Axion inflation, proton decay, and leptogenesis in $SU(5)\times U(1)_{PQ}$

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We implement inflation in a nonsupersymmetric $SU(5)$ model based on a nonminimal coupling of the axion field to gravity. The isocurvature fluctuations are adequately suppressed, axions comprise the dark matter, proton lifetime estimates are of order 8×10^{34}–3×10^{35} yr, and the observed baryon asymmetry arises via nonthermal leptogenesis. The presence of low-scale colored scalars ensures unification of the Standard Model gauge couplings and also helps in stabilizing the electroweak vacuum.

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I. INTRODUCTION

Successful inflation models based on $SU(5)$ grand unified theory (GUT) and employing a Coleman-Weinberg potential with minimal coupling to gravity were constructed some time ago [1] and, for values of the scalar spectral index $n_s\sim0.96$–0.97, the tensor-to-scalar ratio r is $\gtrsim10^{-2}$ [2]. In these models the inflaton field, an $SU(5)$ gauge singlet, evolves from the origin to its nonzero vacuum expectation value, reaching trans-Planckian values near its minimum during the last 60 or so e-foldings. Identifying the $SU(5)$ gauge singlet inflaton with the axion [3,4] field is very attractive and was first done in Ref. [5]. However, it turns out to be not very compelling for this model because of the excessively large value $f_a\sim10^{19}$ GeV imposed on the axion decay constant by inflation [2]. Typically, an axion decay constant $f_a\sim10^{11-12}$ GeV is the desired value for axion dark matter (DM).

It has been known for a while that primordial inflation driven by a scalar quartic potential and based on nonminimal coupling to gravity is fully consistent with the Planck observations [6] for plausible values, say $\xi\lesssim10$, of the dimensionless nonminimal coupling parameter ξ. The inflaton field in this case rolls down from trans-Planckian values to its final minimum which can be sub-Planckian as desired. The scalar quartic coupling λ during inflation in this case turns out to be $\lesssim10^{-8}$. This means that in order to protect λ from unacceptably large radiative corrections, in nonsupersymmetric GUTs the inflaton should again be identified with a gauge singlet scalar field, as was done in the $SU(5)$ model mentioned above.

An axion model needed to resolve the strong CP problem provides a compelling candidate to implement successful inflation in GUTs using nonminimal coupling to gravity. For the sake of simplicity, we will employ a Higgs rather than the Coleman-Weinberg potential. The inflaton (radial component of the axion field) in this case rolls down from trans-Planckian values during inflation to its final value $f_a\sim10^{11-12}$ GeV, thus yielding a viable scenario with axion dark matter.

With trans-Planckian field values during inflation the isocurvature fluctuations are adequately suppressed and observable gravity waves corresponding to $r\sim$ few $\times10^{-3}$ are predicted. The reheating process proceeds via the decay of the inflaton into right-handed (RH) neutrinos. In turn, the latter yield the observed baryon asymmetry via nonthermal leptogenesis.

The realistic GUT model we propose successfully addresses several problems of the standard model (SM) at once, namely the existence and nature of dark matter, the strong CP problem, baryogenesis, the stability of the electroweak vacuum, the origin of the inflationary phase, and the physics behind neutrino masses. All of these issues have been previously studied in the literature in the aim of providing unified schemes which tackle several of them simultaneously; see e.g., Refs. [7–16]. Here we show that a simple nonsupersymmetric GUT model provides an elegant framework to solve all these problems, in addition to providing matter and gauge coupling unification.

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This paper is organized as follows. We describe our model and outline its most salient features in Sec. II. We then analyze the constraints on gauge coupling unification (GCU) and proton decay in Sec. III. These end up predicting the presence of a colored octet scalar not far from the TeV scale. Such a field also plays a critical role in stabilizing the electroweak vacuum which we analyze in Sec. IV. We then outline the main features of inflation based on a quartic potential with nonminimal coupling of the inflaton field to gravity in Sec. V, and derive the predictions for the spectral index n_s, the scalar-to-tensor ratio r, and the running of the spectral index α. We also present constraints on the magnitude of Yukawa couplings involving RH neutrinos and estimate the reheat temperature taking into account the requirement of axion dark matter. Next, in Sec. VI we describe how leptogenesis is implemented in this framework. Finally, in Sec. VII we show how the inflaton coupling to the adjoint **24**-plet ensures $SU(5)$ breaking during inflation so that the superheavy monopoles are inflated away.

