# Pre-Hawking radiation cannot prevent the formation of apparent horizon

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As an attempt to solve the black hole information loss paradox, recently there has been the suggestion that, due to semiclassical effects, a pre-Hawking radiation must exist during the gravitational collapse of matter, which in turn prevents the apparent horizon from forming. Assuming the pre-Hawking radiation does exist, here we argue the opposite. First we note that the stress energy tensor near the horizon for the pre-Hawking radiation is far too small to do anything to the motion of a collapsing shell. Thus the shell will always cross the apparent horizon within a finite proper time. Moreover, the amount of energy that can be radiated must be less than half of the total initial energy (if the particle starts at rest at infinity) before the shell becomes a null shell and cannot radiate any more without becoming tachyonic. We conclude that for any gravitational collapsing process within Einstein gravity and semiclassical quantum field theory, the formation of the apparent horizon is inevitable. Pre-Hawking radiation is therefore not a valid solution to the information paradox.

## I. INTRODUCTION

The origin of the particles and in particular the origin of the energy, in black hole evaporation has been a longstanding issue, which is still not entirely settled. However in the 1970s, calculations of the regularized energy-momentum tensor gave a clear picture of the energy flow due to the quantum particle creation by the black hole [1]. In this picture, the conformal anomaly played a crucial role [2]. The $1 + 1$ dimensional black hole gives the clearest picture. For a massless scalar field in $1 + 1$ dimensions, the classical energy momentum tensor has a zero trace. Going to null coordinates $(u, v)$, the components of the energy momentum tensor $T_{uu}$ and $T_{vv}$ are independently conserved so that $T_{vv,u} = 0$ and $T_{uu,v} = 0$, which implies that the former is constant along the $v = $ const surfaces, while the latter is also constant along the $u = $ const surfaces. The flux of radiation, written, for example, as $\mu l^\mu l^\nu$, where $l^\mu$ is a null vector, is given by $T^{uu}$ and $T^{vv}$, which are related to the conserved components by $T^{uu} = T_{vv}/g_{uv}^2$ and similarly for $T^{vv}$. For the

Schwartzschild metric where $g_{uv} = (1/2)(1 - 2m/r)$, the flux diverges as one nears the horizon.

However, the conformal anomaly, which makes the trace of the regularized energy momentum tensor nonzero, upsets this picture, leading to the picture that for large $r$, the Hawking radiation is a positive energy flux directed along the $u = $ const rays, while near the horizon the energy flux is negative and directed along the $v = $ const rays into the horizon, with no flux along the $u = $ const rays near the horizon. Calculations in $3 + 1$ dimensions also support this picture [3].

Despite this long standing picture, numerous authors regularly believe that the flux of Hawking radiation is not originated from the conformal anomaly, but from the matter which collapses to form the black hole. Since the null rays that reach infinity intersect the infalling matter exponentially, the closer it is to the horizon, the later those rays reach infinity. One therefore has an exponentially increasing density of radiation near the horizon. If this were the picture, then that density of energy could well drastically alter the picture of the horizon and might well result in nonformation of the horizon.

Recently, Barcelo *et al.* discussed what is called pre-Hawking radiation [4]. It is not at all clear to us what this means, but from the calculations it seems to mean that

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matter falling into the black hole decays, emitting a positive energy flux toward infinity, a flux which near infinity looks a lot like the Hawking flux.

While the standard picture given above is the best approximation we have to the actual evaporation of a black hole, based as it is on the semiclassical effect of quantum matter of the gravitational field, it is an interesting question to ask—what would happen if we allowed the infalling matter which is eventually supposed to form a black hole, were to emit outgoing photons to carry away at least a part of the energy of the infalling matter. Could one arrange things so that this outgoing flux would completely deplete the energy of the infalling matter so that no horizon ever formed? Two types of researchers answered this by yes [5,6]. Papers in [5] used the null shell formalism while that in [6] invoked a radiating timelike shell. Note that the authors in [5] claimed that the thin-shell approximation is somehow incomplete (which is incorrect) and that one must use a thick shell. However, the only way to hold up such shell is having a huge tangential pressure far larger than its energy density to stop the shell. This paper, on the other hand, will answer no. The outgoing null radiation cannot carry away all of the energy of the infalling matter. Some will always remain to form a black hole.

