# Disordered $\lambda\varphi^4 + \rho\varphi^6$ Landau-Ginzburg model

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We discuss a disordered $\lambda\varphi^4 + \rho\varphi^6$ Landau-Ginzburg model defined in a $d$-dimensional space. First we adopt the standard procedure of averaging the disorder-dependent free energy of the model. The dominant contribution to this quantity is represented by a series of the replica partition functions of the system. Next, using the replica-symmetry ansatz in the saddle-point equations, we prove that the average free energy represents a system with multiple ground states with different order parameters. For low temperatures we show the presence of metastable equilibrium states for some replica fields for a range of values of the physical parameters. Finally, going beyond the mean-field approximation, the one-loop renormalization of this model is performed, in the leading-order replica partition function.

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## I. INTRODUCTION

The critical behavior of disordered systems has been intensively investigated since the 1970s using numerical simulations and analytical methods [1–7]. Two concepts that are of fundamental importance in statistical disordered systems defined on a spatial lattice are quenched disorder and frustration. In quenched disordered systems, the disorder has slower dynamical evolution than the other dynamical degrees of freedom and, therefore, they can be considered spatially random. Frustration is related to the fact that due to competing interactions there are situations where it is not possible to find an equilibrium state for the first neighbor spins [8,9]. These features are realized in the Edwards-Anderson model for a spin-glass system. The model consists of $N$ Ising spins in a $d$-dimensional lattice with finite range interaction where the exchange bonds are randomly ferromagnetic and antiferromagnetic [10]. The spin-glass phase is characterized by the fact that at low temperatures there are domains where the spins become randomly frozen in different directions in space. As has been emphasized in the literature, disordered systems may have infinitely many local equilibrium states [11]. For instance, in the replica-symmetry-breaking scenario, the free-energy landscape for the infinite-ranged spin glass has a multivalley structure [12–14]. Many different systems beyond the setting of magnetic materials present a spin-glass-like behavior with an unusual free-energy landscape. A fascinating example comes from the field of photonics, in that light presents a glassy behavior when propagating in random nonlinear media where the amplitudes of optical modes play a role similar to that of the spins in a magnetic material [15]. In addition, there have been investigations of dynamical and pure static properties in the random-temperature Landau-Ginzburg model, with special interest on its spin-glass-like behavior [16,17].

Among the large variety of statistical models describing quenched disorder, those formulated in the continuum are especially interesting in view of their close relationship with quantum field theory (see, e.g., Ref. [18]). In these models, the disorder fields can be separated into two groups: those which are random external fields and modify the Gaussian contribution in the replica partition function, and those that lead to a spatial variation of the couplings of the model. In any case, the standard procedure is averaging the disorder-dependent free energy using, for example, the replica method [4]. Rather than computing the disorder average of the logarithm of the partition function, the replica method consists in averaging the $k$th power of the partition function, with the average free energy being obtained in the limit $k \to 0$. In the mean-field theory of spin glasses the concept of replica-symmetry breaking was

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introduced [19] to solve the problem of negative entropy that afflicts the naive replica method, thereby allowing to describe infinitely many pure thermodynamic states of the system.

Recently, an alternative analytic calculation was proposed to compute the disorder-average free energy for the random-source Landau-Ginzburg model [20,21]. The method—which we called the distributional zeta-function method (DZFM)—shares similarities with the conventional replica method. For directed polymers and interfaces in random media (Gaussian models *par excellence*), the DZFM and the conventional replica method give the same result [22] for the replica-symmetric solution. The DZFM was also used to investigated spontaneous symmetry breaking in non-Gaussian models [23,24].

One of the motivations of this paper is to emphasize the differences and similarities between the DZFM and the conventional replica method in the study of disordered systems. Here, we employ the DZFM to explore the free-energy landscape of the $d$-dimensional disordered Landau-Ginzburg $\lambda\varphi^4 + \rho\varphi^6$ model. First, we write the dominant contribution to the average free energy as a series of the replica partition functions of the model. Next, the structure of the replica space is investigated using the saddle-point equations obtained from each replica field theory. Assuming the replica-symmetry ansatz, we prove that the average free energy represents a system with multiple ground states with different order parameters. This situation is similar to the free energy of the spin-glass phase obtained in the Sherrington-Kirkpatrick model, in the replica-symmetry-breaking scenario. Also, for low temperatures we show the presence of metastable equilibrium states for some replica fields in a range of values of the physical parameters. One way to describe the spin-glass behavior in the low-temperature region in the model is to assume that some terms of the series representation for the average free energy describe inhomogeneous domains. At low temperatures, some terms of the series may represent macroscopic regions in space, $\Omega^{(k)}$ with an order parameter $\varphi_0^{(k)}$. Finally, we perform the one-loop renormalization of this model.

This paper is organized as follows. In Sec. II we discuss the $d$-dimensional random-temperature Landau-Ginzburg model. In Sec. III we discuss the DZFM. In Sec. IV we discuss the structure of the replica space using the saddle-point equations of the model. In Sec. V the one-loop renormalization of the disordered model is discussed. Conclusions are given in Sec. VI. We assume that $\hbar = c = k_B = 1$.