II. THE MODEL

The model consists of a simple extension of the original $SU(5)$ model [17]. The SM fermion fields are in the usual **10** (T_L) and $\bar{\mathbf{5}}$ (F_L), and we add the singlet RH neutrinos ν_L^c. The scalar sector of the model involves **5** (H_1), $\mathbf{5}^\star$ (H_2), **24** (Φ), and finally $\mathbf{45}^\star$ (χ, with $\chi_k^{ij} = -\chi_k^{ji}$ for $i, j, k = 1$–5). In addition, we define a global $U(1)_{PQ}$ symmetry to implement the Peccei-Quinn (PQ) mechanism [18] solving the strong CP problem and providing an invisible axion via the Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) mechanism [19,20][1]; see Table I for the $U(1)_{PQ}$ charges of the different fields.

The relevant terms in the scalar potential of the model read

$$V \supset -\frac{1}{2}M_{GUT}^2 \mathrm{Tr}(\Phi^2) - M_\sigma^2 \sigma\sigma^\star + \lambda_\sigma(\sigma^\star\sigma)^2 + \sigma\sigma^\star \sum_\phi \kappa_\phi \phi^\star \phi + \lambda_\nu[(\sigma^\star)^2 H_1 H_2 + \mathrm{H.c.}]. \quad (1)$$

Here $\phi = (H_1, H_2, \Phi, \chi)$ and the dot product $\phi^\star.\phi$ represents the $SU(5) \times U(1)_{PQ}$ invariant contractions. And the Yukawa part of the Lagrangian is given by the following terms (family, gauge, and Lorentz indices are suppressed):

$$\mathcal{L}_{\mathrm{Yuk}} = T_L.\mathbf{Y_{10}}.T_L.H_1 + T_L.\mathbf{Y_5}.F_L.H_2 + T_L.\mathbf{Y_{45}}.F_L.\chi F_L.\mathbf{Y}_\nu.\nu_L^c.H_1 + \frac{1}{2}\mathbf{Y_N}\nu_L^c.\nu_L^c.\sigma + \mathrm{H.c.}, \quad (2)$$

[1]For an early reference on an explicit $SU(5)$ construction solving the strong CP problem we refer to Ref. [21].

TABLE I. Summary of the quantum numbers of the different fields of the model. q is an arbitrary number. $U(1)_X$ is an accidental symmetry of the model when the mixed term $(\sigma^\star)^2 H_1 H_2$ is absent, i.e., in the limit $\lambda_\nu \to 0$.

	T_L	F_L	ν_L^c	H_1	H_2	σ	Φ	χ
$SU(5)$	**10**	$\bar{\mathbf{5}}$	**1**	**5**	$\bar{\mathbf{5}}$	**1**	**24**	$\mathbf{45}^\star$
$U(1)_{PQ}$	$q/2$	$q/2$	$q/2$	$-q$	$-q$	$-q$	0	$-q$
$U(1)_X$	1/5	−3/5	−1	−2/5	2/5	2	0	2/5

where $\mathbf{Y}_i$ are dimensionless 3×3 matrices. The PQ symmetry is tightly related to lepton number in this scheme. We can readily see that if $\lambda_\nu = 0$, the Lagrangian is invariant under $U(1)_{B-L}$ symmetry after the breaking of $SU(5)$ and $U(1)_{PQ}$. Indeed, an accidental $U(1)_X$ symmetry (shown in Table I) combines with the usual hypercharge to leave an unbroken $U(1)$ defined by $X + \frac{4}{5}Y \equiv B - L$. However, this symmetry is explicitly broken in the scalar sector due to the presence of the λ_ν term which is crucial for generating the axion in the DFSZ model and cannot be set to zero. Additionally, $\lambda_\nu \neq 0$ allows us to get rid of the Majoron Goldstone boson [22,23] since lepton number is broken explicitly [7].

The representations involved in the model play crucial roles in different phenomenological sectors.

1. $H_{1,2}$ and χ account for the SM charged fermion masses and mixings in a renormalizable way. The two-Higgs-doublet model consisting of $H_{1,2}$ ensures the implementation of the DFSZ mechanism. The multiplet χ is also crucial for obtaining accurate gauge coupling unification.
2. $\langle\Phi\rangle$ breaks $SU(5)$ to the SM.
3. The phase of σ is the axion which solves the strong CP problem and accounts for the DM of the Universe, and the radial part drives inflation.
4. $\langle\sigma\rangle$ provides large Majorana masses for ν_L^c via the seesaw mechanism. After inflation the latter helps generate the observed baryon asymmetry via nonthermal leptogenesis.

