Remarks on the Z' Drell-Yan cross section

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(Received 4 December 2017; published 20 April 2018)

Many extensions of the standard model contain an extra $U(1)'$ gauge group with a heavy Z' gauge boson. Perhaps the most clear signal for such a Z' would be a resonance in the invariant mass spectrum of the lepton pairs to which it decays. In the absence of such a signal, experiments can set limits on the couplings of such a Z', using a standard formula from theory. We repeat its derivation and find that, unfortunately, the standard formula in the literature is a factor of 8 too small. We briefly explore the implication for existing experimental searches and encourage the high-energy physics community to reexamine analyses that have used this formula.

DOI: 10.1103/PhysRevD.97.075025

I. INTRODUCTION

Many models of physics beyond the standard model (BSM) include an extra $U(1)'$ gauge group with a heavy Z' gauge boson; see for example the review [1]. Perhaps the most clear signal for such a Z' would be a resonance in the invariant mass spectrum of the lepton pairs to which it decays. So far, no such Z' has been observed. See, for example, the recent ATLAS [2] and CMS [3] searches. Such experimental constraints on a Z' can be very useful in BSM model building, *provided* that they are presented in an appropriate form. In practice, experiments usually report their data in three ways.

The first is reporting limits on the Z'/Z cross-section ratio to reduce experimental uncertainties as is done, e.g., in [3,4]. These are not so easy to use, since they require the knowledge of the standard model (SM) Z cross section at NNLO to convert to limits on Z'. Furthermore, results are currently presented only as figures without the detailed numerical values.

The second is to use some "benchmark" models such as the sequential standard model and E_6 models; see [1] for a review. Having fixed the coupling of the Z', experiments can report limits on the mass of these specific Z' gauge bosons, e.g. [2]. These are useful in tracking the progress of the experimental constraints over time, but are not easily translatable to other Z' models.

The most useful approach was presented in [5]. For a given Z' mass and center of momentum energy, the cross section $\sigma(h_1+h_2\to Z'+X\to \ell^+\ell^-+X)$ is expressed as a sum over products of two quantities. One, ω_i, depends on SM parton distribution functions (PDFs) and is independent of the Z' model, and the other, c_i, depends on the couplings of the Z' and is model dependent. See the next section for exact definitions. Experiments obtain constraints on the cross section times the branching ratio, and they can translate them to constraints on the parameters c_i. See, for example, a recent exclusion plot in [4]. The formula given in [5] is

$$\sigma(h_1+h_2\to Z'+X\to \ell^+\ell^-+X) = \frac{\pi}{48s}[c_u w_u(s,M_{Z'}^2)+c_d w_d(s,M_{Z'}^2)]. \qquad (1)$$

We have rederived this formula, and we find that it is too small by a factor of 8; i.e., the denominator should be $6s$. The purpose of this paper is to briefly explore the implications on the Z' experimental limits and bring the issue to the attention of the high-energy physics community.

The rest of the paper is structured as follows. In Sec. II, we rederive the expression for the cross section and compare it to the expressions in Refs. [5,6]. In Sec. III, we explore the implications for the existing experimental constraints. We present our conclusions in Sec. IV.

II. THEORETICAL EXPRESSIONS

A. Z' Drell-Yan cross section at leading order

Consider a Z' flavor-diagonal coupling of the form [1]

$$\mathcal{L} = -g_{z'}Z'_\mu\sum_i \bar{f}_i\gamma^\mu[\epsilon_L^i P_L+\epsilon_R^i P_R]f_i = -\frac{g_{z'}}{2}Z'_\mu\sum_i \bar{f}_i\gamma^\mu[g_V^i-g_A^i\gamma^5]f_i, \qquad (2)$$

where $P_{L,R} = (1 \mp \gamma^5)/2$ and $g^i_{V,A} = \epsilon^i_L \pm \epsilon^i_R$. Averaging over the colors of the initial quarks, the cross section for the process $q\bar{q} \to f_i \bar{f}_i$ is [7]

$$\sigma(q\bar{q} \to Z' \to f_i \bar{f}_i) = \frac{g^4_{z'}}{(Q^2 - M^2_{Z'})^2 + \Gamma^2_{Z'} M^2_{Z'}} \frac{1}{N_c} \frac{1}{3\pi} \frac{Q^2}{64} [(g^q_V)^2 + (g^q_A)^2][(g^f_V)^2 + (g^f_A)^2], \tag{3}$$

where Q^2 is the invariant mass of the $f_i - \bar{f}_i$ pair, $M_{Z'}$ is the Z' mass, $\Gamma_{Z'}$ is the Z' width, and $N_c = 3$.

