Renormalization group approach in a Lifshitz-like Gross-Neveu-Thirring model

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In this work, we use the renormalization group method in the study of the behavior of a quartic fermionic self-interaction in a Gross-Neveu-Thirring model in $2+1$ dimensions, in the context of Hořava-Lifshitz theory. We show that if we include high derivatives in the spatial part of the free Lagrangian density for a critical exponent $z=2$ the model becomes renormalizable by power counting, thus improving the ultraviolet (UV) behavior of theory. We determine the renormalization group (RG) functions at one-loop order and we obtain the fixed points of effective beta function $(\bar{\beta})$. We find that it is asymptotically free for the case $\bar{\beta}<0$.

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I. INTRODUCTION

One of the very well-known symmetries in nature at the scale of observable energy is the Lorentz invariance [1]. However, there recently has been great interest in the possibility that in more fundamental theories small violations are induced to the Lorentz invariance [2,3]. A minimal expected feature of any model that considers violation of Lorentz invariance is the relativistic manifestation of this invariance at low energies, so that the Lorentz symmetry is seen as an emergent scenario.

Another point of great interest that has received much attention lately is the analysis of the possible effects of space-time anisotropy. These studies involve different areas, such as gravity and cosmology [4], string theory [5], and condensed matter systems [6]. In particular, in the context of quantum field theory, this possibility emerges as an alternative for the study of nonrenormalizable theories, once it improves the UV behavior of the perturbative series despite violating the symmetry of Lorentz [7]. The breaking of Lorentz symmetry has been studied in several situations, encompassing commutative models [8], the standard extended model [9] and the physics of graphene [10], which provide conditions for parameters of the Lorentz symmetry breaking [11].

In the context of the Lorentz invariance break, as in Lifshitz-like models, we explore the possibility that a nonrenormalizable model can be renormalized by adjusting the parameter of Lifshitz scaling that measures the degree of anisotropy between space and time. This is due to behavior of Lifshitz-like anisotropic scaling symmetry in which time and space scale differently: $x^0 \to \lambda^z x^0$, $x^i \to \lambda x^i$ [12,13], where the exponent z characterizes the scaling symmetry. Because of this anisotropic scaling, the Lorentz symmetry is explicitly broken for $z \neq 1$. In the sense of the power count, the power in the spatial component of the momentum in the denominator of the free propagator is increased by the factor z. Thus, a suitable choice of z can lead to a better UV behavior of the theory, or in the sense of interactions, make a previously nonrenormalizable theory into a renormalizable one.

The paper is organized as follows. In Sec. II, we present the Lifshitz-type fermionic quartic self-interaction model and the Feynman rules. In Sec. III we show the renormalization group equations and the calculations of renormalization group functions. In Sec. IV we use the Zimmerman's reduction mechanism of the coupling constants through which we will find the effective beta function of the system, and we will investigate the fixed points of the model, as well as the mass renormalization. In Sec. V we review the main results obtained in this paper and in the Appendix we show some details of the calculations.

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II. THE MODEL

The Lagrangian density of the model in $2+1$ space-time dimension is given by

$$\mathcal{L} = \bar{\psi}[i\gamma^0\partial_0 + ib\gamma^i\partial_i + a(i\gamma^i\partial_i)^z - M]\psi + g_1(\bar{\psi}\psi)^2 + g_2(\bar{\psi}\gamma^0\psi)^2 + g_3(\bar{\psi}\gamma^i\psi)^2, \quad (1)$$

where z designates the highest degree of the spatial derivatives, (γ^0, γ^i) are Dirac matrices, and M is the bare mass fermion field. The g_i are the coupling constants associated with each term of four fermions. The parameters a and b correspond to the strong and soft break of the Lorentz symmetry, respectively [14]. The part associated with the Thirring term has been separated into two terms (time and space parts), each generated by its coupling constant. The effective dimension of the Lagrangian density is $z+d$ [14], where d represents spatial dimension. By taking a as a dimensionless parameter, we find for $\mathrm{Dim}[b] = z-1$ and the canonical effective dimension of the fermionic field is $\mathrm{Dim}[\psi] = d/2$. Therefore, for a generic Feynman diagram G, the degree of divergence will be given by

$$d(G) = d + z - \mathrm{Dim}[\psi]N_F - \sum_v (d + z - \mathrm{Dim}[\psi]\mu_v{}^F), \quad (2)$$

where N_F is the number of external fermionic lines and $\mu_v{}^F$ are the fermionic lines joining at the interaction vertex v. For a purely fermionic theory with a quartic nonderivative self-interaction, the renormalizability requires that the value of the critical parameter must be $z = d = 2$. In this situation, Eq. (2) becomes $d(G) = 4 - N_F$, which reflects the fact that with this choice the model is renormalizable perturbatively, i.e., g_1, g_2 and g_3 are dimensionless. Thus, the divergent diagrams have two and four external lines and are quadratic and logarithmically divergent, respectively. In this work we will use a variant of the dimensional regularization to render finite the relevant diagrams, and for that reason we will introduce a new coupling constant $g_i \to \mu^\epsilon g_i$, where μ is a renormalization parameter, $i = 1, 2, 3$ and $\epsilon = 2 - d$ which must be set zero at the end.

