Flavor physics without flavor symmetries

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We quantitatively analyze a quark-lepton flavor model derived from a six-dimensional supersymmetric theory with $SO(10) \times U(1)$ gauge symmetry, compactified on an orbifold with magnetic flux. Two bulk **16**-plets charged under the $U(1)$ provide the three quark-lepton generations whereas two uncharged **10**-plets yield two Higgs doublets. At the orbifold fixed points mass matrices are generated with rank one or two. Moreover, the zero modes mix with heavy vectorlike split multiplets. The model possesses no flavor symmetries. Nevertheless, there exist a number of relations between Yukawa couplings, remnants of the underlying grand unified theory symmetry and the wave function profiles of the zero modes, which lead to a prediction of the light neutrino mass scale, $m_{\nu_1} \sim 10^{-3}$ eV and heavy Majorana neutrino masses in the range from 10^{12} to 10^{14} GeV. The model successfully includes thermal leptogenesis.

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I. INTRODUCTION

The Standard Model (SM) of particle physics is a chiral gauge theory with three copies of a quark-lepton generation containing a quark doublet $q = (u, d)$, a lepton doublet $l = (\nu, e)$ and four singlets, u^c, d^c, e^c and n^c, of Weyl fermions in different representations of the gauge group $G_{\rm SM} = SU(3) \times SU(2) \times U(1)$. This gauge theory has a $U(3)^6$ flavor symmetry which is almost completely broken by 36 complex Yukawa couplings and six complex Majorana mass terms. Only a $\mathbb{Z}_2$ matter parity and the global $U(1)$ of baryon number survive, which is broken by an anomaly. Most of the 84 real parameters are unphysical and can be eliminated by a redefinition of the quark and lepton fields, leaving 25 observables: six quark masses, three charged lepton masses, six Majorana neutrino masses, six mixing angles in the charged current and four CP-violating phases. The traditional goal of flavor physics is to reduce the number of independent input parameters by means of symmetries in order to obtain relations among the various observables. These relations would then shed light on the origin of the Yukawa couplings.

Relations between quark and lepton Yukawa matrices are obtained in grand unified theories (GUTs) where the Standard Model gauge group is embedded in the non-Abelian gauge groups $SU(4) \times SU(2) \times SU(2)$ [1], $SU(5)$ [2], $SO(10)$ [3,4] or flipped $SU(5)$ [5,6]. For example, in $SU(5)$ GUTs the 36 SM Yukawa couplings are reduced to 24 couplings and in $SO(10)$ GUTs with two Higgs **10**-plets only 12 independent couplings are left. However, the obtained relations between Yukawa couplings are only partially successful and in order to account for all measured observables one needs higher-dimensional Higgs representations and/or higher-dimensional operators [see for example Refs. [7–22] for quantitative analyses of the fermion mass spectrum in some $SO(10)$ models].

A partial understanding of the hierarchies among quark and lepton Yukawa couplings can be obtained by means of $U(1)$ flavor symmetries [23] or discrete symmetries [24,25]. Such flavor symmetries have also been derived in string compactifications [26–30]. They are of particular importance in supersymmetric compactifications where they can forbid operators leading to proton decay. Note, however, that none of these flavor symmetries are exact. They are all spontaneously or explicitly broken.

Hierarchical Yukawa couplings can also be obtained in toroidal compactifications of super Yang-Mills theories with magnetic flux in ten or fewer dimensions. The couplings between bulk Higgs and matter fields are calculated as overlap integrals of wave functions that have nontrivial profiles in the magnetized extra dimensions [31]. In a similar way, Yukawa couplings of magnetized toroidal orbifolds have been analyzed [32–37]. The resulting flavor structure depends on the number of pairs of Higgs doublets. In the simplest cases it appears difficult to obtain the measured hierarchies of quark and lepton masses [34,35].

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In this paper we pursue an alternative avenue. Our starting point is the six-dimensional (6D) orbifold GUT model with gauge group $SO(10) \times U(1)$ considered in Ref. [38]. The GUT group $SO(10)$ is broken to different subgroups at the orbifold fixed points where also the Yukawa couplings are generated [39,40]. Abelian magnetic flux generates three quark-lepton families from two bulk **16**-plets, ψ and χ, and, together with two uncharged $\mathbf{16}^*$-plets vectorlike split multiplets. Moreover, the magnetic flux breaks supersymmetry [41]. Two uncharged bulk 10-plets yield two Higgs doublets. The 6D theory has no flavor symmetry. All quarks and leptons arise as zero modes of bulk **16**-plets. But since their wave functions are different, they couple with different strengths to the Higgs fields at the fixed points. As a consequence, also the effective four-dimensional (4D) theory has no flavor symmetries. Nevertheless, the GUT symmetry and the flux compactification lead to a number of relations between the Yukawa matrices. The 36 SM complex Yukawa couplings are reduced to 12 complex couplings. In addition there are nonrenormalizable terms generating the heavy Majorana neutrino masses and mass mixing terms between the chiral quark-lepton generations and the vectorlike multiplets. In the following we shall study to what extent such a structure can quantitatively describe the measured observables, extending the previous work on two quark-lepton generations [42].

The paper is organized as follows. In Sec. II we describe symmetry breaking and zero modes of the model under consideration. Moreover, we list the values of the zero mode wave functions at the various fixed points and work out the Yukawa couplings which determine the flavor spectrum. Section III is devoted to numerical fits of the model to measured observables. In a first fit, light and heavy neutrino masses and the baryon asymmetry are predicted, whereas in a second fit the observed baryon asymmetry is also fitted. A summary and conclusions are given in Sec. IV. Some technical features of numerical fits and results are described in Appendices A and B, respectively.

II. GUT MODEL AND YUKAWA COUPLINGS

In this section we describe the six-dimensional $SO(10)$ GUT model introduced in Ref. [38], extended by a pair of bulk **16**-plets. This allows to account for the flavor structure of three quark-lepton generations, with some predictions for neutrino masses. Two additional **10**-plets, needed to cancel the 6D $SO(10)$ gauge anomalies, do not mix with quarks and leptons and will not be discussed in the following.

The starting point is an $\mathcal{N}=1$ supersymmetric $SO(10) \times U(1)$ gauge theory in six dimensions with vector multiplets and hypermultiplets, compactified on the orbifold $T^2/\mathbb{Z}_2$. One conveniently groups 6D vector multiplets into 4D vector multiplets $A=(A_\mu,\lambda)$ and 4D chiral multiplets $\Sigma=(A_{5,6},\lambda')$, and 6D hypermultiplets into two chiral multiplets, (ϕ,χ) and (ϕ',χ') [43,44], where (ϕ',χ') transform in the complex-conjugate representation compared to (ϕ,χ). The origin $\zeta_{\rm I}=0$ is a fixed point under reflections, $Ry=-y$, where y denotes the coordinates of the compact dimensions. Imposing chiral boundary conditions on the orbifold, 6D $\mathcal{N}=1$ supersymmetry is broken to 4D $\mathcal{N}=1$ supersymmetry, and the chiral superfields Σ and ϕ' are projected out.

The bulk $SO(10)$ symmetry is broken to the Standard Model group by means of two Wilson lines. The fixed points ζ_i, $i=$ PS, GG, fl are invariant under combined lattice translations and reflection: $\hat{T}_i\zeta_i=\zeta_i$ (see, for instance, Ref. [42]). Demanding that gauge fields on the orbifold satisfy the relations

$$P_i A(x,\hat{T}_i y)P_i^{-1}=\eta_i A(x,y), \qquad i=\mathrm{PS},\mathrm{GG}, \quad (1)$$

with appropriately chosen $SO(10)$ matrices P_i and parities $\eta_{\rm PS},\eta_{\rm GG}=\pm$, the gauge group $SO(10)$ is broken to the Pati-Salam (PS) subgroup $G_{\rm PS}=SU(4)\times SU(2)\times SU(2)$ and the Georgi-Glashow (GG) subgroup $G_{\rm GG}=SU(5)\times U(1)_X$ at the fixed points $\zeta_{\rm PS}$ and $\zeta_{\rm GG}$, respectively (see Fig. 1). In four dimensions the SM gauge group arises from the intersection of the Pati-Salam and Georgi-Glashow subgroups of $SO(10)$, $G_{\rm SM'}=G_{\rm PS}\cap G_{\rm GG}=SU(3)\times SU(2)\times U(1)_{\rm Y}\times U(1)_{\rm X}$. Group theory implies that $SO(10)$ is broken to flipped $SU(5)$, $G_{\rm fl}=SU(5)'\times U(1)_{X'}$ at $\zeta_{\rm fl}$.