We emphasize that our performing this calculation does not imply that we believe that this scenario represents in any way the effects of quantum mechanics on the formation of a black hole. The regularized quantum energy, even in the very early stages of collapse, does not give any support to the idea that the energy of the outgoing radiation comes from the infalling matter [7]. The infalling matter has an energy-momentum tensor that is conserved independently of the quantum radiation which rides on top of the classical spacetime. The conservation of the quantum energy comes from things like the radiation of both positive and negative fluxes by the conformal anomaly.

This paper is organized as follows. In Sec. II, we analyze the radiation from a plane in a flat spacetime to demonstrate some fundamental properties that a radiating body must hold even without gravity. In Sec. III, we repeat the analysis for the case of a gravitational collapsing shell and confirm that the radiation must again be turned off at some point during the process. In Sec. IV, we investigate the maximum amount of radiation from the shell, if we impose the condition that the shell should always be timelike or null. Finally, in Sec. V, we briefly comment its implication on the black hole information loss paradox. Throughout this paper, we follow the convention of $c = G = \hbar = 1$.

## II. NULL RADIATION FROM A PLANE SHELL IN FLAT SPACETIME

To investigate the backreaction to a radiating body, let us first consider a radiating plane in flat spacetime. We consider a sheet of pressureless matter with density $\sigma(u)$ moving along the path $x = X(u)$ in a flat plane-symmetric spacetime with the metric

$$ds^2 = du^2 + 2dudx. \tag{1}$$

We further assume that this sheet of matter emits radiation of massless particles in the positive $x$ direction with a positive intensity $\lambda(u)$. Then the energy-momentum tensor is described by

$$T^{uu} = \sigma\delta(x - X), \tag{2}$$

$$T^{ux} = T^{xu} = \sigma\dot{X}\delta(x - X), \tag{3}$$

$$T^{xx} = \sigma\dot{X}^2 + \lambda(u)\Theta(x - X), \tag{4}$$

where $\delta(z)$ is the Dirac delta-function and $\Theta(z)$ the Heaviside step-function defined as $\Theta(z) = 1$ for $z \geq 0$ and $\Theta(z) = 0$ for $z < 0$.

Applying the local energy-momentum conservation law to the system, i.e., $T^{\mu\nu}_{,\nu} = 0$, we obtain

$$T^{uu}_{,u} + T^{ux}_{,x} = 0 \rightarrow \dot{\sigma} = 0, \tag{5}$$

$$T^{xu}_{,u} + T^{xx}_{,x} = 0 \rightarrow \ddot{X} = -\frac{\lambda}{\sigma}. \tag{6}$$

If $\lambda(u)$ is positive even when $\dot{X} = -1/2$, then $\dot{X}$ will become less than $-1/2$, and the length-squared of the tangent vector to the shell will go negative. The shell will become tachyonic. We believe tachyonic matter is unphysical.

It is interesting to note that by using the coordinate $u$ to define the trajectory of the shell, one can describe shells that are always timelike, null or tachyonic. Had one used the proper length along the shell, the description would have become singular as the shell became null, making the analysis more difficult.

## III. NULL RADIATION FROM A GRAVITATIONAL COLLAPSING SHELL

Now we consider a collapsing timelike shell that radiates energy in the out-going direction. After showing several simple calculations, we argue that it cannot continue to radiate indefinitely without becoming spacelike, i.e., tachyonic, which is unphysical.

The spacetime inside the shell is described by the flat Minkowski space in the Eddington-Finkelstein-type coordinates,

$$ds^2_- = -du^2 - 2dudr + r^2d\Omega^2, \tag{7}$$

while the spacetime outside the shell is described by the outgoing Vaidya metric [8]

$$ds_+^2 = -\left(1 - \frac{2m(U)}{r}\right)dU^2 - 2dUdr + r^2d\Omega^2, \quad (8)$$

where $u$ and $U$ are outgoing null directions for inside and outside the shell, respectively, $m(U)$ is the mass function for outside the shell as a function of $U$, and $r$ is the areal radius for inside and outside the shell.

These two metrics should be continuous at the shell. Hence, one can express $ds_+^2$ in terms of the time variable $u$ after introducing a suitable redshift factor. The metric outside the shell becomes

$$ds_+^2 = -\left(1 - \frac{2m(u)}{r}\right)\left(\frac{dU}{du}\right)^2 du^2$$
$$- 2\left(\frac{dU}{du}\right)drdu + r^2d\Omega^2. \quad (9)$$

Note that the $u$ coordinate is finite and regular even across the horizon, which is not true of the coordinate $U$. Using $u$ as our coordinate will thus allow us to describe the behavior of the shell (and of the spacetime) even at the horizon.