In the Feynman rules, the fermion free propagator of the theory is given by

$$S_F(p)_{\alpha_1\alpha_2} = \frac{i[\gamma^0 p_0 + b\gamma^i p_i + a\boldsymbol{p}^2 + M]_{\alpha_1\alpha_2}}{[p_0^2 - (b^2 + 2aM)\boldsymbol{p}^2 - a^2\boldsymbol{p}^4 - M^2]} \quad (3)$$

and the vertices are given by $ig_1\Delta\otimes\Delta$, $ig_2\Gamma^0\otimes\Gamma_0$, and $ig_3\Gamma^i\otimes\Gamma_i$, where

$$\Delta\otimes\Delta = \delta_{\alpha_1\alpha_2}\delta_{\alpha_3\alpha_4} - \delta_{\alpha_1\alpha_4}\delta_{\alpha_3\alpha_2}, \quad (4)$$

$$\Gamma^0\otimes\Gamma_0 = \gamma^0_{\alpha_1\alpha_2}\gamma_{0\alpha_3\alpha_4} - \gamma^0_{\alpha_1\alpha_4}\gamma_{0\alpha_3\alpha_2}, \quad (5)$$

$$\Gamma^i\otimes\Gamma_i = \gamma^i_{\alpha_1\alpha_2}\gamma_{i\alpha_3\alpha_4} - \gamma^i_{\alpha_1\alpha_4}\gamma_{i\alpha_3\alpha_2}, \quad (6)$$

and $\delta_{\alpha\beta}$ means identity matrix.

III. RENORMALIZATION GROUP EQUATION

The equation of the renormalization group obtained from the use of dimensional regularization is the t'Hoft-Weinberg equation given by [15]:

$$\left[\mu\frac{\partial}{\partial\mu} + \delta_M\frac{\partial}{\partial M} + \beta_1\frac{\partial}{\partial g_1} + \beta_2\frac{\partial}{\partial g_2} + \beta_3\frac{\partial}{\partial g_3} + \beta_a\frac{\partial}{\partial a} + \beta_b\frac{\partial}{\partial b} - N_F\gamma_\psi\right]\Gamma^{(N_F)} = 0, \quad (7)$$

where $\Gamma^{(N_F)} = \Gamma^{(N_F)}(p_1, \ldots, p_N)$ represents the renormalized vertices functions and $p_1, \ldots, p_N$ stand for the external momenta. The two-point and four-point vertex functions can be written as

$$\Gamma^{(2)}_{\alpha_1\alpha_2} = i[\gamma^0 p_0 + b\gamma^i p_i - a(\mathbf{p})^2 - M]_{\alpha_1\alpha_2} + I^{(2)}_{\alpha_1\alpha_2} \quad (8)$$

and

$$\Gamma^{(4)}_{\alpha_1\alpha_2\alpha_3\alpha_4} = ig_1\mu^\epsilon\Delta\otimes\Delta + ig_2\mu^\epsilon\Gamma^0\otimes\Gamma_0 + ig_3\mu^\epsilon\Gamma^i\otimes\Gamma_i + I^{(4)}_{\alpha_1\alpha_2\alpha_3\alpha_4}, \quad (9)$$

where $I^{(2)}_{\alpha_1\alpha_2}$ and $I^{(4)}_{\alpha_1\alpha_2\alpha_3\alpha_4}$ represent the quantum corrections. We can renormalize these functions using the following procedure: by considering an amplitude of the form $I^{(N_F)} = \mathrm{polo}^{(N_F)} + \mathrm{finite}^{(N_F)}$, the renormalized amplitude can be obtained by the operation

$$(1-\mathcal{T})\mu^{x\epsilon}I^{(N_F)} = \mathrm{finite}^{(N_F)} + x\ln\mu\,\mathrm{Res}^{(N_F)}, \quad (10)$$

where $x=1$ and $x=2$, for the function of two and four points, respectively, $\mathcal{T}$ is an operator which removes the pole term in the amplitudes, and $\mathrm{Res}^{(N_F)}$ represents the residue of the diagrams with N_F external lines, which are given by the coefficients of the term $1/\epsilon$.