Like the vector multiplets, the hypermultiplets satisfy the relations

$$P_i\phi(x,\hat{T}_i y)=\eta_i\phi(x,y), \qquad i=\mathrm{PS},\mathrm{GG}, \quad (2)$$

where the matrices $P_{\rm PS}$ and $P_{\rm GG}$ now depend on the representation of the hypermultiplet (see Ref. [42]). The $SO(10)$ multiplets ϕ can be decomposed into SM multiplets, $\phi=\{\phi^\alpha\}$. Each of them belongs to a representation of $G_{\rm PS}$ as well as $G_{\rm GG}$ and is therefore characterized by two parities,

$$\phi^\beta(x,\hat{T}_{\rm PS}y)=\eta^\beta_{\rm PS}\phi^\beta(x,y), \quad \phi^\beta(x,\hat{T}_{\rm GG}y)=\eta^\beta_{\rm GG}\phi^\beta(x,y). \quad (3)$$

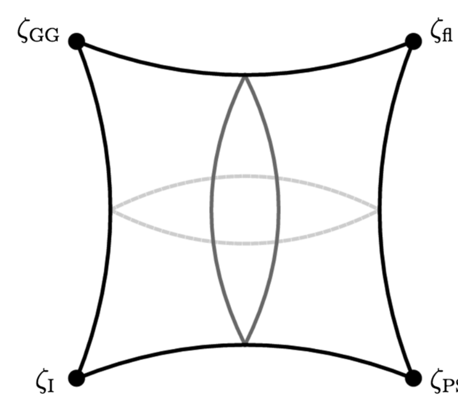

FIG. 1. Orbifold $T^2/\mathbb{Z}_2$ with two Wilson lines and the fixed points $\zeta_{\rm I}$, $\zeta_{\rm PS}$, $\zeta_{\rm GG}$, and $\zeta_{\rm fl}$.

TABLE I. PS and GG parities for bulk **10**-plets, **16**-plets and **16***-plets. The index $i = 1, 2$ labels two quark-lepton families of zero modes.

$SO(10)$				**10**				
G_{PS}	**(1, 2, 2)**		**(1, 2, 2)**		**(6, 1, 1)**		**(6, 1, 1)**	
G_{GG}	$\mathbf{5}^*_{-2}$		$\mathbf{5}_{+2}$		$\mathbf{5}^*_{-2}$		$\mathbf{5}_{+2}$	
Parities	η_{PS}	η_{GG}	η_{PS}	η_{GG}	η_{PS}	η_{GG}	η_{PS}	η_{GG}
H_1	+	−	+	+	−	−	−	+
			H_u					
H_2	+	+	+	−	−	+	−	−
	H_d							
$SO(10)$				**16**				
G_{PS}	**(4, 2, 1)**		**(4, 2, 1)**		**(4*****, 1, 2)**		**(4*****, 1, 2)**	
G_{GG}	$\mathbf{10}_{-1}$		$\mathbf{5}^*_{+3}$		$\mathbf{10}_{-1}$		$\mathbf{5}^*_{+3}$, $\mathbf{1}_{-5}$	
Parities	η_{PS}	η_{GG}	η_{PS}	η_{GG}	η_{PS}	η_{GG}	η_{PS}	η_{GG}
ψ	−	+	−	−	+	+	+	−
	q_i		l_i		u_i^c, e_i^c u_3^c, e_3^c		d_i^c, n_i^c	
χ	−	−	−	+	+	−	+	+
	q_3		l_3		u_4^c, e_4^c		d_3^c, n_3^c d_4^c, n_4^c	
Ψ	−	−	−	+	+	−	+	+
							D^c, N^c	
$SO(10)$				**16***				
G_{PS}	**(4*****, 2, 1)**		**(4*****, 2, 1)**		**(4, 1, 2)**		**(4, 1, 2)**	
G_{GG}	$\mathbf{10}^*_{+1}$		$\mathbf{5}_{-3}$		$\mathbf{10}^*_{+1}$		$\mathbf{5}_{-3}$, $\mathbf{1}_{+5}$	
Parities	η_{PS}	η_{GG}	η_{PS}	η_{GG}	η_{PS}	η_{GG}	η_{PS}	η_{GG}
ψ^c	−	+	−	−	+	+	+	−
					u, e			
χ^c	−	−	−	+	+	−	+	+
							d, n	
Ψ^c	−	−	−	+	+	−	+	+
							D, N	

They can be freely chosen subject to the requirement of anomaly cancellations. A given set of parities then defines a 4D model with the SM gauge group. The model [38] contains two pairs of **16**- and **16***-plets, ψ and ψ^c with parities $\eta_{\mathrm{PS}} = -1, \eta_{\mathrm{GG}} = +1$, and Ψ and Ψ^c with parities $\eta_{\mathrm{PS}} = -1, \eta_{\mathrm{GG}} = -1$. Two **10**-plets contain the Higgs doublets H_u and H_d. We now introduce a third pair of **16**- and **16***-plets, χ and χ^c with parities $\eta_{\mathrm{PS}} = -1, \eta_{\mathrm{GG}} = -1$.

Magnetic flux is generated by a $U(1)$ background gauge field. For a bulk **16**-plet with charge q and magnetic flux $f = -4\pi N/q$ one obtains N left-handed **16**-plets of zero modes. In addition there is a split multiplet of zero modes whose quantum numbers depend on the choice of η_{PS} and η_{GG}. We choose the charges $q = 2$ and $q = 1$ for ψ and χ, respectively, whereas ψ^c, χ^c, Ψ and Ψ^c carry zero $U(1)$ charge.[1] The resulting zero modes are summarized in Table I. Note that the expectation values of N^c and N break $U(1)_X$, and therefore $B - L$.

The zero modes of the charged hypermultiplets have nontrivial wave function profiles. The decomposition of all bulk **16**- and **16***-plets reads

$$\psi = \sum_{i=1,2}\Big[q_i\psi^{(i)}_{-+} + l_i\psi^{(i)}_{--} + (d_i^c + n_i^c)\psi^{(i)}_{+-}\Big] + \sum_{\alpha=1,2,3}(u_\alpha^c + e_\alpha^c)\psi^{(\alpha)}_{++}, \tag{4}$$

$$\chi = q_3\chi^{(1)}_{--} + l_3\chi^{(1)}_{-+} + (u_4^c + e_4^c)\chi^{(1)}_{+-} + \sum_{i=1,2}(d_{i+2}^c + n_{i+2}^c)\chi^{(i)}_{++}, \tag{5}$$

$$\Psi = D^c + N^c, \qquad \psi^c = u + e,$$
$$\chi^c = d + n, \qquad \Psi^c = D + N. \tag{6}$$

Here the chiral multiplet $q = (u, d)$ contains an $SU(2)$ doublet of left-handed up and down quarks, $l = (\nu, e)$

[1] We expect that charged and neutral $SO(10)$ singlets can be added such that all gauge and gravitational anomalies cancel. For the model of Ref. [38] this was recently shown in Ref. [45]. We also neglect the possible effect of zero modes localized at the fixed points, which may be needed to cancel fixed-point anomalies.

