$ZH\eta$ vertex in the simplest little Higgs model

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The issue of deriving $ZH\eta$ vertex in the simplest little Higgs (SLH) model is revisited. Special attention is paid to the treatment of noncanonically-normalized scalar kinetic matrix and vector-scalar two-point transitions. We elucidate a general procedure to diagonalize a general vector-scalar system in gauge theories and apply it to the case of SLH. The resultant $ZH\eta$ vertex is found to be different from those which have already existed in the literature for a long time. We also present an understanding of this issue from an effective field theory viewpoint.

I. INTRODUCTION

The discovery of the 125 GeV Higgs-like boson [1,2] marks a prominent triumph of the standard model (SM). Nevertheless, it is widely believed that this is not the end of the story. The SM in its current form leaves too many unanswered questions, from theoretical ones like the issue of Higgs mass naturalness [3,4], to observational ones like the nature of the dark matter present in the universe [5,6]. Almost all models going beyond the SM (BSM) entail an enlargement of the scalar sector, and consequently forms of interaction which are absent in the SM could be possible. Searching for such kind of new interactions therefore may lead to decisive evidence of the existence of BSM and provide a clue to the nature of the BSM physics.

For example, Lorentz symmetry does not forbid the interaction of one gauge boson (denoted as Z) with two scalar bosons (denoted as H and η) at the dimension-4 level, in the form like

$$Z^\mu(H\partial_\mu\eta - \eta\partial_\mu H) \qquad (1)$$

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The SM has only one Higgs particle and thus cannot accommodate such kind of vector-scalar-scalar (VSS) interactions.[1] Going beyond the SM, the appearance of interactions like Eq. (1) is quite common in models like the two-Higgs-doublet model (2HDM) and supersymmetric models, which may lead to the associated production of two scalar bosons [7,8] or Higgs-to-Higgs cascade decays [9,10] as important collider signatures.

Besides the usual 2HDM and supersymmetric models which contain a linearly-realized scalar sector, VSS interactions have also been studied in the context of nonlinearly-realized scalar sectors. Nonlinearly realized scalar sectors are frequently adopted when building a model in which the Higgs is realized as a pseudo-Goldstone boson of some global symmetry breaking [11], which could be helpful in addressing the hierarchy problem. In principle the derivation of VSS vertices in such models is similar to the linearly realized case: start from the gauge covariant kinetic terms of the scalar fields and then expand the interaction fields into vacuum expectation values and mass eigenstate fields after which the three-point VSS vertices could be extracted. Nevertheless there can be important technical differences in intermediate steps. When the scalar sector is nonlinearly realized, scalar kinetic terms are in general not automatically canonically normalized, and there can be "unexpected" vector-scalar two-point transitions which need to be taken care of. We will show in the following sections that

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[1]Here we mean physical fields. Unphysical fields like Goldstone or ghost can certainly participate in VSS interactions in the SM.

these situations indeed occur for the case of the simplest little Higgs (SLH) model [12], which is proposed as a simple solution to the Higgs mass naturalness problem.

From a more general perspective, the problem we encounter is how to diagonalize a vector-scalar system in gauge field theories. Specifically, the Lagrangian we start with might not be canonically normalized in its kinetic part, and may have some general vector-scalar two-point transitions. To do perturbation theory in the usual manner, we need to first render its kinetic part canonically normalized, which could be done via the usual complete-the-square method. To remove the vector-scalar two-point transitions, strictly speaking we need to choose appropriate gauge-fixing terms. Finally we still need to diagonalize the scalar mass matrix with contribution from both the original scalar mass terms and the gauge-fixing terms. These steps set the stage for the derivation of VSS interactions.

In Sec. II the systematic procedure of diagonalize a general vector-scalar system in gauge field theories will be elucidated. Then in Sec. III we apply this procedure to the SLH model and derive the mass eigenstate $ZH\eta$ vertex[2] to $\mathcal{O}((\frac{v}{f})^3)$. The $ZH\eta$ vertex derived here is found to be different from those which have already existed in the literature [13,14] for a long time. In Sec. IV we present our discussion and conclusion.

II. GENERAL DIAGONALIZATION PROCEDURE

Consider a gauge field theory in which there are n_S real scalar fields G_i, $i = 1, 2, ..., n_S$ and n_M real massive gauge boson fields Z_p^μ, $p = 1, 2, ..., n_M$. If complex fields exist, we can always decompose them into their real components and proceed in a similar manner. The G_i's which we start with neither need to be canonically normalized nor need to have diagonalized mass terms. For simplicity (but without loss of generality) the Z_p's are assumed to have canonically normalized kinetic terms but do not have to be diagonalized in their mass terms. When we say the Z_p's are massive, it means that the eigenvalues of the mass matrix of Z_p's are all positive. Especially, massless gauge bosons like photon are temporarily excluded from discussion. However, generalizing the procedure to theories containing massless gauge bosons is straightforward.

Now suppose the classical Lagrangian of this gauge theory contains the following quadratic parts[3] (summation over repeated indices is implicitly assumed):

$$\mathcal{L}_{\text{quad}} \supset \frac{1}{2}V_{ij}(\partial_\mu G_i)(\partial^\mu G_j) + F_{pi}Z_p^\mu(\partial_\mu G_i)$$
$$- \frac{1}{2}(\mathbb{M}_G^2)_{ij}G_iG_j + \frac{1}{2}(\mathbb{M}_V^2)_{pq}Z_{p\mu}Z_q^\mu \qquad (2)$$

Here V is a real invertible $n_S \times n_S$ symmetric matrix, F is a real $n_M \times n_S$ matrix, $\mathbb{M}_G^2$ is a $n_S \times n_S$ symmetric matrix the rank of which does not exceed $n_E \equiv n_S - n_M$,[4] and $\mathbb{M}_V^2$ is a real $n_M \times n_M$ symmetric matrix which has n_M positive eigenvalues. The elements of the four matrices V, F, $\mathbb{M}_G^2$, $\mathbb{M}_V^2$ depend only on the model parameters, not on field variables. For convenience let us define

$$\tilde{G}_p = F_{pi}G_i, \qquad p = 1, 2, ..., n_M \qquad (3)$$

Then the vector-scalar two-point transition term [the second term on the right-hand side of Eq. (2)] is simply $Z_p^\mu \partial_\mu \tilde{G}_p$.

To carry out perturbation theory, it is preferable to eliminate the vector-scalar two-point transitions, make the scalar kinetic terms canonically normalized and at the same time diagonalize the scalar and vector mass terms. We will see that the procedure involved actually goes hand in hand with the quantization of the theory. Also, the tight structure of the gauge theory greatly facilitates the diagonalization process.

