). \tag{4.5}$$

After substitution of (4.3) to (4.2), using (4.5) and performing the algebraic transformations, one can see, that imaginary terms compensate each other. The final expression for axial current is

$$\begin{aligned}\langle : j^5_\mu : \rangle &= C_1 w_\mu + \mathrm{sgn}(\varpi:\tilde{\varpi})C_2\alpha_\mu, \\ C_1 &= \frac{1}{4\pi^2}\frac{g_2\cosh(g_1)\sin(g_2) + g_1\sinh(g_1)\cos(g_2)}{g_1^2 + g_2^2}(I_1(\xi) + I_1(-\xi)) + \frac{1}{8\pi^2}\frac{g_1\sinh(2g_1) + g_2\sin(2g_2)}{g_1^2 + g_2^2}(I_2(\xi) + I_2(-\xi)), \\ C_2 &= \frac{1}{4\pi^2}\frac{g_2\sinh(g_1)\cos(g_2) - g_1\cosh(g_1)\sin(g_2)}{g_1^2 + g_2^2}(I_1(\xi) + I_1(-\xi)) + \frac{1}{8\pi^2}\frac{g_2\sinh(2g_1) - g_1\sin(2g_2)}{g_1^2 + g_2^2}(I_2(\xi) + I_2(-\xi)),\end{aligned} \tag{4.6}$$

where $I_1(\xi)$, $I_2(\xi)$ denote one-dimensional integrals, which can be calculated numerically:

$$I_1(\xi) = \int \frac{dp\mathbf{p}^2\cosh(\frac{\varepsilon}{T}-\xi)}{(\cosh(\frac{\varepsilon}{T}-\xi-g_1)+\cos(g_2))(\cosh(\frac{\varepsilon}{T}-\xi+g_1)+\cos(g_2))},$$
$$I_2(\xi) = \int \frac{dp\mathbf{p}^2}{(\cosh(\frac{\varepsilon}{T}-\xi-g_1)+\cos(g_2))(\cosh(\frac{\varepsilon}{T}-\xi+g_1)+\cos(g_2))}. \quad (4.7)$$

The expression (4.6) contains two terms. The first one is proportional to w_μ and corresponds to usual expression for CVE [2,11,24]. At the same time, in comparison to the usual formula for CVE, (4.7) also includes "electric" term, proportional to acceleration 4-vector α_μ divided by temperature in the case of global thermodynamic equilibrium. C_1 starts with zero order in vorticity, while C_2 in the lowest order is proportional to $g_1 g_2 \sim \varpi:\tilde{\varpi}$ and includes the powers of vorticity starting from the third order. Thus, the electric term in (4.6) is two orders of magnitude smaller than the magnetic term.

B. Massless limit

Massless limit is of special interest, as it corresponds to chiral invariance manifestation, and also because in this case the integrals (4.7) can be calculated analytically and the expressions (4.6), (4.7) can be significantly simplified. For this purpose it is convenient to come back to the formulae (4.3). Using (4.5), (4.2), (4.3) and passing to the massless limit $\varepsilon = |\mathbf{p}|$, the integrals in (4.2) can be expressed through polylogarithmic functions $\mathrm{Li}_3(z)$

$$\begin{aligned}\langle :j^5_\mu: \rangle = &-\frac{T^3}{4\pi^2(g_1^2+g_2^2)}((g_1+ig_2)\mathrm{Li}_3(-e^{g_1-ig_2-\xi}) - (g_1-ig_2)\mathrm{Li}_3(-e^{-g_1-ig_2-\xi})\\ &+(g_1+ig_2)\mathrm{Li}_3(-e^{g_1-ig_2+\xi}) - (g_1-ig_2)\mathrm{Li}_3(-e^{-g_1-ig_2+\xi}) + \mathrm{c.c.})w_\mu\\ &+\frac{\mathrm{sgn}(\varpi:\tilde{\varpi})T^3}{4\pi^2(g_1^2+g_2^2)}((g_2+ig_1)\mathrm{Li}_3(-e^{-g_1-ig_2-\xi}) - (g_2-ig_1)\mathrm{Li}_3(-e^{g_1-ig_2-\xi})\\ &+(g_2+ig_1)\mathrm{Li}_3(-e^{-g_1-ig_2+\xi}) - (g_2-ig_1)\mathrm{Li}_3(-e^{g_1-ig_2+\xi}) + \mathrm{c.c.})\alpha_\mu. \end{aligned} \quad (4.8)$$

