

Closing in on the large-scale CMB power asymmetry

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Measurements of the cosmic microwave background (CMB) temperature anisotropies have revealed a dipolar asymmetry in power at the largest scales, in apparent contradiction with the statistical isotropy of standard cosmological models. The significance of the effect is not very high, and is dependent on *a posteriori* choices. Nevertheless, a number of models have been proposed that produce a scale-dependent asymmetry. We confront several such models for a physical, position-space modulation with CMB temperature observations. We find that, while some models that maintain the standard isotropic power spectrum are allowed, others, such as those with modulated tensor or uncorrelated isocurvature modes, can be ruled out on the basis of the overproduction of isotropic power. This remains the case even when an extra isocurvature mode fully anticorrelated with the adiabatic perturbations is added to suppress power on large scales.

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I. INTRODUCTION

The standard six-parameter Λ cold dark matter (Λ CDM) cosmological model describes the temperature fluctuations in the cosmic microwave background (CMB) radiation spectacularly well, as demonstrated by the WMAP satellite [1], the Atacama Cosmology Telescope [2], the South Pole Telescope [3], and, especially, the *Planck* satellite [4]. Central assumptions in the Λ CDM model are that the fluctuations are Gaussian and statistically homogeneous and isotropic. Despite the success of the standard model, several “anomalies” have been noticed in the CMB, which apparently violate these assumptions (for reviews, see Refs. [5–8]). The statistical significance of these anomalies is not very high, and is weakened substantially with *a posteriori* (look elsewhere) corrections [5,7,9,10] when those are well defined.

Probably the most intriguing of the anomalies is a very roughly 6% dipolar or hemispherical asymmetry in the large-scale CMB temperature fluctuation power, first noted in the WMAP one-year data [11]. Later analyses showed that the asymmetry is substantially reduced on multipole scales $\ell \gtrsim 100$ [7,12–16]. The significance of the asymmetry is only at the 3σ level, and is sensitive to *a posteriori* choices in the maximum multipole scale [5,7], so it should

perhaps not be considered a great surprise. Nevertheless, an origin to the asymmetry as a physical modulation of the primordial fluctuations would clearly be of fundamental importance for cosmology, and in particular might provide information about inflation, given the large-scale nature of the effect. Therefore it is worthwhile to investigate possible physical explanations. Since concrete inflationary models for modulation are difficult to construct [17], we consider phenomenological models in this study.

While most studies of the dipolar asymmetry have been performed in angular multipole space, any physical model will necessarily be described best in position (or k) space. In Ref. [18] we developed a formalism for describing a spatial modulation and its effect on CMB temperature anisotropies, and for performing Bayesian estimation of the modulation parameters. This formalism was crucial for answering an important question: what does a modulation that fits the temperature data predict for *other* observations, such as CMB lensing [18] and polarization [19]? Given the inconclusive significance level of the asymmetry, probes of modes independent from CMB temperature may be essential in order to confirm or refute a physical origin to the asymmetry. Our formalism is an extension of an approach to describe the effects in the CMB of gradients in cosmological parameters [20]. The effects of various such parameter gradients were discussed in [21].

In this paper we apply our formalism [18,19] for the first time to determine whether any models for modulation can already be ruled out. To do this we point out that some models necessarily increase the statistically *isotropic*

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temperature power over Λ CDM, and so the ordinary power spectra can be considered as “independent probes” to test a physical origin for the asymmetry. We consider purely phenomenological modulations of the ordinary adiabatic fluctuations, as well as a gradient of the scalar spectrum tilt and modulations of tensor and isocurvature contributions, in doing so testing several of the models discussed in [21]. Modulated isocurvature modes were studied in [13], and, in the context of a particular inflationary model [22], in Ref. [23]. In the process we also provide constraints on unmodulated tilted tensor and isocurvature modes using the latest data.