A. Two-point Green's functions

At 1-loop order the diagrams of two points are shown in Fig. 1, and they have an analytical structure given by

$$I^{(2)}_{a_{\alpha_1\alpha_2}} = -4i\mu^\epsilon\int\frac{dk^0}{2\pi}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\{g_1\mathrm{Tr}[S_F(k)]\delta_{\alpha_1\alpha_2} + g_2\mathrm{Tr}[\gamma^0 S_F(k)]\gamma_{0_{\alpha_1\alpha_2}} + g_3\mathrm{Tr}[\gamma^i S_F(k)]\gamma_{i_{\alpha_1\alpha_2}}\}, \quad (11)$$

and

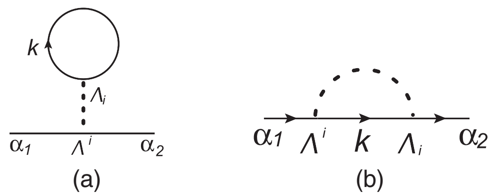

FIG. 1. One loop correction for two-point functions. Figure (a) involves the trace of the Dirac matrices. Diagram (b) is the one loop correction absent from the trace. The dotted line represents the different types of interaction vertices.

$$I^{(2)}_{b_{\alpha_1\alpha_2}} = 4i\mu^{\epsilon}\int\frac{dk^0}{2\pi}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\{g_1[S_F(k)]_{\alpha_1\alpha_2} + g_2[\gamma_0 S_F(k)\gamma^0]_{\alpha_1\alpha_2} + g_3[\gamma_i S_F(k)\gamma^i]_{\alpha_1\alpha_2}\}. \quad (12)$$

By using the method described in the Appendix and $I^{(2)}_{\alpha_1\alpha_2} = I^{(2)}_{a_{\alpha_1\alpha_2}} + I^{(2)}_{b_{\alpha_1\alpha_2}}$, we get

$$I^{(2)}_{\alpha_1\alpha_2} = i\mu^{\epsilon}\left\{g_1\left(\frac{b^2}{2\pi a^2}\frac{1}{\epsilon} + \text{finite}_1\right) - g_2\left(\frac{b^2}{2\pi a^2}\frac{1}{\epsilon} + \text{finite}_2\right) - 2g_3\left(\frac{b^2}{2\pi a^2}\frac{1}{\epsilon} + \text{finite}_3\right)\right\}\delta_{\alpha_1\alpha_2}. \quad (13)$$

B. Four-point Green's functions

The 4-point diagrams are shown in Fig. 2. They are logarithmically divergent in the UV regime. Therefore, to extract the divergent term we can calculate these diagrams by taking the external momenta equal to zero. Thus, we can write

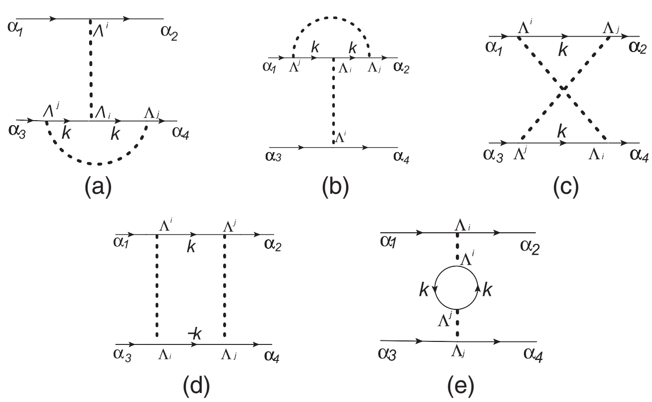

FIG. 2. Diagrams representing the 1-loop corrections of the 4-point vertex functions for external momenta equal to zero. The continuous lines stand for the fermion propagator and the dotted lines indicate all the types of interaction contained in the Lagrangian (1), providing a total of 30 diagrams. Diagram (e) represents the contribution that contains the trace of Dirac matrices, while the other diagrams ((a)–(d)) do not contain the trace operation.

$$I^{(4)}_{\alpha_1\alpha_2\alpha_3\alpha_4}(p_{\text{ext}} = 0) = i\mu^{2\epsilon}\sum_{i,j=1}^{3} g_i g_j (I^{(4)}_{ij})_{\alpha_1\alpha_2\alpha_3\alpha_4}, \quad (14)$$

where,

$$(I^{(4)}_{ij})_{\alpha_1\alpha_2\alpha_3\alpha_4} = I^{(4)}_{a\alpha} + I^{(4)}_{b\alpha} + I^{(4)}_{c\alpha} + I^{(4)}_{d\alpha} + I^{(4)}_{e\alpha}, \quad (15)$$

with

$$I^{(4)}_{a_\alpha} = \int\frac{dk_0}{2\pi}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\{\Lambda^i_{\alpha_1\alpha_2}\otimes[\Lambda^j S(k)\Lambda_i S(k)\Lambda_j]_{\alpha_3\alpha_4} - \Gamma^i_{\alpha_1\alpha_4}\otimes[\Lambda^j S(k)\Lambda_i S(k)\Lambda_j]_{\alpha_3\alpha_2}\}, \quad (16)$$

$$I^{(4)}_{b_\alpha} = \int\frac{dk_0}{2\pi}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\{[\Lambda^j S(k)\Lambda_i S(k)\Lambda_j]_{\alpha_1\alpha_2}\otimes\Lambda^i_{\alpha_3\alpha_4} - [\Lambda^j S(k)\Lambda_i S(k)\Lambda_j]_{\alpha_1\alpha_4}\otimes\Lambda^i_{\alpha_3\alpha_2}\}, \quad (17)$$