At leading order in α_s, the Drell-Yan cross section is given by [8]

$$\sigma(h_1 + h_2 \to Z' + X \to f_i \bar{f}_i + X) = \sum_q \int_0^1 dx_1 \int_0^1 dx_2 [f_{q/h_1}(x_1) f_{\bar{q}/h_2}(x_2) + (x_1 \leftrightarrow x_2, h_1 \leftrightarrow h_2)] \sigma_{q\bar{q} \to Z' \to f_i \bar{f}_i}. \tag{4}$$

To find an expression for $d\sigma(h_1 + h_2 \to Z' + X)/dQ^2$, we insert unity as $1 = \int dQ^2 \delta(Q^2 - x_1 x_2 s)$ and take a derivative with respect to Q^2. Here, $s = (p_1 + p_2)^2$, where p_i is the four-momentum of h_i. We get

$$\begin{aligned} \frac{d\sigma(h_1 + h_2 \to Z' + X \to f_i \bar{f}_i + X)}{dQ^2} &= \sum_q \int_0^1 dx_1 \int_0^1 dx_2 [f_{q/h_1}(x_1) f_{\bar{q}/h_2}(x_2) + (x_1 \leftrightarrow x_2, h_1 \leftrightarrow h_2)] \sigma_{q\bar{q} \to Z' \to f_i \bar{f}_i} \delta(Q^2 - x_1 x_2 s) \\ &= \sum_q \int_0^1 dx_1 \int_0^1 dx_2 [f_{q/h_1}(x_1) f_{\bar{q}/h_2}(x_2) + (x_1 \leftrightarrow x_2, h_1 \leftrightarrow h_2)] \delta\left(1 - \frac{x_1 x_2 s}{Q^2}\right) \\ &\quad \times \frac{g^4_{z'}}{(Q^2 - M^2_{Z'})^2 + \Gamma^2_{Z'} M^2_{Z'}} \frac{1}{N_c} \frac{1}{3\pi} \frac{1}{64} [(g^q_V)^2 + (g^q_A)^2][(g^f_V)^2 + (g^f_A)^2]. \end{aligned} \tag{5}$$

As a first check, we note that for a photon, $M_{Z'} = 0$, $\Gamma_{Z'} = 0$, $g_{z'} = e$, $g^i_A = 0$, and $g^i_V = 2e_i$, where e_i is the fermion electric charge in units of the positron charge. For an $l^+ l^-$ final state, Eq. (5) becomes

$$\frac{d\sigma(h_1 + h_2 \to l^+ l^- + X)}{dQ^2} = \frac{4\pi\alpha^2}{9Q^4} \sum_q e^2_q \int_0^1 dx_1 \int_0^1 dx_2 [f_{q/h_1}(x_1) f_{\bar{q}/h_2}(x_2) + (x_1 \leftrightarrow x_2, h_1 \leftrightarrow h_2)] \delta\left(1 - \frac{x_1 x_2 s}{Q^2}\right), \tag{6}$$

which is a well-known result [9].

As a second check, we consider the case of the SM Z. We have [8] $g^i_V = t^i_{3L} - 2e_i \sin^2\theta_W$, $g^i_A = t^i_{3L}$, and $g_{z'} = e/(\sin\theta_W \cos\theta_W)$, where t^i_{3L} is the weak isospin of fermion i ($+1/2$ for up-type quark and neutrino, $-1/2$ for down-type quark and a charged lepton), and θ_W is the weak angle. For Z decay to $\ell^+ \ell^-$ Eq. (5) becomes

$$\begin{aligned} \frac{d\sigma(h_1 + h_2 \to Z + X \to f_i \bar{f}_i + X)}{dQ^2} &= \sum_q \int_0^1 dx_1 \int_0^1 dx_2 [f_{q/h_1}(x_1) f_{\bar{q}/h_2}(x_2) + (x_1 \leftrightarrow x_2, h_1 \leftrightarrow h_2)] \delta\left(1 - \frac{x_1 x_2 s}{Q^2}\right) \\ &\quad \times \frac{\pi\alpha^2}{\sin^4\theta_W \cos^4\theta_W} \frac{1}{(Q^2 - M^2_Z)^2 + \Gamma^2_Z M^2_Z} \frac{1}{3N_c} \frac{1}{64} [4(t^q_{3L} - 2e_q \sin^2\theta_W)^2 + 4(t^q_{3L})^2][1 + (1 - 4\sin^2\theta_W)^2]. \end{aligned} \tag{7}$$

We will show in Sec. II B that this result agrees with [6].

