Dispersion theoretic calculation of the $H \to Z + \gamma$ amplitude

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We have calculated the W-loop contribution to the amplitude of the decay $H \to Z + \gamma$ in two different methods: (1) in the R_ξ-gauge using dimensional regularization (DimReg), and (2) in the unitary gauge through the dispersion method. Using the dispersion method we have followed two approaches: (i) without subtraction and (ii) with subtraction, the subtraction constant being determined adopting the Goldstone boson equivalence theorem (GBET) at the limit $M_W \to 0$. The results of the calculations in R_ξ-gauge with DimReg and the dispersion method with the GBET completely coincide, which shows that DimReg is compatible with the dispersion method obeying the GBET.

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I. INTRODUCTION

The calculation of the Higgs decay rate into two photons through the W-loop has become the subject of a controversy. After extracting the transverse factor

$$\mathcal{P}_{\mu\nu} = k_{2\mu} k_{1\nu} - (k_1 \cdot k_2) g_{\mu\nu}, \qquad (1)$$

which takes current conservation into account, the invariant amplitude is finite. Since however this amplitude is the sum of individually divergent Feynman diagrams most authors use dimensional regularization (DimReg) for its evaluation. Surprisingly, the DimReg result [1,2][1] differs (by a real additive constant) from the outcome of a direct computation that works with the physical unitary gauge [5,6].

Responding to a criticism, [7], which points out that the delicate cancellation of divergences is ambiguous and thus one needs a regularization, the result of [5] was confirmed in [8] by applying unsubtracted dispersion relations in a calculation that deals only with absolutely convergent integrals. Nevertheless, this calculation was also subsequently criticized in [3]. The origin of the controversy stems from the fact that perturbative amplitudes may be ambiguous even if the corresponding momentum space integrals are convergent: the Feynman rules need to be supplemented by conditions like gauge invariance, or the associated Ward identities, alongside with locality (or causality [9]) which yields the analytic properties in momentum space. The argument for an unsubtracted dispersion relation follows directly from the fact that the only constants that may appear in perturbative calculations should be the coupling constants and masses that are part of the full renormalizable Lagrangian. Thus, the absence of $H\gamma\gamma$-coupling in the SM Lagrangian, implies a zero subtraction in the dispersion integrals for the $H \to \gamma + \gamma$ amplitude. The same argument holds for the $H \to Z + \gamma$ amplitude, as well.

However, since the SM is a spontaneously broken theory and masses are generated through the Higgs mechanism, it was argued that the considered amplitude should obey the boundary condition defined by the Goldstone boson equivalence theorem (GBET) [10,11]. In [12] the amplitude of $H \to \gamma + \gamma$ was calculated in the unitary gauge staying strictly in four dimensions but fulfilling the Goldstone boson equivalence theorem. Their result is the same as in [1]. In [13] it was shown how the amplitude for the decay $H \to \gamma + \gamma$, calculated in the R_ξ gauge and in the unitary gauge, may lead to different results.

These controversial results in the calculations of the amplitude for $H \to \gamma + \gamma$ motivated us to consider the decay $H \to Z + \gamma$. These two processes are similar in a sense that at tree level they are both zero and induced by loop corrections only, the W-loops giving the main

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[1]These are two among many such calculations. The authors of [3] list 13 papers to which one may add still another one, [4], that also concurs with the majority result.

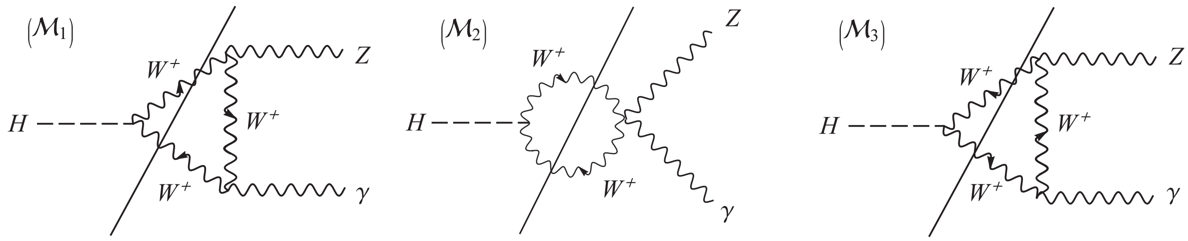

FIG. 1. Feynman diagrams for the W-loop contribution to the decay $H \to Z + \gamma$. The inclined lines indicate the cuts.

contribution. At $M_Z = 0$ the process $H \to Z + \gamma$ should reproduce the results for $H \to \gamma + \gamma$.

In this paper we calculate the one-loop W-contributions to $H \to Z + \gamma$ in two approaches: First we calculate the amplitude using the dispersion relation approach. We consider two cases: (1) we assume the unsubtracted dispersion relation, and (2) we assume a nonzero subtraction constant adopting the GBET in the limit $M_W \to 0$. Next we calculate the same amplitude in the commonly used R_ξ-gauge using the conventional dimensional regularization (DimReg).

The goal of these calculations is to compare the two results: from the dispersion-relation approach, in which we deal with finite quantities only—with and without subtraction, to the result in R_ξ-gauge with DimReg. The dispersion-relation approach can, in fact, be viewed as a general tool for resolving the ambiguities in the regularization scheme in quantum field theory. We show that with the dispersion-relation approach, where no regularization is necessary, and with subtraction determined by the GBET, we get exactly the same result as in R_ξ-gauge with DimReg.

Previously the decay $H \to Z + \gamma$ was calculated using DimReg and R_ξ-gauge by Cahn *et al.* [14], and later a complete analytic expression was obtained by other authors [15,16]. Recently in [17] this calculation was done in the unitary gauge, with the help of dimensional regularization. We completely agree with their results.

Before we go into the details it shall be mentioned that in this study we have used a couple of helpful *Mathematica* packages, [18–23].

II. THE FEYNMAN DIAGRAMS

We consider the contribution of the W-bosons loop-induced amplitude of the decay $H \to Z + \gamma$. We work in the unitary gauge, when only the physical particles contribute. There are two types of diagrams. In Fig. 1 the three W-loop diagrams that contribute to the absorptive part of the amplitude are shown. These are the same diagrams as in the process $H \to \gamma + \gamma$ [5,8], in which one of the final photons is replaced by Z. In the same figure also the unitary cuts, needed for obtaining the absorptive parts of the amplitude are shown.

In Fig. 2 the two additional diagrams that contribute to $H \to Z + \gamma$ are shown. These are $H \to Z + Z^*$ with the subsequent transition $Z^* \to \gamma$ with W^+W^- and W^+ in the loops. Clearly, kinematically their contribution to the absorptive part is zero and we do not consider them further.

The amplitude for the process $\mathcal{M}$ is

$$\mathcal{M} = \mathcal{M}_{\mu\nu}(k_1, k_2)\zeta_1^\mu \zeta_2^\nu, \tag{2}$$

where k_1 and k_2 are the momenta of the Z-boson and the photon, ζ_1, ζ_2 are their polarizations, orthogonal to k_1 and k_2, respectively:

$$k_1^2 = M_Z^2, \qquad k_2^2 = 0, \qquad k_{1\mu}\zeta_1^\mu = 0, \qquad k_{2\nu}\zeta_2^\nu = 0 \tag{3}$$

The contribution to $\mathcal{M}_{\mu\nu}$ of the three diagrams on Fig. 1 is

$$\mathcal{M}_{\mu\nu} = \mathcal{M}_{1\mu\nu} + \mathcal{M}_{2\mu\nu} + \mathcal{M}_{3\mu\nu}, \quad \text{with} \tag{4}$$