$$I^{(4)}_{c_\alpha} = \int\frac{dk_0}{2\pi}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\{[\Lambda^i S(k)\Lambda^j]_{\alpha_1\alpha_2}\otimes[\Lambda^j S(k)\Lambda_i]_{\alpha_3\alpha_4} - [\Lambda^i S(k)\Lambda^j]_{\alpha_1\alpha_4}\otimes[\Lambda^j S(k)\Lambda_i]_{\alpha_3\alpha_2}\}, \quad (18)$$

$$I^{(4)}_{d_\alpha} = \int\frac{dk_0}{2\pi}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\{[\Lambda^i S(k)\Lambda^j]_{\alpha_1\alpha_2}\otimes[\Lambda_i S(k)\Lambda_j]_{\alpha_3\alpha_4} - [\Lambda^i S(k)\Lambda^j]_{\alpha_1\alpha_4}\otimes[\Lambda_i S(k)\Lambda_j]_{\alpha_3\alpha_2}\}, \quad (19)$$

$$I^{(4)}_{e_\alpha} = -\int\frac{dk_0}{2\pi}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\{\Lambda_{i_{\alpha_1\alpha_2}}\otimes\Lambda_{j_{\alpha_3\alpha_4}}\text{Tr}[\Lambda^i S(k)\Lambda^j S(k)] - \Lambda_{i_{\alpha_1\alpha_4}}\otimes\Lambda_{j_{\alpha_3\alpha_2}}\text{Tr}[\Lambda^i S(k)\Lambda^j S(k)]\}, \quad (20)$$

where we have defined $\Lambda^1_{\alpha\beta} = \delta_{\alpha\beta}$, $\Lambda^2_{\alpha\beta} = \gamma^0_{\alpha\beta}$, $\Lambda^3_{\alpha\beta} = \gamma^i_{\alpha\beta}$ and for simplicity, we adopt $I^{(4)}_{a_\alpha} = I^{(4)}_{a_{\alpha_1\alpha_2\alpha_3\alpha_4}}$ and so on. By performing the integrals in Eqs. (16)–(20) (for details see the Appendix) and collecting the results, we obtain

$$I^{(4)}_{\alpha} = i\mu^{2\epsilon}\left\{\left\{g_1^2\left[\frac{4}{\pi a}\frac{1}{\epsilon} + \text{finite}_{11}\right] + g_2^2\left[\frac{4}{\pi a}\frac{1}{\epsilon} + \text{finite}_{22}\right]\right\}\times\Delta\otimes\Delta + \left\{g_3^2\left[\frac{-8}{\pi a}\frac{1}{\epsilon} + \text{finite}_{33}\right] + g_1 g_3\left[\frac{8}{\pi a}\frac{1}{\epsilon} + \text{finite}_{13}\right] + g_2 g_3\left[\frac{-8}{\pi a}\frac{1}{\epsilon} + \text{finite}_{23}\right]\right\}\times\Gamma^i\otimes\Gamma_i + g_1 g_2\left[\frac{8}{\pi a}\frac{1}{\epsilon} + \text{finite}_{12}\right]\times\Gamma^0\otimes\Gamma_0\right\}. \quad (21)$$

From Eqs. (10), (13), and (21) we can easily identify the residues

$$\mathrm{Res}_1 = \frac{ib^2}{2\pi a^2}, \qquad \mathrm{Res}_2 = -\frac{ib^2}{2\pi a^2}, \qquad \mathrm{Res}_3 = -\frac{ib^2}{\pi a^2},$$
$$\mathrm{Res}_{11} = -\frac{8i}{\pi a}, \qquad \mathrm{Res}_{22} = -\frac{8i}{\pi a}, \qquad \mathrm{Res}_{33} = -\frac{16i}{\pi a},$$
$$\mathrm{Res}_{12} = -\frac{16i}{\pi a}, \qquad \mathrm{Res}_{13} = -\frac{16i}{\pi a}, \qquad \mathrm{Res}_{23} = -\frac{16i}{\pi a}.$$

We can determine the functions of the renormalization group inserting into Eq. (7) the following expansions

$$\delta_M = \sum_{ijk} \delta_{ijk} g_1^i g_2^j g_3^k, \tag{22}$$

$$\gamma_\psi = \sum_{ijk} \gamma_{ijk} g_1^i g_2^j g_3^k, \tag{23}$$

and

$$\beta_\xi = \sum_{ijk} \beta_{(\xi)_{ijk}} g_1^i g_2^j g_3^k, \tag{24}$$

where $\xi = 1, 2, 3, a, b$, with the sum restricted to $i + j + k \leq 2$ and then use Eq. (13) and Eq. (21). The coefficients δ_{ijk}, γ_{ijk}, $\beta_{(a)_{ijk}}$, and $\beta_{(b)_{ijk}}$ are given in terms of the residues of the 2-point functions, whereas the coefficients $\beta_{(\xi)_{ijk}}$ ($\xi = 1, 2, 3$) are given in terms of the 4-point residues. In this way, we obtain