Having performed the checks for a photon and the SM Z, we return to the general Z' case. In the narrow width approximation, the expression for the cross section, Eq. (5), can be simplified by using

$$\frac{1}{(Q^2 - M^2_{Z'})^2 + \Gamma^2_{Z'} M^2_{Z'}} \to \frac{\pi}{\Gamma_{Z'} M_{Z'}} \delta(Q^2 - M^2_{Z'}), \tag{8}$$

and [9]

$$\mathrm{BR}(Z' \to \ell^+\ell^-) = \frac{\Gamma_{Z'}(Z' \to \ell^+\ell^-)}{\Gamma_{Z'}} = \frac{g_{Z'}^2[(g_V^\ell)^2 + (g_A^\ell)^2]M_{Z'}}{48\pi\Gamma_{Z'}}. \quad (9)$$

Setting $N_c = 3$, we obtain

$$\sigma(h_1 + h_2 \to Z' + X \to \ell^+\ell^- + X) = \frac{\pi}{6s}[c_u w_u(s, M_{Z'}^2) + c_d w_d(s, M_{Z'}^2)], \quad (10)$$

where

$$c_{u,d} = \frac{1}{2} g_{z'}^2[(g_V^{u,d})^2 + (g_A^{u,d})^2]\mathrm{BR}(Z' \to \ell^+\ell^-), \quad (11)$$

and at leading order in α_s,

$$w_q(s, M_{Z'}^2) = \int_0^1 dx_1 \int_0^1 dx_2[f_{q/h_1}(x_1)f_{\bar{q}/h_2}(x_2) + (x_1 \leftrightarrow x_2, h_1 \leftrightarrow h_2)]\delta\left(\frac{M_{Z'}^2}{s} - x_1x_2\right). \quad (12)$$

Equation (10) is the main result of this paper. Notice the factor of 6 in the denominator compared to Eq. (1).

B. Comparison to Ref. [6]

To check our calculation, we compare it to [6]. In that paper the α_s^2 correction to the Drell-Yan K factor were calculated. We will only need the α_s^0 terms. Equation (2.22) of [6] for a vector boson-quark coupling is

$$Vq_i\bar{q}_j\colon\ ig_V\gamma_\mu(v_i^V + a_i^V\gamma_5). \quad (13)$$

The first two lines of Eq. (A.13) of [6] are

$$v_u^\gamma = \frac{2}{3}, \qquad a_u^\gamma = 0$$
$$v_d^\gamma = -\frac{1}{3}, \qquad a_d^\gamma = 0. \quad (14)$$

Comparing to Eq. (2), we get that for a photon coupling $g_\gamma = -e$.

The last two lines of Eq. (A.13) of [6] are

$$v_u^Z = 1 - \frac{8}{3}\sin^2\theta_W, \qquad a_u^Z = -1$$
$$v_d^Z = -1 + \frac{4}{3}\sin^2\theta_W, \qquad a_d^Z = 1. \quad (15)$$

Comparing to Eq. (2), we get that for a Z coupling,

$$g_{z'} = -\frac{e}{\boldsymbol{4}\sin\theta_W\cos\theta_W}. \quad (16)$$

Notice the factor of 4 compared to the standard expression [8]. It implies that one should be careful in adapting the results of [6] to the case of a Z'. Equation (2.2) of [6] is

$$\frac{d\sigma_V}{dQ^2} = \tau\sigma_V(Q^2, M_V^2)W_V(\tau, Q^2), \qquad \tau = \frac{Q^2}{s}. \quad (17)$$

The $\mathcal{O}(\alpha_s^0)$ expression for $W_{\gamma,Z}(\tau, Q^2)$ from Eq. (A.20) of [6] is

$$W_{\gamma,Z}(\tau, Q^2) = \int_0^1 dx_1 \int_0^1 dx_2\delta(\tau - x_1x_2) \times \sum_{i,j\in Q,\bar{Q}} \delta_{ij}(v_i^2 + a_i^2)q_i(x_1)\bar{q}_j(x_2), \quad (18)$$

where q_i ($\bar{q}_j$) is the quark (antiquark) PDFs.