In the next sections we will investigate in detail all these aspects of the model.

III. FERMIONS MASSES, GAUGE COUPLING UNIFICATION, AND PROTON DECAY

From Eq. (2), we obtain the following mass relations:

$$M_e = \mathbf{Y_5}^T \langle H_2 \rangle - 6\mathbf{Y_{45}}^T \langle \chi \rangle, \quad (3)$$

$$M_d = \mathbf{Y_5} \langle H_2 \rangle + 2\mathbf{Y_{45}} \langle \chi \rangle, \quad (4)$$

$$M_u = 4(\mathbf{Y_{10}} + \mathbf{Y_{10}}^T) \langle H_1 \rangle, \quad (5)$$

$$M_\nu \simeq \mathbf{Y}_\nu{}^T.\mathbf{Y_N}^{-1}.\mathbf{Y}_\nu \frac{\langle H_1 \rangle^2}{\langle \sigma \rangle}. \quad (6)$$

In the last equation we have assumed the seesaw scaling $\langle\sigma\rangle \gg \langle H_1\rangle$ for natural couplings. We define $\langle\chi\rangle \equiv \langle\chi\rangle_1^{15} = \langle\chi\rangle_2^{25} = \langle\chi\rangle_3^{35} = -3\langle\chi\rangle_4^{45}$. It is clear from these expressions that there is enough parameter freedom to fit the fermion masses and cure the wrong predictions of minimal $SU(5)$ [24–26].

Next, we turn to the issue of GCU. In the minimal $SU(5)$ model, the gauge couplings do not properly unify at high energy. However, the χ and Φ multiplets contain representations which can alter the renormalization group (RG) evolution in a favorable way [27]. In particular, we will use $R_8 \equiv (\mathbf{8}, \mathbf{2}, 1/2) \in \chi$ and $R_3 \equiv (\mathbf{3}, \mathbf{3}, -1/3) \in \chi$ to obtain precise GCU. Note that in the absence of the PQ symmetry R_3 can mediate proton decay leading to a lower limit on its mass that was estimated to be around 10^{10} GeV [27,28]. However, in our scenario R_3 cannot induce nucleon decay due to the absence of the couplings $T_L.\chi.H_2$ or $T_L.\chi.H_1^\star$ and thus it can be very light. This significantly enlarges the parameter space consistent with GCU.

We solve the system of RG equations in order to obtain successful GCU. The equations depend on three parameters: M_{R_3}, M_{R_8}, and M_{GUT} (for simplicity, we ignore threshold effects). We require that M_{GUT} is large enough so that gauge-mediated proton decay does not rule out the model, i.e.,

$$\tau_p \sim \alpha_{GUT}^{-2} \frac{M_{GUT}^4}{m_p} \gtrsim 10^{34} \text{ yr}, \tag{7}$$

where m_p is the proton mass and the lower limit is the current experimental bound on $\tau_p(p \to e^+\pi^0)$ [29]. After combining these constraints, we find

$$M_{GUT} \approx \left(\frac{M_{R_8}}{\text{TeV}}\right)^{-0.126} \times 10^{16} \text{ GeV}, \tag{8}$$

$$M_{R_3} \approx \left(\frac{M_{R_8}}{\text{TeV}}\right)^{0.05} \times 6.1 \times 10^7 \text{ GeV}, \tag{9}$$

and

$$M_{R_8} \lesssim 6 \times 10^5 \text{ GeV}. \tag{10}$$

The maximum proton lifetime is achieved for the smallest possible R_8 mass. For 1 TeV mass, we obtain $\tau_p \approx 2.4 \times 10^{35}$ yr, which is around the expected sensitivity of the Hyper-Kamiokande experiment [30]. We display the gauge coupling unification in Fig. 1 for the case where $M_{R_8} = 1$ TeV.