## II. RANDOM-TEMPERATURE LANDAU-GINZBURG MODEL

In this work we are interested in studying disordered systems through statistical field theory defined in the continuum. In the classical statistical mechanics of Hamiltonian systems, any state is a probability measure on the phase space. The expectation value of any observable can be obtained from an average constructed with the Gibbs measure

$$d\mu_{\text{Gibbs}} = \frac{1}{Z} e^{-\beta H} d\mu_{\text{Liouville}}, \tag{1}$$

where $Z$ is the partition function, $H$ is the Hamiltonian, $\beta = 1/T$, $T$ is the absolute temperature, and $d\mu_{\text{Liouville}}$ is the Liouville measure. The partition function is obtained from a normalization procedure. For systems described in the continuum with infinitely many degrees of freedom, this framework can be maintained. For instance, Euclidean functional methods (with functionals of probability measures) introduced classical probabilistic concepts into quantum field theory. The Euclidean correlation functions, i.e., the Schwinger functions, are the analytic continuation of the imaginary-time vacuum expectation values of the Wightman functions [25–28]. For a scalar field, these $n$-point correlation functions, which are the moments of probability measure, are defined by

$$\langle \varphi(x_1)\ldots\varphi(x_k)\rangle = \frac{1}{Z}\int [d\varphi] \prod_{i=1}^{k} \varphi(x_i) \exp\left(-S(\varphi)\right), \tag{2}$$

where $[d\varphi]$ is a formal Lebesgue measure (i.e., a measure in the space of all field configurations) and $S(\varphi)$ is the Euclidean action of the system.

Let us assume a $\lambda\varphi^4 + \rho\varphi^6$ scalar model without disorder defined in $\mathbb{R}^d$. The partition function of the model is defined as

$$Z = \int_{\partial\Omega} [d\varphi] \exp(-H(\varphi)), \tag{3}$$

where the effective Hamiltonian is given by $H(\varphi) = H_0(\varphi) + H_I(\varphi)$. The free-field effective Hamiltonian $H_0(\varphi)$ is given by

$$H_0(\varphi) = \int d^d x \frac{1}{2} \varphi(x)(-\Delta + m_0^2)\varphi(x), \tag{4}$$

where $\Delta$ is the Laplacian in $\mathbb{R}^d$ and $H_I(\varphi)$ is the self-interacting non-Gaussian contribution, defined by

$$H_I(\varphi) = \int d^d x \left(\frac{\lambda_0}{4}\varphi^4(x) + \frac{\rho_0}{6}\varphi^6(x)\right). \tag{5}$$

In Eq. (3), $[d\varphi]$ is the formal Lebesgue measure (i.e., a measure in the space of all field configurations) given by $[d\varphi] = \prod_x d\varphi(x)$, and $\partial\Omega$ in the functional integral means that the field $\varphi(x)$ satisfies some boundary condition. Periodic boundary conditions can be imposed to preserve translational invariance, replacing $\mathbb{R}^d$ by the torus $\mathbb{T}^d$.

In order to generate the correlation functions of the model by functional derivatives, as usual a fictitious source is introduced. Therefore, the generating functional of the correlation functions of the model is

$$Z(j) = \int_{\partial\Omega} [d\varphi] \exp\left(-H(\varphi) + \int d^d x j(x)\varphi(x)\right). \tag{6}$$

The $n$-point correlation functions read

$$\langle\varphi(x_1)\ldots\varphi(x_k)\rangle = Z^{-1}(j)\frac{\delta^k Z(j)}{\delta j(x_1)\ldots\delta j(x_k)}\bigg|_{j=0}. \tag{7}$$

These moments of the probability measure are the sum of all diagrams with $k$ external legs, including disconnected ones, with the exception of the vacuum diagrams. The generating functional of $n$-point connected correlation functions can be obtained by defining $W(j) = \ln Z(j)$. The order parameter of the model without disorder $\langle\varphi(x)\rangle$ is given by

$$\langle\varphi(x)\rangle = Z^{-1}(j)\frac{\delta Z(j)}{\delta j(x)}\bigg|_{j=0}. \tag{8}$$

In the following we are interested in discussing the random-temperature $d$-dimensional Landau-Ginzburg model. In the Landau-Ginzburg Hamiltonian, if $\lambda_0$ and $m_0^2$ are regular functions of the temperature, a random contribution $\delta m_0^2(x)$ added to $m_0^2$ can be considered as a local perturbation in the temperature. In this case the Hamiltonian of the model becomes

$$H(\varphi, \delta m_0^2) = \int d^d x\left[\frac{1}{2}\varphi(x)(-\Delta + m_0^2 - \delta m_0^2(x))\varphi(x) + \frac{\lambda_0}{4}\varphi^4(x) + \frac{\rho_0}{6}\varphi^6(x)\right]. \tag{9}$$

The $\varphi^6$ contribution in the interaction Hamiltonian must be introduced to obtain a Hamiltonian that is bounded from below, as is necessary to correctly describe the critical properties of the model. Brézin and Dominicis [29], studying a random field model, showed that new interactions should be considered. This term is related to the tricritical phenomenon [30,31].

The local minima in the Hamiltonian are the configurations of the scalar field that satisfy the saddle-point equations where the solutions depend on the particular configuration of the random mass. (The terms "random mass"/"random temperature," "false vacuum"/"metastable equilibrium state," and "true vacuum"/"stable equilibrium state" are used interchangeably throughout the text.) The existence of a large number of metastable states in many disordered systems and the loss of translational invariance makes the traditional perturbative expansion formalism quite problematic. As discussed in the literature, averaging the free energy over the disorder field allows us to implement a perturbative approach in a straightforward way.