The shell itself is defined by its path $r = R(u)$. Because of continuity of the angular part of the induced metric on the shell, this same equation must apply to both outside and inside the shell. Note that if the shell is a null ingoing shell, we must have $dR/du = -1/2$.

The condition that the induced metric on the shell be the same from either side of the shell requires

$$1 + 2\frac{dR}{du} = U'^2\left(1 - \frac{2m}{R}\right) + 2U'\frac{dR}{du} \quad (10)$$

(where $U' = dU/du$) or equivalently

$$\frac{dR}{du} = \frac{1 - U'^2(1 - 2m/R)}{2(U' - 1)}. \quad (11)$$

In order to make the full metric continuous across the shell, we replace the radial coordinate $r$ by a new coordinate $z$, such that the shell is located at $z = 0$,

$$r = \begin{cases} R + \frac{z}{U'} & (z > 0), \\ R + z & (z \leq 0). \end{cases} \quad (12)$$

The metric of the spacetime thus becomes

$$ds^2 = -\left[\left(\left(1 - \frac{2m}{R + z/U'}\right)U'^2 + 2U'R' - \frac{2zU''}{U'}\right)\Theta(z)\right.$$
$$\left. + (1 + 2R')(1 - \Theta(z))\right]du^2 - 2dudz$$
$$+ \left(R + \frac{z}{U'}\Theta(z) + z(1 - \Theta(z))\right)^2 d\Omega^2, \quad (13)$$

for which all components of the shell are continuous across the shell.

By assuming that the shell is composed of dust, we can solve the Einstein equations for the motion of the shell. Since the Einstein tensor involves second derivatives of the metric, and since the metric is continuous across the shell, one expects

$$G^{\mu\nu} = G^{\mu\nu}_{\text{bulk}} + G^{\mu\nu}_{\text{shell}}\delta(z). \quad (14)$$

Only the $uu$, $\theta\theta$, and $\phi\phi$ coordinates of $G^{\mu\nu}_{\text{shell}}$ can be nonzero due to spherical symmetry. The dust conditions imply that the angular components of the Einstein tensor must be zero. Calculating the Einstein tensor from the metric, we have

$$G^{uu}_{\text{shell}} = \frac{2(U' - 1)}{RU'}, \quad (15)$$

$$G^{\theta\theta}_{\text{shell}} = \frac{U'^2m + U''R^2 - RU'^2 + RU'}{R^4U'}. \quad (16)$$

Zero tangential stress implies that

$$U'' = -\frac{U'(U'm - RU' + R)}{R^2}. \quad (17)$$

Thus Eqs. (17) and (10) give two first order equations for the variables $U'$ and $R$. We can also write them in terms of a second order equation for $R$ instead. $R''$ can be obtained by differentiating both sides of Eq. (11). One can simplify $R'$ and $U''$ by using Eqs. (11) and (17). Eventually this relation simplifies to

$$R'' = \frac{U'}{2R(U' - 1)}\left\{2m'U' - \left[1 - \left(1 - \frac{2m}{R}\right)U'\right]^2\right\}, \quad (18)$$

where $U'$ can be obtained in terms of $R$ and its derivative from Eq. (10). This implies that if $m' < 0$, then $R''$ is always negative.

Now we define

$$\rho \equiv R'(u) + \frac{1}{2} = -\frac{U'}{2(1 - U')}\left[1 - U'\left(1 - \frac{2m}{R}\right)\right]. \quad (19)$$

This new variable is useful especially if the shell approaches the null direction; as $R'$ approaches $-1/2$, $\rho$ goes to zero. By using this new variable, one can rephrase $R''$ as follows:

$$\rho' = R'' = \frac{m'U'^2}{R(U' - 1)} - \frac{2(U' - 1)}{RU'}\rho^2. \quad (20)$$