In gauge field theories, the vector-scalar two-point transitions are usually eliminated by adding appropriate gauge-fixing terms. If we require the R_ξ gauge-fixing procedure remove all the vector-scalar two-point transitions, then it is natural to consider adding the following gauge-fixing Lagrangian:

$$\mathcal{L}_{gf} = -\sum_{p=1}^{n_M} \frac{1}{2\xi^p}(\partial_\mu Z_p^\mu - \xi^p \tilde{G}_p)^2 \qquad (4)$$

Here ξ^p, $p = 1, 2, ..., n_M$ are gauge parameters. There is freedom in the choice of the gauge-fixing function and the requirement to remove vector-scalar two-point transitions is not sufficient to uniquely determine it. However we will see below there is a theoretically well-motivated choice which facilitates the diagonalization process. After adding the gauge-fixing terms, we have

$$\mathcal{L}_{\text{quad}} + \mathcal{L}_{gf} \supset \frac{1}{2}V_{ij}(\partial_\mu G_i)(\partial^\mu G_j) - \frac{1}{2}\xi^p \tilde{G}_p^2 - \frac{1}{2}(\mathbb{M}_G^2)_{ij}G_iG_j$$
$$- \frac{1}{2\xi^p}(\partial_\mu Z_p^\mu)^2 + \frac{1}{2}(\mathbb{M}_V^2)_{pq}Z_{p\mu}Z_q^\mu. \qquad (5)$$

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[2]By "mass eigenstate" $ZH\eta$ vertex we mean the $ZH\eta$ vertex obtained after rotating Z, H, η fields into their corresponding mass eigenstates. For previous studies related to the η particle in the SLH, we refer the reader to [13–25].

[3]Here we suppress the gauge boson kinetic terms which are assumed to be already canonically normalized.

[4]Here we assume all the Z_p's acquire their masses by eating appropriate Goldstones. In compliance with the fact that n_M massless Goldstones should exist before gauge-fixing, the rank of $\mathbb{M}_G^2$ should not exceed $n_S - n_M$.

The matrix V denotes the scalar kinetic matrix. If it is not the identity matrix, we may simply use the complete-the-square method to diagonalize it and then make the resulting terms canonically normalized. This is in complete analogy to the diagonalization of quadratic forms in linear algebra. Note that the overall transformation employed to render the scalar kinetic terms canonically normalized need not be orthogonal.

Now suppose we have found a transformation of the scalar fields

$$S_i = U_{ij}G_j \tag{6}$$

which renders the scalar kinetic terms diagonalized and canonically normalized:

$$\frac{1}{2}V_{ij}(\partial_\mu G_i)(\partial^\mu G_j) = \frac{1}{2}(\partial_\mu S_i)(\partial^\mu S_i). \tag{7}$$

Here U is a real invertible $n_S \times n_S$ matrix which only needs to satisfy

$$V = U^T U. \tag{8}$$

It is evident that U is not uniquely determined. It is only determined up to an orthogonal transformation. We may take advantage of this freedom to do additional orthogonal transformation to further diagonalize the scalar mass matrix while still keeping scalar kinetic terms in their canonically normalized form.

After the transformation Eq. (6) we obtain

$$\mathcal{L}_{\text{quad}} + \mathcal{L}_{gf} \supset \frac{1}{2}(\partial_\mu S_i)(\partial^\mu S_i) - \frac{1}{2}\xi^p \tilde{G}_p^2$$
$$- \frac{1}{2}((U^{-1})^T \mathbb{M}_G^2 U^{-1})_{ij} S_i S_j$$
$$- \frac{1}{2\xi^p}(\partial_\mu Z_p^\mu)^2 + \frac{1}{2}(\mathbb{M}_V^2)_{pq} Z_{p\mu} Z_q^\mu. \tag{9}$$

In the above equation $\tilde{G}_p$'s can be viewed as linear combinations of S_i's. It should be noted from a physical perspective that the n_S scalar degrees of freedom with which we started could be divided into two categories (after appropriate linear combinations if needed): unphysical scalars and physical scalars. Specifically, n_M unphysical scalars should exist and serve as unphysical Goldstones to be eaten by n_M gauge bosons to make them massive. The remaining $n_E = n_S - n_M$ scalar degrees of freedom then must be physical scalars. By virtue of this observation, there must exist an orthogonal transformation

$$\bar{S}_i = P_{ij}S_j \tag{10}$$

which diagonalizes the $-\frac{1}{2}((U^{-1})^T \mathbb{M}_G^2 U^{-1})_{ij} S_i S_j$ term. Then Eq. (9) becomes

$$\mathcal{L}_{\text{quad}} + \mathcal{L}_{gf} \supset \frac{1}{2}(\partial_\mu \bar{S}_i)(\partial^\mu \bar{S}_i) - \frac{1}{2}\xi^p \tilde{G}_p^2 - \frac{1}{2}\nu_r^2 \bar{S}_r^2$$
$$- \frac{1}{2\xi^p}(\partial_\mu Z_p^\mu)^2 + \frac{1}{2}(\mathbb{M}_V^2)_{pq} Z_{p\mu} Z_q^\mu. \tag{11}$$

The index r ranges from $n_M + 1$ to n_S (this will be assumed whenever we use the index r), and ν_r's depend only on model parameters, not on field variables. With this labeling convention the latter n_E fields in $\bar{S}_i$'s correspond to physical scalars while the remaining ones are unphysical Goldstone bosons. The matrix P and the ν_r's can be made independent of the ξ^p's, because in the course of diagonalizing the $-\frac{1}{2}((U^{-1})^T \mathbb{M}_G^2 U^{-1})_{ij} S_i S_j$ term, the $-\frac{1}{2}\xi^p \tilde{G}_p^2$ term is left untouched.

It is helpful to recall that in Eq. (11) the $\tilde{G}_p$'s can be viewed as linear combinations of $\bar{S}_i$'s. In fact, because n_E physical scalars must exist, the matrix P can be chosen so that the $\tilde{G}_p$'s do not contain the $\bar{S}_r$'s. That is to say, the $\tilde{G}_p$'s can be expressed as linear combinations of $\bar{S}_i$, $i = 1, 2, \ldots, n_M$. Therefore, by examining Eq. (11) it is obvious that in $\mathcal{L}_{\text{quad}} + \mathcal{L}_{gf}$ the n_E physical scalars are clearly separated from the unphysical ones after the orthogonal transformation Eq. (10).

At this stage we need to take a closer look at the unphysical scalar mass term in Eq. (11), which is

$$\mathcal{L}' \equiv -\frac{1}{2}\xi^p \tilde{G}_p^2. \tag{12}$$

Recalling that the $\tilde{G}_p$'s are linear combinations of $\bar{S}_i$, $i = 1, 2, \ldots, n_M$, the next thing we need to do is to find an orthogonal transformation

$$\tilde{S}_i = K_{ij}\bar{S}_j \tag{13}$$

which diagonalizes $\mathcal{L}'$. In Eq. (13) i, j range from 1 to n_S, and K is a $n_S \times n_S$ orthogonal matrix. Nevertheless, to avoid spoiling the already diagonalized physical scalar mass term, it is advisable to consider the following block-diagonal form of K:

$$K = \begin{pmatrix} K_M & \mathbf{0}_{n_M \times n_E} \\ \mathbf{0}_{n_E \times n_M} & I_{n_E \times n_E} \end{pmatrix}. \tag{14}$$

Here $I_{n_E \times n_E}$ is the $n_E \times n_E$ identity matrix, and K_M is a $n_M \times n_M$ orthogonal matrix. With this form of matrix K it is made clear that the $\bar{S}_r$'s actually do not get transformed in this step, however the $-\frac{1}{2}\xi^p \tilde{G}_p^2$ term is diagonalized by K_M.