Let us briefly discuss the $n$-point correlation function associated with a disordered system. The disorder-generating functional for one realization of the disorder is given by

$$Z(\delta m_0^2; j) = \int_{\partial\Omega} [d\varphi] \exp\left(-H(\varphi, \delta m_0^2) + \int d^d x j(x)\varphi(x)\right), \tag{10}$$

where a fictitious source $j(x)$ is introduced. The $n$-point correlation function for one realization of disorder reads

$$\langle\varphi(x_1)\ldots\varphi(x_n)\rangle_{\delta m_0^2} = \frac{1}{Z(\delta m_0^2)}\int [d\varphi]\prod_{i=1}^{n}\varphi(x_i)\exp\left(-H(\varphi, \delta m_0^2)\right), \tag{11}$$

where the disordered functional integral $Z(\delta m_0^2) = Z(\delta m_0^2, j)|_{j=0}$. As in the pure system, one can define a generating functional for one disorder realization, $W_1(\delta m_0^2; j) = \ln Z(\delta m_0^2; j)$. Now, we can define a disorder-averaged correlation function as follows:

$$\mathbb{E}[\langle\varphi(x_1)\ldots\varphi(x_n)\rangle_{\delta m_0^2}] = \int [d\delta m_0^2] P(\delta m_0^2)\langle\varphi(x_1)\ldots\varphi(x_n)\rangle_{\delta m_0^2}, \tag{12}$$

where $\mathbb{E}[\ldots]$ means the average over the ensemble of all the realizations of the quenched disorder, $[d\delta m_0^2]$ is the formal Lebesgue measure, and the probability distribution of the disorder is written as $[d\delta m_0^2]P(\delta m_0^2)$, where $P(\delta m_0^2)$ is given by

$$P(\delta m_0^2) = p_0 \exp\left(-\frac{1}{4\sigma}\int d^d x(\delta m_0^2(x))^2\right). \tag{13}$$

The quantity $\sigma$ is a small parameter that describes the strength of disorder and $p_0$ is a normalization constant. In this case we have a delta-correlated disorder field, i.e., $\mathbb{E}[\delta m_0^2(x)\delta m_0^2(y)] = \sigma\delta^d(x-y)$. A relevant quantity is the disorder-averaged generating functional $W_2(j) = \mathbb{E}[W_1(\delta m_0^2; j)]$:

$$W_2(j) = \int [d\delta m_0^2] P(\delta m_0^2)\ln Z(\delta m_0^2; j). \tag{14}$$

Taking the functional derivative of $W_2(j)$ with respect to $j(x)$, we get

$$\frac{\delta W_2(j)}{\delta j(x)}\bigg|_{j=0} = \int [d\delta m_0^2] P(\delta m_0^2)\left[\frac{1}{Z(h; j)}\frac{\delta Z(h; j)}{\delta j(x)}\right]\bigg|_{j=0}. \tag{15}$$

Since $\langle\varphi(x)\rangle_{\delta m_0^2}$ is the expectation value of the field for a given configuration of the disorder in the Euclidean field theory with random mass, the above quantity is the averaged normalized expectation value of the field. Taking two functional derivatives of $W_2(j)$ with respect to $j(x)$, we get

$$\left.\frac{\delta^2 W_2(j)}{\delta j(x_1)\delta j(x_2)}\right|_{j(x_i)=0} = \mathbb{E}[\langle\varphi(x_1)\varphi(x_2)\rangle_{\delta m_0^2}] - \mathbb{E}[\langle\varphi(x_1)\rangle_{\delta m_0^2}\langle\varphi(x_2)\rangle_{\delta m_0^2}]. \tag{16}$$

That is, contrary to the pure-system case, one finds that $\mathbb{E}[\langle\varphi(x_1)\rangle_{\delta m_0^2}\langle\varphi(x_2)\rangle_{\delta m_0^2}]$ is different from $\mathbb{E}[\langle\varphi(x_1)\rangle_{\delta m_0^2}]\mathbb{E}[\langle\varphi(x_2)\rangle_{\delta m_0^2}]$. This fact shows that taking functional derivatives of $W(j)$ does not lead to connected correlation functions. Indeed, in the disordered system, there is a multivalley structure spoiling the usual perturbative approach, in which the field is expanded around only one minimum [32,33]. One way to tackle this problem is to use the spectral zeta-function method, which is a global approach, i.e., we do not rely upon only one specific minimum. Our aim is to compute

$$W_2(j)|_{j=0} = \int [d\delta m_0^2]P(\delta m_0^2)\ln Z(\delta m_0^2). \tag{17}$$

In the next section we use the DZFM to calculate the average free energy of the system.

## III. DISTRIBUTIONAL ZETA-FUNCTION METHOD

For free fields without disorder, the spectral zeta-function is a way of regularizing the determinant of the Laplace operator with some boundary conditions, and it can be used to calculate the free energy of the system. Here, we are interested in obtaining the average free energy which is directly related to $W_2(j)|_{j=0}$, after introducing the temperature, i.e., $F = -\frac{1}{\beta}W_2(j)|_{j=0}$. Our aim is to compute the disorder-averaged free energy given by

$$F = -\frac{1}{\beta}\int [d\delta m_0^2]P(\delta m_0^2)\ln Z(\delta m_0^2), \tag{18}$$

where again $[d\delta m_0^2]$ is also a formal Lebesgue measure.