As the shell approaches a null shell, Eq. (11) implies that $U'$ approaches $(1 - 2m/R)^{-1}$. Hence, we obtain
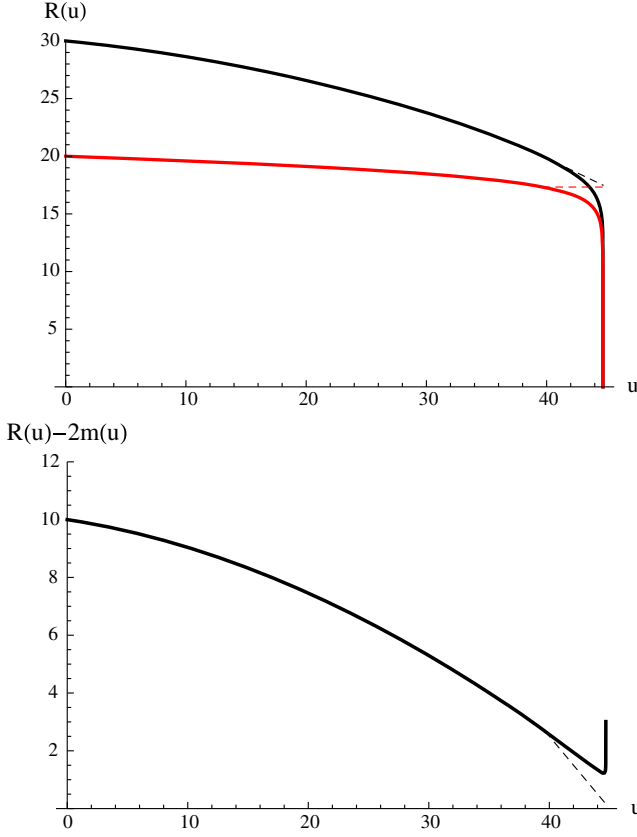
FIG. 1. Numerical evaluations for the coupled system of $R$, $U$, and $m$, where the initial condition is given by $R(0) = 30$, $R'(0) = -0.1$, $m(0) = 10$, $U(0) = 10$ is an arbitrary constant, and $U'(0)$ is given by Eq. (11), where we choose the positive definite solution for $U'$ since both $u$ and $U$ are future directed coordinates. The dashed lines are plots where $m'(u)$ is set to zero when the curve becomes lightlike so that the shell cannot go tachyonic. Upper: $R(u)$ (black) and $2m(u)$ (red, apparent horizon). Lower: The solid lines shows that $R(u) - 2m(u)$ is always positive and the shell does not cross the apparent horizon. The dashed curve does cross. From the upper curve the mass when the apparent horizon forms differs little from the initial mass.

$$\rho' = \frac{m'}{2m - 4m^2/R} - \frac{4m}{R^2}\rho^2. \tag{21}$$

Therefore, if $\rho$ goes to zero while the shell continues to radiate ($m' < 0$), then $\rho$ will become negative, which implies that $R' < -1/2$ and the shell is tachyonic. It is therefore reasonable to conclude that a physical shell cannot emit unrestricted amount of energy; its radiation must be turned off at certain stage of the emission process (i.e., $m'$ should approach zero).

Numerical evaluations for the coupled system of $R$, $U$, and $m$ as functions of $u$ are presented in Figs. 1 and 2. One can solve Eqs. (18) and (17) for $R$ and $U$, respectively. In order to solve those equations we require a definite form for the function $m(u)$. We follow the BMT model [6] and assume that $m(u)$ satisfies
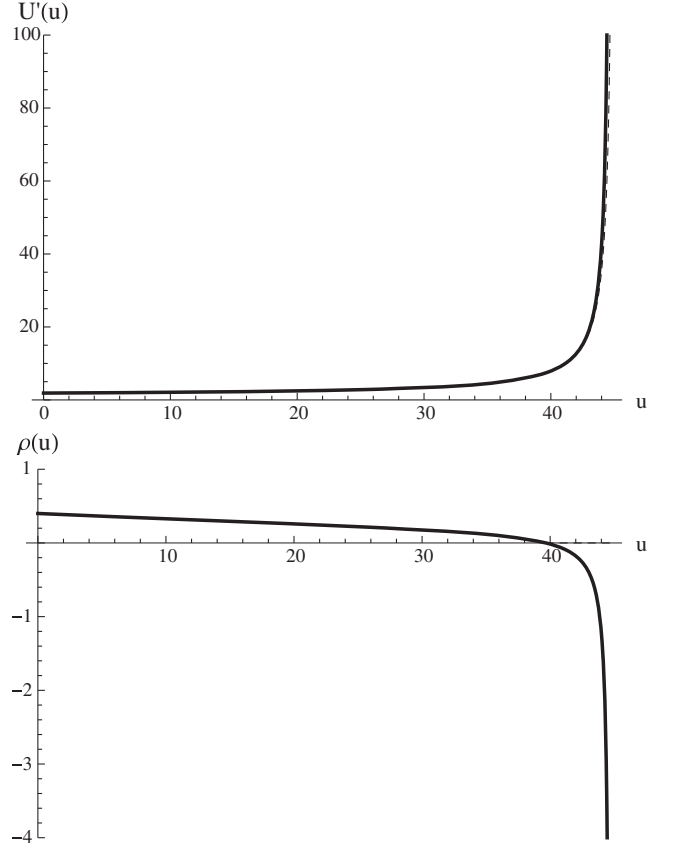