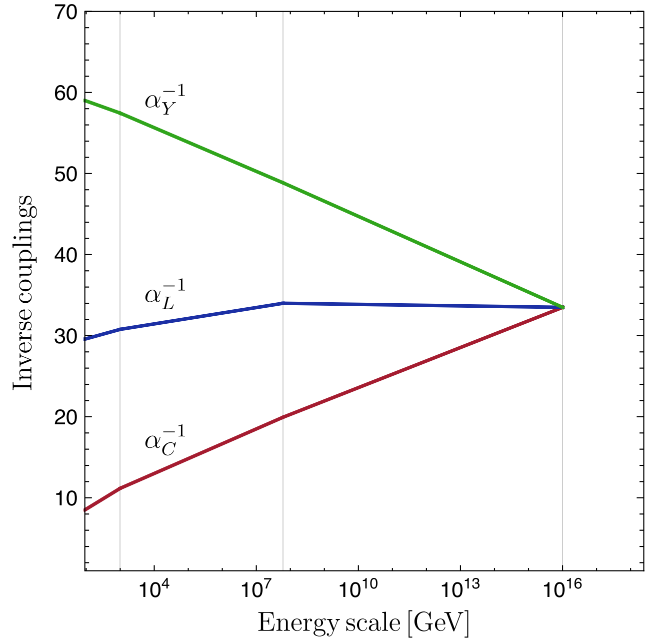

FIG. 1. Evolution of the inverse coupling strengths with energy. Here $M_{R_8} = 1$ TeV, $M_{R_3} \approx 6.1 \times 10^7$ GeV, and $M_{\text{GUT}} \approx 10^{16}$ GeV. The proton lifetime in this case is $\tau_p \approx 2.4 \times 10^{35}$ yr.

IV. VACUUM STABILITY

It is well known that the vacuum instability problem associated with the quartic coupling of the Higgs (see for instance Ref. [31]) can be overcome with new physics around the TeV scale. The GCU analysis in the previous section revealed that the model predicts scalar representations which have to be lighter than $\sim 10^{10}$ GeV, more or less the scale at which the quartic coupling becomes negative. This is remarkable as such scalars will contribute to the running of the quartic coupling and could positively tilt it before it becomes negative. In this section we will analyze the effect of these scalars on the stability of the vacuum. For renormalization energy $\mu < M_{R_8}$, we use the SM RG equations at the two-loop level to calculate the evolution of the Higgs quartic coupling [32–37]. We include the effects from the new particles R_3 and R_8 at the one-loop level. These modify the first-order coefficients b_i of the SM, $\frac{dg_i}{dln\mu} = \frac{b_i}{16\pi^2} g_i^3$, $b_i(\mu) = (\frac{41}{10}, -\frac{19}{6}, -7) + \Theta(\mu - M_{R_8})(2, \frac{4}{3}, \frac{4}{5}) + \Theta(\mu - M_{R_3})(\frac{1}{2}, 2, \frac{1}{5})$. In solving the RGEs, we use the boundary conditions at the top quark pole mass given in Ref. [31]. For the $SU(3)_c$ coupling constant and the top mass, we use $\alpha_s = 0.1184$ and $M_t = 173.34$ GeV [38] respectively. The SM Higgs mass is fixed at $M_h = 125.09$ GeV [39]. We find in particular that R_8, being very light, induces a significant effect on the running of the gauge couplings. The scalar R_3 on the other hand has a negligible effect on the running before the instability scale. As we can see in Fig. 2, the quartic Higgs coupling is indeed prevented from becoming negative by including the

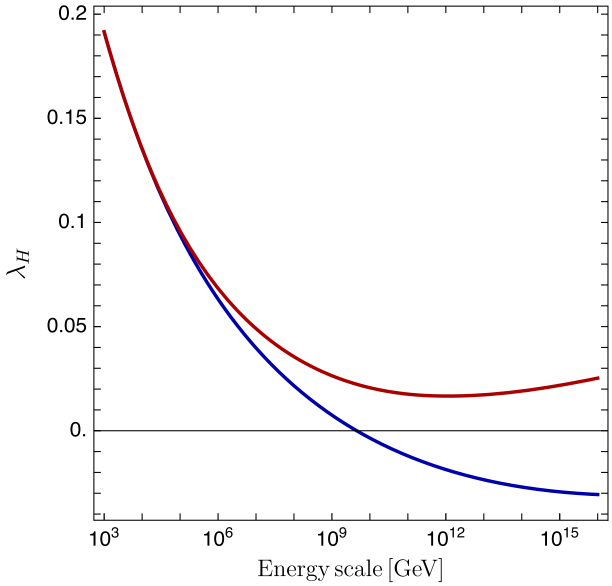

FIG. 2. Vacuum stability of the Higgs potential. The red (upper) line shows the running of the Higgs quartic coupling in the model compatible with Fig. 1. We display in blue (lower line) its running in the SM for comparison.