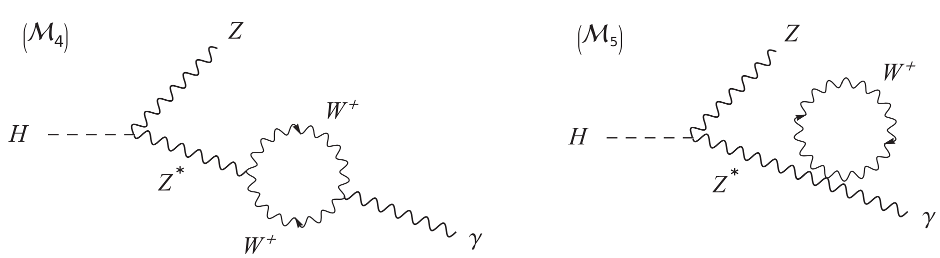

FIG. 2. Feynman diagrams with an intermediate Z^*-boson for the decay $H \to Z + \gamma$. Their contribution to the absorptive part of the amplitude is kinematically forbidden.

$$\mathcal{M}_{1\mu\nu} = \frac{-\mathrm{i}eg^2\cos\theta_W M}{(2\pi)^4}\int \mathrm{d}^4k \frac{V_{\mu\rho\beta}(-k_1,-P_2,P_1)V_{\nu\gamma\sigma}(-k_2,-P_3,P_2)}{D_1D_2D_3} \times \left(g^\beta_\alpha - \frac{P_{1\alpha}P_1^\beta}{M^2}\right)\left(g^{\rho\sigma} - \frac{P_2^\rho P_2^\sigma}{M^2}\right)\left(g^{\alpha\gamma} - \frac{P_3^\alpha P_3^\gamma}{M^2}\right), \quad (5)$$

$$\mathcal{M}_{3\mu\nu} = \mathcal{M}_{1\mu\nu}(\mu\leftrightarrow\nu, k_1\leftrightarrow k_2) = \frac{-\mathrm{i}eg^2\cos\theta_W M}{(2\pi)^4}\int \mathrm{d}^4k \frac{V_{\nu\rho\beta}(-k_2,-\tilde{P}_2,P_1)V_{\mu\gamma\sigma}(-k_1,-P_3,\tilde{P}_2)}{D_1\tilde{D}_2D_3} \times \left(g^\beta_\alpha - \frac{P_{1\alpha}P_1^\beta}{M^2}\right)\left(g^{\rho\sigma} - \frac{\tilde{P}_2^\rho \tilde{P}_2^\sigma}{M^2}\right)\left(g^{\alpha\gamma} - \frac{P_3^\alpha P_3^\gamma}{M^2}\right), \quad (6)$$

$$\mathcal{M}_{2\mu\nu} = \frac{\mathrm{i}eg^2\cos\theta_W M}{(2\pi)^4}\int \mathrm{d}^4k \frac{V_{\gamma\beta\mu\nu}}{D_1D_3}\left(g^\beta_\alpha - \frac{P_{1\alpha}P_1^\beta}{M^2}\right)\left(g^{\alpha\gamma} - \frac{P_3^\alpha P_3^\gamma}{M^2}\right). \quad (7)$$

Here, θ_W is the Weinberg (weak mixing) angle and $M = M_W$ is the mass of the W-boson.

The $WW\gamma$ and WWZ vertices are denoted by $V_{\alpha\beta\gamma}$, the $WWZ\gamma$ vertex is denoted by $V_{\alpha\beta\mu\nu}$, they are given in Appendix A, where all Feynman rules in the unitary gauge are recalled.

We have also used the following brief notations:

$$P_1 = k + \frac{p}{2}, \qquad P_2 = k - \frac{v}{2}, \qquad P_3 = k - \frac{p}{2}, \quad (8)$$

$$D_i = P_i^2 - M^2 + i\epsilon, \qquad (i = 1,2,3), \quad (9)$$

$$\tilde{P}_2 = k + \frac{v}{2}, \qquad \tilde{D}_2 = \tilde{P}_2^2 - M^2 + i\epsilon \quad (10)$$

$$p = k_1 + k_2, \qquad v = k_1 - k_2. \quad (11)$$

Taking into account the transformation properties under the reflection $k \to -k$ of the loop momentum,

$$\tilde{P}_2(k\to -k) = -P_2, \qquad \tilde{D}_2(k\to -k) = D_2. \quad (12)$$

We relate $\mathcal{M}_{3\mu\nu}$ and $\mathcal{M}_{1\mu\nu}$, thus simplifying our calculation:

$$\mathcal{M}_{3\mu\nu}(k\to -k) = \mathcal{M}_{1\mu\nu}. \quad (13)$$

III. ABSORPTIVE PART OF THE AMPLITUDE

We obtain the absorptive part through the Cutkosky rules which set the momenta of the W's on-shell [24]:

$$\frac{1}{p^2 - M^2} \to (2\pi\mathrm{i})\theta(\pm p_0)\delta(p^2 - M^2). \quad (14)$$

The imaginary part is obtained via the cut diagrams, $\mathcal{M}^C_{i\mu\nu}$:

$$\Im m\mathcal{M}_{\mu\nu} = -\frac{\mathrm{i}}{2}(2\mathcal{M}^C_{1\mu\nu} + \mathcal{M}^C_{2\mu\nu}). \quad (15)$$

Obviously, here we have taken into account Eq. (13).

Further we define the invariant absorptive part $\mathcal{A}$ of the amplitude through the imaginary part of the amplitude:

$$\Im m\mathcal{M}_{\mu\nu} = \frac{eg^2\cos\theta_W}{8\pi M}\mathcal{A}(\tau)\mathcal{P}_{\mu\nu}, \qquad \tau = \frac{p^2}{4M^2}, \quad (16)$$

where $\mathcal{P}_{\mu\nu}$ is the transverse-momentum (gauge invariant), given by Eq. (1),

$$k_1^\mu\mathcal{P}_{\mu\nu} = k_2^\nu\mathcal{P}_{\mu\nu} = 0. \quad (17)$$

Then $\mathcal{A}$ is obtained via the expression:

$$\mathcal{A}(\tau)\mathcal{P}_{\mu\nu} = \frac{M^2}{\pi}\int \mathrm{d}^4k\mathcal{I}_{\mu\nu}\theta(P_{10})\theta(-P_{30})\delta(D_1)\delta(D_3), \quad (18)$$

where $\mathcal{I}_{\mu\nu}$ is determined by the Feynman diagrams on Fig. 1. The two delta functions $\delta(D_1)$ and $\delta(D_3)$ in Eq. (18) reduce the one-loop integral to a phase-space integral. In the next section as the second step we will calculate from the absorptive part the real part of the amplitude by applying the dispersion integral technique. One can also inverse the step of computing the absorptive part. Instead of cutting the one-loop amplitude, one can sew appropriate tree-level amplitudes together to form the one-loop amplitude which turns the cutting step around, avoiding the explicit construction of one-loop Feynman diagrams. But then one can rely on evaluating Feynman integrals to do the second step [25]. These are the so-called unitarity cut methods based on [26], see also, e.g., [27,28].

The tensor $\mathcal{I}_{\mu\nu}$ is obtained via straightforward, but rather tedious calculations starting from the expressions (5)–(7).

Also we make use of the following identities, that hold for both the $WW\gamma$ and WWZ vertices:

$$V_{\alpha\beta\gamma}(p_1,p_2,p_3) = -V_{\beta\alpha\gamma}(p_2,p_1,p_3) = V_{\gamma\alpha\beta}(p_3,p_1,p_2), \tag{19}$$

and

$$p_1^\alpha V_{\alpha\mu\gamma}(p_1,-k_1,p_3) = p_3^2 g_{\mu\gamma} - p_{3\mu}p_{3\gamma} - M_Z^2 g_{\mu\gamma}, \tag{20}$$

$$p_1^\alpha V_{\alpha\nu\gamma}(p_1,-k_2,p_3) = p_3^2 g_{\nu\gamma} - p_{3\nu}p_{3\gamma}, \tag{21}$$

$$p_1^\alpha p_3^\gamma V_{\alpha\mu\gamma}(p_1,-k_1,p_3) = -M_Z^2 P_{3\mu}, \tag{22}$$

$$p_1^\alpha p_3^\gamma V_{\alpha\nu\gamma}(p_1,-k_2,p_3) = 0. \tag{23}$$

After rather cumbersome calculations we end up with the following expression for $\mathcal{I}_{\mu\nu}$:

$$\begin{aligned}\mathcal{I}_{\mu\nu} = {} & \frac{8M_Z^2}{M^4 D_2} k^2 \left(k_\mu k_\nu + \frac{k_{2\mu}k_\nu}{2} - \frac{k_\mu k_{1\nu}}{2} - \frac{k_{2\mu}k_{1\nu}}{4}\right) + \frac{-2M_Z^2}{M^4} k^2 g_{\mu\nu} \\ & + \frac{8M_Z^2}{M^2 D_2}\left[-k_\mu k_\nu - \frac{k_{2\mu}k_\nu}{2} + \frac{k_\mu k_{1\nu}}{2} - \frac{k_{2\mu}k_{1\nu}}{8} + \frac{1}{4} g_{\mu\nu} k_1\cdot k_2 - \frac{1}{8} g_{\mu\nu} k\cdot(k_1-k_2)\right] + \frac{M_Z^2}{M^2} g_{\mu\nu} \\ & + \frac{2}{M^2 D_2}[4k_1\cdot k_2 k_\mu k_\nu + 2k^2 k_{2\mu}k_{1\nu} - 4k\cdot k_1 k_{2\mu}k_\nu - 4k\cdot k_2 k_\mu k_{1\nu} \\ & + g_{\mu\nu}(4k\cdot k_1 k\cdot k_2 - 2k^2 k_1\cdot k_2)] \\ & + \frac{2}{D_2}\left[\left(-3k^2 + 3k\cdot k_1 - 3k\cdot k_2 - \frac{9}{2}k_1\cdot k_2 + 3M^2 - \frac{3}{4}M_Z^2\right) g_{\mu\nu} \right. \\ & \left. + 12k_\mu k_\nu + 3k_{1\nu}k_{2\mu} - 6k_\mu k_{1\nu} + 6k_{2\mu}k_\nu\right]. \end{aligned} \tag{24}$$

Now we have to do the integration in (18). We perform it in the rest frame of the decaying Higgs boson, with the z-axis pointing along $\mathbf{k}_1$:

$$p^\alpha = k_1^\alpha + k_2^\alpha = (p, \mathbf{0}), \qquad p \equiv p_0 = 2M\sqrt{\tau}, \tag{25}$$

$$k_1^\alpha = \frac{p}{2\tau}(\tau+a,0,0,\tau-a), \qquad k_2^\alpha = \frac{p}{2\tau}(\tau-a,0,0,a-\tau),$$
$$a = \frac{M_Z^2}{4M^2} = \frac{1}{4\cos^2\theta_W}, \tag{26}$$

$$v^\alpha = k_1^\alpha - k_2^\alpha = \frac{p}{\tau}(a,0,0,\tau-a),$$
$$v^2 = 4M^2(2a-\tau), \qquad (p\cdot v) = M_Z^2. \tag{27}$$

The two δ-functions: $\delta(D_1) = \delta[(k+p/2)^2 - M^2]$ and $\delta(D_3) = \delta[(k-p/2)^2 - M^2]$ immediately determine k_0 and $|\mathbf{k}|$:

$$k^\alpha = (k_0, \mathrm{k}) \Rightarrow k_0 = 0, \qquad |\mathbf{k}|^2 = M^2(\tau-1) = \frac{p^2}{4}\beta^2, \tag{28}$$

where

$$\beta = \sqrt{1-\tau^{-1}}. \tag{29}$$

Thus, we are left only with the 2-dimensional integral over the direction of $\mathbf{k} = |\mathbf{k}|(\sin\theta\cos\phi, \sin\theta\sin\phi)$. For D_2 we obtain:

$$D_2 = -2M^2(\tau-a)(1-\beta\cos\theta). \tag{30}$$

The absorptive part of the amplitude is nonzero at $\tau > 1$ and it reads:

$$\begin{aligned}\mathcal{A}(\tau) = \frac{a}{\tau-a}\Bigg\{ & \left[1 + \frac{1}{\tau-a}\left(\frac{3}{2} - 2a\tau\right)\right]\beta \\ & - \left[1 - \frac{1}{2(\tau-a)}\left(2a - \frac{3}{2\tau}\right) - \frac{3}{2a}\left(1 - \frac{1}{2\tau}\right)\right] \\ & \times \ln\left(\frac{1+\beta}{1-\beta}\right)\Bigg\}, \\ \tau > 1. & \end{aligned} \tag{31}$$

The details of the calculations are presented in Appendix B.

IV. REAL PART OF THE AMPLITUDE

The full invariant amplitude $\mathcal{F}(\tau, a)$ is defined by

$$\mathcal{M}_{\mu\nu} = -\frac{eg^2\cos\theta_W}{8\pi M}\mathcal{F}(\tau,a)\mathcal{P}_{\mu\nu}, \tag{32}$$

where $\mathcal{P}_{\mu\nu}$ is the transverse-momentum factor (1).

The vanishing of the absorptive part of the amplitude at $\tau < 1$ tells us that the invariant amplitude $\mathcal{F}$ at $\tau < 1$, which is the physically interested region, is only real. Following the analytic properties of the amplitude, we define the invariant *unsubtracted* amplitude $\mathcal{F}_{un}(\tau, a)$ in this region, $\tau < 1$, through the convergent dispersion integral:

$$\mathcal{F}_{un}(\tau, a) = \frac{1}{\pi}\int_1^\infty \frac{\mathcal{A}(y)}{y-\tau}\mathrm{d}y, \quad \tau < 1. \tag{33}$$

From the explicit expression for $\mathcal{A}$ and its behavior at $\tau \to \infty$ we obtain that this integral is absolutely convergent. This however does not imply that there are no subtractions in (33): the dispersion integral (33) determines the *full* amplitude $\mathcal{F}(\tau, a)$ up to an additive constant $C(a)$:

$$2\pi\mathcal{F}(\tau, a) = 2\pi\mathcal{F}_{un}(\tau, a) + \mathcal{C}(a)\cdot \tag{34}$$

In order to determine $\mathcal{C}(a)$ we need some additional information about the amplitude—some boundary condition or a physical measurable quantity at some fixed value of τ. In our calculations we fix $\mathcal{C}(a)$ through the Goldstone Boson equivalence theorem (GBET) [10], which fixes the behavior of the amplitude at $\tau \to \infty$.

In accordance with this we calculate the amplitude $\mathcal{F}(\tau, a)$ in two steps:

(1) First we calculate $\mathcal{F}_{un}(\tau, a)$ using the dispersion relation Eq. (33).
(2) We calculate $\mathcal{C}(a)$ using the GBET.

A. The unsubtracted amplitude $\mathcal{F}_{un}(\tau,a)$

The unsubtracted amplitude $\mathcal{F}_{un}(\tau, a)$ is determined by the convergent dispersion integral Eq. (33). The integrals in Eq. (33) are taken analytically—they are given in Appendix C, and we obtain:

$$2\pi\mathcal{F}_{un}(\tau, a) = \frac{3-4a^2}{\tau - a} + \left(6 - 4a - \frac{3-4a^2}{\tau-a}\right)F(\tau, a) - 2a\left(2 + \frac{3-4a\tau}{\tau-a}\right)G(\tau, a), \tag{35}$$

$$F(\tau, a) = \frac{f(\tau) - f(a)}{\tau - a}, \tag{36}$$

$$G(\tau, a) = \frac{g(\tau) - g(a)}{\tau - a}, \tag{37}$$

$$f(x) = \begin{cases} \arcsin^2(\sqrt{x}) & \text{for } x \le 1, \\ -\frac{1}{4}\left(\ln\frac{1+\sqrt{1-x^{-1}}}{1-\sqrt{1-x^{-1}}} - i\pi\right)^2 & \text{for } x > 1, \end{cases} \tag{38}$$

$$g(x) = \begin{cases} \sqrt{\frac{1-x}{x}}\arcsin(\sqrt{x}) & \text{for } x \le 1, \\ \frac{1}{2}\sqrt{\frac{x-1}{x}}\left(\ln\frac{1+\sqrt{1-x^{-1}}}{1-\sqrt{1-x^{-1}}} - i\pi\right) & \text{for } x > 1. \end{cases} \tag{39}$$

The result for $\tau > 1$ in the above formula is obtained via analytic continuation. (The same result may be found if we had set $\tau > 1$ in the integrand and taken the $i\epsilon$ prescription in D_2 into account.)