TABLE II. Wave functions at different fixed points for one flux quantum $N = 1$. $\psi^{(\alpha)}$, $\alpha = 1, 2, 3$, and $\psi^{(i)}$, $i = 1, 2$, are mode functions of the bulk field ψ with $q = 2$; $\chi^{(i)}$, $i = 1, 2$, and $\chi^{(1)}$ are mode functions of the bulk field χ with $q = 1$.

	ζ_{I}	ζ_{PS}	ζ_{GG}	ζ_{fl}
$\psi^{(\alpha)}_{++}$	(1.086, 1.6818, 0.1454)	(−1.086, 1.6818, 0.1454)	(1.0864, 0.1454, 1.6818)	(−1.0864, 0.1454, 1.6818)
$\psi^{(i)}_{+-}$	$(0.7654(1+i), 1.6818)$	$(-0.7654(1+i), 1.6818)$	(0, 0)	(0, 0)
$\psi^{(i)}_{-+}$	(0.4238, 1.9546)	(0, 0)	(1.9546, 0.4238)	(0, 0)
$\psi^{(i)}_{--}$	$(0.2749(1+i), 1.9546)$	(0, 0)	(0, 0)	$(1.3819(1-i), 0.3887i)$
$\chi^{(i)}_{++}$	(1.4195, 0.5880)	(1.4195, −0.5880)	(0.5880, 1.4195)	(0.5880, −1.4195)
$\chi^{(1)}_{+-}$	1.4089	1.4089	0	0
$\chi^{(1)}_{-+}$	1.4089	0	1.4089	0
$\chi^{(1)}_{--}$	1.2920	0	0	$1.2920i$

contains a doublet of a left-handed neutrino and an electron, and the charge-conjugate states of right-handed up and down quarks, neutrinos and electrons are contained in u^c, d^c, n^c and e^c, respectively.

All Yukawa couplings and mass mixing terms depend on the values of the wave functions at the four fixed points. For $\psi^{(a)}_{\eta_{\mathrm{PS,GG}}}$ and $\chi^{(a)}_{\eta_{\mathrm{PS,GG}}}$ we use expressions given in Ref. [42]. For N flux quanta, a wave function $\varphi^{(a)}_{\eta_{\mathrm{PS}},\eta_{\mathrm{GG}}}(y_1, y_2)$ is given as

$$\varphi^{(a)}_{\eta_{\mathrm{PS}},\eta_{\mathrm{GG}}}(y_1, y_2; N) = \mathcal{N} e^{-2\pi N y_2^2} \sum_{n\in\mathbb{Z}} e^{-2\pi N (n-\frac{a}{2N})^2 - i\pi (n-\frac{a}{2N})(ik_{\mathrm{PS}}-k_{\mathrm{GG}})} \times \cos\left[2\pi\left(-2nN + a + \frac{k_{\mathrm{PS}}}{2}(y_1 + iy_2)\right)\right], \tag{7}$$

where $\eta_{\mathrm{PS}} = e^{i\pi k_{\mathrm{PS}}}$, $\eta_{\mathrm{GG}} = e^{i\pi k_{\mathrm{GG}}}$ and $k_{\mathrm{PS}}, k_{\mathrm{GG}} = 0, 1$. For $\eta_{\mathrm{PS}} = \eta_{\mathrm{GG}} = +1$, one gets $N + 1$ massless modes with $a = 0, 1, \ldots, N$. In the remaining cases, one obtains N zero modes with $a = 0, 1, \ldots, N - 1$. We choose the ordering

$$\begin{aligned}
\psi^{(2)}_{\eta_{\mathrm{PS}},\eta_{\mathrm{GG}}}(y_1, y_2) &= \varphi^{(0)}_{\eta_{\mathrm{PS}},\eta_{\mathrm{GG}}}(y_1, y_2; 2),\\
\psi^{(1)}_{\eta_{\mathrm{PS}},\eta_{\mathrm{GG}}}(y_1, y_2) &= \varphi^{(1)}_{\eta_{\mathrm{PS}},\eta_{\mathrm{GG}}}(y_1, y_2; 2),\\
\psi^{(3)}_{++}(y_1, y_2) &= \varphi^{(2)}_{++}(y_1, y_2; 2),\\
\chi^{(1)}_{\eta_{\mathrm{PS}},\eta_{\mathrm{GG}}}(y_1, y_2) &= \varphi^{(0)}_{\eta_{\mathrm{PS}},\eta_{\mathrm{GG}}}(y_1, y_2; 1),\\
\chi^{(2)}_{++}(y_1, y_2) &= \varphi^{(1)}_{++}(y_1, y_2; 1).
\end{aligned} \tag{8}$$

The wave functions evaluated at the different fixed points $\zeta_{\mathrm{I}}: (y_1 = 0, y_2 = 0)$, $\zeta_{\mathrm{PS}}: (y_1 = 1/2, y_2 = 0)$, $\zeta_{\mathrm{GG}}: (y_1 = 0, y_2 = 1/2)$, $\zeta_{\mathrm{fl}}: (y_1 = 1/2, y_2 = 1/2)$ are given in Table II.

The Yukawa interactions arise at the four fixed points in the model. Considering the unbroken symmetries and the corresponding matter multiplets (see Table I) at the different fixed points, one obtains the following Yukawa superpotential from the lowest-dimensional operators[2]:

$$\begin{aligned}
W_{\mathrm{Y}} = {}& \delta_{\mathrm{I}}\left[\left(\frac{1}{2}y^{\mathrm{I}}_{ua}\psi\psi + y^{\mathrm{I}}_{ub}\psi\chi + \frac{1}{2}y^{\mathrm{I}}_{uc}\chi\chi\right)H_1 + \left(\frac{1}{2}y^{\mathrm{I}}_{da}\psi\psi + y^{\mathrm{I}}_{db}\psi\chi + \frac{1}{2}y^{\mathrm{I}}_{dc}\chi\chi\right)H_2 + \left(\frac{1}{2}y^{\mathrm{I}}_{na}\psi\psi + y^{\mathrm{I}}_{nb}\psi\chi + \frac{1}{2}y^{\mathrm{I}}_{nc}\chi\chi\right)\Psi^c\Psi^c\right]\\
&+ \delta_{\mathrm{PS}}\left(\frac{1}{2}y^{\mathrm{PS}}_{na}4^*_\psi 4^*_\psi + y^{\mathrm{PS}}_{nb}4^*_\psi 4^*_\chi + \frac{1}{2}y^{\mathrm{PS}}_{nc}4^*_\chi 4^*_\chi\right)FF\\
&+ \delta_{\mathrm{GG}}\left(\frac{1}{2}y^{\mathrm{GG}}_{ua}10_\psi 10_\psi H_5 + y^{\mathrm{GG}}_{db}10_\psi 5^*_\chi H_{5^*} + y^{\mathrm{GG}}_{\nu c}5^*_\chi 1_\chi H_5 + \frac{1}{2}y^{\mathrm{GG}}_{nc}1_\chi 1_\chi NN\right)\\
&+ \delta_{\mathrm{fl}}\left(y^{\mathrm{fl}}_{ea}\tilde{5}^*_\psi \tilde{1}_\psi H_{\tilde{5}} + y^{\mathrm{fl}}_{ub}\tilde{5}^*_\psi \tilde{10}_\chi H_{\tilde{5}^*} + \frac{1}{2}y^{\mathrm{fl}}_{dc}\tilde{10}_\chi \tilde{10}_\chi H_{\tilde{5}} + \frac{1}{2}y^{\mathrm{fl}}_{nc}\tilde{10}_\chi \tilde{10}_\chi \tilde{T}^*\tilde{T}^*\right),
\end{aligned} \tag{9}$$

where $1_\psi = n^c$ and $\tilde{1}_\chi = e^c$. In addition to the above, the mixing between the ψ, χ and ψ^c, χ^c at various fixed points can be written as[3]

[2]The magnetic flux generates a Stueckelberg mass term for the $U(1)$ vector boson [46]. By means of the corresponding axion the Yukawa couplings can be made invariant with respect to the $U(1)$ symmetry.