The polylogarithms $\mathrm{Li}_3(z)$ in (4.8) have the notable property [27], which eventually leads to compensation of higher order terms in the current

$$\mathrm{Li}_3(-e^{-x}) - \mathrm{Li}_3(-e^{x}) = \frac{\pi^2}{6}x + \frac{1}{6}x^3. \quad (4.9)$$

Using (4.9), the definitions of g_1 and g_2 (4.4) and formulas (2.10) one obtains:

$$\langle :j^5_\mu: \rangle = \left(\frac{T^2}{6}\left[1+\frac{\alpha^2-w^2}{4\pi^2}\right]+\frac{\mu^2}{2\pi^2}\right)Tw_\mu + \frac{T^3}{12\pi^2}(w\cdot\alpha)\alpha_\mu, \quad (4.10)$$

which can be further simplified in the limit of global thermodynamic equilibrium, using (2.6), (4.10):

$$\langle :j^5_\mu: \rangle = \left(\frac{1}{6}\left[T^2+\frac{a^2-\omega^2}{4\pi^2}\right]+\frac{\mu^2}{2\pi^2}\right)\omega_\mu + \frac{1}{12\pi^2}(\omega\cdot a)a_\mu. \quad (4.11)$$

Formula (4.11) contains three parts: thermal vortical current $\langle :j^5_\mu: \rangle_{\mathrm{Tvort}}$, depending on temperature, chemical potential and vorticity and corresponding to usual CVE, purely vortical term $\langle :j^5_\mu: \rangle_{\mathrm{vort}}$, which does not depend on μ and T, and purely acceleration term $\langle :j^5_\mu: \rangle_{\mathrm{acc}}$, expressed only through vorticity and acceleration

$$\langle :j^5_\mu: \rangle = \langle :j^5_\mu: \rangle_{\mathrm{Tvort}} + \langle :j^5_\mu: \rangle_{\mathrm{vort}} + \langle :j^5_\mu: \rangle_{\mathrm{acc}},$$
$$\langle :j^5_\mu: \rangle_{\mathrm{Tvort}} = \left(\frac{T^2}{6}+\frac{\mu^2}{2\pi^2}\right)\omega_\mu, \qquad \langle :j^5_\mu: \rangle_{\mathrm{vort}} = \frac{a^2-\omega^2}{24\pi^2}\omega_\mu,$$
$$\langle :j^5_\mu: \rangle_{\mathrm{acc}} = \frac{1}{12\pi^2}(\omega\cdot a)a_\mu. \quad (4.12)$$

In the limit $T,\mu \to 0$ the first term $\langle :j^5_\mu: \rangle_{\mathrm{Tvort}}$ vanishes, while the last two terms in (4.12) form nonzero contribution, which is of the third order in the vorticity and acceleration. Such a structure of a current might be related to the vacuum effects in accelerated frames [28].

Let us notice, that the introduction of complex vectors (2.11) allows one to diagonalize the expression (4.11):

$$\langle :j^5_\mu: \rangle = 2\pi \mathrm{Im}\left[\left(\frac{1}{6}(T^2+\varphi^2)+\frac{\mu^2}{2\pi^2}\right)\varphi_\mu\right]. \quad (4.13)$$

The divergence of (4.11) in the case of global thermodynamic equilibrium can be calculated in the straightforward way, using the equations, in particular (2.8), from [24]. For different components of axial current, introduced in (4.12), one obtains

$$\partial^\mu \langle :j^5_\mu: \rangle_{\rm Tvort} = 0, \qquad \partial^\mu \langle :j^5_\mu: \rangle_{\rm vort} = 0,$$
$$\partial^\mu \langle :j^5_\mu: \rangle_{\rm acc} = \frac{1}{6\pi^2}(\omega \cdot a)(a^2+\omega^2), \tag{4.14}$$

so that the divergence of full current is

$$\partial^\mu \langle :j^5_\mu: \rangle = \partial^\mu \langle :j^5_\mu: \rangle_{\rm acc} = \frac{1}{6\pi^2}(\omega \cdot a)(a^2+\omega^2). \tag{4.15}$$

The contribution to the right-hand side of (4.15) comes from the third order acceleration-dependent term $\langle :j^5_\mu: \rangle_{\rm acc}$. This contribution is purely topological being proportional to $(\omega \cdot a)$, it is of the fourth order over the vorticity and it vanishes in the limit $a \to 0$. Then, in fact, from (4.15) it follows, that axial current, calculated with the use of relativistic distribution functions (3.4), does not conserve, which was not expected for the free theory without external fields in global equilibrium case (while could appear in local equilibrium).

