Exclusive vector meson photoproduction with a leading baryon in photon-hadron interactions at hadronic colliders

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Exclusive vector meson photoproduction associated with a leading baryon ($B = n, \Delta^+, \Delta^0$) in pp and pA collisions at RHIC and LHC energies is investigated using the color dipole formalism and taking into account nonlinear effects in the QCD dynamics. In particular, we compute the cross sections for ρ, ϕ and J/Ψ production together with a Δ and compare the predictions with those obtained for a leading neutron. Our results show that the $V + \Delta$ cross section is almost 30% of the $V + n$ one. Our results also show that a future experimental analysis of these processes is, in principle, feasible and can be useful to study leading particle production.

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A good description of particle production at forward rapidities and high energies is fundamental to our understanding of collider and cosmic ray physics [1]. Recent results indicate that an accurate knowledge of the leading particle momentum spectrum and its energy dependence is crucial for the interpretation of cosmic ray data (See e.g. Ref. [2]). Additionally, as at forward rapidities and high energies we probe the small Bjorken—x components of the target wave function, particle production in this kinematical range is directly connected to the QCD nonlinear dynamics at high energies [3].

The successful operation of the HERA ep collider at DESY from 1991 to 2007 and, more recently, of the hadronic colliders RHIC and LHC greatly helped us to improve our understanding of many aspects of QCD dynamics. However, as several questions remain without answer, these experimental studies must be continued and more observables must be investigated. One of the most promising observables to constrain QCD dynamics at high energies is exclusive vector meson photoproduction (EVMP) in hadronic collisions [4–23]. This process is characterized by two rapidity gaps and two intact hadrons in the final state, with the cross sections being proportional to the square of the target gluon distribution (in leading logarithmic approximation [4]) or, equivalently, to the square of the dipole—target forward scattering amplitude in the color dipole formalism [6]. Consequently, EVMP is strongly sensitive to non-linear effects associated to the high gluonic density in the target, which are expected to contribute significantly to the QCD dynamics at high energies [3].

In collisions involving a proton, processes in which the proton dissociates (becoming a neutron, for example) are very important. In ep collisions they can significantly affect EVMP. In addition to the main reaction $e + p \to e + V + p$, a non-negligible fraction of vector mesons V may come from the reaction with proton dissociation $ep \to e + V + X$. In the latter reaction the proton dissociation would reduce the rapidity gap expected in the former reaction. We are thus facing two important challenges: the experimental identification of EVMP [1] (especially for the Run 2 LHC energies due to the large pileup) and the quantitative estimate of the contribution of EVMP with the proton dissociation [24,25]. Both subjects are intrinsically related. In order to identify exclusive processes without the measurement of rapidity gaps it is necessary to tag the protons in the final state. If the proton dissociates, this signature of the event will be destroyed. Therefore, more detailed studies of proton dissociative process and/or alternative final states that can be used to tag the EVPM are important and timely.

We have recently studied [26] one of the possible proton dissociation processes, where the proton dissociates into a leading neutron and a pion, with the former carrying a large fraction of the proton momentum. In principle, the presence

of a leading neutron in EVMP can be used to tag the event using the zero degree calorimeters (ZDC) already installed in several of the colliders detectors [27]. Our results indicated that the associated cross sections are non-negligible and that an experimental analysis is feasible. One of the advantages of the approach proposed in Ref. [26] is that it has a strong predictive power. It is based on the same assumptions used to successfully describe the EVMP without proton dissociation [20] as well as inclusive and exclusive γp interactions with a leading neutron at HERA [28,29]. Therefore, the experimental measurement of this process will impose stringent constraints on the theoretical description of leading neutron and of EVMP as well.