[3]Note that the structure of the mixing terms is considerably simpler than the one found in Ref. [42]. This is due to the fact that no mixings with color triplets from bulk 10-plets have to be taken into account to obtain satisfactory flavor mixings.

$$W_{\mathrm{mix}} = \sum_{p=\mathrm{I,PS,GG,fl}} \delta_p (\mu_a^p \psi^c \psi + \mu_b^p \psi^c \chi + \mu_c^p \chi^c \chi + \mu_d^p \chi^c \psi). \tag{10}$$

For simplicity, we assume universal mass terms at fixed points and set $\mu_i^{\mathrm{I}} = \mu_i^{\mathrm{PS}} = \mu_i^{\mathrm{GG}} = \mu_i^{\mathrm{fl}} \equiv \mu_i$ for $i = a, b, c, d$.

After the electroweak symmetry breaking the mass Lagrangian for up-type quarks obtained from Eqs. (9) and (10) can be written as

$$\begin{aligned} -\mathcal{L}_m^{\mathrm{up}} = v_u \Bigg[& \sum_{p=\mathrm{I,GG}} y_{ua}^p (\psi_{-+}^{(i)} \psi_{++}^{(\alpha)}) \Big|_p u_i u_\alpha^c + y_{ub}^{\mathrm{I}} (\psi_{-+}^{(i)} \chi_{+-}^{(1)}) \Big|_{\mathrm{I}} u_i u_4^c + \sum_{p=\mathrm{I,fl}} y_{ub}^p (\chi_{--}^{(1)} \psi_{++}^{(\alpha)}) \Big|_p u_3 u_\alpha^c + y_{uc}^{\mathrm{I}} (\chi_{--}^{(1)} \chi_{+-}^{(1)}) \Big|_{\mathrm{I}} u_3 u_4^c \Bigg] \\ & + \sum_{p=\mathrm{I,PS,GG,fl}} \mu_a \psi_{++}^{(\alpha)} \Big|_p u u_\alpha^c + \sum_{p=\mathrm{I,PS}} \mu_b \chi_{+-}^{(1)} \Big|_p u u_4^c + \mathrm{H.c.}, \end{aligned} \tag{11}$$

where $i, j = 1, 2$ and $\alpha = 1, 2, 3$. The mass Lagrangian for the down-type quarks can be obtained from Eq. (9) in the same way. We obtain

$$\begin{aligned} -\mathcal{L}_m^{\mathrm{down}} = v_d \Bigg[& y_{da}^{\mathrm{I}} (\psi_{-+}^{(i)} \psi_{+-}^{(j)}) \Big|_{\mathrm{I}} d_i d_j^c + \sum_{p=\mathrm{I,GG}} y_{db}^p (\psi_{-+}^{(i)} \chi_{++}^{(j)}) \Big|_p d_i d_{j+2}^c + y_{db}^{\mathrm{I}} (\chi_{--}^{(1)} \psi_{+-}^{(i)}) \Big|_{\mathrm{I}} d_3 d_i^c + \sum_{p=\mathrm{I,fl}} y_{dc}^p (\chi_{--}^{(1)} \chi_{++}^{(j)}) \Big|_p d_3 d_{j+2}^c \Bigg] \\ & + \sum_{p=\mathrm{I,PS}} \mu_d \psi_{+-}^{(i)} \Big|_p d d_i^c + \sum_{p=\mathrm{I,PS,GG,fl}} \mu_c \chi_{++}^{(j)} \Big|_p d d_{j+2}^c + \mathrm{H.c.} \end{aligned} \tag{12}$$

Similarly, the charged lepton mass terms are given by

$$\begin{aligned} -\mathcal{L}_m^{\mathrm{cl}} = v_d \Bigg[& \sum_{p=\mathrm{I,fl}} y_{ea}^p (\psi_{--}^{(i)} \psi_{++}^{(\alpha)}) \Big|_p e_i e_\alpha^c + y_{eb}^{\mathrm{I}} (\psi_{--}^{(i)} \chi_{+-}^{(1)}) \Big|_{\mathrm{I}} e_i e_4^c + \sum_{p=\mathrm{I,GG}} y_{eb}^p (\chi_{-+}^{(1)} \psi_{++}^{(\alpha)}) \Big|_p e_3 e_\alpha^c + y_{ec}^{\mathrm{I}} (\chi_{-+}^{(1)} \chi_{+-}^{(1)}) \Big|_{\mathrm{I}} e_3 e_4^c \Bigg] \\ & + \sum_{p=\mathrm{I,PS,GG,fl}} \mu_a \psi_{++}^{(\alpha)} \Big|_p e e_\alpha^c + \sum_{p=\mathrm{I,PS}} \mu_b \chi_{+-}^{(1)} \Big|_p e e_4^c + \mathrm{H.c.}, \end{aligned} \tag{13}$$

where $y_{ea}^{\mathrm{I}} = y_{da}^{\mathrm{I}}$, $y_{eb}^{\mathrm{I}} = y_{db}^{\mathrm{I}}$, $y_{ec}^{\mathrm{I}} = y_{dc}^{\mathrm{I}}$ and $y_{eb}^{\mathrm{GG}} = y_{db}^{\mathrm{GG}}$. For the Dirac-type neutrino mass terms one obtains from Eq. (9)

$$\begin{aligned} -\mathcal{L}_m^{\mathrm{Dirac}} = v_u \Bigg[& y_{\nu a}^{\mathrm{I}} (\psi_{--}^{(i)} \psi_{+-}^{(j)}) \Big|_{\mathrm{I}} \nu_i n_j^c + \sum_{p=\mathrm{I,fl}} y_{\nu b}^p (\psi_{--}^{(i)} \chi_{++}^{(j)}) \Big|_p \nu_i n_{j+2}^c + y_{\nu b}^{\mathrm{I}} (\chi_{-+}^{(1)} \psi_{+-}^{(i)}) \Big|_{\mathrm{I}} \nu_3 n_i^c + \sum_{p=\mathrm{I,GG}} y_{\nu c}^p (\chi_{-+}^{(1)} \chi_{++}^{(j)}) \Big|_p \nu_3 n_{j+2}^c \Bigg] \\ & + \sum_{p=\mathrm{I,PS,GG,fl}} \mu_c \chi_{++}^{(j)} \Big|_p n n_{j+2}^c + \sum_{p=I,\mathrm{PS}} \mu_d \psi_{+-}^{(i)} \Big|_p n n_i^c + \mathrm{H.c.}, \end{aligned} \tag{14}$$

where $y_{\nu a}^{\mathrm{I}} = y_{ua}^{\mathrm{I}}$, $y_{\nu b}^{\mathrm{I}} = y_{ub}^{\mathrm{I}}$, $y_{\nu c}^{\mathrm{I}} = y_{uc}^{\mathrm{I}}$ and $y_{\nu b}^{\mathrm{fl}} = y_{ub}^{\mathrm{fl}}$. Note that the mass mixing terms μ_a and μ_b decouple one linear combination of u_α^c, u_4^c and e_α^c, e_4^c from the low-energy effective theory whereas μ_c and μ_d decouple one linear combination of d_i^c, d_{i+2}^c.