Let us notice, that the correctness of the equation (4.12) for low temperature and density, when $\langle :j^5_\mu: \rangle_{\rm vort}$ and $\langle :j^5_\mu: \rangle_{\rm acc}$ begin dominate, is under discussion, because (as it is pointed in [5]) boundary effects in the accelerated rotating system can be essential in this case and, strictly speaking, we cannot conclude that $\langle :j^5_\mu: \rangle \neq 0$ for $\mu = T = 0$.

Using current conservation in the absence of external fields in global equilibrium as a criterion, we could exclude the term $\frac{1}{12\pi^2}(\omega \cdot a)a_\mu$ from (4.11) (while additional contribution to vortical conductivity quadratic in the acceleration and vorticity remains), and the conserved current would be

$$\langle :j^5_\mu: \rangle_{\rm cons} = \left(\frac{1}{6}\left[T^2 + \frac{a^2-\omega^2}{4\pi^2}\right] + \frac{\mu^2}{2\pi^2}\right)\omega_\mu. \tag{4.16}$$

Equations (4.10), (4.11) were obtained as a result of the exact integration of the distribution functions in (4.2) beyond the limits of perturbation theory. As a result, we derived formulas (4.10), (4.11), which contain terms of only a few first orders. This interesting fact means that terms of higher orders compensate each other, beginning with some order. The origin of this fact can be seen from the formulas (4.9). For example, there are the terms of the form $-(g_1+ig_2)\mathrm{Li}_3(-e^{-g_1+ig_2+\xi})$ and $(g_1+ig_2)\mathrm{Li}_3(-e^{g_1-ig_2-\xi})$ in (4.8). Both of these two terms can be expanded into infinite Taylor series in the vorticity and chemical potential divided to temperature. But from (4.9) one can see, that these two terms compensate each other starting from the fourth order. So the correspondence of (4.10), (4.11) to the low order expansion over the vorticity and chemical potential is the result of compensation between different contributions from the traces in (4.8).

The corrections to CVE, contained in the expression (4.11), can be compared to the existing results. In particular, in [4,5] axial current of right handed massless fermions was calculated for the system with constant rotation speed on the axis of rotation. In the limit $a_\mu = 0$ (4.11) exactly corresponds to Eq. (27) from [4] and Eq. (83) in [5], where the term independent of T, μ was first identified. However, comparing with [4] we find the term along a_μ, which in [4] is equal to zero, because rotation speed is perpendicular to centripetal acceleration. Also, we find the additional quadratic contribution $\sim a^2$ in vortical conductivity.

V. CONCLUSION

In this paper we have investigated the consequences of the ansatz of relativistic distribution functions for particles with spin, introduced in [14]. Analytic formulas for axial current were obtained both for massive (4.6) and massless fermions (4.10), (4.11) in the approximation of weak interaction and small inhomogeneities. We reproduce the standard expression for CVE in the linear approximation in vorticity, using distribution functions [14], thereby confirming a similar calculation of [24]. The result obtained is in full accordance with all known theoretical calculations of the CVE.

We also hypothesize and apply the formalism beyond its currently formulated region of applicability to investigate higher order contributions to axial current. For massless fermions it is possible to simplify the expression for current due to compensation of higher order terms. It is shown, that axial current contains a new topological component along the acceleration vector α_μ and additional quadratic contribution to vortical term. We propose a simple method of diagonalizing an expression for an axial current by introducing complex combinations of acceleration and vorticity vectors, and the resulting expression reflects explicitly the symmetry between acceleration and rotation speed.

The divergence of axial current was calculated (4.15) and it remains nonzero even in global equilibrium.

Finally, as we show, that CVE follows from the kinetic approach, we may conclude that various approaches to baryon polarizations are closely related, and that found earlier numerical similarity between them [12] may be not an occasional one.

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APPENDIX: TRACE CALCULATION

The present goal is to obtain formula for the trace (4.3). For this purpose it is necessary to expand $X(x,p)$ to Taylor series, take the trace in each term and then sum the traces back. Function $\frac{1}{1+x}$ can be expanded to Taylor series

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^{t(n+l)}, \quad t, l \Rightarrow \begin{cases} t = 1, l = 0 & \text{if } |x| < 1, \\ t = -1, l = 1 & \text{if } |x| > 1. \end{cases} \tag{A1}$$