There are several important physical consequences for this amplitude.

(1) The amplitude at threshold, $\tau = a$, is finite. We have:

$$\lim_{\tau\to a} 2\pi\mathcal{F}_{un}(\tau, a) = \frac{1}{2a}\left[3 - 4a + 4a^2 - \frac{(3-16a+12a^2)}{1-a}g(a)\right]. \tag{40}$$

The absence of singularities in the amplitude is in accordance with the required analytic properties of $\mathcal{F}_{un}(\tau, a)$, necessary for the validity of the dispersion relations.

(2) In the asymptotic limit $\tau \to \infty$, which implies $M_H^2 \gg M^2$ at fixed a, we obtain:

$$\lim_{\tau\to\infty}\mathcal{F}_{un}(\tau, a) = 0. \tag{41}$$

(3) In the limit of $a \to 0$, we have to recover the corresponding invariant amplitude $\mathcal{F}^{\gamma\gamma}(\tau)$ for the $H \to \gamma + \gamma$ process:

$$\mathcal{M}_{\mu\nu}^{\gamma\gamma} = \frac{-e^2 g}{8\pi M}\mathcal{P}_{\mu\nu}\mathcal{F}^{\gamma\gamma}(\tau), \tag{42}$$

where $\mathcal{P}_{\mu\nu}$ is the same transverse bilinear combination as Eq. (1) with the (on shell) photon momenta k_1, k_2. We obtain:

$$\lim_{a\to 0} 2\pi\mathcal{F}_{un}(\tau, a) = 3\tau^{-1}[1 + (2-\tau^{-1})f(\tau)], \tag{43}$$

which is exactly the result for $F_{un}^{\gamma\gamma}(\tau)$, obtained in the unitary gauge, both, with direct calculations without renormalization in [5], and using the dispersive relations approach without subtraction in [8].

(4) We calculated also the amplitude of the process in the commonly used R_ξ-gauge using DimReg. The calculation was done with the help of the automatic tools FeynArts [18] and FormCalc [19]. There are 20 Feynman triangle vertex graphs, 6 Feynman vertex graphs with a four-point interaction and 10 graphs with self-energies from $Z^* - \gamma$ transition. It is checked that the result is UV finite and independent of ξ and it coincides with the one, obtained earlier in [15]. However, the result for the amplitude $\mathcal{F}_{\mathrm{DimReg}}(\tau)$, obtained using dimensional regularization, differs by a real additive constant from our result for $\mathcal{F}_{un}(\tau)$:

$$2\pi\mathcal{F}_{\rm DimReg}(\tau,a) = 2\pi\mathcal{F}_{un}(\tau,a) + 2(1-2a), \quad (44)$$

which leads to a nonvanishing asymptotic behavior at $\tau \to \infty$.

B. The charged ghost contribution and the constant $\mathcal{C}(a)$

We determine the constant $\mathcal{C}(a)$ through the charged ghost contribution adopting the GBET, which implies that at $M_W \to 0$, i.e. at $\tau \to \infty$, the $SU(2)\times U(1)$ symmetry of the SM is restored and the longitudinal components of the physical $W^\pm$-bosons are replaced by the physical Goldstone bosons $\phi^\pm$. In the following $\mathcal{M}^\phi_{\mu\nu}$ denotes the amplitude of $H \to Z+\gamma$ in which the $W^\pm$ are replaced by their Goldstone bosons $\phi^\pm$. The GBET implies [10]:

$$\lim_{\tau\to\infty}\mathcal{M}_{\mu\nu}(\tau,a) = \lim_{\tau\to\infty}\mathcal{M}^\phi_{\mu\nu}(\tau,a). \quad (45)$$

We calculate the charged ghost contribution in two different ways: through direct calculations and via the dispersion integral. Both calculations lead to the same result.

(i) There are 3 vertex graphs and 2 self-energy graphs, shown in Figs. 3 and 4, that possibly can contribute. We denote the contribution from the vertex diagrams by $\mathcal{M}^\phi_{1+2+3,\mu\nu}$. Following the Feynmann rules for the $\phi^\pm$-vertices, given in Appendix A, with direct calculations using DimReg we learn that the self-energy graphs do not contribute, the result is finite and gauge invariant, as expected:

$$\begin{aligned}\lim_{\tau\to\infty}\mathcal{M}^\phi_{\mu\nu} &= \lim_{\tau\to\infty}\mathcal{M}^\phi_{1+2+3,\mu\nu}\\ &= \lim_{\tau\to\infty}\frac{eg^2\cos\theta_W}{8\pi M}\frac{1}{2\pi}\frac{4a\tau-2\tau}{\tau-a}\mathcal{P}_{\mu\nu}\\ &= -\frac{eg^2\cos\theta_W}{8\pi M}\frac{1}{2\pi}2(1-2a)\mathcal{P}_{\mu\nu}. \quad (46)\end{aligned}$$

Following the GBET, Eq. (45), Eqs. (34), (41) and (46) determine the constant $\mathcal{C}(a)$:

$$\mathcal{C}(a) = 2(1-2a). \quad (47)$$

The details of the calculations are given in Appendix D.

Thus, our result for the invariant amplitude $\mathcal{F}$, Eqs. (34), (35) and ([47]), completely coincides with the result for the same amplitude $\mathcal{F}_{\rm DimReg}$ obtained in R_ξ-gauge with DimReg, Eq. (44).

(ii) However, as the goal of our approach with the dispersion integrals is to obtain the amplitude using only finite quantities, we shall obtain the Goldstone-boson contribution by using the dispersion method.

Analogously to Eq. (32), we single out the coupling constants (see the Feynman rules in Appendix A) and define the invariant part $\mathcal{F}^\phi$ of the decay amplitude $\mathcal{M}^\phi_{\mu\nu}$ in the Higgs-Goldstone boson scalar theory:

$$\mathcal{M}^\phi_{\mu\nu}(\tau,a) = -\frac{eg^2\cos\theta_W}{8\pi M}\frac{M_H^2}{4M^2}\mathcal{F}^\phi(\tau,a)\mathcal{P}_{\mu\nu}. \quad (48)$$

We shall apply the dispersive approach (without subtraction) to the function $\mathcal{F}^\phi(\tau,a)$. In order to obtain the form factor $\tau\mathcal{F}^\phi(\tau,a)$ that enters the amplitude $\mathcal{M}^\phi_{\mu\nu}$,

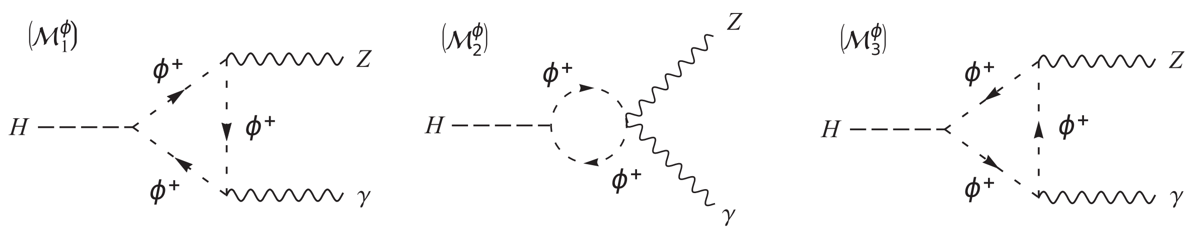

FIG. 3. The vertex Feynman diagrams for the charged Higgs ghost contribution to the decay $H \to Z+\gamma$. The Cutkosky cuts are analagous to those shown in Fig. 1.