In this work we will extend and complement our previous study [26] on EVMP and consider processes where the proton splits into $\Delta\pi$ states. These are, after $p \to n\pi$, the next most important proton dissociation process and their influence on the leading neutron longitudinal momentum (x_L) spectrum measured at HERA has been investigated by the H1 and ZEUS collaborations with the help of their standard event generators. The conclusion, presented in Ref. [30], is that the presence of Δ intermediate states leads to significantly softer leading neutron spectra. In fact, neutrons coming from the process $p \to n\pi$ peak at $x_L \simeq 0.7$ and neutrons coming from the $p \to \Delta\pi \to n\pi\pi$ sequential dissociation peak at $x_L < 0.5$. In an analogous calculation, here we compute the rapidity distributions and total cross sections for ρ, ϕ, and J/Ψ production associated with a leading baryon ($B = \Delta^+, \Delta^0$) in pp and pA collisions at RHIC and LHC energies.

We start our analysis presenting a brief review of the formalism needed to describe EVMP associated with a leading particle in photon-induced interactions at hadronic collisions. The process is represented in Fig. 1 for a generic hadronic $h_1 h_2$ collision considering that a leading baryon is produced in association with the vector meson. It will be characterized by one rapidity gap associated to the photon exchange and another between the vector meson and the meson M due to the diffractive interaction. The basic assumption in the description of photon-induced interactions at hadronic colliders is that the corresponding hadronic cross sections can be factorized in terms of the equivalent flux of photons and the photon-target cross section. As a consequence, the rapidity distribution of mesons produced in EVMP in association with a leading baryon is given by [26]

$$\frac{d\sigma[h_1 + h_2 \to h_3 \otimes V \otimes \pi + B]}{dY} = \left[\omega \frac{dN}{d\omega}\Big|_{h_1} \sigma_{\gamma h_2 \to V \otimes \pi + B}(\omega)\right]_{\omega_L} + \left[\omega \frac{dN}{d\omega}\Big|_{h_2} \sigma_{\gamma h_1 \to V \otimes \pi + B}(\omega)\right]_{\omega_R} a \quad (1)$$

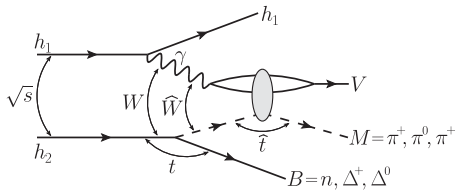

FIG. 1. Typical diagram of exclusive vector meson photoproduction in association with a leading baryon.

where h_3 corresponds to the initial hadron (h_1 or h_2) which has emitted the photon, $Y = \ln(2\omega/M_V)$ is the rapidity of a vector meson with mass M_V and $(dN/d\omega)_h$ denotes the equivalent photon spectrum of the incident hadron h, with the flux of a nucleus being enhanced by a factor Z^2 in comparison to the proton one. As in Refs. [20,26] we will assume that the photon flux associated to the proton and nucleus can be described by the Drees—Zeppenfeld [31] and the relativistic pointlike charge [32] models, respectively. Moreover, the symbol $\otimes$ in Eq. (1) represents a rapidity gap in the final state and $\omega_L (\propto e^{-Y})$ and $\omega_R (\propto e^{Y})$ are energies of the photons emitted by the h_1 and h_2 hadrons, respectively. Finally, the photon-target cross section is given by $\sigma_{\gamma h}$, which depends on the photon energy ω in the collider frame. Following Refs. [26,29], we will describe the photon-target interaction in terms of the pion splitting function and of the color dipole scattering amplitude, such that the photon-target interaction can be seen as a sequence of three factorizable subprocesses: (i) the photon emitted by one of the hadrons fluctuates into a quark-antiquark pair (the color dipole), (ii) the color dipole interacts diffractively with the pion emitted by the proton (the other hadron), and (iii) the vector meson and the leading particle are formed. Moreover, as in Refs. [28,29] we will assume that the absorptive corrections associated to soft rescatterings [33,34] can be approximated by a constant factor $\mathcal{K}$. The total cross section for the process $\gamma p \to V \otimes \pi + B$ can be expressed by

$$\sigma_{\gamma p \to V \otimes \pi + B}(W^2) = \mathcal{K} \cdot \int dx_L dt f_{\pi/p}(x_L, t) \cdot \sigma_{\gamma\pi \to V \otimes \pi}(\hat{W}^2) \quad (2)$$