It remains to find the $n_M \times n_M$ orthogonal matrix K_M. We note that $\mathcal{L}'$ written in the form of Eq. (12) is highly suggestive, because it has already completed the square. Therefore it seems natural to guess that the transformation we need is simply

$$\tilde{S}_p = \alpha_p \tilde{G}_p, \qquad p = 1, 2, ..., n_M \text{ (no summation over } p)$$
$$\tag{15}$$

Here the α_p's are constants chosen to make the transformed fields canonically normalized. Because the $\tilde{G}_p$'s can be expressed as linear combinations of $\bar{S}_i$, $i = 1, 2, ..., n_M$, Eq. (15) effectively leads to a transformation from $\bar{S}_i$, $i = 1, 2, ..., n_M$ to $\tilde{S}_i$, $i = 1, 2, ..., n_M$, from which the matrix K_M can be inferred.

There is one remaining potential loophole that we need to deal with. It is necessary to ensure that the matrix K_M inferred from Eq. (15) is indeed an orthogonal matrix, otherwise we will not be able to keep the scalar kinetic terms in their diagonalized and canonically normalized form.

To help determine whether the matrix K_M inferred from Eq. (15) is orthogonal we denote the real vector space spanned by G_i, $i = 1, 2, ..., n_S$ as $\mathbb{L}$ and introduce an inner product in $\mathbb{L}$, defined by

$$\langle S_i | S_j \rangle \equiv \delta_{ij}, \quad i, j = 1, 2, ..., n_S. \tag{16}$$

This means the S_i's constitute an orthonormal basis in $\mathbb{L}$. The inner product of any two elements in $\mathbb{L}$ can then be calculated by virtue of the linearity property of the inner product. It is obvious that the $\bar{S}_i$'s also form an orthonormal basis in $\mathbb{L}$. Based on simple algebraic knowledge the problem of judging whether K_M is orthogonal reduces to judging whether $\tilde{S}_p$, $p = 1, 2, ..., n_M$ form an orthonormal basis in the subspace spanned by themselves.

As long as all the $\tilde{G}_p$'s have positive norm, we may always adjust the α_p's so that

$$\langle \tilde{S}_p | \tilde{S}_p \rangle = 1, \quad \forall \ p = 1, 2, ..., n_M. \tag{17}$$

Therefore the question becomes whether $\langle \tilde{S}_p | \tilde{S}_q \rangle = 0$ holds when $p, q = 1, 2, ..., n_M$ and $p \neq q$. According to Eq. (15) we only need to check whether $\langle \tilde{G}_p | \tilde{G}_q \rangle = 0$ holds when $p, q = 1, 2, ..., n_M$ and $p \neq q$.

Fortunately, when the scalar fields are canonically normalized in their kinetic part, the vector-scalar two-point transitions in a gauge theory has the form [26]

$$i \sum_{nm\alpha} \partial_\mu \phi'_n t^\alpha_{nm} A^\mu_\alpha v_m. \tag{18}$$

Here ϕ'_n is the shifted scalar field with zero vacuum expectation value, v_m is the vacuum expectation value of the original scalar fields. t^α denotes the generator matrix with α being the adjoint index and A^μ_α is the corresponding gauge field. On the other hand, the elements of the gauge boson mass matrix are [26]

$$\mu^2_{\alpha\beta} = -\sum_{nml} t^\alpha_{nm} t^\beta_{nl} v_m v_l \tag{19}$$

Compare Eq. (18) and Eq. (19) it is easy to find for our case the useful property

$$\langle \tilde{G}_p | \tilde{G}_q \rangle = (\mathbb{M}^2_V)_{pq}, \quad \forall \ p, q = 1, 2, ..., n_M \tag{20}$$

A nonlinearly-realized scalar sector does not introduce additional difficulty in arriving at Eq. (20), because compared to the linearly realized case, the relevant differences begin from quadratic terms in the field expansion and do not affect Eq. (18) and Eq. (19).

Equation (20) suggests that if the gauge bosons are already in their mass eigenstates, then the related Goldstone boson vectors must be orthogonal to each other, which is exactly what we desire. Physically this implies that massive gauge bosons eat their corresponding Goldstone bosons along the directions dictated by their mass eigenstates. Therefore it would be desirable we rotate the gauge boson fields to their mass eigenstates before adding the gauge-fixing terms Eq. (4). This offers great convenience for the diagonalization of scalar mass matrix afterwards.

On the other hand, if the gauge-fixing terms in Eq. (4) are added when Z^μ_p's are not mass eigenstate fields, although this way of gauge-fixing is also legitimate, it would cause further inconveniences. First, after rotation to gauge boson mass eigenstates, the term $-\frac{1}{2\xi^p}(\partial_\mu Z^\mu_p)^2$ will induce kinetic mixing between gauge bosons in a general R_ξ gauge, spoiling the diagonalization of gauge boson kinetic terms. Second, from Eq. (20) it is obvious that now the $\tilde{G}_p$'s are not orthogonal to each other. Therefore the diagonalization of scalar mass terms would not be straightforward. Due to the above considerations in the following we adopt the procedure in which gauge-fixing terms Eq. (4) are added after rotating gauge boson fields to their mass eigenstates.

Suppose the gauge boson mass matrix $\mathbb{M}^2_V$ can be diagonalized as follows

$$R \mathbb{M}^2_V R^{-1} = \mathbb{M}^2_{DV} \equiv \text{diag}\{\mu^2_1, \mu^2_2, ..., \mu^2_{n_M}\} \tag{21}$$

Here R is a $n_M \times n_M$ orthogonal matrix, and $\mu^2_1, \mu^2_2, ..., \mu^2_{n_M}$ are positive. Let us define

$$G^m_p \equiv \frac{R_{pq}}{\mu_p} \tilde{G}_q = \frac{(RF)_{pi}}{\mu_p} G_i,$$
$$p = 1, 2, ..., n_M \text{ (no summation over } p) \tag{22}$$

(superscript m denotes canonically-normalized mass eigenstates). Now we can check with the help of Eq. (20)(no summation over p, q)

$$\langle G_p^m | G_q^m \rangle = \frac{1}{\mu_p \mu_q}(R\mathbb{M}_V^2 R^T)_{pq} = \delta_{pq}, \quad \forall \, p,q = 1,2,\ldots,n_M \tag{23}$$

We could further extend the definition of G_p^m to the states $\bar{S}_r$, $r = n_M + 1, \ldots, n_S$ which we have already obtained. According to our diagonalization of physical scalar mass term, $\bar{S}_r$ can be expressed as

$$\bar{S}_r = (PU)_{ri}G_i, \qquad r = n_M + 1, \ldots, n_S \tag{24}$$

where the matrix U and P are introduced in Eq. (6) and Eq. (10), respectively. Finally we can express G_i^m as follows

$$G_i^m = Q_{ij}G_j, \qquad i = 1,2,\ldots,n_S \tag{25}$$

where the $n_S \times n_S$ matrix Q is defined by (no summation over i)

$$Q_{ij} = \begin{cases} \frac{(RF)_{ij}}{\mu_i}, & i = 1,2,\ldots,n_M, \\ (PU)_{ij}, & i = n_M + 1, \ldots, n_S. \end{cases} \tag{26}$$

With the transformation matrix R and Q at our hand it will then be straightforward to derive any three-point or four-point interaction that we are interested in.