Reference [6] calls σ_V the "pointlike cross section," although it is the pointlike $d\sigma_V/dQ^2$. Equation (A.1) of [6] is

$$\sigma_\gamma(Q^2) = \frac{4\pi\alpha^2}{3Q^4}\frac{1}{N_c}. \quad (19)$$

Combining it with (18) in (17), we get our Eq. (6).

Combining Eqs. (A.2) and (A.5) of [6], we have

$$\sigma_Z(Q^2, M_Z^2) = \frac{\pi\alpha}{4M_Z\sin^2\theta_W\cos^2\theta_W}\frac{1}{N_c}\frac{1}{(Q^2 - M_{Z'}^2)^2 + \Gamma_{Z'}^2M_{Z'}^2} \times \frac{\alpha M_z[1 + (1 - 4\sin^2\theta_W)^2]}{48\sin^2\theta_W\cos^2\theta_W}. \quad (20)$$

Combining this with (18) in (17), we get our Eq. (7).

In summary we find agreement between our expressions and [6] for the $\mathcal{O}(\alpha_s^0)$ expression for γ and Z Drell-Yan processes.

C. Comparison to Ref. [5]

We now compare our results to Ref. [5], which first presented (1). Equation (2.8) of [5] is

$$\sum_f z_f g_z Z'_\mu \bar{f}\gamma^\mu f, \quad (21)$$

"…where $f = e_R^j,\ l_L^j,\ u_R^j,\ d_R^j,\ q_L^j$ are the usual lepton and quark fields in the weak eigenstate basis; $l_L^j = (\nu_L^j, e_L^j)$ and $q_L^j = (u_L^j, d_L^j)$ are the $SU(2)_W$ doublet fermions. The index j labels the three fermion generations. Altogether there are 15 fermion charges, z_f [5]."

Equation (3.1) of [5] is

$$\frac{d\sigma(p\bar{p}\to Z'+X\to l^+l^-+X)}{dQ^2} = \frac{1}{s}\sigma(Z'\to l^+l^-)W_{Z'}(s,Q^2), \tag{22}$$

where we ignore the interference of the Z' with the Z and the photon. According to Eq. (3.2) of [5],

$$\sigma(Z'\to l^+l^-) = \frac{g_z^2}{4\pi}\left(\frac{z_{l_j}^2+z_{e_j}^2}{288}\right)\frac{Q^2}{(Q^2-M_{Z'}^2)^2+\Gamma_{Z'}^2M_{Z'}^2}. \tag{23}$$

At $\mathcal{O}(\alpha_s^0)$ and for generation-independent Z' coupling,

$$W_{Z'}(s,Q^2) = g_z^2[(z_q^2+z_u^2)w_u(s,Q^2)+(z_q^2+z_d^2)w_u(s,Q^2)], \tag{24}$$

where

$$w_{u(d)} = \sum_{q=u,c,(d,s,b)}\int_0^1 dx_1\int_0^1 dx_2[f_{q/P}(x_1)f_{\bar{q}/\bar{P}}(x_2) + (x_1\leftrightarrow x_2, P\leftrightarrow\bar{P})]\delta\left(\frac{Q^2}{s}-x_1x_2\right), \tag{25}$$

and we ignore the scale dependence of the PDFs at $\mathcal{O}(\alpha_s^0)$.