where $W = (2\omega\sqrt{s})^{\frac{1}{2}}$ is the center-of-mass energy of the photon-proton system, x_L is the proton momentum fraction carried by the leading particle and t is the square of the four-momentum of the exchanged pion. Moreover, $f_{\pi/p}$ is the flux of virtual pions emitted by the proton and $\sigma_{\gamma\pi \to V \otimes \pi}(\hat{W}^2)$ is the cross section of the interaction between the photon and the pion at center-of-mass energy $\hat{W}$, which is given by $\hat{W}^2 = (1 - x_L)W^2$. Following Refs. [26,28,29], we will assume that the pion flux is given by [35]

$$f_{\pi/p}(x_L, t) = \frac{g^2_{p\pi B}}{16\pi^2} \frac{\mathcal{B}(t, m_p, m_B)}{(t - m_\pi^2)^2} \times (1 - x_L)^{1-2t} \exp\left[2b(t - m_\pi^2)\right] \quad (3)$$

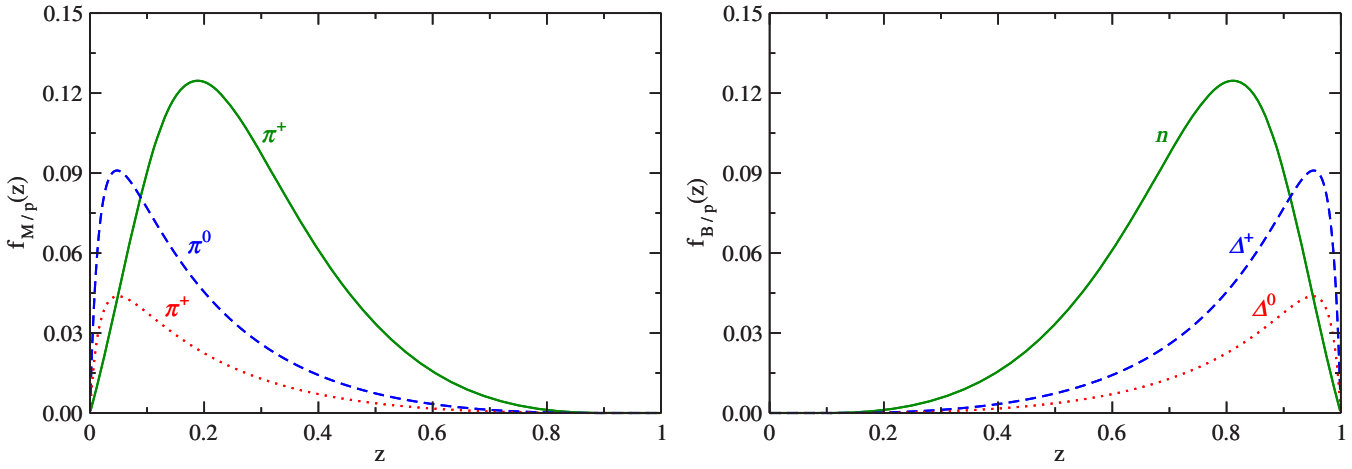

FIG. 2. Distribution of the proton fractional momentum (z) carried by the meson (left) and by the baryon (right).

where $g_{p\pi B}$ is the proton-pion-baryon coupling constant, m_π is the pion mass and $b = 0.3$ GeV^{-2} is related to the $p\pi B$ form factor. The term $\mathcal{B}$ depends of the produced baryon:

$$\mathcal{B}(t, m_p, m_B) = \begin{cases} -t + (m_n - m_p)^2, & \text{for } B = n, \\ \frac{[(m_\Delta + m_p)^2 - t]^2[(m_\Delta - m_p)^2 - t]}{12 m_p^2 m_\Delta^2}, & \text{for } B = \Delta, \end{cases} \tag{4}$$

where m_p, m_n and m_Δ are the respective masses of the proton, neutron and delta. In our analysis we will assume that $g_{p\pi^+ n} = 19.025$, $g_{p\pi^+\Delta^0} = 11.676$ and $g_{p\pi^0\Delta^+} = 16.512$ [36]. Additionally, the $\gamma\pi \to V \otimes \pi$ cross section will be expressed by