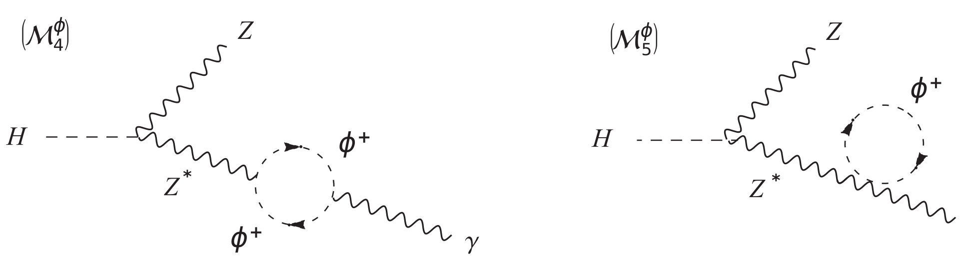

FIG. 4. The self-energy Feynman diagrams for the charged Higgs ghost contribution with an intermediate Z-boson (in the unitary gauge) for the decay $H \to Z+\gamma$. Their contribution to the absorptive part of the amplitude is zero, being kinematically forbidden.

Eq. (48), we must multiply the result for $\mathcal{F}^{\phi}(\tau, a)$ by τ. (The same strategy was elaborated for the $H \to \gamma + \gamma$ process in [3].)

In general, a constant term can, of course, be always added and in order to fix the subtraction constant some additional physical boundary conditions are required. In contrast to the SM, where the GBET is a boundary condition that fixes the subtraction constant, in the Higgs-Goldstone scalar theory there are no asymptotic theorems one could refer to.

However, the GBET allows to define a boundary condition for $\mathcal{F}^{\phi}(\tau, a)$, as well. According to the GBET, the constant $\mathcal{C}(a)$ is obtained as the large-τ limit, Eq. (45), which in terms of the form factors reads:

$$\lim_{\tau\to\infty} 2\pi\mathcal{F}(\tau, a) = \lim_{\tau\to\infty} 2\pi[\tau\mathcal{F}^{\phi}(\tau, a)] = \mathcal{C}(a). \quad (49)$$

Since $\mathcal{C}(a)$ is a finite quantity, the structure of Eq. (48) and more precisely the presence of the factor M_H^2 in the coupling, implies that the large-τ behavior of the function $\mathcal{F}^{\phi}(\tau, a)$ is of the form $\mathcal{F}^{\phi}(\tau, a) \sim \mathcal{O}(\tau^{-x})$, with $x \geq 1$. Therefore, the value of the integral $(1/\pi)\int_{ARC} \mathrm{d}y \mathcal{F}^{\phi}(y, a)/(y - \tau)$ over the infinite arc in the complex τ-plane, is zero. This, and the fact that the dispersion integral [see Eq. (51) bellow] is convergent, implies that the dispersion relation applied for $\mathcal{F}^{\phi}(\tau, a)$ does not need a subtraction.

The absorptive part $\mathcal{A}^{\phi}(\tau, a)$ of the function $\mathcal{F}^{\phi}(\tau, a)$ is obtained via the Cutkosky rules from the cut diagrams in Fig. 3. Evidently the self-energy graphs, see Fig. 4, have no absorptive parts. We obtain (see Appendix D):

$$\Im m\mathcal{M}^{\phi}_{\mu\nu}(\tau, a) = -\frac{eg^2\cos\theta_W}{8\pi M}\frac{M_H^2}{4M^2}\mathcal{A}^{\phi}(\tau, a)\mathcal{P}_{\mu\nu}, \quad (50)$$

with

$$\mathcal{A}^{\phi}(\tau, a) = (1 - 2a)\frac{2\alpha\beta - \ln\frac{1+\beta}{1-\beta}}{2(\tau - a)^2}.$$

The expression for the function $\mathcal{F}^{\phi}(\tau, a)$, valid in the whole τ-interval, is obtained via the dispersion integral:

$$\mathcal{F}^{\phi}(\tau, a) = \frac{1}{\pi}\int\frac{\mathcal{A}^{\phi}(y, a)}{y - \tau}\mathrm{d}y = \frac{1 - 2a}{2\pi}(4aI_2(\tau, a) - 2J_2(\tau, a)), \quad (51)$$

where $I_2(\tau, a)$ and $J_2(\tau, a)$ are convergent and given in Appendix C.

In the limit $\tau \to \infty$ $(M \to 0)$ we have $I_2(\tau, a) \to 1/(2a(\tau - a))$ and $J_2(\tau, a) \to \infty$, and we obtain:

$$\lim_{\tau\to\infty}\mathcal{F}^{\phi}(\tau, a) = \frac{2(1 - 2a)}{2\pi(\tau - a)}. \quad (52)$$

Thus, our final result in the limit $M \to 0$ $(\tau \to \infty)$ is

$$\lim_{\tau\to\infty}\mathcal{M}^{\phi}_{\mu\nu} = \lim_{\tau\to\infty}\frac{eg^2\cos\theta_W}{8\pi M}\frac{4a\tau - 2\tau}{2\pi(\tau - a)}\mathcal{P}_{\mu\nu} = -\frac{eg^2\cos\theta_W}{8\pi M}\frac{2(1 - 2a)}{2\pi}\mathcal{P}_{\mu\nu}, \quad (53)$$

which completely coincides with Eq. (46).

V. THE DECAY WIDTH OF $H \to Z + \gamma$

A good approximation for the total width of the Higgs decay into $Z + \gamma$ is given by the contributions of the W-boson and the top-quark loops (cf. [15]):

$$\Gamma(H \to Z + \gamma) = \frac{M_H^3}{32\pi}\left(1 - \frac{M_Z^2}{M_H^2}\right)^3\left[\frac{eg^2}{(4\pi)^2M}\right]^2 \times\left| -\cos\theta_W[2\pi\mathcal{F}_W(\tau)] + \frac{2(3 - 8\sin^2\theta_W)}{3\cos\theta_W}[2\pi\mathcal{F}_t(\tau_t)]\right|^2, \quad (54)$$

where $\mathcal{F}_t(\tau_t)$ stands for the sum of the t-quark one-loop diagrams:

$$2\pi\mathcal{F}_t(\tau_t) = \frac{1}{2(\tau_t - a_t)} \times [1 - (1 - \tau_t + a_t)F(\tau_t, a_t) + 2a_tG(\tau_t, a_t)], \quad (55)$$

$$\tau_t = \frac{M_H^2}{4m_t^2}, \qquad a_t = \frac{M_Z^2}{4m_t^2}, \quad (56)$$

and $\mathcal{F}_W(\tau)$ stands for the sum of the W-boson one-loop diagrams.

Further, we identify $\mathcal{F}_W(\tau)$ with the amplitude obtained with the dispersion integral, Eq. (34), in which the unsubtracted part is given in (35) and $\mathcal{C}(a)$ in (47): $\mathcal{F}_W(\tau) = \mathcal{F}(\tau, a)$. This implies that at the measured value for the Higgs mass $M_H = 125.09$ GeV, using $m_t = 172.44$ GeV for the mass of the top-quark, we obtain the following value for the expected decay width:

$$\Gamma(H \to Z + \gamma) = 8.1 \text{ KeV}. \quad (57)$$

If, however, $\mathcal{F}_W(\tau)$ was identified to the unsubtracted amplitude $\mathcal{F}_W(\tau) = \mathcal{F}_{\text{un}}(\tau, a)$, Eq. (35), the value for the decay width of $H \to Z + \gamma$ would be about 20% smaller which, as we showed, is not the correct result.

VI. CONCLUDING REMARKS

We have calculated the W-boson induced corrections to the decay $H \to Z + \gamma$ in the Standard Model in the unitary gauge using the dispersion-relation approach. This approach is very attractive as it deals only with finite quantities and thus does not involve any uncertainties related to regularization. However, the problem with the dispersion method is that it determines the amplitude merely up to an additive subtraction constant.

In accordance with this arbitrariness, we calculate the amplitude in two approaches: (1) without subtraction and (2) with subtraction. We use the zero-mass limit at $M_W \to 0$ as determined by the GBET, to determine the subtraction constant. In this latter case we perform the calculations in two ways: (i) through direct calculations of the amplitude determined by the GBET, using DimReg, and (ii) via the dispersion method, starting from the absorptive part of the amplitude, and thus using only finite quantities. The two completely different calculations lead us to the same subtraction constant.