Comparing (21) to (2), we have $g_z = -g_{z'}$, and $z_{l_j} = z_{e_L^j} = (g_V^{e_j}+g_A^{e_j})/2$, $z_{e_j} = z_{e_R^j} = (g_V^{e_j}-g_A^{e_j})/2$, $z_{u_L^j} = z_q = (g_V^u+g_A^u)/2$, $z_{d_L^j} = z_q = (g_V^d+g_A^d)/2$, $z_{u,d} = (g_V^{u,d}-g_A^{u,d})/2$. As a result, see Eq. (2),

$$\begin{aligned}
z_{l_j}^2+z_{e_j}^2 &= \frac{(g_V^{e_j}+g_A^{e_j})^2}{4}+\frac{(g_V^{e_j}-g_A^{e_j})^2}{4} \\
&= \frac{(g_V^{e_j})^2+(g_A^{e_j})^2}{2} = (\epsilon_L^{e_j})^2+(\epsilon_R^{e_j})^2 \\
z_q^2+z_u^2 &= \frac{(g_V^u+g_A^u)^2}{4}+\frac{(g_V^u-g_A^u)^2}{4} \\
&= \frac{(g_V^u)^2+(g_A^u)^2}{2} = (\epsilon_L^u)^2+(\epsilon_R^u)^2 \\
z_q^2+z_d^2 &= \frac{(g_V^d+g_A^d)^2}{4}+\frac{(g_V^d-g_A^u)^2}{4} \\
&= \frac{(g_V^d)^2+(g_A^d)^2}{2} = (\epsilon_L^d)^2+(\epsilon_R^d)^2.
\end{aligned} \tag{26}$$

Altogether, we find

$$\frac{d\sigma(p\bar{p}\to Z'+X\to l^+l^-+X)}{dQ^2} = \sum_{q=u,d}\int_0^1 dx_1\int_0^1 dx_2[f_{q/P}(x_1)f_{\bar{q}/\bar{P}}(x_2)+(x_1\leftrightarrow x_2, P\leftrightarrow\bar{P})] \times\delta\left(1-\frac{x_1x_2s}{Q^2}\right)\times\frac{g_z^4}{(Q^2-M_{Z'}^2)^2+\Gamma_{Z'}^2M_{Z'}^2}\frac{1}{N_c}\frac{1}{3\pi}\frac{1}{\mathbf{512}}[(g_V^q)^2+(g_A^q)^2][(g_V^e)^2+(g_A^e)^2]. \tag{27}$$

In other words, the result of [5] is 8 times too small compared to our Eq. (7).

Using the narrow width approximation, [5] obtained

$$\sigma(h_1+h_2\to Z'+X\to\ell^+\ell^-+X) = \frac{\pi}{48s}[c_uw_u(s,M_{Z'}^2)+c_dw_d(s,M_{Z'}^2)]. \tag{28}$$

Again this expression is 8 times too small compared to our Eq. (10). The wrong Eq. (1) also appears in the "Z'-boson searches" review by the PDG [8], and in [10], often cited together with [5] by ATLAS and CMS papers on Z' searches.

Interestingly, there are two papers, [11,12], that have the correct numerical factor but do not note the discrepancy. In general, the expression for the cross section is meaningful only if one also defines $c_{u,d}$ in terms of the Z' charges and $w_{u,d}$ in terms of the PDFs. Furthermore, one has to define the Z' charges by presenting the Lagrangian since, unlike the PDFs, there is more than one convention for them in the literature. These three conditions were met in [11]. Equation (6.5) of the published version of that paper has the right numerical factor, but the authors state in a footnote, "We note that the analysis of Ref. [5] absorbs a factor of 8 in their PDFs contained within the function, defined as $W_{Z'}$" [11]. We checked the definition of $w_{u,d}$ in [11], where it is called $\mathcal{W}_{\{AB(q\bar{q})\}}$, and it is the same[1] as the $\mathcal{O}(\alpha_s^0)$ expression for $w_{u,d}$ in [5]. Only two of the conditions were met in [12] that define $c_{u,d}$ and the Z' charges, but not

[1]Daniel Feldman informed us [13] that they meant to imply that, numerically, Ref. [5] appeared to include the factor of 8, e.g. in the figures of [5], but it was missing in the analytical expression of [5].