$$\begin{aligned}
&\gamma_\psi = \beta_a = \beta_b = 0, \\
&\delta_M = g_1 \mathrm{Res}_1 + g_2 \mathrm{Res}_2 + g_3 \mathrm{Res}_3, \\
&\beta_1 = g_1^2 \mathrm{Res}_{11} + g_2^2 \mathrm{Res}_{22}, \\
&\beta_2 = g_1 g_2 \mathrm{Res}_{12}, \\
&\beta_3 = g_3^2 \mathrm{Res}_{33} + g_1 g_3 \mathrm{Res}_{13} + g_2 g_3 \mathrm{Res}_{23},
\end{aligned} \tag{25}$$

and thus

$$\delta_M = \frac{b^2}{2\pi a^2}(g_1 - g_2 - 2g_3), \tag{26}$$

$$\beta_1 = -\frac{8}{\pi a}(g_1^2 + g_2^2), \tag{27}$$

$$\beta_2 = -\frac{16}{\pi a} g_1 g_2, \tag{28}$$

$$\beta_3 = -\frac{16}{\pi a}(g_3^2 + g_1 g_3 + g_2 g_3), \tag{29}$$

which are the functions of the renormalization group of the model at one-loop order. We note that the beta functions for the parameters a and b are zero, which means that up to this order both remain constant independent of the renormalization point.

IV. REDUCTION OF COUPLING CONSTANTS

The structure of the beta functions of the model makes the analysis of the fixed points very complicated, since we have three beta functions, each with three coupling constants to be analyzed simultaneously. Then, for the analysis of the fixed points, we will use the Zimmermann's reduction formalism of coupling constants [16]. The idea of this formalism is to reduce the three constants to an effective coupling constant, which, in principle, can be any of the three (g_1, g_2 or g_3). Such a scheme has been applied in a variety of situations including the cases of nonrenormalizable models, treated as effective theories, and also in massive theories [17]. Following Zimmermann's formalism, we consider one of the constants of the model to be effective, for instance g_1, such that

$$g_2 = \rho_1 g_1, \qquad g_3 = \rho_2 g_1. \tag{30}$$

The relationship between the effective coupling constant and the original coupling constant satisfies the following differential equation [16],

$$\beta_j = \frac{\partial g_j}{\partial g_i} \beta_i$$

with

$$\lim_{g_i \to 0} g_j = 0,$$

and $i = 1, 2, 3$ are indices which corresponds to the effective beta function that *a priori* can be any of the three of the model and $j = 1, 2, 3$ ($j \neq i$) are indices for the original beta function. So

$$\beta_2 = \rho_1 \beta_1, \qquad \beta_3 = \rho_2 \beta_1. \tag{31}$$

Since we are taking β_1 as the effective beta function, the beta functions given by Eqs. (27), (28), and (29) are replaced in the previous expressions. We replace the Eqs. (30) giving a system of equations of the three original beta functions as a function of the effective coupling constant

$$\begin{aligned}
&\rho_1 = \pm 1, \\
&\rho_2 = \frac{\rho_1^2 - 2\rho_1 - 1}{2},
\end{aligned}$$

where ρ_1 and ρ_2 are the coefficients of the reduction.

We will consider the analysis by taking $g_1 = \bar{g}$ as the effective coupling constant since the nature of the fixed points of the reduced system remains the same for both values of ρ_1 ($+1$ or -1). In this way, the effective beta function of the reduced system is of the form

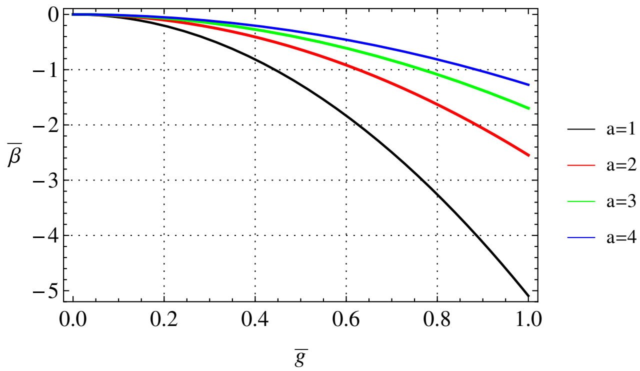

FIG. 3. The behavior of the negative beta function.

$$\bar{\beta}(\bar{g}) = -\frac{16}{\pi a}\bar{g}^2, \tag{32}$$

and we note that effective beta function has a single fixed point in $\bar{g} = 0$, called a trivial fixed point (see Fig. 3).