III. THE CASE OF SLH

A. Preparation for the calculation

The SLH model was proposed as a simple solution to the Higgs mass naturalness problem, making use of the collective symmetry breaking mechanism [27]. Its electroweak gauge group is enlarged to $SU(3)_L \times U(1)_X$, and two scalar triplets are introduced to realize the global symmetry breaking pattern

$$[SU(3)_1 \times U(1)_1] \times [SU(3)_2 \times U(1)_2]$$
$$\rightarrow [SU(2)_1 \times U(1)_1] \times [SU(2)_2 \times U(1)_2] \tag{27}$$

The scalar sector of the SLH model is usually written in a nonlinearly realized form. In this paper we follow the convention of [28] and parametrize the two scalar triplets as follows

$$\Phi_1 = \exp\left(\frac{i\Theta'}{f}\right) \exp\left(\frac{it_\beta\Theta}{f}\right) \begin{pmatrix} 0 \\ 0 \\ fc_\beta \end{pmatrix} \tag{28}$$

$$\Phi_2 = \exp\left(\frac{i\Theta'}{f}\right) \exp\left(-\frac{i\Theta}{ft_\beta}\right) \begin{pmatrix} 0 \\ 0 \\ fs_\beta \end{pmatrix}. \tag{29}$$

Here we introduced the shorthand notation $s_\beta \equiv \sin\beta$, $c_\beta \equiv \cos\beta$, $t_\beta \equiv \tan\beta$. f is the Goldstone decay constant which is supposed to be at least a few TeV. Θ and Θ' are 3×3 matrix fields, defined by

$$\Theta = \frac{\eta}{\sqrt{2}} + \begin{pmatrix} \mathbf{0}_{2\times2} & h \\ h^\dagger & 0 \end{pmatrix}, \qquad \Theta' = \frac{\zeta}{\sqrt{2}} + \begin{pmatrix} \mathbf{0}_{2\times2} & k \\ k^\dagger & 0 \end{pmatrix} \tag{30}$$

where h and k are parametrized as ($v \approx 246$ GeV denotes the vacuum expectation value of the Higgs doublet)

$$h = \begin{pmatrix} h^0 \\ h^- \end{pmatrix}, \qquad h^0 = \frac{1}{\sqrt{2}}(v + H - i\chi) \tag{31}$$

$$k = \begin{pmatrix} k^0 \\ k^- \end{pmatrix}, \qquad k^0 = \frac{1}{\sqrt{2}}(\sigma - i\omega) \tag{32}$$

The covariant derivative in the electroweak sector can be written as

$$D_\mu = \partial_\mu - igA_\mu^a T^a + ig_x Q_x B_\mu^x, \qquad g_x = \frac{gt_W}{\sqrt{1 - t_W^2/3}}. \tag{33}$$

Here $t_W \equiv \tan\theta_W$. A_μ^a and B_μ^x denote the $SU(3)_L$ and $U(1)_X$ gauge fields, respectively. The $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge quantum number of Φ_1, Φ_2 is $(\mathbf{1}, \mathbf{3})_{-\frac{1}{3}}$, therefore for Φ_1, Φ_2, $Q_x = -\frac{1}{3}$, and $A_\mu^a T^a$ can be written as

$$A_\mu^a T^a = \frac{A_\mu^3}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{A_\mu^8}{2\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$
$$+ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & W_\mu^+ & Y_\mu^0 \\ W_\mu^- & 0 & X_\mu^- \\ Y_\mu^{0\dagger} & X_\mu^+ & 0 \end{pmatrix}. \tag{34}$$

The gauge kinetic terms for Φ_1, Φ_2 are

$$\mathcal{L}_{gk} = (D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2). \tag{35}$$

The first order (in $\frac{v}{f}$) gauge boson mixing for A^3, A^8, B_x takes the form

$$\begin{pmatrix} A^3 \\ A^8 \\ B_x \end{pmatrix} = \begin{pmatrix} 0 & c_W & -s_W \\ \sqrt{1 - \frac{t_W^2}{3}} & \frac{s_W t_W}{\sqrt{3}} & \frac{s_W}{\sqrt{3}} \\ -\frac{t_W}{\sqrt{3}} & s_W\sqrt{1 - \frac{t_W^2}{3}} & c_W\sqrt{1 - \frac{t_W^2}{3}} \end{pmatrix} \begin{pmatrix} Z' \\ Z \\ A \end{pmatrix}. \tag{36}$$

We note that Z', Z are not the ultimate mass eigenstate fields. For future convenience we split the Y^0 field into real and imaginary parts

$$Y_\mu^0 \equiv \frac{1}{\sqrt{2}}(Y_{R\mu} + iY_{I\mu}), \qquad Y_\mu^{0\dagger} \equiv \frac{1}{\sqrt{2}}(Y_{R\mu} - iY_{I\mu}) \quad (37)$$

In this paper we intend to focus on the neutral sector, in which there are six scalar degrees of freedom: $\eta, \zeta, H, \chi, \sigma, \omega$. Four degrees of freedom will be eaten to give mass to massive neutral gauge bosons and are unphysical. The remaining two are physical and need to play the role of the observed Higgs-like boson and the pseudoaxion which has been discussed a lot in the literature. The pseudoaxion actually corresponds to the Goldstone boson of a spontaneously broken global $U(1)$ symmetry in the SLH. To give it a mass, the so-called "μ term" needs to be introduced

$$\mathcal{L}_\mu = \mu^2(\Phi_1^\dagger \Phi_2 + \text{H.c.}) \qquad (38)$$

The observed Higgs-like boson will acquire its mass from the Coleman-Weinberg potential (however the μ term will also contribute to its potential). Because $\mathcal{L}_{gk}$, $\mathcal{L}_\mu$ and the Coleman-Weinberg potential conserve CP, it will be convenient to group the neutral bosons into the CP-even and CP-odd sectors: H, σ, Y_R belong to the CP-even sector, while $\eta, \zeta, \chi, \omega, Z', Z, Y_I, A$ belong to the CP-odd sector. There are no two-point transitions between these two sectors.

Some comments concerning the parametrization of Φ_1, Φ_2 in Eq. (28) and Eq. (29) are in order. First, we have chosen to retain the heavy sector fields in Θ', rather than omitting them from the beginning. Apparently the omission of Θ' can be justified by doing a $SU(3)_L$ gauge transformation. This justification is valid, and in the more precise language of Faddeev-Popov gauge-fixing, the omission of Θ' actually corresponds to a certain choice of the gauge-fixing function. However, this omission could lead to future inconvenience, since as we will show, $\mathcal{L}_{gk}$ contains two-point transitions between heavy sector gauge bosons and the pseudo-axion. Θ' can be rotated away by a gauge transformation but heavy sector gauge bosons cannot. This means that when doing perturbation theory we need to always carry those two-point vector-scalar transitions, which are quite inconvenient. Nevertheless, the omission of Θ' and heavy sector gauge bosons can indeed be convenient if we only need to obtain the $\mathcal{O}(\frac{v}{f})$ coefficient of the mass eigenstate $ZH\eta$ vertex, since the effect of those omitted two-point vector-scalar transitions will be suppressed due to the heavy gauge boson masses. Second, we have chosen to parametrize Φ_1, Φ_2 with two exponentials for each, rather than use a single exponential like

$$\Phi_{1,SE} = \exp\left[\frac{i}{f}(\Theta' + t_\beta \Theta)\right] \begin{pmatrix} 0 \\ 0 \\ fc_\beta \end{pmatrix} \qquad (39)$$

Also, in Eq. (28) and Eq. (29) the exponential of Θ' has been put to the left of the exponential of Θ. For non-commutative matrices the single exponential parametrization is not mathematically equivalent to the double exponential parametrization. Moreover, the double exponential parametrization will depend on the order of the two exponentials. However, these parametrizations are related to each other by field redefinition and should thus be physically equivalent. Which one to use is a matter of convenience. We choose the double exponential parametrization in Eq. (28) and Eq. (29) because it does not introduce mass mixing between heavy and light sector scalars in $\mathcal{L}_\mu$ and will thus facilitate the mass diagonalization.