R_8 field at 1 TeV and R_3 at 6.1×10^7 GeV. Remarkably, the same fields which allow us to implement GCU also stabilize the effective potential of the SM at high energies.

Finally, we can use this analysis to constrain the mass of R_8. Indeed, the heavier it is the less important is its effect on the Higgs quartic coupling, and so we expect an upper bound not far from the TeV region. We find that

$$M_{R_8} \lesssim 10^4 \text{ GeV}, \tag{11}$$

which is more constraining than the upper bound derived from GCU considerations only. Using Eq. (8), this upper bound translates as $\tau_p \gtrsim 7.8 \times 10^{34}$ yr.

V. AXION INFLATION

We assume that inflation is driven by the radial part of the complex singlet σ, $\rho \equiv \sqrt{2}\mathcal{R}e(\sigma)$. Without loss of generality, we take $\mathbf{Y_N} = \mathrm{diag}(\mathrm{Y_{N_1}}, \mathrm{Y_{N_2}}, \mathrm{Y_{N_3}})$, with Y_{N_i} real and positive. The relevant terms of the Lagrangian of the model are

$$\mathcal{L}_{inf} = \left(\sum_{i=1}^{3} \frac{Y_{N_i}}{2\sqrt{2}} \rho \nu_{iL}^c \nu_{iL}^c + \text{H.c.}\right) - V_{inf}, \tag{12}$$

$$V_{inf} = \frac{1}{4}\lambda_\sigma(\rho^2 - f_a^2)^2 + \frac{1}{2}\rho^2 \sum_\phi \kappa_\phi \phi^\star . \phi, \tag{13}$$

where $f_a \equiv \sqrt{2}\langle\sigma\rangle$ and for simplicity, we only consider real couplings. We enforce $\kappa_{H_{1,2}} > 0$ to ensure that inflation is driven by ρ. The couplings of the inflaton with ν_L^c and the scalar fields induce quantum corrections that can have significant effects on the inflationary observables. These effects induce an additional contribution to the potential V_{inf} denoted as $V^{(1)}$,

$$V^{(1)} = \frac{\beta}{16\pi^2}\rho^4 \ln\frac{\rho^2}{\mu^2}, \tag{14}$$

where [40]

$$\beta = 20\lambda_\sigma^2 + 2\sum_\phi \kappa_\phi^2 + 2\lambda_\sigma \sum y_{Ni}^2 - \sum y_{Ni}^4.$$

We require that these radiative corrections are not significant, i.e., $|\beta| \ll (16\pi^2)|\lambda_\sigma|$. The most conservative limit we can set on the couplings is [defining max $(Y_{N_i}) = Y_N$]:

$$y_N \lesssim 6 \times 10^{-2}\left(\frac{\lambda_\sigma}{10^{-7}}\right)^{\frac{1}{4}}. \tag{15}$$

For the rest of the paper we will suppose that $\kappa_\phi \ll y_N$ and impose Eq. (15).

A. ρ^4 inflation with nonminimal coupling to gravity

We consider a scenario where ρ has a nonminimal coupling to gravity. For simplicity, we assume that all other scalars, including the SM Higgs, have quasiminimal couplings. In the Jordan frame, the action of nonminimal ρ^4 inflation is given by

$$S_J^{\text{tree}} = \int d^4x\sqrt{-g}\left[-\left(\frac{1}{2}(1+\xi\rho^2)\right)\mathcal{R} + \frac{1}{2}(\partial\rho)^2 - \frac{\lambda_\sigma}{4}\rho^4\right]. \tag{16}$$

In the Einstein frame one finds,

$$S_E = \int d^4x\sqrt{-g_E}\left[-\frac{1}{2}\mathcal{R}_E + \frac{1}{2}(\partial S)^2 - V_E(S(\rho))\right], \tag{17}$$

where the canonically normalized scalar field S is written in terms of the original scalar as

$$\left(\frac{dS}{d\rho}\right)^{-2} = \frac{(1+\xi\rho^2)^2}{1+(6\xi+1)\xi\rho^2}. \tag{18}$$