$$\sigma(\gamma\pi \to V \otimes \pi) = \int_{-\infty}^{0} \frac{d\sigma}{d\hat{t}} d\hat{t} = \frac{1}{16\pi} \int_{-\infty}^{0} |\mathcal{A}^{\gamma\pi \to V\pi}(\hat{x}, \Delta)|^2 d\hat{t}, \tag{5}$$

where the scattering amplitude is given in the dipole formalism by

$$\mathcal{A}^{\gamma\pi \to V\pi}(\hat{x}, \Delta) = i \int d\alpha d^2\boldsymbol{r} d^2\boldsymbol{b} e^{-i[\boldsymbol{b} - (1-\alpha)\boldsymbol{r}]\cdot\boldsymbol{\Delta}} (\Psi^{V*}\Psi) 2\mathcal{N}_\pi(\hat{x}, \boldsymbol{r}, \boldsymbol{b}) \tag{6}$$

with $\hat{x} = M_V^2/\hat{W}^2$ being the scaled Bjorken variable, $\hat{t} = -\Delta^2$ denotes the transverse momentum lost by the outgoing pion, the variable α $(1-\alpha)$ is the longitudinal momentum fraction of the quark (antiquark) whereas the variable $\boldsymbol{b}$ is the transverse distance from the center of the target to the center of mass of the $q\bar{q}$ dipole. Finally, $(\Psi^{V*}\Psi)$ denotes the overlap between the real photon and exclusive final state wave functions, which we assume to be given by the Gauss-LC model described in Ref. [28], and $\mathcal{N}^\pi(\hat{x}, \boldsymbol{r}, \boldsymbol{b})$ is the imaginary part of the forward amplitude of the scattering between a small dipole (a colorless quark-antiquark pair) and a pion, at a given rapidity interval, which is directly related to the QCD dynamics at high energies [3]. As in Ref. [26], we will assume that $\mathcal{N}^\pi$ can be expressed in terms of the dipole-proton scattering amplitude $\mathcal{N}^p$, usually probed in the inclusive and exclusive processes at HERA, as follows

$$\mathcal{N}^\pi(\hat{x}, \boldsymbol{r}, \boldsymbol{b}) = R_q \cdot \mathcal{N}^p(\hat{x}, \boldsymbol{r}, \boldsymbol{b}), \tag{7}$$

with R_q being a constant. Moreover, we will assume that $\mathcal{N}^p(\hat{x}, \boldsymbol{r}, \boldsymbol{b})$ is given by the bCGC model proposed in Ref. [37] and recently updated in Ref. [38]. It is important to emphasize that this model reproduces quite well the HERA data on exclusive ρ and J/Ψ production [39] as well as the EVMP without proton dissociation in hadronic collisions [20]. As the formalism that will be used in our study of the EVMP associated with a leading baryon is the same extensively discussed in our previous papers [26,28,29] for the leading neutron production, we refer the reader to these references for a more detailed analysis about the dependence of our predictions on the meson flux and on the photon-hadron scattering amplitude.

In order to estimate the rapidity distributions and total cross sections for EVMP associated with a leading baryon we need to specify a model for the absorptive corrections, represented by the $\mathcal{K}$ factor in Eq. (2), as well as a value for the R_q factor in Eq. (7). The range of possible values for $\mathcal{K}$ was fixed in Ref. [29] using HERA data [40] for the $\sigma(\gamma p \to \rho \otimes \pi + n)$ process, being given by $(\mathcal{K}_{\min}, \mathcal{K}_{\text{med}}, \mathcal{K}_{\max}) = (0.152, 0.179, 0.205)$. In what follows we will perform our calculations of the rapidity distributions and total cross sections assuming that $\mathcal{K} = \mathcal{K}_{\text{med}}$. Moreover, as in Refs. [26,29], we will assume that $R_q = 2/3$, as expected from the additive quark model, which allows us to describe the inclusive and exclusive HERA data for leading neutron production. As a consequence of these assumptions, there is no parameter to be