The aim of this section is to derive the mass eigenstate $ZH\eta$ vertex in the SLH. With the current double exponential parametrization it is possible to demonstrate that H does not mix with σ, and the scalar kinetic terms are already canonically-normalized in the CP-even sector. Also, the μ term gives η a mass but does not introduce mass mixing between η and other fields. According to our argument in the previous section this means that after all the diagonalization procedure is completed, the whole effect on η is supposed to be a simple rescaling. This offers great convenience for the derivation of the mass eigenstate $ZH\eta$ vertex. The needed rescaling factor can be easily computed. Going back to the notation of Sec. II, the inner product between two Goldstone bosons G_i and G_j in Eq. (2) satisfies

$$\begin{aligned} \langle G_i | G_j \rangle &= (U^{-1})_{ik}(U^{-1})_{jl}\langle S_k | S_l \rangle \\ &= (U^{-1})_{ik}(U^{-1})_{jl}\delta_{kl} = (U^{-1})_{ik}(U^{-1})_{jk} \\ &= (V^{-1})_{ij}. \end{aligned} \qquad (40)$$

We employ the convention that $\eta, \zeta, \chi, \omega$ correspond to indices 1,2,3,4 respectively, therefore

$$\langle \eta | \eta \rangle = (V^{-1})_{11}. \qquad (41)$$

Consequently, the ultimate mass eigenstate field η^m is related to η through

$$\eta = \sqrt{(V^{-1})_{11}}\, \eta^m. \qquad (42)$$

To obtain the mass eigenstate $ZH\eta$ vertex, we also need to know the component of η^m in ζ, χ, ω. For the case of the SLH, let us denote the CP-odd sector elements of the matrix F introduced in Eq. (2) as

$$F = \begin{pmatrix} F_{Z\eta} & F_{Z\zeta} & F_{Z\chi} & F_{Z\omega} \\ F_{Z'\eta} & F_{Z'\zeta} & F_{Z'\chi} & F_{Z'\omega} \\ F_{Y\eta} & F_{Y\zeta} & F_{Y\chi} & F_{Y\omega} \end{pmatrix}. \qquad (43)$$

(We assume for the *CP*-odd sector gauge boson mass matrix, the first, second, and third row/column correspond to Z, Z', Y_I, respectively.) In the third row, $F_{Y\eta}$ denotes the coefficient of the two-point transition $Y_I^\mu \partial_\mu \eta$ (similar for $F_{Y\zeta}$, $F_{Y\chi}$, $F_{Y\omega}$). Due to *CP*-conservation there is no two-point transition between Y_R^μ and the *CP*-odd scalars, therefore no confusion would arise. The photon field A^μ does not have two-point transition with scalars. We would like to denote the submatrix formed by the second, third, and fourth column of F as $\tilde{F}$

$$\tilde{F} \equiv \begin{pmatrix} F_{Z\zeta} & F_{Z\chi} & F_{Z\omega} \\ F_{Z'\zeta} & F_{Z'\chi} & F_{Z'\omega} \\ F_{Y\zeta} & F_{Y\chi} & F_{Y\omega} \end{pmatrix}. \tag{44}$$

Now the application of Eq. (25) and Eq. (26) to the *CP*-odd scalar sector of the SLH leads to

$$\begin{pmatrix} \zeta^m \\ \chi^m \\ \omega^m \end{pmatrix} = \mathbb{M}_{DV}^{-1} R \left[\begin{pmatrix} F_{Z\eta} \\ F_{Z'\eta} \\ F_{Y\eta} \end{pmatrix} \eta + \tilde{F} \begin{pmatrix} \zeta \\ \chi \\ \omega \end{pmatrix} \right]. \tag{45}$$

As before the superscript m denotes canonically normalized mass eigenstate fields. Inverting Eq. (45) and using Eq. (42) will lead to

$$\begin{pmatrix} \zeta \\ \chi \\ \omega \end{pmatrix} = \tilde{F}^{-1} R^T \mathbb{M}_{DV} \begin{pmatrix} \zeta^m \\ \chi^m \\ \omega^m \end{pmatrix} - \sqrt{(V^{-1})_{11}} \tilde{F}^{-1} \begin{pmatrix} F_{Z\eta} \\ F_{Z'\eta} \\ F_{Y\eta} \end{pmatrix} \eta^m. \tag{46}$$

We define the four-component column vector

$$\Upsilon \equiv \begin{pmatrix} \sqrt{(V^{-1})_{11}} \\ -\sqrt{(V^{-1})_{11}} \tilde{F}^{-1} \begin{pmatrix} F_{Z\eta} \\ F_{Z'\eta} \\ F_{Y\eta} \end{pmatrix} \end{pmatrix} \tag{47}$$

and denote the first row of R as $\mathbb{R}_1$

$$\mathbb{R}_1 = (R_{11} \quad R_{12} \quad R_{13}) \tag{48}$$

where R_{ij} represents the $(i;j)$ element of R. We will also need the coefficient matrices

$$\mathbb{C}^{dH} = \begin{pmatrix} C_{Z\eta}^{dH} & C_{Z\zeta}^{dH} & C_{Z\chi}^{dH} & C_{Z\omega}^{dH} \\ C_{Z'\eta}^{dH} & C_{Z'\zeta}^{dH} & C_{Z'\chi}^{dH} & C_{Z'\omega}^{dH} \\ C_{Y\eta}^{dH} & C_{Y\zeta}^{dH} & C_{Y\chi}^{dH} & C_{Y\omega}^{dH} \end{pmatrix},$$

$$\mathbb{C}^{Hd} = \begin{pmatrix} C_{Z\eta}^{Hd} & C_{Z\zeta}^{Hd} & C_{Z\chi}^{Hd} & C_{Z\omega}^{Hd} \\ C_{Z'\eta}^{Hd} & C_{Z'\zeta}^{Hd} & C_{Z'\chi}^{Hd} & C_{Z'\omega}^{Hd} \\ C_{Y\eta}^{Hd} & C_{Y\zeta}^{Hd} & C_{Y\chi}^{Hd} & C_{Y\omega}^{Hd} \end{pmatrix}. \tag{49}$$

Here $C_{Z\eta}^{dH}$ denotes the coefficient of $Z^\mu \eta \partial_\mu H$, while $C_{Z\eta}^{Hd}$ denotes the coefficient of $Z^\mu H \partial_\mu \eta$, and so on. If we have calculated the matrices $\mathbb{C}^{dH}$, $\mathbb{C}^{Hd}$ and the vectors Υ and $\mathbb{R}_1$, then the coefficient of mass eigenstate antisymmetric $ZH\eta$ vertex ($Z^\mu(\eta\partial_\mu H - H\partial_\mu \eta)$ with all fields understood to be mass eigenstate fields) can be obtained as

$$c_{ZH\eta}^{as} = \frac{\mathbb{R}_1 \mathbb{C}^{dH} \Upsilon - \mathbb{R}_1 \mathbb{C}^{Hd} \Upsilon}{2} \tag{50}$$

while the coefficient of mass eigenstate symmetric $ZH\eta$ vertex ($Z^\mu(\eta\partial_\mu H + H\partial_\mu \eta)$ with all fields understood to be mass eigenstate fields) can be obtained as

$$c_{ZH\eta}^{s} = \frac{\mathbb{R}_1 \mathbb{C}^{dH} \Upsilon + \mathbb{R}_1 \mathbb{C}^{Hd} \Upsilon}{2}. \tag{51}$$

Here we remark that we divide a general VSS vertex into its antisymmetric and symmetric parts because they exhibit distinct features in physical processes. For example, the symmetric VSS vertex does not contribute when the involved vector boson is on shell. Therefore, only the antisymmetric $ZH\eta$ vertex is expected to contribute at tree level to decay processes $H \to Z\eta$ (or $\eta \to ZH$ if η is heavy) where Z is supposed to be on shell.