Then for $X(x, p)$ using (A1) we will have

$$X = \sum_{n=0}^{\infty} (-1)^n \exp\left[t(n+l)\left(\beta \cdot p - \xi - \frac{1}{2}\varpi : \Sigma\right)\right] = \sum_{n=0}^{\infty} (-1)^n \exp[t(n+l)(\beta \cdot p - \xi)] \times \sum_{m=0}^{\infty} \frac{1}{m!}\left(t(n+l)\left(-\frac{1}{2}\varpi : \Sigma\right)\right)^m. \tag{A2}$$

The product $\Sigma^{\alpha\beta}\Sigma^{\gamma\delta}$ can be decomposed to the basis of the space of 4×4 matrices

$$\Sigma^{\alpha\beta}\Sigma^{\gamma\delta} = \frac{1}{4}(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma})I + \frac{i}{4}\epsilon^{\alpha\beta\gamma\delta}\gamma^5 - \frac{i}{2}(g^{\beta\delta}\Sigma^{\alpha\gamma} + g^{\alpha\gamma}\Sigma^{\beta\delta} - g^{\alpha\delta}\Sigma^{\beta\gamma} - g^{\beta\gamma}\Sigma^{\alpha\delta}). \tag{A3}$$

From (A3) the product $(\varpi : \Sigma)^2$ can be defined. After extraction of chiral projective operators one obtains

$$(\varpi : \Sigma)^2 = \eta \frac{1+\gamma^5}{2} + \theta \frac{1-\gamma^5}{2}, \quad \eta = \frac{1}{2}\varpi : \varpi + \frac{i}{2}\varpi : \tilde{\varpi}, \quad \theta = \eta^* = \frac{1}{2}\varpi : \varpi - \frac{i}{2}\varpi : \tilde{\varpi}. \tag{A4}$$

Then the even power of $(\varpi : \Sigma)$ can be calculated

$$(\varpi : \Sigma)^{2k} = \eta^k \frac{1+\gamma^5}{2} + \theta^k \frac{1-\gamma^5}{2}, \quad k = 0, 1, 2.... \tag{A5}$$

From (A5) and (A3) one has

$$\mathrm{tr}((\varpi : \Sigma)^{2k+1}\Sigma^{\nu\beta}) = (\varpi^{\nu\beta} + i\tilde{\varpi}^{\nu\beta})\eta^k + (\varpi^{\nu\beta} - i\tilde{\varpi}^{\nu\beta})\theta^k, \quad \mathrm{tr}((\varpi : \Sigma)^{2k}\Sigma^{\nu\beta}) = 0, \quad k = 0, 1, 2.... \tag{A6}$$

The traces in (4.2) now can be defined using decomposition (A2) and (A6)

$$\mathrm{tr}(X\Sigma^{\nu\beta}) = \sum_{n=0}^{\infty} (-1)^n \exp[t(n+l)(\beta \cdot p - \xi)] \times \sum_{k=0}^{\infty} \frac{1}{(2k+1)!}\left[\left(-\frac{1}{2}\right)t(n+l)\right]^{2k+1} \times \{(\varpi^{\nu\beta} + i\tilde{\varpi}^{\nu\beta})\eta^k + (\varpi^{\nu\beta} - i\tilde{\varpi}^{\nu\beta})\theta^k\}. \tag{A7}$$

Square roots of η and θ are

$$\sqrt{\eta} = 2g_1 + 2i\,\mathrm{sgn}(\varpi : \tilde{\varpi})g_2, \quad \sqrt{\theta} = 2g_1 - 2i\,\mathrm{sgn}(\varpi : \tilde{\varpi})g_2.$$

Here g_1 and g_2 are given by (4.4). Now, the series over k in (A7) can be summed to sine functions

$$\mathrm{tr}(X\Sigma^{\nu\beta}) = \sum_{n=0}^{\infty} (-1)^n \exp[t(n+l)(\beta \cdot p - \xi)] \times i\left\{\frac{1}{\sqrt{\eta}}\sin\left(\frac{i}{2}t(n+l)\sqrt{\eta}\right)[\varpi^{\nu\beta} + i\tilde{\varpi}^{\nu\beta}] + \frac{1}{\sqrt{\theta}}\sin\left(\frac{i}{2}t(n+l)\sqrt{\theta}\right)[\varpi^{\nu\beta} - i\tilde{\varpi}^{\nu\beta}]\right\}. \tag{A8}$$

Sines in (A8) can be decomposed to exponents and after that the series over n can be summed back using (A1). Also we notice, that $\mathrm{sgn}(\varpi : \tilde{\varpi})$ can be extracted as a factor of $\tilde{\varpi}$. After that (A8) transforms to (4.3).

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