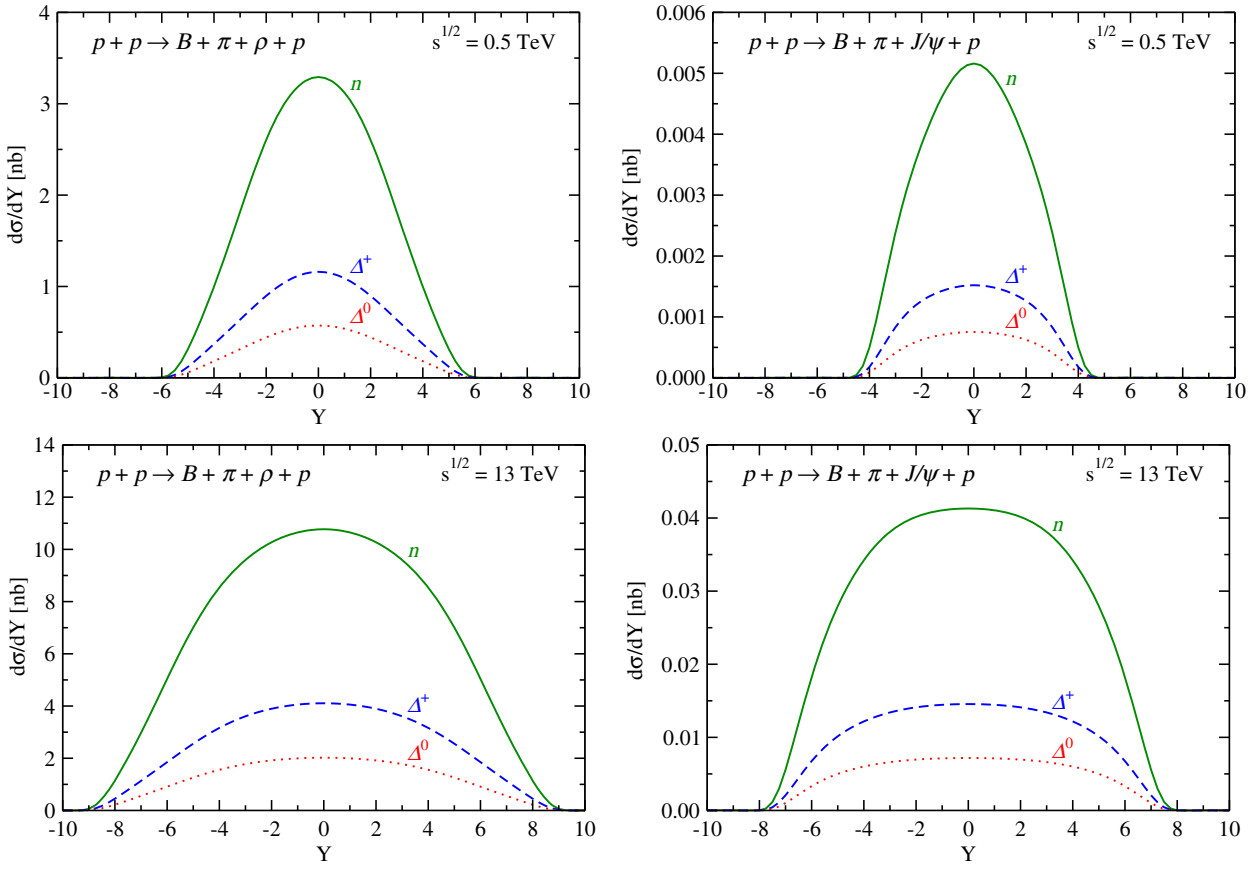

FIG. 3. Vector meson rapidity distributions in EVMP associated with a leading baryon in pp collisions. Upper and panels: $\sqrt{s} =$ 0.5 TeV and 13 TeV respectively. Left and right panels: ρ and J/ψ respectively.

fixed and we can make predictions which can be confronted with data.