The "effective coupling constant" satisfies

$$\frac{\partial \bar{g}(g,t)}{\partial t} = \bar{\beta}(\bar{g}), \tag{33}$$

with the condition $\bar{g}(g, t = 0) = g_1$ (from now $g_1 \equiv g$) and t is a logarithmic energy scale. Thus, the solution which comes from Eq. (32) is given by

$$\bar{g}(g,t) = \frac{g}{1 + \frac{16}{\pi a} gt}. \tag{34}$$

As shown in Fig. 4, $\bar{g}(g,t) \to 0$ when $t \to \pm\infty$ besides $\bar{g}$ is finite at any finite value of the momentum. This corresponds to a UV stable fixed point of $\bar{\beta}$ and ensures that the perturbation theory in the vicinity of this fixed point could be used, and its behavior at high moments exhibiting an asymptotic freedom. Also, we must take into account the existence of a pole at $t_p = -\frac{\pi a}{16 g}$. For $t > t_p$ the running coupling of the reduced system changes signal, however, the nature of the fixed point remains unchanged. Other choices for the effective coupling constant (such as g_2 or g_3) are possible, however, the trivial fixed point becomes infrared stable, and the perturbation theory in the vicinity of this point could not be investigated.

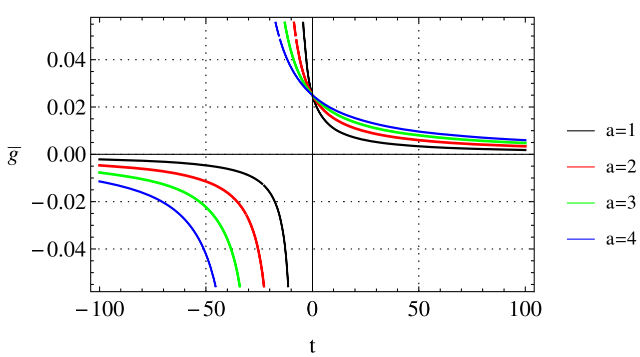

FIG. 4. The general behavior of the effective coupling constant, corresponds to the solution of $\bar{g}$ with respect to $t \to \pm\infty$, considering the particular case where $g = 0.025$, for different values of parameter a.

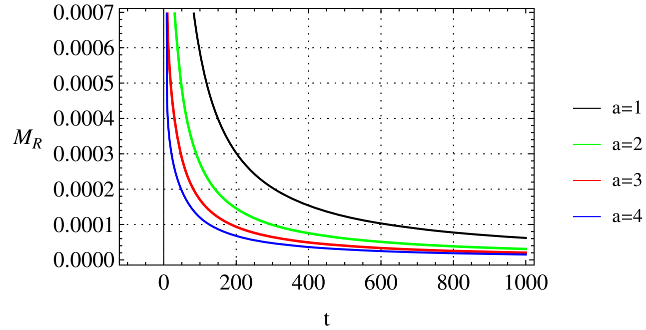

FIG. 5. The general behavior of renormalized mass for different values of a and fix $b = 1$ and $g = 0.025$.

For δ_M we have

$$\delta_M(g,t) = \frac{b^2}{\pi a^2}\bar{g}(g,t), \tag{35}$$

such that

$$\delta_M(g) - M_R = \frac{\partial M_R}{\partial t}. \tag{36}$$

Solving the Eq. (36), we obtain

$$M_R(g,t) = M\exp(-t) + \frac{b^2}{16a}\exp\left(\frac{-a\pi}{16g} - t\right) \times \left(Ei\left(\frac{a\pi}{16g} + t\right) - Ei\left(\frac{a\pi}{16g}\right)\right), \tag{37}$$

being,

$$E_i(x) = \sum_{k=1}^{\infty}\frac{(x)^k}{kk!} + \gamma + \frac{1}{2}\left(\mathrm{Log}[x] - \mathrm{Log}\left[\frac{1}{x}\right]\right),$$

where γ corresponds to the Euler-Mascheroni constant and we have considered $M_R(g, t = 0) = M$. From Eq. (37) we see that the mass asymptotically disappears with $t \to \infty$ and that it presents a divergence when the momentum tends to $-\infty$, as we see in Fig. 5.

V. CONCLUSIONS

In this paper we investigate the renormalization group functions in the Gross-Neveu-Thirring model in $(2 + 1)$ space-time at one loop order in the context of the Hořava-Lifshitz theory. We use a variant of dimensional regularization prescription to render finite the Feynman amplitudes. This model presents an explicit Lorentz symmetry break, in which time scales with z leading to an effective dimension

$d + z$, where d is spatial dimension and z in critical exponent which characterizes space-time anisotropy. The presence of the term that represents the stronger Lorentz symmetry break in Lagrangian given by Eq. (1) makes the model have a better behavior in the UV regime, and in the specific case which $z = 2$, the model becomes renormalizable in the coupling constants. As expected, the beta functions of the parameters a and b are null, as well as the anomalous dimension of the fermionic field since the external momentum does not flow through the fermionic loop. Thus, up to this order, it is not possible to verify in which scale of energy the restoration of the Lorentz symmetry could occur. Using the Zimmermann procedure for the reduction of the coupling constants, and adopting g_1 as the effective coupling constant, we show that the theory exhibits a UV fixed point at origin, which means that theory is asymptotically free. With this choice of effective coupling, we note that the nature of the fixed points of the theory does not change for the values $\rho_1 = 1$ or $\rho_1 = -1$. In addition, the Lorentz symmetry breaking contained in the interactions of four-fermions are restored to the two possible choices of ρ_1 (or ρ_2). On the other hand, the choice $\rho_1 = -1$ implies that $\delta_M = 0$, and therefore, $M_R(t) = M \exp^{-t}$, which is independent of four-fermions interactions, contrary for the case $\rho_1 = 1$ which gives the renormalized mass shown in Eq. (37).