Recall that a measure space $(\Omega, \mathcal{W}, \eta)$ consists in a set $\Omega$, a $\sigma$ algebra $\mathcal{W}$ in $\Omega$, and a measure $\eta$ on this $\sigma$ algebra. Given a measure space $(\Omega, \mathcal{W}, \eta)$ and a measurable $f$: $\Omega \to (0,\infty)$, we define the associated generalized $\zeta$ function as

$$\zeta_{\eta,f}(s) = \int_\Omega f(\omega)^{-s} d\eta(\omega)$$

for those $s \in \mathbb{C}$ such that $f^{-s} \in L^1(\eta)$, where in the above integral $f^{-s} = \exp(-s\log(f))$ is obtained using the principal branch of the logarithm. This formalism contains some well-known examples of zeta functions, such as the classical Riemann zeta function [34,35], the prime zeta function [36–39], the families of super-zeta functions [40], and the spectral zeta functions [41]. The usual approach is to define zeta functions in terms of a countable collection of numbers, such as prime numbers, length of closed paths, etc. Here we use the definition of the distributional zeta function $\Phi(s)$, inspired by the spectral zeta function, as

$$\Phi(s) = \int [d\delta m_0^2]P(\delta m_0^2)\frac{1}{Z(\delta m_0^2)^s} \tag{19}$$

for $s \in \mathbb{C}$, where this function is defined in the region where the above integral converges. The average free energy can be written as

$$F = \frac{1}{\beta}(d/ds)\Phi(s)|_{s=0^+}, \qquad \mathrm{Re}(s) \geq 0, \tag{20}$$

where $\Phi(s)$ is well defined. To proceed, we use Euler's integral representation for the gamma function,

$$\frac{1}{Z(\delta m_0^2)^s} = \frac{1}{\Gamma(s)}\int_0^\infty dt t^{s-1}e^{-Z(\delta m_0^2)t}, \quad \text{for } \mathrm{Re}(s) > 0. \tag{21}$$

Although the above Mellin integral converges only for $\mathrm{Re}(s) > 0$, as $Z(\delta m_0^2) > 0$, we will show how to obtain from the above expression a formula for the free energy valid for $\mathrm{Re}(s) \geq 0$. Substituting Eq. (21) into Eq. (19), we get

$$\Phi(s) = \frac{1}{\Gamma(s)}\int [d\delta m_0^2]P(\delta m_0^2)\int_0^\infty dt t^{s-1}e^{-Z(\delta m_0^2)t}. \tag{22}$$

We already know that the distributional zeta function $\Phi(s)$ is defined for $\mathrm{Re}(s) \geq 0$. Now we will use the above expression to compute its derivative at $s = 0$ using analytical tools. We assume (when necessary) the commutativity of the disorder average, differentiation, and integration.

To continue, we take $a > 0$ and write $\Phi = \Phi_1 + \Phi_2$, where

$$\Phi_1(s) = \frac{1}{\Gamma(s)}\int [d\delta m_0^2]P(\delta m_0^2)\int_0^a dt t^{s-1}e^{-Z(\delta m_0^2)t} \tag{23}$$

and

$$\Phi_2(s) = \frac{1}{\Gamma(s)}\int [d\delta m_0^2]P(\delta m_0^2)\int_a^\infty dt t^{s-1}e^{-Z(\delta m_0^2)t}, \tag{24}$$

where $a$ is a dimensionless parameter, whose interpretation will be discussed in the next section. The average free energy can be written as

$$F = \frac{1}{\beta}\frac{d}{ds}\Phi_1(s)\Big|_{s=0^+} + \frac{1}{\beta}\frac{d}{ds}\Phi_2(s)\Big|_{s=0}. \quad (25)$$

Let us define the integer moment of the partition function $\mathbb{E}[(Z(\delta m_0^2))^k] \equiv \mathbb{E}[Z^k]$, where

$$\mathbb{E}[(Z(\delta m_0^2))^k] = \int [d\delta m_0^2] P(\delta m_0^2)(Z(\delta m_0^2))^k. \quad (26)$$

The integral $\Phi_2(s)$ defines an analytic function defined in the whole complex plane. The contribution of $\Phi_1(s)$ reads

$$\Phi_1(s) = \frac{a^s}{\Gamma(s+1)} + \frac{1}{\Gamma(s)}\sum_{k=1}^{\infty}\frac{(-1)^k a^{k+s}}{k!(k+s)}\mathbb{E}[Z^k], \quad (27)$$

which is valid for $\mathrm{Re}(s) \geq 0$. The function $\Gamma(s)$ has a pole at $s = 0$ with residue 1, and therefore

$$\frac{d}{ds}\Phi_1(s)|_{s=0^+} = \sum_{k=1}^{\infty}\frac{(-1)^k a^k}{k!k}\mathbb{E}[Z^k] + f(a), \quad (28)$$

where

$$f(a) = \frac{d}{ds}\left(\frac{a^s}{\Gamma(s+1)}\right)\Big|_{s=0} = (\log a + \gamma), \quad (29)$$

and $\gamma$ is Euler's constant 0.577… The derivative of $\Phi_2$ in Eq. (24) is given by

$$\frac{d}{ds}\Phi_2(s)|_{s=0} = \int [d\delta m_0^2] P(\delta m_0^2)\int_a^{\infty}\frac{dt}{t}e^{-Z(\delta m_0^2)t} = R(a). \quad (30)$$

Hence, integrating over the disorder, the average free energy can be represented by

$$F = \frac{1}{\beta}\left[\sum_{k=1}^{\infty}\frac{(-1)^k a^k}{k!k}\mathbb{E}[Z^k] + \log a + \gamma + R(a)\right]. \quad (31)$$