The inflation potential now reads

$$V_E(S(\rho)) = \frac{\frac{1}{4}\lambda_\sigma(t)\rho^4}{(1+\xi\rho^2)^2}, \tag{19}$$

and the inflationary slow-roll parameters [41,42] in terms of ρ are expressed as

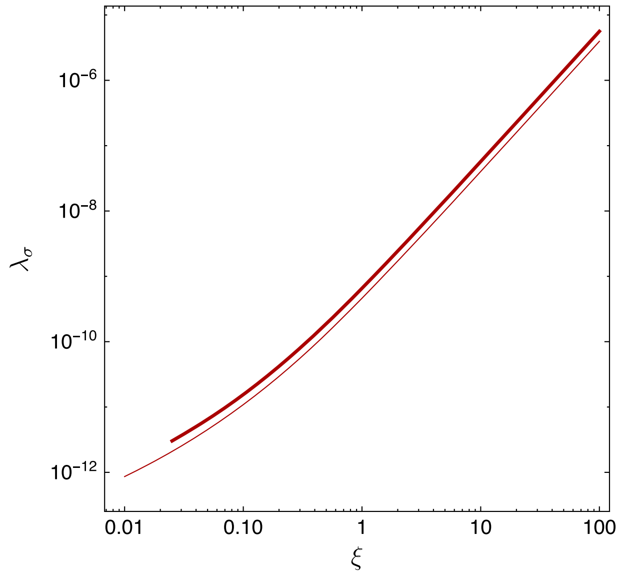

FIG. 3. Correlation between the inflaton's quartic coupling λ_σ and the nonminimal coupling ξ for $N = 50$ e-foldings (thick line) and $N = 60$ e-foldings (thin line). n_s is within the 68% confidence level of Planck's measurement.

$$\epsilon(\rho) = \frac{1}{2}\left(\frac{V_E'}{V_E S'}\right)^2,$$
$$\eta(\rho) = \frac{V_E''}{V_E (S')^2} - \frac{V_E' S''}{V_E (S')^3},$$
$$\frac{\zeta(\rho)}{\sqrt{2\epsilon(\rho)}} = \frac{V_E'''}{V_E (S')^3} - \frac{3V_E'' S''}{V_E (S')^4} + \frac{3V_E' (S'')^2}{V_E (S')^5} - \frac{V_E' S'''}{V_E (S')^4},$$

where a prime denotes a derivative with respect to ρ, and we use units where the reduced Planck mass, $M_{\rm Pl} \simeq 2.4 \times 10^{18}$ GeV, is equal to unity unless otherwise stated. The number of e-folds is given by

$$N = \frac{1}{\sqrt{2}} \int_{\rho_e}^{\rho_0} \frac{d\rho}{\sqrt{\epsilon(\rho)}} \left(\frac{dS}{d\rho}\right). \qquad (20)$$

The inflationary predictions for the scalar spectral index n_s, the tensor-to-scalar ratio r, and the running of the spectral index $\alpha = \frac{dn_s}{d\ln k}$ are obtained after fixing N and ξ. The quartic coupling λ_σ can be fixed using the amplitude of density perturbations at some pivot scale [43],

$$\Delta_{\mathcal{R}}^2 = \frac{V_E}{24\pi^2 \epsilon}\bigg|_{k^\star} = 2.196 \times 10^{-9}|_{0.05\ \rm Mpc^{-1}}. \qquad (21)$$

In Fig. 3 we show the predicted values of the quartic coupling as a function of the minimal coupling ξ for $N = 50$ and $N = 60$ e-foldings. n_s is constrained to be within the 68% confidence level of Planck's measurement [43].

For $\xi \gtrsim 0.1$, the predicted values of n_s, r, and α quickly converge toward

	n_s	$r \times 10^3$	$-\alpha \times 10^4$
$N = 50$	0.962	4	7.5
$N = 60$	0.968	3	5.3

This implies that the Hubble expansion rate at the end of inflation is

$$H_I \simeq 2\pi \times 10^{13}\ \text{GeV}. \qquad (22)$$

B. Reheating

As can be seen in Eq. (15), the coupling of the inflaton with ν_L^c can be sizable, and this can be used to reheat the Universe via the decays $\rho \to 2\nu_L^c$. This is the dominant process because our assumption that $\kappa_\phi \ll 1$ makes reheating via the scalars inefficient. In order to do so, the mass of at least one of the RH neutrinos $M_N = y_N f_a/\sqrt{2}$ must be smaller than half the inflaton's mass $m_\rho = B f_a$, with $B \simeq \sqrt{2\lambda_\sigma}$. This translates as

$$y_N < \sqrt{2\lambda_\sigma}. \qquad (23)$$

Note that this condition is more stringent than the one obtained in Eq. (15). Assuming an instantaneous conversion of the inflaton's energy density into radiation, at the time when $H(t) \approx \Gamma_\rho$ (decay rate of ρ), we can define the reheating temperature as