Furthermore, we also calculated the amplitude in the commonly used R_ξ-gauge class using dimensional regularization as regularization scheme and compared the result to the one obtained via the dispersion method. The R_ξ-gauge result completely coincides with the dispersion method together with the subtraction term determined by the GBET.

Thus, we have shown that the dispersion-relation approach, with a subtraction determined by the GBET, presents an alternative method for calculating the $H \to Z + \gamma$ amplitude (and for $H \to \gamma + \gamma$ as also shown in [4]) to the commonly used R_ξ-gauge technique. However, the dispersion method has two important advantages: (1) it deals only with finite quantities and thus is free of uncertainties related to the choice of regularization and (2) it is much simpler—working in the unitary gauge effectively we deal with only 3 Feynman diagrams, while in the R_ξ-gauge one has to consider more than 20 graphs.

ACKNOWLEDGMENTS

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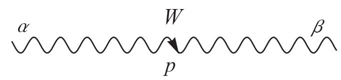
APPENDIX A: FEYNMAN RULES

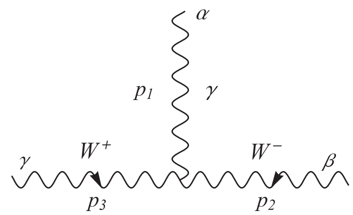
The Feynman rules for building $\mathcal{M}_{\mu\nu}$, Eq. (4), are presented in the subsections A 1, A 2. The Feynman rules needed for the calculation of the constant $\mathcal{C}(a)$, defined by (34), are presented in the subsections A 2, A 3.

In all vertex Feynman diagrams it is assumed that all momenta flow into the vertex.

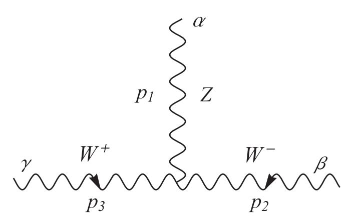
1. Feynman rules involving W-boson in the unitary gauge



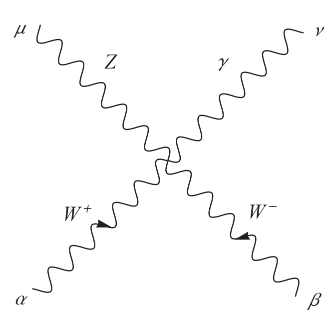
$$\frac{\mathrm{i}}{p^2 - M^2 + \mathrm{i}\epsilon}\left(-g^{\alpha\beta} + \frac{p^\alpha p^\beta}{M^2}\right) \tag{A1}$$



$$\mathrm{i}eV_{\alpha\beta\gamma}(p_1,p_2,p_3) = \mathrm{i}e\left[(p_2-p_3)_\alpha g_{\beta\gamma} + (p_3-p_1)_\beta g_{\gamma\alpha} + (p_1-p_2)_\gamma g_{\alpha\beta}\right] \tag{A2}$$

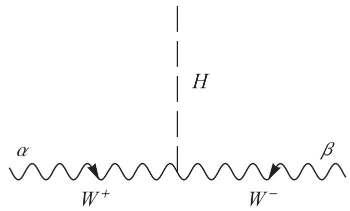


$$\mathrm{i}g\cos\theta_W V_{\alpha\beta\gamma}(p_1,p_2,p_3) = \mathrm{i}g\cos\theta_W\left[(p_2-p_3)_\alpha g_{\beta\gamma} + (p_3-p_1)_\beta g_{\gamma\alpha} + (p_1-p_2)_\gamma g_{\alpha\beta}\right] \tag{A3}$$



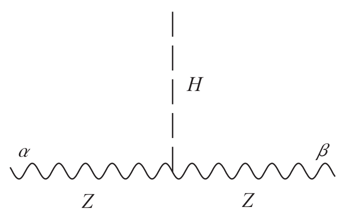
(A4)

$$-\,\mathrm{i}eg\cos\theta_W V_{\alpha\beta\mu\nu} = -\,\mathrm{i}eg\cos\theta_W\left[2g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}\right]$$



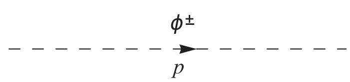
$$\mathrm{i}gM g_{\alpha\beta} \tag{A5}$$

2. Feynman vertex rule for the triple Higgs—Z-boson interaction

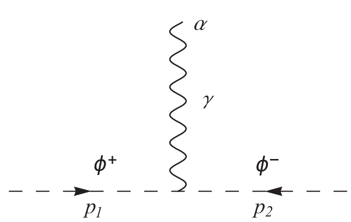


$$\mathrm{i}\,\frac{g}{\cos\theta_W}\,M_Z g_{\alpha\beta} \tag{A6}$$

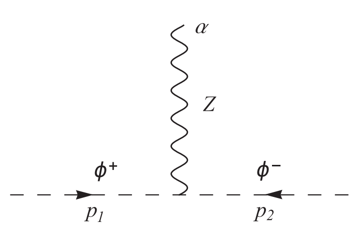
3. Feynman rules involving the charged Higgs ghost in the R_ξ gauge



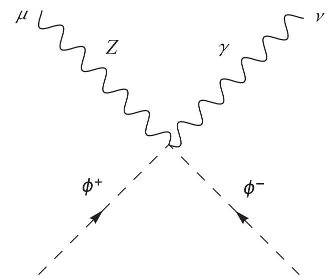
$$\frac{\mathrm{i}}{p^2 - \xi_W M^2 + \mathrm{i}\epsilon} \tag{A7}$$



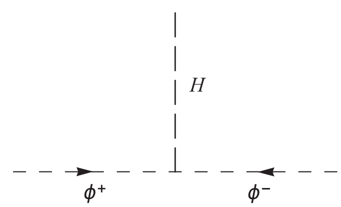
$$\mathrm{i}e(p_1 - p_2)_\alpha \tag{A8}$$



$$-\mathrm{i}\,g\,x_W\,(p_1 - p_2)_\alpha, \qquad x_W = -\frac{\cos 2\theta_W}{2\cos\theta_W} \tag{A9}$$



$$-2\,\mathrm{i}\,e\,g\,x_W\,g_{\mu\nu} \tag{A10}$$



$$-\mathrm{i}\,g\,\frac{M_H^2}{2M} \tag{A11}$$

APPENDIX B: INTEGRALS FOR THE ABSORPTIVE PART $\mathcal{A}(\tau)$

Here we give the integrals involved in computation of the absorptive part $\mathcal{A}(\tau)$ of the amplitude.

The calculations are done in the rest frame of the Higgs boson, with z-axis taken along k_1, the kinematics as given in Sec. III. We have used also the following relations:

$$k^2 = -|\mathrm{k}|^2 = -M^2(\tau - 1), (k \cdot p) = 0,$$
$$(k \cdot v) = -2M^2(\tau - a)\beta\cos\theta. \tag{B1}$$

The evaluation of $\mathcal{A}(\tau)$ is reduced to the following integrals:

$$1.\ \Im m \int \frac{d^4k}{(2\pi)^4}\frac{\mathrm{i}}{D_1 D_3} = -\frac{\beta}{8\pi}, \tag{B2}$$

$$2.\ \Im m \int \frac{d^4k}{(2\pi)^4}\frac{\mathrm{i}}{D_1 D_2 D_3} = \frac{\beta}{32\pi M^2(\tau - a)} I, \tag{B3}$$

$$I = \int_{-1}^{+1} \frac{dx}{1 - \beta x} = \frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right), \tag{B4}$$

$$3.\ \Im m \int \frac{d^4k}{(2\pi)^4}\frac{\mathrm{i}k_\mu}{D_1 D_2 D_3} = -\frac{\beta^2\tau}{64\pi M^2(\tau - a)^2}\left(\frac{a}{\tau}p_\mu - v_\mu\right) J, \tag{B5}$$

$$J = \int_{-1}^{+1} \frac{x\mathrm{d}x}{1-\beta x} = \frac{1}{\beta}(I-2), \tag{B6}$$