Notice that $R(a)$ vanishes as long as $a \to \infty$. Indeed, in the following we discuss the asymptotic behavior of $R(a)$, which is related to the incomplete gamma function, defined as [42]

$$\Gamma(\alpha, x) = \int_x^{\infty} e^{-t}t^{\alpha-1}dt. \quad (32)$$

The asymptotic representation for $|x| \to \infty$ and $-3\pi/2 < \arg x < 3\pi/2$ reads

$$\Gamma(\alpha, x) \sim x^{\alpha-1}e^{-x}\left[1 + \frac{\alpha-1}{x} + \frac{(\alpha-1)(\alpha-2)}{x^2} + \cdots\right]. \quad (33)$$

To conclude this section, we note that the spectral zeta regularization is sometimes used in quantum and statistical field theories to regularize the determinants of operators. Although inspired by the spectral zeta function, the DZFM is not a regularization procedure: the representation is used to compute the free energy of disordered systems in terms of replica partition functions. To regularize ultraviolet divergences, dimensional regularization is employed, which is free from the problems known as "multiplicative anomalies" [43,44] that accompany the zeta regularization method. Nevertheless, for technical reasons there are some similarities between the DZFM and zeta regularization, e.g., the $\log a$ contribution appearing in Eq. (31) is analogous to the contribution to the free energy $\zeta(0)\log\mu^2$ that appears in the latter, where $\mu$ is a parameter with dimensions of mass introduced when performing analytic continuations. We note that in the present case, the $\log a$ term does not change the thermodynamics.

## IV. THE GLASS-LIKE PHASE IN THE DISORDERED MODEL

In this section we will discuss the glass-like phase in the disordered model. From the series representation of the average free energy we have to calculate the integer moments of the partition function $\mathbb{E}[Z^k]$. Using the probability distribution for the disorder and the Hamiltonian of the model, this quantity is given by

$$\mathbb{E}[Z^k] = \int\prod_{i=1}^{k}[d\varphi_i]e^{-H_{\mathrm{eff}}(\varphi_i)}, \quad (34)$$

where the effective Hamiltonian $H_{\mathrm{eff}}(\varphi_i)$ is written as

$$H_{\mathrm{eff}}(\varphi_i) = \int d^dx\left[\frac{1}{2}\sum_{i=1}^{k}\varphi_i(x)(-\Delta + m_0^2)\varphi_i(x) + \frac{1}{4}\sum_{i,j=1}^{k}g_{ij}\varphi_i^2(x)\varphi_j^2(x) + \frac{\rho_0}{6}\sum_{i=1}^{k}\varphi_i^6(x)\right], \quad (35)$$

where the replica-symmetric coupling constants $g_{ij}$ are given by $g_{ij} = (\lambda_0\delta_{ij} - \sigma)$. The saddle-point equations derived from each replica partition function read

$$(-\Delta + m_0^2)\varphi_i(x) + \lambda_0\varphi_i^3(x) + \rho_0\varphi_i^5(x) - \sigma\varphi_i(x)\sum_{j=1}^{k}\varphi_j^2(x) = 0. \quad (36)$$

Using the replica-symmetry ansatz $\varphi_i(x) = \varphi_j(x)$, the above equation becomes

$$(-\Delta + m_0^2)\varphi_i(x) + (\lambda_0 - k\sigma)\varphi_i^3(x) + \rho_0\varphi_i^5(x) = 0. \quad (37)$$

In the replica method, using the simplest possible replica-symmetric ansatz in each replica partition function, we obtain the saddle-point equations of systems without disorder. The replica-symmetry-breaking scheme was introduced to take into account the presence of many different local minima in the disorder Hamiltonian of the original model. For instance, a manifestation of this replica-symmetry breaking appears in ferromagnetic systems with random spin bonds. There is a low-temperature regime with frustrated spin domains nucleated in a ferromagnetic background [45]. As we will see, it is possible to obtain a structure with different order parameters in the scenario constructed using the DZFM, where we do not follow the standard replica-symmetry-breaking arguments. Indeed, consider a generic term of the series given by Eq. (31) with a replica partition function given by $\mathbb{E}[Z^l]$ [see also Eqs. (34) and (35)]. We are led to the following choice for the structure of the fields in each replica partition function:

$$\begin{cases} \varphi_i^{(l)}(x) = \varphi^{(l)}(x) & \text{for } l = 1, 2, \ldots, N, \\ \varphi_i^{(l)}(x) = 0 & \text{for } l > N, \end{cases} \quad (38)$$

where for the sake of simplicity we still employ the same notation for the field. Therefore, the average free energy becomes

$$F = \frac{1}{\beta}\sum_{k=1}^{N}\frac{(-1)^k a^k}{k!k}\mathbb{E}[Z^k] + \cdots. \quad (39)$$

In Eq. (31), the free energy is independent of $a$. However, the entire approach relies on the fact that $a$ can be chosen large enough so that $R(a)$ can be neglected in practice. In this case, the free energy is described by a series which is $a$ dependent. As we will see, this series is able to describe a system with multiple ground states with different order parameters. A whole class of amorphous systems will be described by changing this dimensionless parameter.