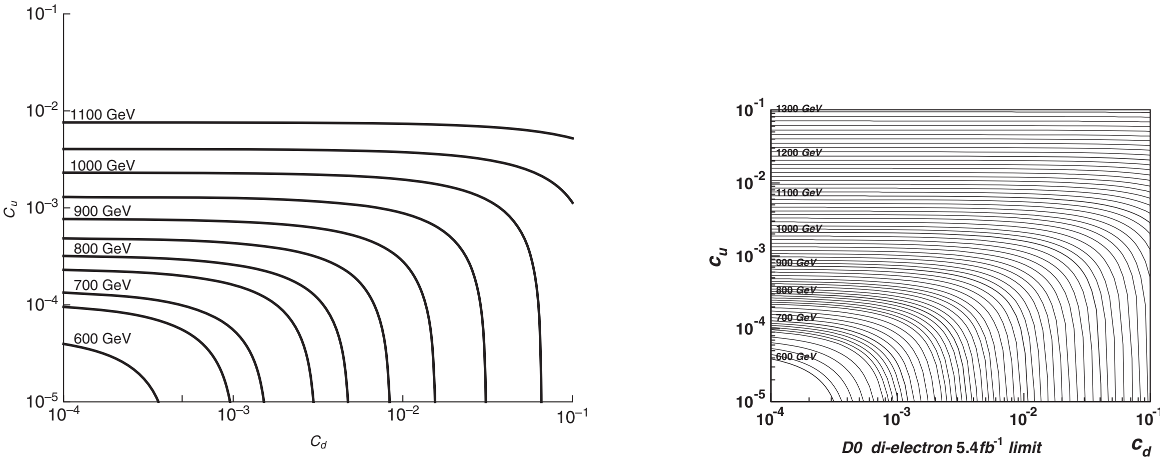

FIG. 1. Left-hand side: Our $c_u - c_d$ exclusion plot based on the D0 data [20] and Eq. (10). Right-hand side: Reference [10] $c_u - c_d$ exclusion plot based on the D0 data [20] and Eq. (1).

$w_{u,d}$ for which they only say, "Here, $w(s, p^2)$ are model-independent functions that depend on the collision center-of-mass energy s and the dilepton invariant mass…" [12]. If we assume that they are the same[2] as in [5], the numerical factor in their Eq. (3.8) is correct. The authors of [12] do not comment on the discrepancy between their Eq. (3.8) and Eq. (3.8) of [5].

Similarly, there are expressions in the literature in which the correct differential Z' cross section is given, but not in the exact form of our Eq. (10); see [15,16]. One can, in principle, derive our Eq. (10) from them. In practice, the wrong equation is the one that was used in [5,8,10].

D. Summary

Perhaps because of the nonstandard definition of the Z boson coupling in (16), the results of [6] seem to have been transcribed incorrectly to the case of a Z' in [5]. In any case, the quoted result for the cross section is a factor 8 too small. How has that affected Z' searches?

III. EFFECT ON EXPERIMENTAL SEARCHES

A. Implementation in PYTHIA

Experimental searches for Z' resonances use PYTHIA, so the implementation of Z' models in Pythia is important. To the best of our knowledge, the first PYTHIA documentation to discuss Z' is [17]. It predates [5], and we assume that it uses the SM Z calculations. See also [18] for a detailed discussion of Z' model implementation in PYTHIA.

B. Z' constraints in Ref. [5]

Reference [5] has used preliminary unpublished data from the CDF and D0 experiments to present the first $c_u - c_d$ exclusion plot. See Refs. [26] and [27] in Ref. [5]. In particular, they presented an exclusion plot using the CDF data presented at SUSY 2004. This talk is not available on the conference web site [19]. The proceedings contribution that is available on the web site does not include the data [19]. Therefore, we cannot check the plot presented in [5].

C. Z' constraints in Ref. [10]

In 2010, Ref. [10] presented a detailed study of the prospects for setting limits on Z' using early LHC data [10]. In particular, they have used D0 data published in [20] to present a $c_u - c_d$ exclusion plot in Fig. 7 of [10].

As in [5], they list the wrong expression for the cross section in Eq. (3.7) of the journal version of [10] (Eq. (II.8) in the arXiv.org version). On the other hand, they say in section III.B of the journal version of [10] (Sec. II.C in the arXiv.org version): "In the following, we take into account QCD NNLO effects as implemented in the WZPROD program [54–56]… We have adopted this package for simulating the Z' production, and have linked it to an updated set of parton density functions (PDF's)." The references they cite ([54–56] above) include [6]. If adapted correctly, they might have the correct expression for the cross section.