The two mass mixing terms in the Dirac neutrino mass matrix for n, n_i^c and n, n_{i+2}^c are comparable to the large Majorana mass terms for n_i^c and n_{i+2}^c. From Eq. (9) one obtains for the Majorana mass terms generated by the $B-L$ breaking vacuum expectation value (VEV) $v_{B-L} = \langle \Psi^c \rangle$

$$-\mathcal{L}_m^N = \frac{v_{B-L}^2}{M_P} \left(\frac{1}{2} \sum_{p=\mathrm{I,PS}} y_{na}^p (\psi_{+-}^{(i)} \psi_{+-}^{(j)}) \Big|_p n_i^c n_j^c + \sum_{p=\mathrm{I,PS}} y_{nb}^p (\psi_{+-}^{(i)} \chi_{++}^{(j)}) \Big|_p n_i^c n_{j+2}^c + \frac{1}{2} \sum_{p=\mathrm{I,PS,GG,fl}} y_{nc}^p (\chi_{++}^{(i)} \chi_{++}^{(j)}) \Big|_p n_{i+2}^c n_{j+2}^c \right) + \mathrm{H.c.} \tag{15}$$

Here $M_P = 2 \times 10^{17}$ GeV is the reduced 6D Planck scale. The eigenvalues of the corresponding 4×4 matrix M_n are $\mathcal{O}(v_{B-L}^2/M_P)$. Together, Eqs. (14) and (15) yield an 8×8 neutrino mass matrix,

$$\mathcal{M}_{\nu,n} = \left(\begin{array}{c|cc} 0_{3\times3} & v_u(Y_D)_{3\times4} & 0_{3\times1} \\ \hline v_u(Y_D^T)_{4\times3} & (M_n)_{4\times4} & (\mu_D^T)_{4\times1} \\ 0_{1\times3} & (\mu_D)_{1\times4} & 0 \end{array} \right), \qquad (16)$$

where $v_u(Y_D)_{3\times4}$ connects ν_i, ν_3 with n_i^c, n_{i+2}^c, and μ_D connects n with n_i^c, n_{i+2}^c. We denote the lower right 5×5 block of the matrix by M_N, which has five Majorana mass eigenstates. $M_D = (v_u(Y_D)_{3\times4}, 0_{3\times1})$ is a 3×5 Dirac neutrino mass matrix. Integrating out the five heavy Majorana neutrinos one obtains the seesaw formula for the 3×3 light neutrino mass matrix,

$$M_\nu = -M_D M_N^{-1} M_D^T, \qquad (17)$$

from which we can extract the relevant neutrino observables.

The above mass matrices contain the complete information about the flavor spectrum of quarks and leptons. In the following section, we shall study in detail the viability of Eqs. (11)–(17) in reproducing the experimentally observed fermion spectrum and the predictions for neutrino masses and the baryon asymmetry via leptogenesis [47].

It is tempting to speculate that a fit of quark and lepton mass matrices with the expressions in Eqs. (11)–(17) is straightforward, given the large number of free parameters. However, this is not the case since the flavor structure of the matrices is determined by the wave function profiles, with matrix elements of $\mathcal{O}(1)$, which naively is at variance with hierarchical quark and charged lepton masses. In fact, in the model of Ref. [42], which has only one bulk **16**-plet, a successful fit turned out to be impossible, despite many parameters. One quark-lepton generation always remained massless. The reason is, that before mass mixings, the mass matrices are generically rank one. In addition, there are relations between Yukawa couplings, which reflect the different unbroken GUT groups at the different fixed points. For example, at the $SO(10)$ fixed point there are several relations [see Eqs. (11)–(14)],

$$\begin{aligned} y_{ea}^{\mathrm{I}} &= y_{da}^{\mathrm{I}}, & y_{eb}^{\mathrm{I}} &= y_{db}^{\mathrm{I}}, & y_{ec}^{\mathrm{I}} &= y_{dc}^{\mathrm{I}}, \\ y_{\nu a}^{\mathrm{I}} &= y_{ua}^{\mathrm{I}}, & y_{\nu b}^{\mathrm{I}} &= y_{ub}^{\mathrm{I}}, & y_{\nu c}^{\mathrm{I}} &= y_{uc}^{\mathrm{I}}, \end{aligned} \qquad (18)$$

and at the Georgi-Glashow and flipped $SU(5)$ fixed points one has

$$y_{eb}^{\mathrm{GG}} = y_{db}^{\mathrm{GG}}, \qquad y_{\nu b}^{\mathrm{fl}} = y_{ub}^{\mathrm{fl}}. \qquad (19)$$

Note that the $SO(10)$ relation for $y_{\nu a}^{\mathrm{I}}$, $y_{\nu b}^{\mathrm{I}}$ and $y_{\nu c}^{\mathrm{I}}$ implies that $B-L$ has to be broken at the GUT scale in order to generate a viable mass scale for the SM neutrinos. Considering these interrelationships between the quark and lepton sectors, it is not guaranteed that one can correctly reproduce all the observables using Eqs. (11)–(17) despite having a substantial number of parameters.

The magnetic flux is quantized in units of the inverse volume V_2^{-1} of the compact dimensions. This leads to scalar quark and lepton masses of GUT scale size [41],

$$m_{\tilde{q}}^2 = m_{\tilde{l}}^2 \sim \frac{4\pi}{V_2} \sim (10^{15}\ \mathrm{GeV})^2. \qquad (20)$$

An analysis of supersymmetry breaking and moduli stabilization shows that also gravitinos and gauginos are heavy (see Refs. [42,48]),

$$m_{\tilde{g}} \sim m_{\tilde{W}} \sim m_{\tilde{B}} \sim m_{3/2} \sim 10^{14}\ \mathrm{GeV}. \qquad (21)$$

One is therefore left with an extension of the Standard Model where, depending on radiative corrections, only two Higgs doublets and Higgsinos can be light. It is interesting that such a model can be consistent with gauge coupling unification, which imposes constraints on $\tan\beta$ and the Higgs boson masses [49].

The presented model assumes that all the quarks and leptons arise as zero modes of bulk fields, caused by magnetic flux. This is the standard picture of flux compactifications in field and string theory. Of course, in principle there could also be "twisted sectors," i.e. matter localized at fixed points. Matter from bulk fields and twisted sectors has previously been considered in orbifold GUTs (see, for example, Ref. [50]) and heterotic string compactifications (see, for example, Ref. [51]). However, in all these models magnetic flux has not been included. An analysis of flux compactifications containing twisted sectors remains a challenging question for further research.

III. NUMERICAL ANALYSIS OF THE FLAVOR SPECTRUM

As described in the previous section, Eqs. (11)–(17) determine the masses and mixing parameters of the SM fermions. In order to check whether the model correctly describes the known fermion spectrum, we perform a χ^2 test. For this we construct a χ^2 function

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{O_i^{\mathrm{th}}(x_1, x_2, \ldots, x_m) - O_i^{\mathrm{exp}}}{\sigma_i^{\mathrm{exp}}} \right)^2, \qquad (22)$$

where $O_i^{\mathrm{th}}(x_1, x_2, \ldots, x_m)$ are the observables estimated from Eqs. (11)–(17). They depend on the various parameters of the model denoted as x_j. The O_i^{exp} are the experimentally measured values of the corresponding observables and σ_i^{exp} are the standard deviations. As of now, 18 of these observables are directly measured in various experiments. They include nine charged fermion masses, two neutrino mass differences, three mixing angles and a phase in the Cabibbo-Kobayashi-Maskawa (CKM) or quark mixing matrix and three mixing angles in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) or lepton mixing matrix [52]. There also exists preliminary and indirect information about the Dirac CP phase in the lepton sector through global fits of neutrino oscillation data [53–55].

The spectrum computed from Eqs. (11)–(17) holds at the GUT scale. We therefore choose the GUT scale extrapolated values of the various observables as O_i^{exp} for consistency. The flux compactification also breaks supersymmetry and leads to a two-Higgs-doublet model (2HDM) of type-II below the GUT scale [38].[4] For this reason, we use the GUT scale values of charged fermion masses extrapolated in the 2HDM with $v_u/v_d = \tan\beta = 10$ from the latest analysis [56] as an example set of data for our analysis. The effects of the renormalization group equations (RGE) are known to be very small in the case of the CKM parameters, and therefore we use their low scale values from Ref. [52]. The RGE effects are small also in the case of neutrino masses and mixing angles if the light neutrino masses are hierarchical and follow normal ordering. Therefore we use the low scale values of solar and atmospheric mass-squared differences and leptonic mixing angles from the recent global fit of neutrino oscillation data performed in Ref. [53]. In order to account for RGE effects, various threshold corrections and uncertainties due to neglecting next-to-leading-order corrections in the theoretical estimations of flavor observables, we adopt a conservative approach and consider 30% standard deviation in the masses of the light quarks (up, down and strange) and the electron and 10% standard deviation in the remaining quantities instead of using the extrapolated experimental values of standard deviations in Eq. (22). Further, we assume normal ordering for the neutrino mass spectrum. The various O_i^{exp} we use are listed in the third column of Table III.