II. FORMALISM

Our goal is to construct physical, position-space models for a temperature dipolar asymmetry, which is confined mostly to large scales. We apply the formalism developed in Refs. [18,19], which captures scale dependence by employing two fluctuation components. The first, $\tilde{Q}^{\text{lo}}(\mathbf{x})$, is restricted mainly to large scales (low k) and is maximally linearly spatially modulated:

$$\tilde{Q}^{\text{lo}}(\mathbf{x}) = Q^{\text{lo}}(\mathbf{x}) \left(1 + \frac{\mathbf{x} \cdot \hat{\mathbf{d}}}{r_{\text{LS}}} \right), \quad (1)$$

where $Q^{\text{lo}}(\mathbf{x})$ is statistically isotropic with power spectrum $\mathcal{P}^{\text{lo}}(k)$, $\hat{\mathbf{d}}$ is the direction of modulation, and r_{LS} is the comoving distance to last scattering. The second component, $Q^{\text{hi}}(\mathbf{x})$, is statistically isotropic with spectrum $\mathcal{P}^{\text{hi}}(k)$. The two fields are taken to be uncorrelated, i.e., $\langle Q^{\text{lo}}(\mathbf{k}) Q^{\text{hi}*}(\mathbf{k}') \rangle = 0$. We attempt to be agnostic as to the origin of the modulation; the isotropic Q^{hi} component is adiabatic, while for the modulated component, Q^{lo} , we consider adiabatic, CDM isocurvature, and tensor fluctuations.

Note, importantly, that while we take the first fluctuation component to be maximally modulated according to Eq. (1), the amount of modulation in the total sky will be determined by a free amplitude parameter, A , inside the definition of $\mathcal{P}^{\text{lo}}(k)$. This convention differs from that used in [18,19], in which the modulation amplitude parameter A appeared explicitly multiplying the dipole term in Eq. (1). Nevertheless, the two conventions are equivalent in terms of observable quantities.

The total temperature anisotropies due to these two fields will be to a very good approximation [18]

$$\delta T(\hat{\mathbf{n}}) = \delta T^{\text{lo}}(\hat{\mathbf{n}})(1 + \hat{\mathbf{n}} \cdot \hat{\mathbf{d}}) + \delta T^{\text{hi}}(\hat{\mathbf{n}}), \quad (2)$$

where δT^{lo} , with power spectrum C_ℓ^{lo} (called the “asymmetry spectrum”), is produced by $\mathcal{P}^{\text{lo}}(k)$, while δT^{hi} , with spectrum C_ℓ^{hi} , is produced by $\mathcal{P}^{\text{hi}}(k)$. These anisotropies lead to the lowest-order spherical harmonic multipole covariance [7,18,20,24]

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = C_\ell \delta_{\ell \ell'} \delta_{mm'} + \frac{\delta C_{\ell \ell'}}{2} \sum_M d_M \xi_{\ell m \ell' m'}^M, \quad (3)$$

where $\delta C_{\ell \ell'} \equiv 2(C_\ell^{\text{lo}} + C_{\ell'}^{\text{lo}})$ and d_M is the spherical harmonic decomposition of $\hat{\mathbf{n}} \cdot \hat{\mathbf{d}}$. The coefficients $\xi_{\ell m \ell' m'}^M$ couple modes ℓ to $\ell \pm 1$:

$$\xi_{\ell m \ell' m'}^M \equiv \sqrt{\frac{4\pi}{3}} \int Y_{\ell' m'}(\hat{\mathbf{n}}) Y_{1M}(\hat{\mathbf{n}}) Y_{\ell m}^*(\hat{\mathbf{n}}) d\Omega. \quad (4)$$

Crucially, the modulated component will also contribute to the total isotropic power, via

$$C_\ell = C_\ell^{\text{lo}} + C_\ell^{\text{hi}}. \quad (5)$$

Therefore a model that produces sufficient asymmetry to fit the temperature data may overproduce isotropic power at large scales and hence be inconsistent with experiments such as *Planck*.