In order to decide which is the case, we created our own $c_u - c_d$ exclusion plot based on the D0 data published in [20]. We use the "LO" MSTW2008 PDFs [21] obtained from [22]. To match [10] as closely as possible, we include their K_{NNLO} factors in $\sigma^{\mathrm{NNLO}}_{\ell^+\ell^-} \simeq K_{\mathrm{NNLO}} \sigma^{\mathrm{LO}}_{\ell^+\ell^-}$ to set the limits. The values of K_{NNLO} are listed in the appendix of [10]. We use Table III corresponding to MSTW 2008.

Our results are presented in Fig. 1. On the left-hand side of Fig. 1, we show the $c_u - c_d$ contours corresponding to Z' masses of 600–1100 GeV in 50 GeV increments, as reported by D0. On the right-hand side, we show the corresponding plot from Ref. [10], taken from the

[2]Manuel Perez-Victoria informed us [14] that $w_{u,d}$ in [12] are the same as in [5].

right-hand side of Fig. 7 in [10]. Notice that [10] has also extrapolated the D0 data to higher masses. The original D0 data only report exclusions up to 1100 GeV. Although not stated explicitly, [10] has presumably interpolated D0 data to generate a much denser plot.

Despite these, it is easy to see that the two exclusion plots agree in the 600–1100 GeV range. This implies that despite claiming to have used Eq. (1), Ref. [10] got the correct exclusion plot, presumably by using [6]. We have checked that using Eq. (1) and/or omitting K_{NNLO} gives a plot that is different from the one presented in [10].

D. LHC Z' constraints

To the best of our knowledge, only CMS has produced $c_u - c_d$ exclusion plots for LHC data. Such plots were given for 7 TeV data in [23,24] and 8 TeV data in [4] LHC data. Unlike the D0 data [20] that included limits on the cross section, CMS has only produced exclusion plots. As a result, we cannot check whether they are using the correct expression for the cross section. Since these CMS analyses [4,23,24] cite [5,10] as the theoretical basis, CMS should check the effect of the wrong factor in (1) on their limits.

IV. CONCLUSIONS

A heavy $U(1)'$ gauge boson, commonly referred to as a Z', is predicted in many BSM models. If it decays to leptons, such a Z' can appear very clearly in the di-lepton invariant mass spectrum. So far, experimental searches have not found such a signal and only limits on the cross section were set. If such experimental data are properly presented, it can be very useful in constraining and improving Z' BSM models.

The most useful way to present such data was suggested in [5], where the cross section is "factorized" into a sum over products of Z' model-dependent c_i and Z' model-independent w_i that depend on the PDFs. Since the latter can be calculated, the cross section can be translated into constraints on the model-dependent c_i without the need to resort to specific models. This method was used to analyze Tevatron data in [5,10] and LHC data in [4,23,24].

We have repeated the derivation of [5] in Sec. II, and we find that the expression given there is 8 times too small. Thus, instead of Eq. (1) used by [5], one should use Eq. (10) presented here. We confirmed our calculations by comparing them to the known cases of the photon [9] and the SM Z [6] Drell-Yan. It should be noted that [6] uses an unusual normalization for the SM Z coupling, and one should be careful in generalizing [6] to the case of a Z'.

Taken at face value, our result might imply that the experimental constraints that use the wrong formula are off by a factor of 8. To test that, we have tried to check the existing exclusion plots in Sec. III. The original paper [5] has used CDF data that was only shown at a conference [19] and is not available online or in the proceedings. As a result, we cannot check the exclusion plot in [5]. A later paper, [10], has claimed to use Eq. (1) and D0 data [20] to produce an exclusion plot. In this case, we can redo their analysis. As Fig. 1 shows, the exclusion plot of [10] agrees with our result. This implies that they have not actually used (1) in their analysis. Finally, for the case of LHC data, exclusion plots were produced by CMS in [4,23,24]. These do not contain enough information that will allow us to reproduce them. Therefore, we cannot determine whether they are correct.

In summary, considering the important implications, we encourage the high-energy physics community to re-examine past analyses that have used the wrong expression for the cross section.

ACKNOWLEDGMENTS

This work was supported by Department of Energy Grant No. DE-SC0007983.

Note added.—After this paper was completed, Alexander Belyaev informed us [25] that both [10] and the CMS analyses [4,23,24] may list the wrong expression (1), but not actually use it. The coefficients in front of c_u and c_d in the cross section were found numerically for all of these papers. This confirms our observation in Sec. IIIC and implies that the exclusion plots in [4,23,24] are valid.

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