Initially, let us analyze the behavior of the meson (baryon) flux as a function of the fractional momentum z which the meson (baryon) takes away from the proton. The results are presented in Fig. 2. As it can be seen, baryons carry the largest fraction of the proton momentum, whereas pions populate the low z region. Due to energy-momentum conservation, the position of the peak of the meson distribution depends on the distribution of the associated baryon. In particular, the pion associated with the neutron has its maximum at $z \sim 0.2$, while pions associated with the delta states reach their peak at $z \sim 0.05$. As a consequence, we expect that a leading Δ carries a larger momentum than a leading neutron. Additionally, the typical center-of-mass energies $\hat{W}$ of the photon-meson interactions in the case of a leading Δ will be smaller than those in the process with a leading neutron. As it will be seen, this aspect has a direct implication on our predictions for EVMP associated with a leading baryon.

In Fig. 3 we present our predictions for the exclusive ρ and J/Ψ photoproduction associated with a leading baryon in pp collisions at RHIC ($\sqrt{s} = 0.5$ TeV) and LHC ($\sqrt{s} = 13$ TeV) energies. The predictions for the leading n, Δ^+ and Δ^0 are presented separately. As both incident protons act as photon sources, we have rapidity distributions that are symmetric with respect to $Y = 0$. The predictions for midrapidities $Y \approx 0$ increase with $\sqrt{s}$ and decrease with M_V. Additionally, the growth with the energy is faster for J/Ψ than for ρ production. Such behavior is expected from the non-linear QCD dynamics [3], which predicts a larger contribution of these effects for processes dominated by larger dipole sizes, as is the case of the ρ production in comparison to the J/Ψ one. We predict that the rapidity distributions of particles produced in association with a leading Δ are smaller than those for a leading neutron. This difference is directly related to the distinct magnitude of the coupling constants and to the different range of center-of-mass photon-meson energies probed in the two processes. As already observed in Fig. 2, in the process with a leading neutron the typical values of z, and consequently $\hat{W}^2$, are larger than those present with a leading Δ. As the $\gamma\pi$ cross section increases with the energy, this implies that the cross sections for the processes with a leading neutron will be larger. Moreover, we observe that the predictions for the leading Δ^0 are smaller than the leading Δ^+ by a factor ~2 at central rapidities, which can be traced back to the fact that $g_{p\pi^0\Delta^+} > g_{p\pi^+\Delta^0}$.