B. Results

In principle the derivation of mass eigenstate $ZH\eta$ vertex with no expansion on the $\frac{v}{f}$ can be carried out manually.[5] However, after obtaining V, F and $\mathbb{M}_V^2$, the calculation of R and the inverse matrices can become extremely cumbersome. Therefore we choose to compute the mass eigenstate $ZH\eta$ vertex to $\mathcal{O}((\frac{v}{f})^3)$, which makes the results easier to obtain and display. For brevity we define $\xi \equiv \frac{v}{f}$ in the following.

Let us first find the scalar kinetic matrix V and vector-scalar transition matrix F for the SLH. They are computed to be

---

[5]In practice, they can be more readily obtained with the help of *Mathematica*.

$$
V = \begin{pmatrix}
1 & 0 & \frac{\sqrt{2}}{t_{2\beta}}\xi - \frac{7c_{2\beta}+c_{6\beta}}{6\sqrt{2}s_{2\beta}^3}\xi^3 & -\sqrt{2}\xi + \frac{5+3c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3 \\
0 & 1 & -\frac{1}{\sqrt{2}}\xi + \frac{5+3c_{4\beta}}{12\sqrt{2}s_{2\beta}^2}\xi^3 & -\frac{2\sqrt{2}}{3t_{2\beta}}\xi^3 \\
\frac{\sqrt{2}}{t_{2\beta}}\xi - \frac{7c_{2\beta}+c_{6\beta}}{6\sqrt{2}s_{2\beta}^3}\xi^3 & -\frac{1}{\sqrt{2}}\xi + \frac{5+3c_{4\beta}}{12\sqrt{2}s_{2\beta}^2}\xi^3 & 1 - \frac{5+3c_{4\beta}}{12s_{2\beta}^2}\xi^2 & \frac{2}{3t_{2\beta}}\xi^2 \\
-\sqrt{2}\xi + \frac{5+3c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3 & -\frac{2\sqrt{2}}{3t_{2\beta}}\xi^3 & \frac{2}{3t_{2\beta}}\xi^2 & 1
\end{pmatrix} + \mathcal{O}(\xi^4) \tag{52}
$$

$$
F = gf \begin{pmatrix}
\frac{1}{\sqrt{2}c_W t_{2\beta}}\xi^2 & -\frac{1}{2\sqrt{2}c_W}\xi^2 & \frac{1}{2c_W}\xi - \frac{5+3c_{4\beta}}{24c_W s_{2\beta}^2}\xi^3 & \frac{1}{3c_W t_{2\beta}}\xi^3 \\
\frac{\rho}{t_{2\beta}}\xi^2 & \frac{\sqrt{2}}{\sqrt{3-t_W^2}} - \frac{1+2c_{2W}}{2\sqrt{2}c_W^2\sqrt{3-t_W^2}}\xi^2 & \kappa\xi - \frac{\kappa(5+3c_{4\beta})}{12s_{2\beta}^2}\xi^3 & -\frac{1}{3c_W^2\sqrt{3-t_W^2}t_{2\beta}}\xi^3 \\
-\xi + \frac{5+3c_{4\beta}}{6s_{2\beta}^2}\xi^3 & -\frac{2}{3t_{2\beta}}\xi^3 & \frac{\sqrt{2}}{3t_{2\beta}}\xi^2 & \frac{1}{\sqrt{2}}
\end{pmatrix} + \mathcal{O}(\xi^4) \tag{53}
$$

where we defined

$$
\rho \equiv \sqrt{\frac{1+2c_{2W}}{1+c_{2W}}}, \qquad \kappa \equiv \frac{c_{2W}}{2c_W^2\sqrt{3-t_W^2}}. \tag{54}
$$

It is obvious from Eq. (52) that the scalar kinetic terms in the original η, ζ, χ, ω are not canonically normalized, and also obvious from Eq. (53) that there are general vector-scalar two-point transitions. Especially, the two-point $Z\eta$ transition appears at $\mathcal{O}(\xi^2)$, only one order of ξ relatively suppressed when compared to $Z\chi$ transition.[6] The appearance of these non-canonically normalized kinetic terms and unexpected[7]; vector-scalar transitions is the exact reason for introducing the systematic procedure in Sec. II.

The Υ vector is computed to be

$$
\Upsilon = \begin{pmatrix}
1 + \frac{1}{s_{2\beta}^2}\xi^2 + \mathcal{O}(\xi^4) \\
-\frac{1}{t_{2\beta}}\xi^2 + \mathcal{O}(\xi^4) \\
-\frac{\sqrt{2}}{t_{2\beta}}\xi - \frac{3-c_{4\beta}}{\sqrt{2}s_{2\beta}^2 t_{2\beta}}\xi^3 + \mathcal{O}(\xi^5) \\
\sqrt{2}\xi + \frac{3-c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3 + \mathcal{O}(\xi^5)
\end{pmatrix}. \tag{55}
$$

---

[6]Although the two-point $Z\eta$ transition appears at $\mathcal{O}(\xi^2)$, the elimination of this part require an $\mathcal{O}(\xi)$ field redefinition, due to the fact that the relative suppression of $Z\eta$ transition to $Z\chi$ transition is $\mathcal{O}(\xi)$. The $ZH\chi$ coupling is $\mathcal{O}(1)$. Therefore, the removal of $Z\eta$ transition could lead to an $\mathcal{O}(\xi)$ change in the derived $ZH\eta$ vertex.

[7]By unexpected we refer to the fact that η is considered physical, yet there exist two-point transitions such as $Z^\mu \partial_\mu \eta$ in $\mathcal{L}_{gk}$.