4. $$\Im m \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{\mathrm{i}k_\mu k_\nu}{D_1 D_2 D_3} = L_1\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + L_2\left(\frac{a}{\tau}p_\mu - v_\mu\right)\left(\frac{a}{\tau}p_\nu - v_\nu\right), \tag{B7}$$

where

$$L_1 = -\frac{\beta^3\tau}{64\pi(\tau-a)}(I-K), \tag{B8}$$

$$L_2 = -\frac{\beta^3\tau^2}{256\pi M^2(\tau-a)^3}(I-3K), \tag{B9}$$

$$K = \int_{-1}^{+1} \frac{x^2\mathrm{d}x}{1-\beta x} = \frac{1}{\beta^2}(I-2). \tag{B10}$$

We recall the notation:

$$D_1 = \left(k+\frac{p}{2}\right)^2 - M^2, \qquad D_2 = \left(k-\frac{v}{2}\right)^2 - M^2,$$
$$D_3 = \left(k-\frac{p}{2}\right)^2 - M^2. \tag{B11}$$

APPENDIX C: INTEGRALS FOR THE REAL PART OF $\mathcal{F}(\tau)$

The invariant amplitude $\mathcal{F}(\tau)$, Eq. (35), is a linear combination of the dispersion integrals I_i and J_i:

$$2\pi\mathcal{F}(\tau) = 4aI_1(\tau,a) + 6aI_2(\tau,a) - 8a^2I_3(\tau,a) + 2(3-2a)J_1(\tau,a) + 4a^2J_2(\tau,a) - 3J_3(\tau,a) - 3aJ_4(\tau,a), \tag{C1}$$

which we list below. We distinguish two types of integrals:

(1) Integrals with β:

$$\beta \equiv \sqrt{1-y^{-1}}. \tag{C2}$$

They are expressed in terms of the integral $I_0(x)$, or equivalently of the elementary function $g(x)$:

$$I_0(\tau) = \frac{1}{2}\int_1^\infty \frac{\beta}{(y-\tau)y}\mathrm{d}y = \frac{1}{\tau}[1-g(\tau)], \tag{C3}$$

where

$$g(\tau) \equiv \sqrt{\tau^{-1}-1}\arcsin\sqrt{\tau}. \tag{C4}$$

For the other integrals we have:

$$I_1(\tau,a) = \frac{1}{2}\int_1^\infty \frac{\beta}{(y-\tau)(y-a)}\mathrm{d}y = \frac{g(a)-g(\tau)}{\tau-a}, \tag{C5}$$

$$I_2(\tau,a) = \frac{1}{2}\int_1^\infty \frac{\beta}{(y-\tau)(y-a)^2}\mathrm{d}y = \frac{\partial}{\partial a}I_1(\tau,a) = \frac{1}{\tau-a}\left\{I_1(\tau,a) - \frac{1}{2(1-a)}[1-I_0(a)]\right\}, \tag{C6}$$

$$I_3(\tau,a) = \frac{1}{2}\int_1^\infty \frac{\beta y}{(y-\tau)(y-a)^2}\mathrm{d}y = I_1(\tau,a) + aI_2(\tau,a); \tag{C7}$$

(2) Integrals with the logarithm l_β:

$$l_\beta \equiv \ln\frac{1+\beta}{1-\beta}. \tag{C8}$$

They are expressed in terms of the integral $J_0(x)$, or equivalently of the elementary function $f(x)$:

$$J_0(\tau) = \frac{1}{2}\int_1^\infty \frac{l_\beta}{(y-\tau)y}\mathrm{d}y = \frac{f(\tau)}{\tau}, \tag{C9}$$

where

$$f(x) \equiv \arcsin^2\sqrt{x}. \tag{C10}$$

We have:

$$J_1(\tau,a) = \frac{1}{2}\int_1^\infty \frac{l_\beta}{(y-\tau)(y-a)}\mathrm{d}y = \frac{f(\tau)-f(a)}{\tau-a}, \tag{C11}$$

$$J_2(\tau,a) = \frac{1}{2}\int_1^\infty \frac{l_\beta}{(y-\tau)(y-a)^2}\mathrm{d}y = \frac{\partial}{\partial a}J_1(\tau,a) = \frac{1}{\tau-a}\left[J_1(\tau,a) - \frac{g(a)}{1-a}\right], \tag{C12}$$

$$J_3(\tau,a) = \frac{1}{2}\int_1^\infty \frac{l_\beta}{(y-\tau)(y-a)y}\mathrm{d}y = \frac{1}{\tau-a}[J_0(\tau) - J_0(a)], \tag{C13}$$

$$J_4(\tau,a) = \frac{1}{2}\int_1^\infty \frac{l_\beta}{(y-\tau)(y-a)^2y}\mathrm{d}y = \frac{\partial}{\partial a}J_3(\tau,a) = \frac{1}{a}[J_2(\tau,a) - J_3(\tau,a)] \tag{C14}$$

Below we give the expansions used to determine the limits at $a \to 0$ and $\tau \to a$. At $|a| < 1$ we have:

$$f(a) = a + \frac{1}{3}a^2 + \frac{8}{45}a^3 + O(a^4), \tag{C15}$$

$$g(a) = 1 - \frac{1}{3}a - \frac{2}{15}a^2 - \frac{8}{105}a^3 + O(a^{7/2}). \tag{C16}$$

When $|\tau - a| < 1$ for the functions $F(a,\tau)$ and $G(a,\tau)$, that enter the amplitude (35), we have:

$$\begin{aligned} F(a,\tau) &= \frac{f(\tau) - f(a)}{\tau - a} \\ &= \frac{1}{1-a}g(a) + \frac{1}{4a(1-a)}\left[1 - \frac{(1-2a)}{1-a}g(a)\right](\tau - a) \\ &+ \frac{1}{8a^2(1-a)^2}\left[2a - 1 + \frac{(8a^2 - 8a + 3)}{3(1-a)}g(a)\right](\tau - a)^2 \\ &+ O((\tau - a)^3). \end{aligned} \tag{C17}$$

$$\begin{aligned} G(a,\tau) &= \frac{g(\tau) - g(a)}{\tau - a} = \frac{1}{2a}\left[1 - \frac{1}{1-a}g(a)\right] \\ &- \frac{1}{8a^2(1-a)^2}[2a^2 - 5a + 3 + (4a - 3)g(a)](\tau - a) \\ &+ \frac{1}{48(1-a)^3a^3}[-8a^3 + 34a^2 - 41a + 15 \\ &- 3(8a^2 - 12a + 5)g(a)](\tau - a)^2 \\ &+ O((\tau - a)^3). \end{aligned} \tag{C18}$$

APPENDIX D: THE CHARGED HIGGS GHOST CONTRIBUTION TO $H \to Z + \gamma$

In Appendix A 3 all involved Feynman rules are given in the R_ξ gauge. Here we calculate the charged Higgs ghosts $\phi^\pm$ contribution, according to the GBET. We use the unitary gauge—the diagrams in Figs. 1 and 2, in which the virtual W-bosons are replaced by the physical scalars $\phi^\pm$—Figs. 3 and 4. Their propagators are that of a scalar, with mass of the W-boson, $\frac{\mathrm{i}}{p^2 - M^2 + \mathrm{i}\epsilon}$. All couplings in Appendix A 3 are ξ-independent and therefore we can take them directly.

(i) First we calculate the constant part of this contribution evaluating the intergrals by using Feynman parametrization. The occurring UV divergent integrals are regularized with dimensional regularization.