To proceed, the mean-field theory corresponds to a saddle-point approximation in each replica partition function. A perturbative approach give us the fluctuation corrections to mean-field theory. Hence, to implement a perturbative scheme, it is necessary to investigate fluctuations around the mean-field equations. Imposing the replica-symmetric ansatz, the replica partition function and the effective Hamiltonian for each replica partition function read

$$\mathbb{E}[Z^k] = \frac{1}{k!}\int\prod_{i=1}^{k}[d\varphi_i^{(k)}]e^{-H_{\mathrm{eff}}(\varphi_i^{(k)})}, \quad (40)$$

and

$$H_{\mathrm{eff}}(\varphi_i^{(k)}) = \int d^dx\sum_{i=1}^{k}\left[\frac{1}{2}\varphi_i^{(k)}(x)(-\Delta + m_0^2)\varphi_i^{(k)}(x) + \frac{1}{4}(\lambda_0 - k\sigma)(\varphi_i^{(k)}(x))^4 + \frac{\rho_0}{6}(\varphi_i^{(k)}(x))^6\right]. \quad (41)$$

Note that a factor of $\frac{1}{k!}$ was absorbed into $\mathbb{E}[Z^k]$, which can be interpreted as representing an ensemble of $k$-identical replica fields. Also, the fields in each replica partition function are different since each field has the quartic coefficient $(\lambda_0 - k\sigma)$. Up to now, we have followed the approach developed in Refs. [23,24], where we considered only the leading term in the series representation for the averaged free energy. However, in order to access the glass-like phases that characterize a disordered system, we have to consider the contributions of all of the terms in the series given by Eq. (39). In the following we show that each term in the series in Eq. (39) describes a field theory with different order parameters. Therefore, a single order parameter is insufficient to describe the low-temperature phase of the disordered system.

Here we follow the discussion for the tricritical phenomenon presented in Ref. [46]. Let us define a critical $k_c$ for each temperature given by

$$k_c = \left\lfloor\frac{\lambda_0(T)}{\sigma} - \frac{4}{\sigma}\sqrt{\frac{m_0^2(T)\rho_0}{3}}\right\rfloor, \quad (42)$$

where $\lfloor x\rfloor$ means the integer part of $x$. Note that $k_c$ is a function of $\sigma$, $m_0$, $\lambda_0$, and $\rho_0$. For simplicity, we consider the case where $m_0^2(T) > 0$. Possible functional forms for the squared mass and coupling constant are $m_0^2(T) = \mu^{2-\gamma}T^\gamma$ and $\lambda_0(T) = \mu^{d-4-\alpha}T^\alpha$, where $\mu$ is an arbitrary parameter with dimensions of mass. The region in the parameter space for which $k \leq k_c$ corresponds to the situation where metastability is absent, as the replica fields in each replica partition function fluctuate around zero-value, stable equilibrium states. For $k > k_c$, the zero value for the replica fields is a metastable equilibrium state. For these replica partition functions there are first-order phase transitions. The existence of domains with different order parameters can be most easily understood using an analogy with a dynamical phase transition induced by a deep temperature quenching [47]. Specifically, a system initially in a stable high-temperature equilibrium state will develop spatially inhomogeneous domains when quenched to sufficiently low temperatures. The dynamical evolution stops when the system reaches a new equilibrium state. The nature of the inhomogeneities depends on the equilibrium free energy landscape. In the present case, the inhomogeneities appear in the form of bubble nucleation due to the form of the replica free energy in Eq. (41), which signals first-order phase transitions.

In the series representation for the free energy, each replica partition function is defined by a functional space where the replica fields are different. As already discussed, the contribution to the free energy that we are interested is $a$ dependent, and therefore this structure with multiple ground states with different order parameters depends on $a$, whose specific value depends on the physical system under consideration. Different classes of amorphous systems are characterized by different values of $a$: when one changes the value of $a$, one chooses a subset of replica partition functions that will be the relevant ones in the series. This is a quite interesting situation, in which the structure of the vacuum states is modified by changing this dimensionless parameter. We claim that the series representation for the average free energy leads to a natural interpretation of describing inhomogeneous systems. The average free energy (39) can be written as

$$F = F_1 + \frac{1}{\beta}\sum_{k=k_c}^{N}\frac{(-1)^k a^k}{k}\mathbb{E}[Z^k] + \cdots, \tag{43}$$

where $F_1$ is the contribution to the average free energy for replica fields which oscillate around the true vacuum, i.e., $\varphi_0^{(k)} = 0$, for $k \leq k_c$. $\mathbb{E}[Z^k]$ is defined in Eqs. (40) and (41).

One interpretation for this series is that each term describes macroscopic homogeneous domains. Each domain $\Omega^{(k)}$ has at least one order parameter $\varphi_0^{(k)}$. An important question is the size of the domains in the model. The size of each domain is characterized by the correlation length $\xi^{(k)}$, which can be estimated from the renormalized correlation functions. Therefore, in the next section we will perform the one-loop renormalization of the model.

## V. ONE-LOOP RENORMALIZATION IN THE DISORDERED MODEL

To proceed we will go beyond the mean-field approximation by implementing the one-loop renormalization in this model. For the sake of simplicity, we consider the leading replica partition function. This partition function is described by a large-$N$ Euclidean replica field theory [23]. Notice that all of the calculations can be performed in a generic replica partition function. The leading replica partition function is written as

$$\mathbb{E}[Z^N] = \frac{1}{N!}\int\prod_{i=1}^{N}[d\varphi_i]e^{-H_{\mathrm{eff}}(\varphi_i)}, \tag{44}$$

where

$$H_{\mathrm{eff}}(\varphi_i) = \int d^dx\sum_{i=1}^{N}\left[\frac{1}{2}\varphi_i(x)(-\Delta + m_0^2)\varphi_i(x) + \frac{1}{4}(\lambda_0 - N\sigma)\varphi_i^4(x) + \frac{\rho_0}{6}\varphi_i^6(x)\right], \tag{45}$$

where for simplicity $\varphi_i^{(N)} = \varphi_i$. Let us define $g_0 = \lambda_0 - f_0$, where $f_0 = N\sigma$. We maintain $f_0$ fixed while $N \to \infty$ and $\sigma \to 0$. Since in the Landau-Ginzburg scenario $\lambda_0$ depends on the temperature, $g_0$ is not positive definite for sufficiently low temperatures. For simplicity we assume that $m_0^2$ is a positive quantity. In this situation we have $N$ replicas with true and false vacua. Vacuum transitions in this theory with $N$ replicas can be described in the following way. Lowering the temperature, each replica field has a false vacuum and two degenerate true vacuum states. The transition from the false vacuum to the true one will nucleate bubbles of the true vacuum [48–50]. One way to proceed is to calculate the transition rates in the diluted instanton approximation. This is a standard calculation that can be found in the literature. Instead of this, our goal is to perform the one-loop renormalization of the model.