A compact expression for Υ valid to all orders in ξ can also be obtained. It is

$$
\Upsilon = \begin{pmatrix}
c_{\gamma+\delta}^{-1} \\
-c_{\gamma+\delta}^{-1}(s_\delta^2 t_\beta - s_\gamma^2 t_\beta^{-1}) \\
\frac{v}{\sqrt{2}f}c_{\gamma+\delta}^{-1}(c_{2\delta}t_\beta - c_{2\gamma}t_\beta^{-1}) \\
\frac{1}{2}c_{\gamma+\delta}^{-1}(s_{2\delta}t_\beta + s_{2\gamma}t_\beta^{-1})
\end{pmatrix} \tag{56}
$$

where

$$
\gamma \equiv \frac{vt_\beta}{\sqrt{2}f}, \qquad \delta \equiv \frac{v}{\sqrt{2}ft_\beta}. \tag{57}
$$

Expanding the above expression to $\mathcal{O}(\xi^3)$, Eq. (55) can be recovered. The above expression for the Υ vector is very useful in derivation of exact results of tree level vertices involving the η particle. The $\mathbb{C}^{dH}$ matrix is computed to be

$$
\mathbb{C}^{dH} = \begin{pmatrix}
0 & 0 & -\frac{g}{2c_W} + \frac{g(5+3c_{4\beta})}{24c_W s_{2\beta}^2}\xi^2 + \mathcal{O}(\xi^4) & 0 \\
0 & 0 & -\frac{g(1-t_W^2)}{2\sqrt{3-t_W^2}} + \frac{g\kappa(5+3c_{4\beta})}{12s_{2\beta}^2}\xi^2 + \mathcal{O}(\xi^4) & 0 \\
0 & 0 & -\frac{\sqrt{2}g}{3t_{2\beta}}\xi + \frac{g(7c_{2\beta}+c_{6\beta})}{30\sqrt{2}s_{2\beta}^3}\xi^3 + \mathcal{O}(\xi^5) & 0
\end{pmatrix}. \tag{58}
$$

The $\mathbb{C}^{Hd}$ matrix is computed to be

$$\mathbb{C}^{Hd} = \begin{pmatrix} \frac{\sqrt{2}g}{c_W t_{2\beta}}\xi - \frac{g(7c_{2\beta}+c_{6\beta})}{3\sqrt{2}c_W s_{2\beta}^3}\xi^3 & -\frac{g}{\sqrt{2}c_W}\xi + \frac{g(5+3c_{4\beta})}{6\sqrt{2}c_W s_{2\beta}^2}\xi^3 & \frac{g}{2c_W} - \frac{g(5+3c_{4\beta})}{8c_W s_{2\beta}^2}\xi^2 & \frac{g}{c_W t_{2\beta}}\xi^2 \\[2mm] \frac{2g\rho}{t_{2\beta}}\xi - \frac{g\rho(7c_{2\beta}+c_{6\beta})}{3s_{2\beta}^3}\xi^3 & -g\rho\xi + \frac{g\rho(5+3c_{4\beta})}{6s_{2\beta}^2}\xi^3 & g\kappa - \frac{g\kappa(5+3c_{4\beta})}{4s_{2\beta}^2}\xi^2 & -\frac{g}{c_W^2\sqrt{3-t_W^2}t_{2\beta}}\xi^2 \\[2mm] -g + \frac{g(5+3c_{4\beta})}{2s_{2\beta}^2}\xi^2 & -\frac{2g}{t_{2\beta}}\xi^2 & \frac{2\sqrt{2}g}{3t_{2\beta}}\xi - \frac{\sqrt{2}g(7c_{2\beta}+c_{6\beta})}{15s_{2\beta}^3}\xi^3 & 0 \end{pmatrix} + \mathcal{O}(\xi^4). \quad (59)$$

The matrix R can be computed as

$$R = \begin{pmatrix} 1 + \mathcal{O}(\xi^4) & -\frac{c_{2W}(1+2c_{2W})}{8c_W^5\sqrt{3-t_W^2}}\xi^2 + \mathcal{O}(\xi^4) & -\frac{\sqrt{2}}{3c_W t_{2\beta}}\xi^3 + \mathcal{O}(\xi^5) \\[2mm] \frac{c_{2W}(1+2c_{2W})}{8c_W^5\sqrt{3-t_W^2}}\xi^2 + \mathcal{O}(\xi^4) & 1 + \mathcal{O}(\xi^4) & -\frac{\sqrt{2}(1+2c_{2W})}{3c_W^2\sqrt{3-t_W^2}t_{2\beta}}\xi^3 + \mathcal{O}(\xi^5) \\[2mm] \frac{\sqrt{2}}{3c_W t_{2\beta}}\xi^3 + \mathcal{O}(\xi^5) & \frac{\sqrt{2}(1+2c_{2W})}{3c_W^2\sqrt{3-t_W^2}t_{2\beta}}\xi^3 + \mathcal{O}(\xi^5) & 1 + \mathcal{O}(\xi^6) \end{pmatrix}. \quad (60)$$

With this precision it is feasible to obtain $c_{ZH\eta}^{as}$ and $c_{ZH\eta}^{s}$ via Eq. (50) and Eq. (51) to $\mathcal{O}(\xi^3)$, the results of which are

$$c_{ZH\eta}^{as} = -\frac{g}{4\sqrt{2}c_W^3 t_{2\beta}}\xi^3 + \mathcal{O}(\xi^5) \quad (61)$$

$$c_{ZH\eta}^{s} = \frac{g}{\sqrt{2}c_W t_{2\beta}}\xi + \frac{g}{24\sqrt{2}c_W s_{2\beta}}\left[\frac{8}{s_{2\beta}t_{2\beta}} + 3c_{2\beta}\left(8 + \frac{6}{c_W^2} - \frac{1}{c_W^4}\right)\right]\xi^3 + \mathcal{O}(\xi^5). \quad (62)$$

Therefore we arrive at the conclusion that the symmetric $ZH\eta$ vertex appear at $\mathcal{O}(\xi)$, while the antisymmetric $ZH\eta$ vertex does not appear until $\mathcal{O}(\xi^3)$. The coefficients of these two vertices are presented in Eq. (62) and Eq. (61), respectively. We note that this conclusion differs from what has been derived and used in the literature [13,14] for a long time. In the intermediate steps, one important discrepancy between our results and Ref. [13] is that in a footnote Ref. [13] claims that choosing the η generator to be the identity matrix would remove the kinetic mixing between η and unphysical Goldstone bosons, while in our derivation Eq. (52) shows there still exists $\mathcal{O}(\xi)$ kinetic mixing of such kind, which we have checked by various means. It is then not clear whether Refs. [13,14] have made appropriate field redefinitions to diagonalize the SLH vector-scalar system.

C. Effective field theory analysis

The fact that the mass eigenstate antisymmetric $ZH\eta$ vertex does not appear until $\mathcal{O}(\xi^3)$ can be understood from an effective field theory (EFT) point of view. Let us focus on the bosonic sector of the SLH, and integrate out heavy sector fields X, Y, Z' and their Goldstones. We are then interested in the EFT formed with the remaining fields, namely the SM and η, which are classified according to gauge transformation properties. Especially, η is a singlet under the SM gauge symmetries. Let us suppose at this moment we have not added the gauge-fixing terms yet. It is obvious that at dimension-four level no gauge-invariant

operator can deliver a $ZH\eta$ vertex. We are then forced to consider higher-dimensional operators. At dimension-five level, let us consider

$$\mathcal{O}_1 = (\partial^\mu\eta)[ih^\dagger(D_\mu - \overleftarrow{D_\mu})h] \quad (63)$$

where $h^\dagger\overleftarrow{D_\mu}h \equiv (D_\mu h)^\dagger h$ and D_μ denotes the SM covariant derivative for the Higgs doublet. We may denote its coefficient as $\frac{c_1}{f}$, in which c_1 is a dimensionless constant. Then we could find in the Lagrangian the following terms

$$\mathcal{L} \supset (D_\mu h)^\dagger(D^\mu h) + \frac{1}{2}(\partial_\mu\eta)^2 + \frac{c_1}{f}\mathcal{O}_1$$

$$\supset \frac{1}{2}(\partial_\mu H)^2 + \frac{1}{2}(\partial_\mu\chi)^2 + \frac{1}{2}(\partial_\mu\eta)^2 + \frac{v}{f}c_1(\partial^\mu\eta)(\partial_\mu\chi)$$

