Reply to "Comment on 'How the huge energy of quantum vacuum gravitates to drive the slow accelerating expansion of the Universe'"

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We reply to Mazzitelli and Trombetta's comment [1] on our cosmological constant paper [2].

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I. INTRODUCTION

Mazzitelli and Trombetta's comment [1] on our paper [2] follows the usual approach of regularization. However, we are not following the usual approach for multiple reasons. The usual approach has many problems, and thus it is not applicable at least to the cosmological constant problem. This is one of the reasons for us to develop a different approach. In this Reply, we first clarify some common misunderstandings (not necessarily those of the authors of Ref. [1]) about our unconventional approach and then point out the problems of the usual approach.

II. CLARIFICATION OF OUR APPROACH

(i) We specifically do not couple gravity to expectation values, but rather to the actual value of the stress-energy tensor from point to point. What we use is essentially the stochastic gravity approach: the quantum stress-energy tensor is taken to be a classical field with fluctuations dictated by the quantum field fluctuations [3]. The stress-energy tensor is not Lorentz or Poincaré invariant on small scales, but it fluctuates wildly with correlated fluctuations, which lead to our results.

(ii) Lorentz invariance: Although neither the stress-energy tensor nor the metric is Lorentz invariant, the long-wavelength theory seems to be Lorentz invariant in the cosmological sense. The result on longer (physical) time and space scales looks like a slowly expanding homogeneous universe with a tiny cosmological constant. The Lorentz invariance of the large-scale metric is emergent—rather than assumed—in our approach.

(iii) Ω^2 is positive: This is a crucial assumption of our approach. Mode by mode, Ω^2 is positive and we assume that this feature is preserved for any cutoff scheme we are using. It is true that the violation of this assumption will destroy our mechanism. Therefore, the renormalization schemes that can lead to negative values of Ω^2 are not allowed unless the probability of such negative values is so small that they will not occur in our observable Universe.

III. PROBLEMS OF THE USUAL APPROACH

The key issue that worries the authors of Ref. [1] is about the sign of Ω^2. In our paper [2], we considered the contribution to Ω^2 from a massless scalar field ϕ as an example. In this case, $\Omega^2 = 8\pi G\dot{\phi}^2/3$. From the basic principles of quantum mechanics, the measured value of the square of any Hermitian operator on any quantum state including vacuum must be non-negative. So there should be no doubt that $\dot{\phi}^2$ is positive. This of course makes sense only if we keep it finite by introducing a cutoff.

When calculating $\langle\dot{\phi}^2\rangle$, one gets a divergent result without a cutoff. The traditional way of dealing with this divergence is to regularize it. This regularization procedure often does not preserve the sign of $\dot{\phi}^2$.

One point of confusion arises from the usually assumed vacuum equation of state for the matter stress-energy tensor of

$$\langle T_{\mu\nu}\rangle = -\langle\rho\rangle g_{\mu\nu}, \tag{1}$$

where $g_{\mu\nu}$ is the metric for a general curved spacetime. Following this, one immediately has

$$\langle\Omega^2\rangle = -\frac{8\pi G\langle\rho\rangle}{3} < 0 \tag{2}$$

if one requires $\langle\rho\rangle > 0$.

The assumption (1) is based on the argument that vacuum is Lorentz invariant and thus every inertial observer would see the same properties for this state. However, there is no well-defined vacuum state in a general curved spacetime: a Lorentz-invariant vacuum state simply does not exist. So the state we used in Ref. [2] is in fact not a Lorentz-invariant vacuum state, although we have called the state "vacuum" for convenience. Since it is not Lorentz invariant, it does not need to satisfy Eq. (1), which is based on the Lorentz-invariant hypothesis. The huge inhomogeneity and anisotropy of the stress tensor also makes Lorentz invariance problematic as a requirement.

The usual approach of regularization presumes that zero-point fluctuations satisfy the vacuum equation of state (1) which looks like a cosmological constant. This approach gives unphysical results; as pointed out by the authors of Ref. [1], because "different cutoffs produce ambiguous results for the sign of $\langle\dot{\phi}^2\rangle$ and of the vacuum energy, the physical meaning of the regularized quantities in this context is doubtful". This is exactly one of the reasons why we seek a different approach.

In our approach, we take the unregulated stress-energy tensor seriously. Both ρ and P_i are positive since they are squares of derivatives of the scalar field and so is Ω^2. As pointed out by the authors of Ref. [1], in our approach $\langle T_{\mu\nu}\rangle$ does not describe a cosmological constant, but rather a radiation fluid. This is just our major point: zero-point energy does not gravitate as a cosmological constant.

We admit that the method we used in Sec. IX A of Ref. [2] can be problematic. The name "Pauli-Villars" in that section is confusing. We did not carry out the usual Pauli-Villars regularization, since we excluded the auxiliary negative energy fields which contribute to Ω^2. What we did in Sec. IX A was a way to evaluate the divergent integral of $\dot{\phi}^2$ by introducing "Pauli-Villars"-type cutoffs. This is why we obtained $\langle\rho\rangle > 0$ there, while (as indicated by the authors of Ref. [1]) the Pauli-Villars approach may give $\langle\rho\rangle < 0$.

One important comment is that the cutoff Λ we used in Ref. [2] is a physical cutoff, representing the energy scale of our effective theory. The high-energy cutoff Λ is not covariant. Whether a Lorentz-invariant cutoff is possible for the highly inhomogeneous fluctuations in the energy-momentum tensor is dubious.

IV. THE PROBLEM WITH FERMION FIELDS

There is another serious problem not mentioned in the Comment [1]. In principle, Ω^2 receives contributions from all fundamental fields. But negative contributions to Ω^2 with naive cutoffs would be expected from fermionic fields. There is even the possibility that the net contribution to $T^{\mu\nu}$ is dominantly negative.

One solution is to include a large negative bare cosmological constant in the Einstein equations. This would bias Ω^2 to positive values. While this looks superficially like another fine-tuning problem, we only need this constant to be large enough to make Ω^2 dominantly positive. (A very small probability for Ω^2 to take negatives values in this case does not lead to a disaster for the model.)

We are currently studying this extension to our model. Not only are the concerns in the Comment [1] no longer a problem, but other problems are also alleviated. Our original model could also be problematic for positive but very small values of Ω^2. We are currently writing up this new model and hope to publish it soon.

[1] F. D. Mazzitelli and L. G. Trombetta, Comment on "How the huge energy of quantum vacuum gravitates to drive the slow accelerating expansion of the Universe", Phys. Rev. D **95**, 068301 (2018).

[2] Q. Wang, Z. Zhu, and W. G. Unruh, How the huge energy of quantum vacuum gravitates to drive the slow accelerating expansion of the universe, Phys. Rev. D **95**, 103504 (2017).

[3] B. L. Hu and E. Verdaguer, Stochastic gravity: Theory and applications, Living Rev. Relativity **11**, 3 (2008).