At this point, let us introduce an external source $J_i(x)$ in replica space linearly coupled with each replica. Considering only the leading term in the series representation for the average free energy, and absorbing the dimensionless quantity $a$ in the functional measure, we are able to define the generating functional of all correlation functions for a large-$N$ Euclidean field theory as $\mathbb{E}[Z^N(J)] = \mathcal{Z}(J)$. To proceed, we follow Ref. [51]. Accordingly, this generating functional of all correlation functions of this Euclidean field theory is given by

$$\mathcal{Z}(J) = \frac{1}{N!}\int\prod_{i=1}^{N}[d\varphi_i]e^{-H_{\mathrm{eff}}(\varphi_i) + \int d^dx\sum_{i=1}^{N}J_i\varphi_i}. \tag{46}$$

It is possible to define the generating functional of connected correlation functions $\mathcal{W}(\mathcal{J}) = \ln\mathcal{Z}(\mathcal{J})$. For simplicity we assume that we have one replica field. Since in the large-$N$ approximation all of the replica fields are equal, this procedure is identical for all of the fields. The generating functional of one-particle irreducible correlations (vertex functions) $\Gamma[\bar{\phi}]$ is obtained by taking the Legendre transform of $\mathcal{W}(\mathcal{J})$,

$$\Gamma[\bar{\phi}] + \mathcal{W}(\mathcal{J}) = \int d^dx(\mathcal{J}(x)\bar{\phi}(x)), \tag{47}$$

where

$$\bar{\phi}(x) = \frac{\delta\mathcal{W}(\mathcal{J})}{\delta\mathcal{J}}\bigg|_{\mathcal{J}=0}. \tag{48}$$

For the sake of completeness we discuss the one-loop renormalization of the corresponding theory. First, a vertex expansion for the effective action is given by

$$\Gamma[\bar{\phi}] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} d^d x_i \Gamma^{(n)}(x_1, \ldots, x_n) \bar{\phi}(x_1) \ldots \bar{\phi}(x_n), \tag{49}$$

where the expansion coefficients $\Gamma^{(n)}$ correspond to the one-particle irreducible (1PI) proper vertex. Writing the effective action in powers of momentum around the point where all external momenta vanish, we have

$$\Gamma[\bar{\phi}] = \int d^d x V(\bar{\phi}) + \cdots \tag{50}$$

The term $V(\bar{\phi})$ is called the effective potential which takes into account the fluctuation in the model. Let us define the Fourier transform of the 1PI proper vertex. We get

$$\Gamma^{(n)}(x_1, \ldots, x_n) = \frac{1}{(2\pi)^n} \int \prod_{i=1}^{n} d^d k_i (2\pi)^d \times \delta(k_1 + \cdots + k_n) e^{i(k_1 x_1 + \cdots + k_n x_n)} \tilde{\Gamma}^{(n)}(k_1, \ldots, k_n). \tag{51}$$

Now, we assume that the field $\bar{\phi}(x) = \phi$ is uniform. This condition is similar to the diluted instanton approximation. In this case, we can write

$$\Gamma[\phi] = \int d^d x \sum_{n=1}^{\infty} \frac{1}{n!} [\tilde{\Gamma}^{(n)}(0, \ldots, 0) \phi^n + \cdots]. \tag{52}$$

The effective potential can be written as

$$V(\phi) = \sum_{n} \frac{1}{n!} \tilde{\Gamma}^{(n)}(0, \ldots, 0) \phi^n. \tag{53}$$

From the above discussion it is possible to write the effective potential for each replica field in the leading replica partition function as $V(\phi) = V_1(\phi) + V_2(\phi)$, where

$$V_1(\phi) = \frac{1}{2}(m_0^2 + \delta m_0^2)\phi^2 + \frac{1}{4}(g_0 + \delta g_0)\phi^4 + \frac{1}{4}(\rho_0 + \delta\rho_0)\phi^6, \tag{54}$$

$\delta m_0^2$, $\delta g_0$, and $\delta\rho_0$ are the counterterms that have to be introduced to remove divergent terms, and

$$V_2(\phi) = \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} \ln\left[1 + \frac{1}{p^2 + m_0^2}(3g_0\phi^2 + 5\rho_0\phi^4)\right]. \tag{55}$$

The calculation that we present here takes into account the corrections due to the fluctuations around the saddle-point of each replica partition function. To proceed, we are interested in implementing the one-loop renormalization in each replica field theory. The contribution to the effective potential given by $V_2(\phi)$ can be written as

$$V_2(\phi) = \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{2s} (3g_0\phi^2 + 5\rho_0\phi^4)^s I(s, d), \tag{56}$$

where $I(s, d)$ is given by

$$I(s, d) = \int \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2 + m_0^2)^s} = \frac{1}{(2\sqrt{\pi})^d} \frac{\Gamma(s - \frac{d}{2})}{\Gamma(s)} (m_0^2)^{\frac{d}{2} - s}. \tag{57}$$