$$- m_Z Z_\mu \partial^\mu\left(\chi + \frac{v}{f}c_1\eta\right) + \frac{m_Z}{v}Z_\mu(\chi\partial^\mu H - H\partial^\mu\chi)$$

$$- \frac{2m_Z}{f}c_1 H Z_\mu \partial^\mu\eta. \quad (64)$$

The appearance of scalar kinetic mixing $(\partial^\mu\eta)(\partial_\mu\chi)$ and vector-scalar two-point transition $Z_\mu\partial^\mu\eta$ signal the need for a further field redefinition in the scalar sector. Up to $\mathcal{O}(\xi)$, the transformation is easily found:

$$\tilde{\chi} = \chi + \frac{v}{f}c_1\eta, \quad (65)$$

$$\tilde{\eta} = \eta. \tag{66}$$

The Lagrangian can be written with the transformed fields

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu H)^2 + \frac{1}{2}(\partial_\mu \tilde{\chi})^2 + \frac{1}{2}(\partial_\mu \tilde{\eta})^2 - m_Z Z_\mu \partial^\mu \tilde{\chi}$$
$$+ \frac{m_Z}{v} Z_\mu (\tilde{\chi}\partial^\mu H - H\partial^\mu \tilde{\chi}) - c_1 \frac{m_Z}{f} Z_\mu (\tilde{\eta}\partial^\mu H + H\partial^\mu \tilde{\eta}). \tag{67}$$

The two-point vector-scalar transition $-m_Z Z_\mu \partial^\mu \tilde{\chi}$ can be eliminated by an appropriate R_ξ gauge-fixing term. From the above expression we see that at $\mathcal{O}(\xi)$, only symmetric mass eigenstate $ZH\eta$ vertex could survive while the antisymmetric counterpart is removed after the transition to mass eigenstate. This is similar to the situation considered in Ref. [29] which also concluded for the case of the SM plus a singlet scalar S that the dimension-five operator cannot give rise to tree-level $S \to ZH$ decay.

At dimension-six level, let us consider the operator

$$\mathcal{O}_2 = (h^\dagger D^\mu h)(h^\dagger D_\mu h). \tag{68}$$

This operator should have a coefficient of $\mathcal{O}(\frac{1}{f^2})$. Apparently it does not contain η. However, if $\mathcal{O}_1$ is also present, then a field redefinition like Eq. (66) needs to be performed, after which $\mathcal{O}_2$ could lead to a mass eigenstate antisymmetric $ZH\eta$ vertex. Since the field redefinition implies an $\mathcal{O}(\xi)$ η component in χ, the resultant mass eigenstate antisymmetric $ZH\eta$ vertex should appear at $\mathcal{O}(\xi^3)$.

We may also consider operators with even higher dimension, but of course they cannot lead to $\mathcal{O}(\xi)$ or $\mathcal{O}(\xi^2)$ mass eigenstate antisymmetric $ZH\eta$ vertex.

Other bosonic operators (containing Z) at dimension-five or six level can be considered, for example

$$\mathcal{O}_3 = \eta(D_\mu h)^\dagger (D^\mu h) \tag{69}$$

$$\mathcal{O}_4 = \partial^\mu (h^\dagger h)[ih^\dagger (D_\mu - \overleftarrow{D_\mu})h] \tag{70}$$

However, these operators do not have the correct CP property. Furthermore, in our parametrization η has a shift symmetry $\eta \to \eta + c$ where c is a constant, which also forbids the appearance of $\mathcal{O}_3$.

Therefore from an EFT analysis, we also arrive at the conclusion that in the SLH, mass eigenstate antisymmetric $ZH\eta$ vertex cannot appear until $\mathcal{O}(\xi^3)$ while symmetric $ZH\eta$ vertex can appear at $\mathcal{O}(\xi)$,[8] consistent with our explicit calculation in the previous subsection. It is important to

note that all of the EFT derivation is based on the field content SM + η (η is a CP-odd singlet[9]), with no additional particles leading to further mass mixings, which could alter the conclusion.

IV. DISCUSSION AND CONCLUSION

In this paper we revisited the issue of deriving the mass eigenstate $ZH\eta$ vertex in the SLH. We found that the scalar kinetic terms are not canonically normalized in the usual parametrization and there are unexpected vector-scalar two-point transitions that need to be taken care of. We formulated the problem in a generic setting as the diagonalization of a vector-scalar system in gauge field theories. Especially we proved that the scalar mass terms coming from the R_ξ gauge-fixing procedure will be automatically orthogonal to each other if the corresponding gauge fields are rotated to their mass eigenstate prior to gauge-fixing.[10] This fact greatly simplifies the diagonalization procedure.

For the SLH model, we found that the double exponential parametrization of scalar triplets, as shown in Eq. (28) and Eq. (29) is convenient for the derivation of $ZH\eta$ vertex, since in this parametrization the η field is only subject to a simple rescaling in the diagonalization procedure, with which we could display in a simple form the η^m component contained in the original $\eta, \zeta, \chi, \omega$ fields we started with, as shown in Eq. (55).

In principle the derivation of mass eigenstate $ZH\eta$ vertex could be worked out to all order in $\xi \equiv \frac{v}{f}$, however the intermediate results are too lengthy and we find it convenient to display the derivation and results to $\mathcal{O}(\xi^3)$. The final results of antisymmetric and symmetric $ZH\eta$ vertices are shown in Eq. (61) and Eq. (62). Contrary to what has existed in the literature [13,14] (which claims an $\mathcal{O}(\xi)$ antisymmetric $ZH\eta$ vertex) for a long time, we found that the coefficient of the antisymmetric $ZH\eta$ vertex $c_{ZH\eta}^{as}$ does not show up until $\mathcal{O}(\xi^3)$. This result is also understood from an EFT point of view. Based on these results we expect that the exotic Higgs decay $H \to Z\eta$ (or $\eta \to ZH$ if η is heavy) and the associated production of h and η at hadron or lepton colliders will be much more difficult to observe due to the $\mathcal{O}(\xi^3)$ suppression in the antisymmetric $ZH\eta$ vertex. On the other hand, the symmetric $ZH\eta$ vertex already appears at $\mathcal{O}(\xi)$, however the investigation of its effect involves some subtleties, which will be treated in a follow-up paper.

The procedure elucidated in this paper can be applied to other models containing a gauged nonlinearly-realized scalar sector as well. From the experience with the SLH we find it important to examine the quadratic part of

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[8]According to Ref. [30], a similar situation occurs for the $ZH\phi_0$ vertex in the left-right twin Higgs model, where ϕ_0 denotes a neutral pseudoscalar. This is consistent with our EFT analysis here, since ϕ_0 does not mix with other physical fields due to an imposed discrete symmetry.

[9]Reference [31] studied the composite two-Higgs-doublet model which contains $\mathcal{O}(1)$ antisymmetric ZHA vertex since the pseudoscalar A is not a singlet.

[10]We refer the reader to Ref. [32] for another example in the littlest Higgs with T-parity.

the Lagrangian in these models, which could contain non-canonically normalized scalar kinetic terms and unexpected vector-scalar two-point transitions. Moreover, finding a convenient parametrization for the exponentials in these models could be very helpful in the diagonalization procedure. We expect to investigate these issues and their phenomenological implications in the future.

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