FIG. 2. As the shell collapses, $U'(u)$ rapidly increases (upper). For the tachyonic case, $\rho(u)$ (lower) also goes large and negative, indicating a strongly tachyonic shell. For the dotted case, $\rho(u)$ is zero (lightlike shell) near the apparent horizon. Note that the dotted curve for $\rho(u)$ coincides with the 0 axis.

$$\frac{dm}{du} = -U'\frac{\alpha}{m^2}, \tag{22}$$

where $\alpha$ is a numerical constant that depends on the number of fields that contribute to the Hawking radiation (we choose $\alpha = 1$ for numerical calculations). For initial conditions, we choose arbitrary constants for $R(0)$, $m(0)$, and $U(0)$, though $R(0) > 2m(0)$ so as to satisfy the condition that the shell is initially outside the apparent horizon. $R'(0)$ that represents the initial velocity of the shell is also arbitrary as long as $R'(0) > -1/2$. Once a set of these initial conditions is chosen, $U'(0)$ is determined by Eq. (11), where the positive solution is taken.

In our solution we do not assume that the shell remains timelike or null and allow it to become tachyonic.

Figures 1 and 2 show the typical behavior of $R$ and $m$. As the shell approaches the apparent horizon, $U'$ increases rapidly, and hence the radiation of the shell increases correspondingly. At first look, in Fig. 1 we see that a horizon never forms. The radius of the shell $R(u)$ always remains larger than $2m(u)$. However it is also clear from Fig. 1 that $R'(u)$ becomes much smaller than $-1/2$, i.e., the shell becomes tachyonic. If we set $m(u)' = 0$ for $u$ larger

than the point where $R' = -1/2$ [i.e., $m(u)$ thereafter remains constant], then $R'(u)$ remains equal to $-1/2$ and $R(u)$ will rapidly equal $2m(u)$ and continues to $R(u) = 0$, as shown by the dashed lines in the graphs.

## IV. MAXIMUM RADIATION BY A PARTICLE

One can get a feeling for the results of this section by looking at a massive particle decaying to two oppositely directed photons. If the initial massive particle is at rest, the two photons will each carry away half of the rest-mass energy. If the massive particle is traveling in one direction, then the photon emitted in that direction will have more than half of the energy, and if the massive particle's velocity approaches that of light, then the energy in the photon moving in the opposite direction will approach zero. We will show that the same happens for the gravitating shell.

Let us define the quantity

$$S \equiv 4\frac{U' - 1}{U'}R^2\left[1 - U'\left(1 - \frac{2m}{R}\right)\right]. \qquad (23)$$

This is the square of the energy density in the shell for timelike matter and would be expected to be conserved if $m' = 0$. It is however also defined and conserved in the absence of radiation for null and spacelike radiation as well.

Using the equations for $R'$ and $U'$, we find

$$S' = 8m'(U' - 1)R, \qquad (24)$$

which means $S$ is conserved if the radiation $m'$ is zero. If the shell is null, $S = 0$, and it is negative for tachyonic matter.

We now evaluate $S$ under various situations. Let us first consider the situation where a shell located at $R_0$ is at rest when it begins to collapse. Since the shell satisfies $R' = 0$ at the starting point, from the equation for $R'$ we have

$$0 = 1 - U_0'^2\left(1 - \frac{2m_0}{R_0}\right), \qquad (25)$$

or

$$U_0' = \frac{1}{\sqrt{1 - 2m_0/R_0}}, \qquad (26)$$

where the subscript 0 denotes the initial condition of the shell. Then we obtain

$$S_0 = 4R_0^2\left(\frac{U_0' - 1}{U_0'}\right)^2 = \frac{16m_0^2}{(1 + \sqrt{1 - 2m_0/R_0})^2}. \qquad (27)$$

If $R_0 \gg 2m_0$, i.e., the shell falls in from a distance that is far from the putative horizon associated with $2m_0$, then the initial condition reduces to

$$S_0 = 4m_0^2. \qquad (28)$$

If $S$ becomes negative, the shell turns spacelike, then it cannot represent physical matter. Thus if $S$ approaches zero, then $m'$ should reduce to zero correspondingly for the system to remain physical, as we argued earlier.