$$T_{RH} = \left(\frac{45}{4\pi^3 g_\star}\right)^{1/4} \sqrt{\Gamma_\rho} \sim 0.1\sqrt{\Gamma_\rho}, \qquad (24)$$

where

$$\Gamma_\rho = \frac{3\, y_N^2}{64\pi} m_\rho. \qquad (25)$$

Using Eq. (23) we can derive the bound

$$T_{RH} \lesssim 3 \times 10^8\ \text{GeV} \sqrt{\left(\frac{\lambda_\sigma}{10^{-7}}\right)\left(\frac{f_a}{10^{12}\ \text{GeV}}\right)} \equiv T_{RH}^{\max}. \qquad (26)$$

C. Nonadiabatic primordial fluctuations of axions

Since inflation is driven by the radial part of the axion field, the PQ symmetry is always broken during inflation and the axion acquires isothermal (more precisely isocurvature) fluctuations [44–49]. In general, these are given by [50]

$$\beta_{\rm iso} = \left(1 + \frac{\pi f_{a,\star}^2 \overline{\theta_i^2}}{\epsilon(\rho)}\right)^{-1} \leq 0.038, \qquad (27)$$

with $f_{a,\star}$ being the effective scale of PQ symmetry breaking and $\overline{\theta_i}$ is the spatially averaged misalignment angle. The upper bound is the current experimental limit (95% confidence level) at $k = 0.05\ \mathrm{Mpc}^{-1}$ [43]. Assuming that axion DM is produced via the misalignment mechanism [51–53], $\overline{\theta_i}$ enters as well in the expression of the axion relic density:

$$\Omega_a h^2 = 0.1199\left(\frac{\overline{\theta_i^2}}{0.28}\right)\left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{7/6}. \qquad (28)$$

In the standard scenario where inflation is unrelated to axions $f_{a,\star} \equiv f_a$, and the bound Eq. (27) favors a large PQ breaking scale. However in our case the effective scale is given by the inflaton field value during inflation, $\rho_\star$, which is trans-Planckian for $\xi \lesssim 10^2$. Given that $\rho_\star \gg f_a$, f_a does not have a direct impact on the isocurvature perturbations and enters only indirectly via Eq. (28). Using Eqs. (27) and (28) we obtain un upper bound on f_a for a given ξ. In Fig. 4 we depict the predicted values of the maximal allowed value of f_a as a function of the minimal coupling ξ for $N = 50$ and $N = 60$ e-foldings. As in Fig. 3, n_s is constrained to be within the 68% confidence level of Planck's measurement [43]. The obtained limit on f_a is compatible with the natural parameter space of axion DM. Finally, note that for our choice of parameters, namely $f_a \sim 10^{12}$ GeV, the PQ symmetry is not restored at the end of reheating since $T_{RH} \ll f_a$; see Eq. (26).

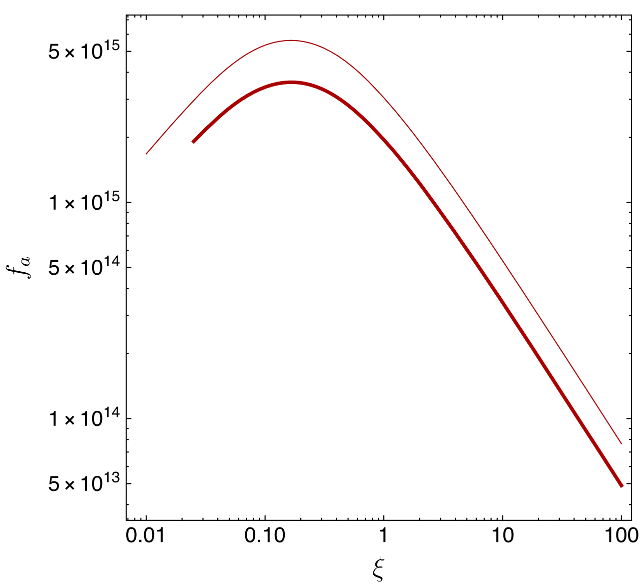

FIG. 4. Upper bound on the PQ scale f_a as a function of the nonminimal coupling ξ due to isocurvature and DM relic abundance constraints. The two lines are for $N = 50$ e-foldings (thick) and $N = 60$ e-foldings (thin). n_s is within the 68% confidence level of Planck's measurement.