At this point, let us use an analytic regularization procedure that has been used in field theory [52] and also to obtain the renormalized vacuum energy of a quantum field in the presence of boundaries [53–55]. Using the well-known result that in the neighborhood of the pole $z = -n$ $(n = 0, 1, 2, \ldots)$ and for $\varepsilon \to 0$ the gamma function has the representation

$$\Gamma(-n + \varepsilon) = \frac{(-1)^n}{n!} \left[\frac{1}{\varepsilon} + \psi(n + 1)\right] \tag{58}$$

[where $\psi(n + 1)$, the digamma function, is the regular part in the neighborhood of the pole], and using the renormalization conditions

$$\frac{d^2}{d\phi^2} V(\phi)|_{\phi=0} = m_R^2, \quad \frac{d^4}{d\phi^4} V(\phi)|_{\phi=0} = g_R, \quad \frac{d^6}{d\phi^6} V(\phi)|_{\phi=0} = \rho_R, \tag{59}$$

we obtain renormalized physical quantities. Note that the normalization conditions are chosen in the metastable vacuum state. It is possible to choose another normalization condition such as in the true minimum of the effective potential. We would like to stress that all of the renormalization conditions are equivalent after one establishes the correspondence between them [56]. For the sake of simplicity, we consider the case where $d = 4$:

$$m_R^2 = m_0^2 \left[1 - \frac{3g_0\psi(2)}{16\pi^2}\right], \tag{60}$$

$$g_R = 6g_0 + \frac{9\psi(1)}{4\pi^2} g_0^2 - \frac{15\psi(2)}{4\pi^2} \rho_0 m_0^2, \tag{61}$$

$$\rho_R = \rho_0 \left[120 - \frac{675\psi(1)}{2\pi^2} g_0\right] + \frac{369}{4\pi^2} \frac{g_0^3}{m_0^2}, \tag{62}$$

where $\psi(1) = -\gamma$ and $\psi(2) = -\gamma + 1$.

In conclusion, the DZFM allows us to write a series representation for the average free energy where each term is given by a replica partition function. In the leading-order approximation we get only one replica partition function with $N$ identical fields. In this case, it is sufficient to work with only one replica field. In order to renormalize this theory we used the effective potential approach. Combining an analytic regularization procedure to regularize the theory and the standard renormalization conditions, we obtained a finite theory. Notice that in order to describe an amorphous system, we had to take into account many replica partition functions in the series representation for the free energy. In this case, the renormalization procedure can be implemented by the same token as the case discussed above.

## VI. CONCLUSIONS

How the mathematical formalism of the statistical mechanics of disordered systems differs from that of homogeneous systems is a fundamental question. In homogeneous systems the study of the low-temperature phase can be simplified making use of the many spatial symmetries that such systems have. In principle, in quenched disordered systems these symmetries are absent. The first step to recover at least the translational symmetry in disordered systems is averaging over the quenched disorder. Here, in order to partially answer the above question, we would like to discuss a few disordered models that have been investigated on the lattice and also using statistical field theory defined in the continuum.

One of the simplest models of spin glass is the Edwards-Anderson model. The spin-glass phase of this model is characterized by the absence of orientational localized magnetic-moment ordering in space at low temperatures. This indicates that the system does not have a unique ground state. These multiple vacuum states appear in an infinite-range spin model: the Sherrington-Kirkpatrick model. In this model, a replica-symmetry-breaking mechanism is introduced in order to prevent the emergence of unphysical results, i.e., a negative entropy at low temperatures, which would arise with the assumption of a replica-symmetric solution in such a model. In the replica-symmetry-breaking scheme for this fully connected model, the mean-field approximation predicts a unusual structure for the free energy: the existence of a multivalley structure in the free-energy landscape. Studying a statistical field theory defined in the continuum, some authors using the replica method with a replica-symmetry-breaking mechanism have discussed the spin-glass-like behavior and the possibility of the existence of infinitely many ground states in the random-temperature Landau-Ginzburg model.

One of the motivations of this paper was to emphasize the differences and similarities between the DZFM and the conventional replica method by discussing a disordered $\lambda\varphi^4 + \rho\varphi^6$ Landau-Ginzburg model defined in a $d$-dimensional space. First, we adopted the standard procedure by averaging the disorder-dependent free energy using the DZFM. We showed that the dominant contribution to the average free energy of this system is written as a series of the replica partition functions of the model. In a generic replica partition function, the structure of the replica space was investigated using the saddle-point equations. In each replica partition function we imposed the replica-symmetry ansatz. We proved that the average free energy represents a system with multiple ground states with different order parameters. This situation is quite similar to the one obtained in a fully connected mean-field model in a replica-symmetry-breaking scenario. For low temperatures, we also showed the existence of metastable equilibrium states for some replica fields. In the low-temperature regime, one way to proceed is to consider the possibility that the series representation for the average free energy describes inhomogeneous domains, i.e., macroscopic regions in a sample $\Omega^{(k)}$ with at least one proper characteristic order parameter $\varphi_0^{(k)}$.

Finally, we discussed the leading term in the series representation for the average free energy. This leading term of this series expansion is a large-$N$ Euclidean replica field theory. In this leading-order replica partition function, the one-loop renormalization of this model was performed. It is important to point out that it is possible to go beyond the one-loop approximation using the composite field operator formalism [57–62], where an infinite number of leading diagrams is summed. This technique deals with the effective action formalism for composite operators. One must consider a generalization of the effective action where the scalar field is coupled linearly and quadratically to sources. This generalization is under investigation by the authors.

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