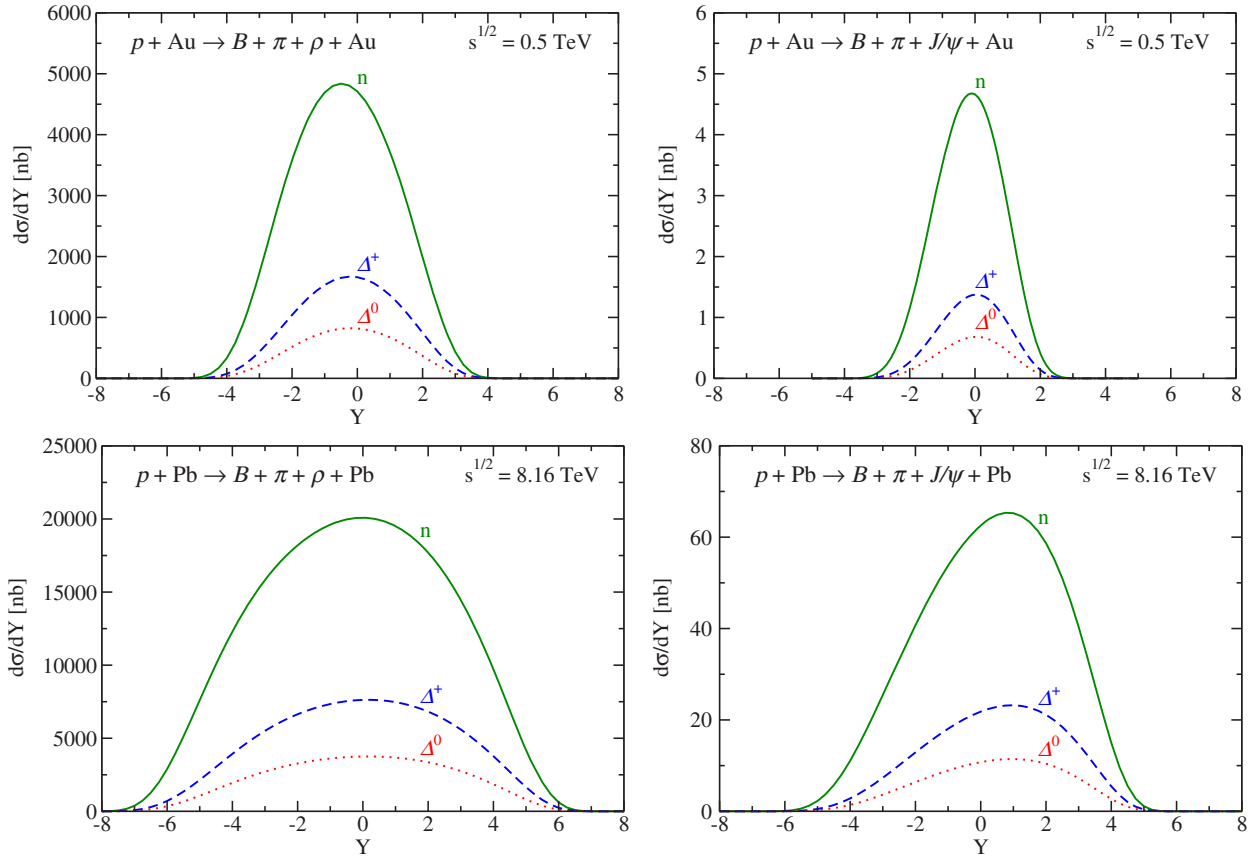

FIG. 4. Vector meson rapidity distributions in EVMP associated with a leading baryon in pA collisions. Upper and panels: $\sqrt{s} =$ 0.5 TeV and 8.16 TeV respectively. Left and right panels: ρ and J/ψ respectively.

The results for exclusive ρ and J/Ψ photoproduction associated with a leading baryon in pA collisions at RHIC ($\sqrt{s} = 0.5$ TeV) and LHC ($\sqrt{s} = 8.1$ TeV) energies are presented in Fig. 4. In our calculations we have assumed that $A = 197$ (208) for RHIC (LHC). As the nuclear photon flux is enhanced by a factor Z^2 in comparison to the proton one, we obtain asymmetric rapidity distributions. Similar enhancement is predicted in the magnitude of the rapidity distributions at $Y \sim 0$. As in the pp case, the EVMP associated with a leading neutron is a factor $\gtrsim 3(5)$ larger than those associated to a leading $\Delta^+(\Delta^0)$.

Let us now estimate the total cross sections for pp collisions at $\sqrt{s} = 0.2, 0.5, 8$ and 13 TeV, pAu collisions at $\sqrt{s} = 0.2$ and 0.5 TeV and pPb collisions at $\sqrt{s} = 5.02$ and 8.16 TeV. As in Refs. [26,29,40], we will assume that $\sqrt{|t|} < 0.2$ GeV. Moreover, we will also present our predictions for exclusive ϕ photoproduction. We have verified that the rapidity distributions for this final state are similar to those for the ρ production, but smaller in magnitude. In Table I we present our predictions. As expected from the analysis of the rapidity distributions, the cross sections increase with the energy and decrease with the mass of the vector meson. Moreover, their magnitude is enhanced in pA collisions in comparison with pp collisions. For the exclusive ρ photoproduction associated with a leading neutron, we predict values of the order of 10^2 (10^5) nb in pp (pPb) collisions at LHC energies.