VI. BARYOGENESIS

In Eq. (26) we found that the maximum reheat temperature obtained from the inflaton's decays to RH neutrinos is of order 10^8–10^9 GeV. This allows us to implement baryogenesis via nonthermal leptogenesis [54]. Assuming hierarchical RH neutrino masses, the lepton-to-entropy ratio [55] is given by

$$\eta_L \simeq -10^{-5}\left(\frac{T_{\mathrm{RH}}}{10^9\ \mathrm{GeV}}\right)\left(\frac{M_N}{m_\rho}\right), \qquad (29)$$

and the observed baryon asymmetry is related to η_L via the usual relation $\eta_B \simeq 10^{-10} \simeq -\frac{8}{23}\eta_L$. This leads to

$$M_N \simeq 0.3\left(\frac{10^7\ \mathrm{GeV}}{T_{\mathrm{RH}}}\right)m_\rho. \qquad (30)$$

Since $m_\rho \simeq B f_a$, we find that for $\xi \sim 1$ and $T_{\mathrm{RH}} \sim 10^7$ GeV, the heavy RH neutrino with mass of the order of 10^8 GeV can give rise to the observed baryon asymmetry via nonthermal leptogenesis.

VII. GUT MONOPOLES

The $SU(5)$ symmetry breaks to the SM when the effective mass-squared term of Φ, $-M_{\mathrm{GUT}}^2 + \kappa_\Phi \rho^2$, in the effective potential becomes of order $-T_H^2$, with T_H being the Hawking-Gibbons temperature, $T_H \equiv H/(2\pi)$. We want to make sure that Φ is pushed away from its origin during inflation. This must occur at the early stage of inflation to ensure that monopoles are adequately inflated away [1]. In the limit $\kappa_\Phi \to 0$, this translates as a lower bound on the unification scale,

$$M_{\mathrm{GUT}} > (1 + \xi\rho_i^2)T_H \qquad (31)$$

where ρ_i is the starting value of the inflaton field. $\xi\rho_i^2$ varies from ≈0 to ≈70 in the range $\xi \in [0, 100]$. Using the result in Eq. (22), we find that the equation above yields $M_{\mathrm{GUT}} \gtrsim 7 \times 10^{14}$ GeV, which is less constraining than the limit from the proton lifetime. If $\kappa_\Phi < 0$ then this condition is even easier to satisfy. However, in the case where $\kappa_\Phi > 0$ we will have an upper limit on κ_Φ to ensure that $(M_{\mathrm{GUT}}^2 - |\kappa_\Phi|\rho_i^2) > T_H^2$ and monopoles are properly inflated away. For realistic M_{GUT} values, we find that $\kappa_\Phi \lesssim 10^{-7}$ for $\xi = 10^{-2}$ and $\kappa_\Phi \lesssim 10^{-5}$ for $\xi = 10^2$, which is in agreement with our initial assumptions on the smallness of κ_ϕ.

VIII. SUMMARY AND CONCLUSIONS

We have presented a realistic grand unified theory based on $SU(5) \times U(1)_{PQ}$ which consistently addresses multiple outstanding beyond-the-SM problems. Fermion masses and mixings were accounted for in a renormalizable fashion, and precise gauge coupling unification was achieved. With the unification scale predicted to lie around

10^{16} GeV, the proton lifetime is predicted to be in the range 8×10^{34}–3×10^{35} yr and should be accessible in the next-generation detectors. The effective Higgs potential is automatically stabilized thanks to the physics used to implement gauge coupling unification.

The QCD axion is our candidate for the dark matter in the Universe. The axion field plays several roles in our model. The radial component of the axion field drives inflation by exploiting a nonminimal coupling to gravity, reheating proceeds from the axion field coupling to RH neutrinos, and the observed baryon asymmetry arises via nonthermal leptogenesis. The coupling with RH neutrinos induces small neutrino masses via the seesaw mechanism. The isocurvature fluctuations are adequately suppressed and the axion decay constant f_a lies in the desired range of 10^{11}–10^{12} GeV. The model predicts a tensor-to-scalar ratio r in an observable range, $r = \text{few} \times 10^{-3}$. We finally comment that the discussion can be extended to realistic $SO(10)$ models with a suitable intermediate scale such as $SU(4) \times SU(2) \times SU(2)$ or $SU(3) \times SU(2) \times SU(2) \times U(1)$.

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