III. MODELS

We employ the same models as described in Ref. [19] to describe a large-scale modulation. First, we consider the adiabatic tanh model, with k -space asymmetry spectrum

$$\mathcal{P}^{\text{lo}}(k) = \frac{A_{\text{tanh}}}{2} \mathcal{P}^{\Lambda\text{CDM}}(k) \left[1 - \tanh\left(\frac{\ln k - \ln k_c}{\Delta \ln k}\right) \right], \quad (6)$$

where

$$\mathcal{P}^{\Lambda\text{CDM}}(k) = A_s \left(\frac{k}{k_0} \right)^{n_s - 1} \quad (7)$$

describes the usual Λ CDM power-law primordial comoving curvature perturbation spectrum. The parameters $\Delta \ln k$ and k_c describe the width and position of a small-scale cutoff and $A_{\text{tanh}} \leq 1$ is the amplitude of the modulation. Next we consider an adiabatic power-law model (abbreviated “ad.-PL”):

$$\mathcal{P}^{\text{lo}}(k) = A_{\text{PL}} \mathcal{P}^{\Lambda\text{CDM}}(k_0^{\text{lo}}) \left(\frac{k}{k_0^{\text{lo}}} \right)^{n_s^{\text{lo}} - 1}, \quad (8)$$

where n_s^{lo} and $A_{\text{PL}} \leq 1$ are the modulation tilt and amplitude, and $k_0^{\text{lo}} = 1.5 \times 10^{-4} \text{ Mpc}^{-1}$ is a pivot scale. For both of these adiabatic models we fix $\mathcal{P}^{\text{hi}}(k)$ via the constraint

$$\mathcal{P}^{\text{lo}}(k) + \mathcal{P}^{\text{hi}}(k) = \mathcal{P}^{\Lambda\text{CDM}}(k) \quad (9)$$

(and hence $C_\ell^{\text{lo}} + C_\ell^{\text{hi}} = C_\ell^{\Lambda\text{CDM}}$), so that the isotropic power is automatically consistent with standard Λ CDM. This constraint will be satisfied, for example, in the scenario of [25].

Next we consider a single-component adiabatic model with a linear gradient in the tilt, n_s , of the primordial power spectrum (“ n_s -grad” for short). In this case we can directly write the asymmetry spectrum as [18,19]

$$C_\ell^{\text{lo}} = -\frac{\Delta n_s}{2} \frac{dC_\ell^{\Lambda\text{CDM}}}{dn_s}. \quad (10)$$

Here we have used a linear approximation for the effect of the gradient, which will be well justified by our results. The modulation amplitude is specified by the increment in tilt, Δn_s , from modulation equator to pole. Note that this modulation will depend implicitly on the pivot scale for A_s .

Finally we consider three models that naturally produce contributions on large scales. The first is a modulation of the standard Λ CDM integrated Sachs-Wolfe (ISW) contribution with amplitude $A_{\text{ISW}} \leq 1$ [26]. This phenomenological model automatically satisfies *isotropic* CMB constraints and C_ℓ^{lo} is simply the contribution of the ISW effect to the total power C_ℓ . The second is a modulated CDM density isocurvature component,

$$\mathcal{P}^{\text{lo}}(k) = \frac{\alpha_{k_*}}{1 - \alpha_{k_*}} \mathcal{P}^{\Lambda\text{CDM}}(k_*) \left(\frac{k}{k_*}\right)^{n_T - 1}, \quad (11)$$

and the third is a modulated tensor component,

$$\mathcal{P}^{\text{lo}}(k) = r_{k_*} \mathcal{P}^{\Lambda\text{CDM}}(k_*) \left(\frac{k}{k_*}\right)^{n_t}. \quad (12)$$