The details of our procedure of extracting physical observables from Eqs. (11)–(17) are described in Appendix A. For an estimation of O_i^{th} in the case of charged fermions, we first integrate out the heavy vector-like states and obtain effective 3×3 matrices for each flavor. In the case of neutrinos, five Weyl fermions, namely n in Eq. (14) and n_i^c, n_{i+2}^c, $i = 1, 2$ in Eqs. (15) and (17) form a 5×5 Majorana mass matrix M_N with GUT scale eigenvalues. The mass matrix of three light neutrinos is then given by the seesaw mass formula (17). The various fermion masses and the CKM and PMNS matrices are obtained using the diagonalization procedure described in Appendix A. The elements of the CKM matrix are denoted as V_{ij} while we use the Particle Data Group [52] convention for the parametrization of the PMNS matrix to represent its elements in terms of the mixing angles θ_{ij}.

[4] This feature automatically suppresses the contribution of dimension-five operators in proton decay.

TABLE III. Fit without leptogenesis: the results obtained for the best fit corresponding to $\chi^2 = 0.5$.

Observables	O^{th}	O^{exp}	Deviations (in %)
m_u [GeV]	0.00048	0.00048	0
m_c [GeV]	0.23	0.23	0
m_t [GeV]	74.0	74.1	0
m_d [GeV]	0.0011	0.0011	0
m_s [GeV]	0.018	0.021	−16
m_b [GeV]	1.19	1.16	3
m_e [GeV]	0.00043	0.00044	−2
m_μ [GeV]	0.093	0.093	0
m_τ [GeV]	1.60	1.61	−1
m^2_{sol} [eV^2]	0.000075	0.000075	0
m^2_{atm} [eV^2]	0.0025	0.0025	0
V_{us}	0.22	0.23	−3
V_{cb}	0.041	0.041	0
V_{ub}	0.0036	0.0036	0
$\sin^2\theta_{12}$	0.31	0.31	0
$\sin^2\theta_{23}$	0.44	0.44	0
$\sin^2\theta_{13}$	0.022	0.022	0
J^Q_{CP}	0.000031	0.000030	1
δ_{MNS} [^{o}]	281	261	8
Predictions			
α_{21} [^{o}]	273	M_{N_1} [GeV]	1.8×10^{10}
α_{31} [^{o}]	215	M_{N_2} [GeV]	6.3×10^{10}
m_{ν_1} [eV]	0.0043	M_{N_3} [GeV]	1.1×10^{11}
$m_{\beta\beta}$ [eV]	0.0004	M_{N_4} [GeV]	1.7×10^{12}
m_β [eV]	0.0098	M_{N_5} [GeV]	2.7×10^{13}
η_B	5.2×10^{-12}		

The function χ^2 is numerically minimized in order to check the viability of the model in different cases. The model contains a large number of free parameters [20 complex couplings in Eq. (9), four real mass parameters in Eq. (10) and a real VEV v_{B-L}]. For simplicity, we first assume that all couplings in Eq. (9) are real. This leads to $m = 25$ real parameters to account for $n = 19$ observed quantities. We the find that one can correctly reproduce the entire fermion spectrum with vanishing leptonic Dirac CP phase. The reason for this can be understood as follows. In the case of real couplings in Eq. (9), the CP violations in the quark and lepton sectors arise entirely from the complex profile factors given in Table II. By choosing an appropriate basis, it can be shown that the CP violation in the lepton sector due to the profile factors can be completely rotated away while the same cannot be done for the quarks. It turns out that the model can still successfully account for the observed CP violation in the quark sector while it leads to a CP-conserving lepton sector.

The recent T2K data [57] and the global fits of neutrino oscillation data show a mild preference for maximal Dirac CP violation, $\sin\delta_{\mathrm{MNS}} \sim -1$. Moreover, in order to account for the observed baryon asymmetry of the Universe through leptogenesis, the model would require CP phases in the lepton sector. Motivated by this, we shall consider more general Yukawa couplings in Eq. (9). Since CP violation in the quark sector is already explained without complex couplings, we consider the minimal case in which only the Yukawa couplings of SM singlet fermions are complex, i.e. $y^p_{na}, y^p_{nb}, y^p_{nc}$ with $p = \mathrm{I}, \mathrm{PS}$ and $y^{\mathrm{GG}}_{nc}, y^{\mathrm{fl}}_{nc}$. This introduces eight new parameters in the model. In the following, we discuss two different χ^2 fits obtained for this case.

A. Predicting neutrino masses and baryon asymmetry

For the above choice of couplings the χ^2 function includes $n = 19$ observables as functions of $m = 33$ real parameters. We minimize χ^2 numerically in order to find solutions for the parameters which can reproduce the data. We find a very good fit corresponding to $\chi^2 = 0.5$ at the minimum. The results of this fit are listed in Table III. It is remarkable that all observables are fitted to their experimental values with very small deviations. The maximum deviation is found in the strange quark mass which is still smaller than the allowed 30% deviation from its experimental value extrapolated at the GUT scale. The fitted values of parameters are listed in Appendix B.

At the bottom of Table III we show predictions for various quantities that can be estimated from the fitted values of the parameters. These include the Majorana phases (α_{21}, α_{31}), the mass of the lightest SM neutrino m_{ν_1}, the effective neutrinoless double beta decay mass $m_{\beta\beta}$, the mass measured in standard beta decay m_β and the masses of the heavy neutrinos M_{N_α} with $\alpha = 1, \ldots, 5$. As a comparison with the subsequent fit will show, the order of magnitude of the absolute neutrino mass scale, i.e. m_{ν_1}, is a robust prediction whereas the remaining quantities can change significantly if the fit is slightly varied.

The baryon asymmetry generated by decays and inverse decays of the lightest singlet neutrino can be written as [58,59]

$$\eta_B = 0.96 \times 10^{-2} \epsilon_1 \kappa_f, \tag{23}$$

where the CP asymmetry is given by [60]

$$\epsilon_1 = -\frac{3}{16\pi \tilde{m}_1} \mathrm{Im}[(h^\dagger M_\nu h^*)_{11}], \tag{24}$$

and washout processes are taken into account by the efficiency factor

$$\kappa_f \simeq 2 \times 10^{-2} \times \left(\frac{0.01\ \mathrm{eV}}{\tilde{m}_1}\right)^{1.1}. \tag{25}$$

CP asymmetry and washout processes depend on the effective neutrino mass

$$\tilde{m}_1 = \frac{v_u^2}{M_{N_1}} (h^\dagger h)_{11}. \tag{26}$$

In Eqs. (24) and (26), h denotes the Dirac neutrino Yukawa matrix in a basis where the mass matrix of the heavy neutrinos is diagonal, i.e. $h = Y_D U_N$ with $U_N^T M_N U_N = \mathrm{diag}(M_{N_1}, \ldots, M_{N_5})$. In order to obtain the expression (24) for the CP asymmetry, a summation over lepton flavors in the final state has to be carried out.

Using the parameters of the fit, one obtains for the baryon asymmetry generated from N_1, $\eta_B \simeq 5.2 \times 10^{-12}$, which is 2 orders of magnitude smaller than the observed value $\eta_B \simeq (6.10 \pm 0.04) \times 10^{-10}$ [52]. However, for the heavy Majorana masses given in Table III, the baryon asymmetry calculated from Eqs. (23)–(25) can be modified by flavor effects of charged leptons and other heavy neutrinos by more than an order of magnitude [61,62]. To obtain a realistic estimate of the baryon asymmetry, the flavor effects of charged leptons and in particular the contributions of the heavier Majorana neutrinos have to be taken into account.