Since the dispersion relation of this model is the same as the electrons in graphene bilayer in the Bernal-Stacking configuration [18,19], the four fermion interactions could simulate other microscopic interactions [20], beyond the long-range Coulombian interaction between electrons in graphene [21]. Thus, it is possible that the approach used in this work has some relevance in the study of two-dimensional systems.

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APPENDIX: SOME DETAILS OF THE CALCULATIONS

This appendix aims to detail the calculation of integrals in the anisotropic case. All integrals, considering a break of Lorentz symmetry up to a second derivative in the spatial part of the Lagrangian, have the following general structure [14,22]

$$J(x,y,z) = \int \frac{dk_0}{2\pi} \frac{d^d\mathbf{k}}{(2\pi)^d} \frac{k_0^x |\mathbf{k}|^y}{(k_0^2 - b^2\mathbf{k}^2 - a^4\mathbf{k}^4 - M^2)^z}, \tag{A1}$$

and using the Schwinger parametrization [23], the final result of the integral will be (for the details of this calculation check the Appendix of Ref. [14])

$$\begin{aligned} J(x,y,z) = {} & \frac{i^{-z} 2^{-4-d}}{3a^2\Gamma(\frac{d}{2})} (-(-1)^{\frac{3}{4}})^{-x-1} (1 + (-1)^x)(ia^2)^{-\frac{d}{4}-\frac{y}{4}-\frac{1}{2}} (iM^2)^{\frac{d}{4}+\frac{x}{2}+\frac{y}{4}-z-1} \pi^{-\frac{d}{2}-1} \\ & \times \Gamma\left(\frac{x+1}{2}\right) \left\{ 6a^2b^2M^2 \Gamma\left(\frac{d}{4}+\frac{y}{4}+\frac{1}{2}\right) \Gamma\left(-\frac{d}{4}-\frac{y}{4}-\frac{x}{2}+z\right) + b^6 \Gamma\left(\frac{d}{4}+\frac{y}{4}+\frac{3}{2}\right) \Gamma\left(-\frac{d}{4}-\frac{y}{4}-\frac{x}{2}+z+1\right) \right. \\ & \left. - 3aM \left[2a^2M^2 \Gamma\left(\frac{d}{4}+\frac{y}{4}\right) \Gamma\left(-\frac{d}{4}-\frac{y}{4}-\frac{x}{2}+z-\frac{1}{2}\right) + b^4 \Gamma\left(\frac{d}{4}+\frac{y}{4}+1\right) \Gamma\left(-\frac{d}{4}-\frac{y}{4}-\frac{x}{2}+z+\frac{1}{2}\right) \right] \right\}. \end{aligned} \tag{A2}$$

In order to find the solution we must assign the values of x, y, z, $d(\epsilon)$, a, b e M in the above expression. After the identification of these variables, we expand it to ϵ small and then we can determine the pole term Res/ϵ.

Now, consider Fig. 1(b) given by equation

$$\begin{aligned} I^{(2)}_{b_{\alpha_1\alpha_2}} = {} & 4i\mu^\epsilon \int \frac{dk^0}{2\pi} \int \frac{d^d\mathbf{k}}{(2\pi)^d} \{ g_1 [S_F(k)]_{\alpha_1\alpha_2} \\ & + g_2 [\gamma_0 S_F(k) \gamma^0]_{\alpha_1\alpha_2} + g_3 [\gamma_i S_F(k) \gamma^i]_{\alpha_1\alpha_2} \}, \end{aligned} \tag{A3}$$

where the propagator Eq. (3) is replaced, after operating the matrices of Dirac and writing the denominator as $A(k) = k_0^2 - b_1^2\vec{k}^2 - a^4\vec{k}^4 - M^2$, being $b_1^2 = b^2 + 2aM$ allows us to consider that the vertex function of two points is given by

$$\begin{aligned} I^{(2)}_{b_{\alpha_1\alpha_2}} = {} & 4i\mu^\epsilon \delta_{\alpha_1\alpha_2} \left\{ \int \frac{dk_0}{2\pi} \int \frac{d^d\mathbf{k}}{(2\pi)^d} \left[\frac{a\mathbf{k}^2 + M}{A(k)} \right] \right. \\ & \left. \times (g_1 + g_2 + 2g_3) \right\}. \end{aligned} \tag{A4}$$