In the experimental analysis, it may be possible to reconstruct the Δ. Then the predictions shown in Figs. 3 and 4 can be directly compared to data. If the reconstruction of Δ is not possible, then we have to remember that the $\Delta \to N\pi$ decay channel corresponds to 99.4% of the delta decays [41]. Then the experimentally measured EVPM cross sections and rapidity distributions and also the associated leading neutron momentum distributions will receive a significant contribution from processes with delta resonances. Indeed, our results indicate that the channels $\Delta^0 \to n\pi^0$ and $\Delta^+ \to n\pi^+$ together have a cross section which is about 20–28% of the direct leading neutron production cross section. The secondary Δ decay into $n\pi$ will not affect the rapidity distribution of the vector meson, but it will give a contribution to the neutron spectrum which is softer than the one coming from the primary $p \to n\pi$ splitting.

In comparison with the usual EVMP [20], EVMP with a leading baryon is smaller by approximately two (three) orders of magnitude in the case of a leading neutron (delta).

TABLE I. Total cross sections of EVMP associated with a leading baryon in pp and pA collisions at different center-of-mass energies. In the case of pA collisions we consider $A = 197$ (208) for RHIC (LHC) energies.

	pp				pA			
VM	$\sqrt{s}$/TeV	$\sigma(n\pi^+)$/nb	$\sigma(\Delta^0\pi^+)$/nb	$\sigma(\Delta^+\pi^0)$/nb	$\sqrt{s}$/TeV	$\sigma(n\pi^+)/\mu$b	$\sigma(\Delta^0\pi^+)/\mu$b	$\sigma(\Delta^+\pi^0)/\mu$b
ρ	0.2	10.41	1.74	3.53	0.2	7.71	1.18	2.39
ρ	0.5	21.00	3.64	7.39	0.5	21.20	3.47	7.03
ρ	8.0	97.85	17.99	36.53	5.02	124.03	22.09	44.82
ρ	13.0	121.34	22.45	45.59	8.16	163.12	29.35	59.56
ϕ	0.2	1.71	0.28	0.57	0.2	1.09	0.16	0.33
ϕ	0.5	3.65	0.63	1.27	0.5	3.37	0.54	1.09
ϕ	8.0	18.66	3.41	6.93	5.02	22.43	3.96	8.03
ϕ	13.0	23.39	4.30	8.74	8.16	29.99	5.35	10.86
J/ψ	0.2	0.01	0.001	0.002	0.2	1.63×10^{-3}	0.21×10^{-3}	0.42×10^{-3}
J/ψ	0.5	0.03	0.004	0.008	0.5	12.66×10^{-3}	1.80×10^{-3}	3.63×10^{-3}
J/ψ	8.0	0.32	0.056	0.114	5.02	0.25	0.04	0.08
J/ψ	13.0	0.45	0.079	0.160	8.16	0.37	0.06	0.13

However, it is important to emphasize that these events will be characterized by very forward baryons, which can be used to tag the events.

To conclude: recent results on photon-induced interactions at hadronic colliders have demonstrated that the analysis of these processes is feasible at RHIC and LHC, and that it is possible to use the resulting experimental data to investigate e.g. the nuclear effects in the gluon distribution, the QCD dynamics at high energies and several other issues that still lack a satisfactory understanding.

This possibility has stimulated the improvement of the theoretical description of these processes as well as the proposal of complementary processes that also probe the QCD dynamics and are more easily tagged in collisions with a high pileup. Along this line, we have recently proposed the study of EVMP associated with a leading neutron in γp interactions at pp and pA collisions and obtained large values for the total cross sections and event rates. This result motivated the analysis performed in the present work, where we have extended the study to other leading particles, with higher mass, which also generate a neutron in the final state through its decay. We have estimated the exclusive ρ, ϕ and J/Ψ photoproduction associated with a leading neutron and a leading Δ in pp and pA collisions at RHIC and LHC energies. We have found that the production associated with a leading Δ is non-negligible, being about 30% of the one with a leading neutron. Our results indicate that the experimental analysis of these process is, in principle, feasible. In particular, if a combined analysis of the events using central and forward detectors is performed, as those expected to occur using the CMS-TOTEM Precision Proton Spectrometer [42] and ATLAS + LHCf experiments [43]. We expect thus that our results motivate a future experimental analysis of EVMP associated with a leading baryon in hadronic collisions at RHIC and LHC colliders.

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