From Eqs. (17) and (23)–(26) one can easily read off how a rescaling of couplings may lead to a baryon asymmetry enlarged by 2 orders of magnitude. Rescaling h by a factor of 10 while keeping the neutrino masses constant, i.e. rescaling M_N by a factor of 100, enhances ϵ_1 by a factor of 100, leaving $\tilde{m}_1$ and κ_f unchanged. Hence, η_B is indeed enlarged by a factor of 100. It is not clear, however, whether such a rescaling can be made consistent with a description of the quark sector since the Dirac neutrino Yukawa couplings and the up-quark Yukawa couplings are related.

B. Predicting neutrino masses

We now perform a fit including the baryon asymmetry η_B in the χ^2 function in order to check the viability of the model in reproducing the correct baryon asymmetry together with the flavor spectrum. The number of input parameters is the same as the before. The results are displayed in Table IV. We obtain the minimal $\chi^2 = 0.95$ which is slightly higher compared to the previous case but it can be still considered a very good fit. The resulting input parameters are listed in Appendix B.

Compared to the first fit the Majorana phases α_{21} and α_{31} have changed by about 50%. The order of magnitude of the light neutrino masses has remained the same whereas the heavy neutrino masses have increased by 2 orders of magnitude, as expected. Correspondingly, the $B - L$ breaking VEV has increased by a factor of 10. The increase of the heavy Majorana masses has the interesting effect that the baryon asymmetry is now indeed dominated by decays and inverse decays of the Majorana neutrino N_1. Since $M_{N_2} \ldots M_{N_5} \sim 10^{14}$ GeV, they are likely not to be produced from the thermal bath and therefore they have no effect on

TABLE IV. Fit with leptogenesis: the results obtained for the best fit corresponding to $\chi^2 = 0.95$.

Observables	O^{th}	O^{exp}	Deviations (in %)
m_u [GeV]	0.00048	0.00048	0
m_c [GeV]	0.23	0.23	0
m_t [GeV]	74.1	74.1	0
m_d [GeV]	0.00096	0.00113	−15
m_s [GeV]	0.018	0.021	−18
m_b [GeV]	1.16	1.16	0
m_e [GeV]	0.00051	0.00044	16
m_μ [GeV]	0.094	0.093	1
m_τ [GeV]	1.61	1.61	0
m^2_{sol} [eV^2]	0.000075	0.000075	0
m^2_{atm} [eV^2]	0.0025	0.0025	0
V_{us}	0.23	0.23	0
V_{cb}	0.041	0.041	0
V_{ub}	0.0035	0.0035	0
$\sin^2\theta_{12}$	0.31	0.31	0
$\sin^2\theta_{23}$	0.44	0.44	0
$\sin^2\theta_{13}$	0.022	0.022	0
J^Q_{CP}	0.000030	0.000030	0
δ_{MNS} [^{o}]	279	261	7
η_B	6.1×10^{-10}	6.1×10^{-10}	0
Predictions			
α_{21} [^{o}]	129	M_{N_1} [GeV]	1.3×10^{12}
α_{31} [^{o}]	353	M_{N_2} [GeV]	2.0×10^{14}
m_{ν_1} [eV]	0.0017	M_{N_3} [GeV]	3.5×10^{14}
$m_{\beta\beta}$ [eV]	0.0026	M_{N_4} [GeV]	3.7×10^{14}
m_β [eV]	0.0089	M_{N_5} [GeV]	4.6×10^{14}

the baryon asymmetry. Moreover, the enhanced mass $M_{N_1} \sim 10^{12}$ GeV now lies in the unflavored regime where flavor effects of charged leptons can be neglected. For the effective light neutrino mass we find

$$\tilde{m}_1 = 0.023 \text{ eV}, \tag{27}$$

lying precisely in the mass range

$$\sqrt{m^2_{\mathrm{sol}}} < \tilde{m}_1 < \sqrt{m^2_{\mathrm{atm}}}. \tag{28}$$

Hence, leptogenesis takes place in the preferred strong washout regime where the final asymmetry is independent of initial conditions. For this value of $\tilde{m}_1$ the heavy Majorana neutrino mass has to satisfy the lower bound $M_1 > 10^{11}$ GeV (see Fig. 10 in Ref. [58]), which is also satisfied. We conclude that the estimation of the baryon asymmetry and the fit to the fermion spectrum are self-consistent.

It is instructive to reconstruct from the fitted values of the input parameters given in Table V how the description of the flavor spectrum and baryogenesis is accomplished. The mixing of the zero modes of ψ and χ via the heavy vectorlike multiplets is difficult to disentangle but it is clear that the largest up-type and down-type Yukawa couplings scale as one expects for the heaviest generation,

$$y^{\mathrm{I}}_{uc} \sim y^{\mathrm{fl}}_{ub} \sim \frac{m_t}{m_b}\frac{y^{\mathrm{I}}_{dc}}{\tan\beta} \sim \frac{m_t}{m_b}\frac{y^{\mathrm{fl}}_{dc}}{\tan\beta}. \tag{29}$$

The relations at the $SO(10)$ fixed point $y^{\mathrm{I}}_{ea} = y^{\mathrm{I}}_{da}$ and $y^{\mathrm{I}}_{\nu c} = y^{\mathrm{I}}_{uc}$ are also very important [see Eqs. (13) and (14)]. The last one implies that $B - L$ is broken at the GUT scale and therefore $m_{\nu_1} \sim 10^{-3}$ eV.

The Yukawa couplings vary over a range comparable to the range in the Standard Model. This, together with mass mixings with vectorlike states and wave function values differing by an order of magnitude leads to a successful fit of the measured observables.

IV. SUMMARY AND CONCLUSIONS

Six-dimensional supersymmetric theories with GUT gauge symmetries are an attractive intermediate step towards embedding the Standard Model in string theory. We have analyzed the structure of Yukawa couplings and mass mixings that occur in an orbifold compactification of a 6D $SO(10)$ GUT model with Abelian magnetic flux. Three quark-lepton generations are generated as zero modes of bulk **16**-plets together with two Higgs doublets obtained from two bulk **10**-plets and further vectorlike split multiplets. Although all quarks and leptons have the same origin, they have different wave functions in the compact dimensions and therefore different couplings to the Higgs fields at the orbifold fixed points.

The underlying GUT symmetry and the wave function profiles of the zero modes imply a number of relations between the various Yukawa couplings. In a minimal setup the model has 33 real parameters. It is nontrivial that a good fit is possible to quark and lepton masses and mixings, CP-violating phases and the baryon asymmetry via leptogenesis (20 observables). Due to $SO(10)$ relations between up-quark and Dirac neutrino Yukawa couplings, $B - L$ is broken at the GUT scale. The smallest neutrino mass is predicted to be $m_{\nu_1} \sim 10^{-3}$ eV and also the neutrino masses m_β and $m_{\beta\beta}$, to be measured in standard beta decay and neutrinoless double beta decay, are very small. Heavy Majorana neutrino masses are predicted in the range from 10^{12} to 10^{14} GeV, and the effective light neutrino mass is $\tilde{m}_1 = 0.023$ eV. Hence, the baryon asymmetry is indeed dominated by decays and inverse decays of the lightest GUT scale Majorana neutrino and flavor effects on the generated asymmetry are negligible. It is remarkable that all light neutrino masses lie in the neutrino mass window 10^{-3} eV $< m_{\nu_i} < 0.1$ eV where thermal leptogenesis works best.

The model presented in this paper addresses the question of flavor physics in flux compactifications, but it is incomplete in several respects. First of all, the vacuum expectation values $\langle H_u \rangle$, $\langle H_d \rangle$ and $\langle N \rangle$ correspond to flat directions of the model. Hence, the determination of the scales of electroweak breaking and $B - L$ breaking require further interactions and parameters which remain to be specified. Another important point concerns the effect of the large mass mixing terms on the zero mode profiles (for a recent discussion, see Ref. [37]). In principle, one has to analyze numerically the differential equations for the bulk wave functions including the mixing terms. This may lead to $\mathcal{O}(1)$ effects on the wave functions at the fixed points. However, since the values of the wave functions at fixed points are already $\mathcal{O}(1)$, we expect no qualitative change of our discussion, but rather a quantitative change in the numerical values of the free parameters. These questions will be studied in detail in a future analysis.