The absence of k_0 and k_i in the above expression is due to the fact that their contributions are zero. Using the integral (A1), we get that $I^{(2)}_{b_{\alpha_1\alpha_2}}$ is

$$I^{(2)}_{b_{\alpha_1\alpha_2}} = 4i\mu^\epsilon \delta_{\alpha_1\alpha_2}\{aJ(0,2,1) + MJ(0,0,1)[g_1+g_2+2g_3]\}, \tag{A5}$$

where $J(0,2,1)$ and $J(0,0,1)$ corresponds to Eq. (A1) with the values for x, y and z respectively, after replacing them in Eq. (A2) getting

$$J(0,2,1) = -i\frac{b_1^2}{8\pi a^3}\frac{1}{\epsilon} + \text{Finite Terms},$$
$$J(0,0,1) = i\frac{1}{4\pi a}\frac{1}{\epsilon} + \text{Finite Terms}.$$

So that the contribution given by Fig. 1(b) is

$$I^{(2)}_{b_{\alpha_1\alpha_2}} = -i\mu^\epsilon \delta_{\alpha_1\alpha_2}[g_1+g_2+2g_3]\frac{b^2}{2\pi a^2}\frac{1}{\epsilon}. \tag{A6}$$

As for the four-point function, consider Fig. 2(e) given by

$$I^{(4)}_{e_\alpha} = -i^2\mu^{2\epsilon}\sum_{i,j=1}^{3} g_i g_j \int \frac{dk_0}{2\pi}\int \frac{d^d\mathbf{k}}{(2\pi)^d}\{[\Lambda_{i_{\alpha_1\alpha_2}}\otimes\Lambda_{j_{\alpha_3\alpha_4}} - \Lambda_{i_{\alpha_1\alpha_4}}\otimes\Lambda_{j_{\alpha_3\alpha_2}}]\mathrm{Tr}[\Lambda^i S(k)\Lambda^j S(k)]\}. \tag{A7}$$

The procedure to solve $I^{(4)}_{e_\alpha}$ is to replace the propagator and Dirac matrices respectively. Let us take the case where $i=j=1$ and after multiplying Dirac matrices we obtain

$$-i^2\mu^\epsilon g_1^2 \int \frac{dk_0}{2\pi}\int \frac{d^d\mathbf{k}}{(2\pi)^d}\frac{\Delta\otimes\Delta}{A^2(k)}\mathrm{Tr}[k_0^2 + b^2 k_l k_m \gamma^l\gamma^m + (a\mathbf{k}^2+M^2)^2].$$

Repeating this procedure in the 6 possible cases and performing the standard procedure in the trace of the Dirac matrices in the 2×2 representation, we arrive at a structure similar to (A4). Recognizing that the integrals present in $I^{(4)}_{e_\alpha}$ have the form of (A1), we can write $I^{(4)}_{e_\alpha}$ as

$$\begin{aligned} I^{(4)}_{e_\alpha} = -i^2\mu^{2\epsilon}\{ & g_1^2\Delta\otimes\Delta[J(2,0,2) + b_2^2 J(0,2,2) + a^2 J(0,4,2) + M^2 J(0,0,2)] + g_2^2\Delta\otimes\Delta[J(2,0,2) - b_2^2 J(0,2,2) \\ & + a^2 J(0,4,2) + M^2 J(0,0,2)] + g_3^2\Gamma^i\otimes\Gamma_i[-J(2,0,2) + b_2^2 J(0,2,2) + a^2 J(0,4,2) + M^2 J(0,0,2)] \\ & + g_1 g_2 \Gamma^0\otimes\Gamma_0[aJ(0,2,2) + a^2 J(0,4,2) + M^2 J(0,0,2)] + g_1 g_3\Gamma^i\otimes\Gamma_i[aJ(0,2,2) + a^2 J(0,4,2) + M^2 J(0,0,2)] \\ & + g_2 g_3 \Gamma^i\otimes\Gamma_i[aJ(0,2,2) + a^2 J(0,4,2) + M^2 J(0,0,2)]\}, \end{aligned} \tag{A8}$$

where $b_2^2 = b^2 + 2aM^2$. Replacing the corresponding x, y, and z values in (A2) we have:

$$J(2,0,2) = i\frac{1}{8\pi a}\frac{1}{\epsilon} + \text{Finite Terms},$$
$$J(0,2,2) = \text{Finite Terms},$$
$$J(0,4,2) = -i\frac{1}{8\pi a^3}\frac{1}{\epsilon} + \text{Finite Terms},$$
$$J(0,0,2) = \text{Finite Terms}.$$

Given Eq. (A8) and the previous results, we obtain that the contribution of $I^{(4)}_{e_\alpha}$ to the vertex function of four points is

$$I^{(4)}_{e_\alpha} = i\mu^{2\epsilon} g_3^2 \Gamma^i\otimes\Gamma_i\frac{1}{2\pi a}\frac{1}{\epsilon}. \tag{A9}$$

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