By integrating Eq. (24) and assuming that the minimum value of $S$ is 0, we find

$$-\Delta S \le \frac{16m_0^2}{(1 + \sqrt{1 - 2m_0/R_0})^2} \qquad (29)$$

and

$$\frac{16m_0^2}{(1 + \sqrt{1 - 2m_0/R_0})^2} \ge -8\int(U' - 1)R\frac{dm}{du}du, \qquad (30)$$

where $\Delta S = \int S'du$ for a given integration domain and $m'$ is assumed to be negative. Since

$$\frac{d(U' - 1)R}{du} = \frac{(U' - 1)^2}{2} \ge 0, \qquad (31)$$

the multiplier of $m'$ is increasing and reaches its minimum at $R_0$. Therefore, $-\int(U' - 1)Rdm \ge -(U_0' - 1)R_0\int dm$, and we have

$$-\Delta m \le \frac{m_0}{2}\frac{2\sqrt{1 - 2m_0/R_0}}{1 + \sqrt{1 - 2m_0/R_0}} \le \frac{m_0}{2}. \qquad (32)$$

Note that the closer the shell approaches the horizon, the smaller the $\Delta m$ becomes. Also, from Eq. (30), the closer to the horizon where the radiation is emitted, the smaller the change in mass must be. Thus, the maximum amount of mass that can be radiated must be less than $1/2$ of the total, and it must take place far from the horizon (where quantum effects would predict a negligible amount of radiation). Evidently, the shell cannot radiate away its entire gravitational mass to prevent the formation of the apparent horizon.

Would a nonzero initial inward velocity at infinity of the shell help to ameliorate the situation? The answer is no. Using the equation for $R'$ again, we have

$$R_0' = \frac{1 - U_0'^2(1 - 2m_0/R_0)}{2(U_0' - 1)}. \qquad (33)$$

Assuming $m_0 \ll R_0$, we expand $U_0'$ as $U_0' = 1 + \alpha m_0/R_0 + \mathcal{O}(m_0/R_0)^2$ and $m_0 \ll R_0$, and find

$$\alpha = \frac{1}{1 + R_0'} \qquad (34)$$

and

$$S_0 = 4m_0^2 \alpha(2 - \alpha) = 4m_0^2 \left( \frac{1 + 2R_0'}{1 + R_0'} \right). \qquad (35)$$

Hence, we obtain the following inequality:

$$-8 \int (U' - 1) Rm' du \leq -\left( \frac{8m_0}{1 + R_0'} \right) \Delta m. \qquad (36)$$

Thus we have

$$-\Delta m \leq \left( \frac{1 + 2R_0'}{2} \right) m_0 \leq \frac{m_0}{2}, \qquad (37)$$

since $-1/2 \leq R_0' \leq 0$. Again, the maximum amount of radiation that can be radiated is half of the initial Schwarzschild mass, which corresponds to the situation where $R_0' = 0$ at infinity.

If the shell becomes null, it has no rest mass. Thus the radiation can cause the rest mass of the shell to go to zero. However, it is the energy of the shell, not its rest mass, that determines its Schwarzschild mass, and the gravitational mass of the shell cannot go to zero without the shell going tachyonic.

## V. CONCLUSION

In this paper, we critically examined the assertion that a radiating and collapsing shell can never cross the apparent horizon. Some authors refer to such a radiation as pre-Hawing radiation. Our analysis has resulted in two major conclusions. First, the collapsing shell may emit radiation before the horizon is formed, but it must be weak unless the shell becomes spacelike. Second, the maximum amount of the radiated energy is bounded, where our estimation shows that it cannot be larger than half of its initial energy. Therefore either the radiation would stop or the shell would become tachyonic. Since the latter is unphysical, it is inevitable that the radiation stops at some point during the collapse. Note that we do not consider the possibility that there are internal pressures in the shell because such pressures would have to be huge to counteract the radiation pressure.

Although our analysis does not rely on the quantum mechanical nature of the pre-Hawking radiation, authors of the recent work [7] indeed show that quantum stress-energy tensor cannot play an important role during gravitational collapse, which is consistent with our result. In a different setup, authors of [9] calculate a 2D model of the quantum radiation from a massless scalar field for a collapsing shell that stops (due to transverse stresses) just outside the horizon. They show that the quantum emission is not sufficient to stop the system from almost having a horizon very near the expected value associated with the mass that began the collapse. Their conclusion is consistent with ours.

We conclude that the assertion that pre-Hawking radiation can prevent the formation of the apparent horizon is based on unphysical assumptions. Black holes do form.

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