Our results provide a nonstandard perspective on the flavor problem. Traditionally, one searches for flavor symmetries to understand the hierarchies of fermion masses and mixings. In the considered model with flux compactification the quarks and leptons of the three generations have different internal wave functions and therefore different couplings to the Higgs fields. As a consequence, there is no fundamental flavor symmetry. The effective 6D theory still contains unexplained Yukawa couplings which may be related to geometry and fluxes if the orbifold singularities are resolved in a ten-dimensional theory. The presented model illustrates that in string compactifications flavor symmetries are not fundamental, although they may occur as approximate accidental symmetries in specific compactifications.

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APPENDIX A: EXTRACTION OF MASSES AND MIXING PARAMETERS

In this appendix we discuss our method of extracting physical observables from Eqs. (11)–(17). For the charged fermions, $f = u,\ d,\ e$, Eqs. (11), (12) and (13) can generally be written as

$$-\mathcal{L}_m^f = (\,f_1 \;\; f_2 \;\; f_3 \;\; f\,) M_f \begin{pmatrix} f_1^c \\ f_2^c \\ f_3^c \\ f_4^c \end{pmatrix} + \text{H.c.}, \quad \text{(A1)}$$

where

$$M_f = \begin{pmatrix} v_f Y_f \\ \mu_\alpha^f \end{pmatrix}, \quad \text{(A2)}$$

$v_e = v_d = v\cos\beta$, $v_u = v\sin\beta$ and $v = 174$ GeV. Y_f is a 3×4 Yukawa coupling matrix and μ_α^f, $\alpha = 1, \ldots, 4$, are the GUT scale mass mixing terms. We then obtain a Hermitian matrix

$$H_f \equiv M_f M_f^\dagger = \left(\begin{array}{c|c} v_f^2 Y_f Y_f^\dagger & v_f (Y_f)_{i\alpha} \mu_\alpha^* \\ \hline v_f (Y_f^*)_{i\alpha} \mu_\alpha & \tilde{\mu}_f^2 \end{array}\right), \quad \text{(A3)}$$

with $\tilde{\mu}_f^2 = \sum_\alpha |\mu_\alpha^f|^2$. One typically finds $(H_f)_{44} \gg (H_f)_{i4} \gg (H_f)_{ij}$ with $i = 1, 2, 3$. One linear combination of f_1, f_2 and f_3 forms together with f a Dirac fermion with GUT scale mass and decouples from the low-energy spectrum. After integrating it out, we obtain an effective 3×3 matrix $\tilde{H}_f$ for the three families of SM fermions,

$$\begin{aligned} (\tilde{H}_f)_{ij} &= v_f^2 (Y_f Y_f^\dagger)_{ij} - \frac{1}{\tilde{\mu}_f^2} (H_f)_{i4} (H_f^*)_{j4} \\ &= v_f^2 (Y_f Y_f^\dagger)_{ij} - v_f^2 (Y_f)_{i\alpha} (Y_f^*)_{j\beta} \frac{\mu_\alpha^{f*} \mu_\beta^f}{\tilde{\mu}_f^2}. \end{aligned} \quad \text{(A4)}$$

In the case of the three families of light neutrinos we similarly construct $\tilde{H}_\nu = M_\nu M_\nu^\dagger$ using the 3×3 Majorana neutrino mass matrix M_ν obtained from Eq. (17). The Hermitian matrices $\tilde{H}_f$ obtained for $f = u, d, e, \nu$ are then diagonalized using $U_f^\dagger \tilde{H}_f U_f = \text{Diag}(m_{f_1}^2, m_{f_2}^2, m_{f_3}^2)$ where m_{f_i} are the physical masses of corresponding fermions. The CKM and PMNS mixing matrices are constructed using $V = U_u^\dagger U_d$ and $U = U_l^\dagger U_\nu$, respectively.

The effective masses for standard beta decay and neutrinoless double beta decay denoted by m_β and $m_{\beta\beta}$, respectively, are obtained using

$$m_\beta = \sqrt{(M_{\nu f} M_{\nu f}^\dagger)_{ee}} \quad \text{and} \quad m_{\beta\beta} = |(M_{\nu f})_{ee}|, \quad \text{(A5)}$$

where $M_{\nu f}$ is the neutrino mass matrix in the diagonal basis of charged leptons and is given by $M_{\nu f} = U_l^\dagger M_\nu U_l^*$.

APPENDIX B: FITTED VALUES OF PARAMETERS

We list in Table V the values of input parameters of the model defined in Eqs. (9) and (10) obtained from the two fits. The GUT scale mixing parameters $\mu_{a,b,c,d}$ are given in units of the reduced Plank scale, $M_P = 2\times 10^{17}$ GeV.

TABLE V. The fitted values of input parameters obtained for Fit 1 and Fit 2 displayed in Tables III and IV, respectively.

Parameters	Fit 1 (Table III)	Fit 2 (Table IV)
y^{I}_{ua}	-0.5115×10^{-3}	0.1148×10^{-2}
y^{I}_{da}	0.4472×10^{-4}	-0.1314×10^{-3}
y^{I}_{na}	$(0.8090+0.9252i)\times 10^{-3}$	$(0.2728+0.0583i)\times 10^{-3}$
y^{I}_{ub}	0.7538×10^{-2}	0.2691×10^{-1}
y^{I}_{db}	-0.2982×10^{-2}	0.8655×10^{-3}
y^{I}_{nb}	$(-0.0829-0.3117i)\times 10^{-2}$	$(-0.6072+0.6514i)\times 10^{-2}$
y^{I}_{uc}	-0.2341	-0.2311
y^{I}_{dc}	0.4630×10^{-1}	-0.3160×10^{-1}
y^{I}_{nc}	$-0.4134-0.6924i$	$(-0.3838+0.2643i)\times 10^{-1}$
y^{GG}_{ua}	-0.9908×10^{-6}	-0.9914×10^{-6}
y^{GG}_{db}	0.1278×10^{-3}	0.1186×10^{-3}
$y^{\mathrm{GG}}_{\nu c}$	0.2355×10^{-1}	0.2220
y^{GG}_{nc}	$(0.0951-0.1447i)\times 10^{-1}$	$0.1422-0.0589i$
y^{fl}_{ea}	0.1756×10^{-2}	-0.2330×10^{-1}
y^{fl}_{ub}	-0.1058×10^{-1}	-0.1616
y^{fl}_{dc}	-0.5899×10^{-2}	0.1149×10^{-1}
y^{fl}_{nc}	$(0.2005-0.2579i)\times 10^{-1}$	$(-0.3909+0.9498i)\times 10^{-2}$
y^{PS}_{na}	$(-0.0362+0.2929i)\times 10^{-3}$	$(0.0469+0.4710i)\times 10^{-3}$
y^{PS}_{nb}	$(0.4811+0.2618i)\times 10^{-2}$	$(0.3124+0.5919i)\times 10^{-2}$
y^{PS}_{nc}	$(0.2737+0.0246i)\times 10^{-1}$	$(0.8301-0.3691i)\times 10^{-1}$
μ_a	$0.9625\ M_P$	$0.1095\times 10^{-3}\ M_P$
μ_b	$0.2191\times 10^{-2}\ M_P$	$0.3401\times 10^{-2}\ M_P$
μ_c	$0.2228\times 10^{-4}\ M_P$	$0.7488\times 10^{-2}\ M_P$
μ_d	$0.2071\times 10^{-4}\ M_P$	$0.9124\times 10^{-1}\ M_P$
v_{B-L}	$0.8360\times 10^{-2}\ M_P$	$0.8522\times 10^{-1}\ M_P$

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