Based on Fig. 3 we get the matrix elements

$$\mathcal{M}^\phi_{1\mu\nu} = \mathrm{i}eg^2 x_W \frac{M_H^2}{2M}\frac{1}{(2\pi)^4}\int \mathrm{d}^4k \frac{(P_1 + P_2)_\mu (P_2 + P_3)_\nu}{D_1 D_2 D_3} \tag{D1}$$

$$\mathcal{M}^\phi_{3\mu\nu} = \mathrm{i}eg^2 x_W \frac{M_H^2}{2M}\frac{1}{(2\pi)^4} \times \int \mathrm{d}^4k \frac{(\tilde{P}_2 + P_3)_\mu (P_1 + \tilde{P}_2)_\nu}{D_1 \tilde{D}_2 D_3} \tag{D2}$$

$$\mathcal{M}^\phi_{2\mu\nu} = -\mathrm{i}eg^2 x_W \frac{M_H^2}{2M}\frac{1}{(2\pi)^4}\int \mathrm{d}^4k \frac{2g_{\mu\nu}}{D_1 D_3}. \tag{D3}$$

Here we use $D_i = P_i^2 - M^2$, $P_1 = k$, $P_2 = k - k_1$, $P_3 = k - k_1 - k_2$, $\tilde{P}_2 = k - k_2$. Furthermore, similar to Eq. (4) we can write the sum of the three vertex amplitudes as $2\mathcal{M}^\phi_{1\mu\nu} + \mathcal{M}^\phi_{2\mu\nu}$, which is

$$\mathcal{M}^\phi_{1+2+3\mu\nu} = \mathrm{i}eg^2 x_W \frac{M_H^2}{M}\frac{1}{(2\pi)^4}\int \mathrm{d}^4k \frac{T}{D_1 D_2 D_3}, \tag{D4}$$

with

$$\begin{aligned} T &= (P_1 + P_2)_\mu (P_2 + P_3)_\nu - g_{\mu\nu} D_2 \\ &= 4k_\mu k_\nu - 4k_\mu k_{1\nu} + (2k.k_1 + M^2 - M_Z^2 - k^2)g_{\mu\nu}. \end{aligned} \tag{D5}$$

Using the formula for Feynman parametrization,

$$\frac{1}{D_1 D_2 D_3} = 2\int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \times \frac{1}{(x_1 D_1 + x_2 D_2 + (1 - x_1 - x_2)D_3)^3}, \tag{D6}$$

and by the substitution $k_\mu \to l_\mu + (1 - x_1)k_{1\mu} + (1 - x_1 - x_2)k_{2\mu}$ we get

$$\begin{aligned} \frac{1}{D_1 D_2 D_3} &= \int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \frac{2}{(l^2 - \Delta)^3}, \\ \Delta &= M^2 + 2k_1.k_2 x_1(x_1 + x_2 - 1) \\ &+ M_Z^2 x_1(x_1 - 1). \end{aligned} \tag{D7}$$

The two necessary integrals over l are

$$\int \mathrm{d}^4l \frac{1}{(l^2 - \Delta_i)^3} = -\mathrm{i}\pi^2 \frac{1}{2\Delta_i}, \tag{D8}$$

$$\int \mathrm{d}^4l \frac{l^2}{(l^2 - \Delta_i)^3} = \mathrm{i}\pi^2\left(\Delta^{\mathrm{UV}} - \frac{1}{2}\right) \quad \text{with} \quad \Delta^{\mathrm{UV}} = \frac{1}{\epsilon} + \text{const}. \tag{D9}$$

All odd powers of l vanish due to the symmetric integration and thus will be dropped. Applying

$l_\mu l_\nu = \frac{l^2}{d}$ with the dimension parameter $d = 4 - 2\epsilon$ we get

$$T \to 4k_{1\nu}k_{2\mu}x_1(x_1 + x_2 - 1) + \left(M^2 - 2k_1.k_2x_1(x_1 + x_2 - 1) - M_Z^2x_1^2 + \epsilon\frac{l^2}{2}\right)g_{\mu\nu}. \quad \text{(D10)}$$

Integrating over l and neglecting terms of the order M^2/M_H^2 and M_Z^2/M_H^2 we obtain:

$$-\frac{\mathrm{i}}{\pi^2}\int \mathrm{d}^4k\frac{T}{D_1D_2D_3} = \frac{1}{2}g_{\mu\nu} - \int_0^1 \mathrm{d}x_1\int_0^{1-x_1}\mathrm{d}x_2 \times \frac{-4k_{1\nu}k_{2\mu}x_1x_2 + (M^2 + 2k_1.k_2x_1x_2 - M_Z^2x_1^2)g_{\mu\nu}}{M^2 - 2k_1.k_2x_1x_2 + M_Z^2x_1(x_1 - 1)} = -\frac{1}{k_1.k_2}\mathcal{P}_{\mu\nu} + \ldots. \quad \text{(D11)}$$

with $\mathcal{P}_{\mu\nu}$ given by Eq. (1). By inserting this result into Eq. (D4) we receive for the leading term of the vertex graphs with the charged ghost:

$$\mathcal{M}^\phi_{1+2+3\mu\nu} = eg^2x_W\frac{M_H^2}{M}\frac{\pi^2}{(2\pi)^4}\frac{P_{\mu\nu}}{k_1.k_2} = \frac{eg^2\cos\theta_W}{8\pi M}\frac{1}{2\pi}\left(\frac{M_Z^2}{M^2} - 2\right)\frac{M_H^2}{M_H^2 - M_Z^2}P_{\mu\nu} = \frac{eg^2\cos\theta_W}{8\pi M}\frac{1}{2\pi}\frac{4a\tau - 2\tau}{\tau - a}P_{\mu\nu}. \quad \text{(D12)}$$

We have used $2k_1.k_2 = M_H^2 - M_Z^2$, and $x_W/\cos\theta_W = \frac{1}{2}(\frac{M_Z^2}{M^2} - 2) = (2a - 1)$ after inserting $\tau = \frac{M_H^2}{4M^2}$ and $a = \frac{M_Z^2}{4M^2}$. Explicit calculations show that the sum of the two self-energy graphs, given by Fig. 4 vanishes in the unitary gauge using dimensional regularization.

(ii) Now we derive the imaginary part of the amplitude $\mathcal{M}^\phi_{\mu\nu}$ by applying the Cutkosky cuts to Fig. 3. We have

$$\Im m\mathcal{M}^\phi_{\mu\nu} = -\frac{\mathrm{i}}{2}(2\mathcal{M}^{\phi C}_{1\mu\nu} + \mathcal{M}^{\phi C}_{2\mu\nu}). \quad \text{(D13)}$$

where "C" denotes the cut diagrams. Then $\Im m\mathcal{M}^\phi_{\mu\nu}$ can be written as

$$\Im m\mathcal{M}^\phi_{\mu\nu} = eg^2x_W\frac{M_H^2}{2M}\times\Im m\int\frac{\mathrm{d}^4k}{(2\pi)^4} \times \mathrm{i}\left(\frac{(P_1 + P_2)_\mu(P_2 + P_3)_\nu}{D_1D_2D_3} - \frac{g_{\mu\nu}}{D_1D_3}\right), \quad \text{(D14)}$$

with the momenta and denominators defined in Sec. II, with the substitution for D_1 and D_3 following Eq. (14), and

$$(P_1 + P_2)_\mu(P_2 + P_3)_\nu = 4k_\mu k_\nu - 2k_\mu k_{1\nu} + 2k_\nu k_{2\mu} - k_{1\nu}k_{2\mu}. \quad \text{(D15)}$$

By using Appendix B we get the result

$$\Im m\mathcal{M}^\phi_{\mu\nu} = eg^2x_W\frac{M_H^2}{2M}\frac{2a\beta + \tau(\beta^2 - 1)\ln\frac{1+\beta}{1-\beta}}{32\pi M^2(\tau - a)^2}\mathcal{P}_{\mu\nu}. \quad \text{(D16)}$$

With $x_W = \cos\theta_W(2a - 1)$ and $\beta^2 - 1 = -1/\tau$ we get the result

$$\Im m\mathcal{M}^\phi_{\mu\nu} = \frac{eg^2\cos\theta_W}{16\pi M}\frac{M_H^2}{4M^2}(2a - 1)\frac{2a\beta - \ln\frac{1+\beta}{1-\beta}}{(\tau - a)^2}\mathcal{P}_{\mu\nu}. \quad \text{(D17)}$$

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