In these latter two cases the models are described by two parameters, a primordial power ratio (α_{k_*} or r_{k_*} , evaluated at scale $k_* = 0.002 \text{ Mpc}^{-1}$) and a tilt (n_T or n_t). Since these components are maximally modulated, these power ratios also determine the modulation amplitudes. For both isocurvature and tensor models we set $\mathcal{P}^{\text{hi}}(k) = \mathcal{P}^{\Lambda\text{CDM}}(k)$, so that, with respect to isotropic power, we simply have Λ CDM plus isocurvature or tensor modes. This will give us an extra constraint for these models over the adiabatic cases. This is reasonable since it would require a very contrived adiabatic scalar large-scale power deficit that, when combined with the isocurvature or tensor spectrum, resulted in the usual Λ CDM spectrum. For the tensor model we also consider an unmodulated isocurvature component that is fully (anti-)correlated with the adiabatic scalars. Anticorrelated isocurvature modes would decrease power on large scales, potentially allowing for a larger contribution of modulated tensors. This inclusion adds one extra parameter, which is simply the amplitude of perturbations for the new mode.

IV. MODULATION ESTIMATOR

For a full-sky, noise-free measurement of the temperature multipoles, we can write down an estimator for the modulation amplitude $\Delta X_M \equiv A d_M$ as [7,18,20]

$$\Delta \hat{X}_M = \frac{1}{4A} \sigma_X^2 \sum_{\ell m \ell' m'} \frac{\delta C_{\ell\ell'}}{C_\ell C_{\ell'}} \frac{\varepsilon_{\ell m \ell' m'}^M}{\varepsilon_{\ell m \ell' m'}} a_{\ell m}^* a_{\ell' m'}, \quad (13)$$

where $A = A_{\text{tanh}}, A_{\text{PL}}, \Delta n_s, A_{\text{ISW}}, \alpha_{k_*}/(1 - \alpha_{k_*})$, or r_{k_*} , depending on the model, and where the cosmic variance of the estimator is given by

$$\sigma_X^2 = 12A^2 \left(\sum_{\ell} (\ell + 1) \frac{\delta C_{\ell\ell+1}^2}{C_\ell C_{\ell+1}} \right)^{-1}. \quad (14)$$

The presence of noise and incomplete sky coverage modifies the above relations. We use a C -inverse filter approach that accounts for noise, and, optimally, for the mask (as described in Refs. [27,28]). Masking and residuals in the data will induce a mean-field value for ΔX_M that can be estimated with simulations. Further details of the full estimator we use can be found in Appendix C of Ref. [7].

For fixed modulation parameters the maximum likelihood is

$$\ln \mathcal{L} = \sum_M \frac{\Delta \hat{X}_M^2}{2\sigma_X^2}. \quad (15)$$

We can then build the rest of the likelihood by sampling on a grid of values for the k -space parameters (see Ref. [18]). For the tensor and isocurvature models we assign a uniform prior on A , in order to obtain consistency with the *isotropic* likelihood results. For all other models we use a prior uniform in the individual ΔX_M .

V. RESULTS

Our dipole asymmetry constraints come from *Planck* TT data using the SMICA solution [29]. The best-fit asymmetry spectra for all of our models are illustrated in Fig. 1, where we see the expected large-scale character of the asymmetry. The corresponding full posteriors for $\alpha_{0.002}$ and $r_{0.002}$ and their tilts are shown in Fig. 2 (orange contours), where we can see that large values of $\alpha_{0.002}$ or $r_{0.002}$ are needed to explain the asymmetry. [Recall that the power ratios $\alpha_{0.002}$ and $r_{0.002}$ also fix the modulation amplitude for the case of maximal modulation in Eq. (1).]

For the isocurvature and tensor models we can also obtain constraints from the isotropic power spectra described in Table I; we will refer to these as *isotropic* constraints. These were obtained with a version of COSMOMC [30] modified to accommodate uncorrelated isocurvature modes. For the isotropic constraints all six of the Λ CDM parameters were varied, in addition to the isocurvature or tensor fractions. For these models Fig. 2 also shows the isotropic posteriors for $\alpha_{0.002}$ and $r_{0.002}$ and their tilts (blue contours), as well as the joint constraints, with the assumption that the isotropic and asymmetry likelihoods are independent (recall that they arise from diagonal and off-diagonal elements of the multipole covariance, respectively). Figure 2 shows that, for both

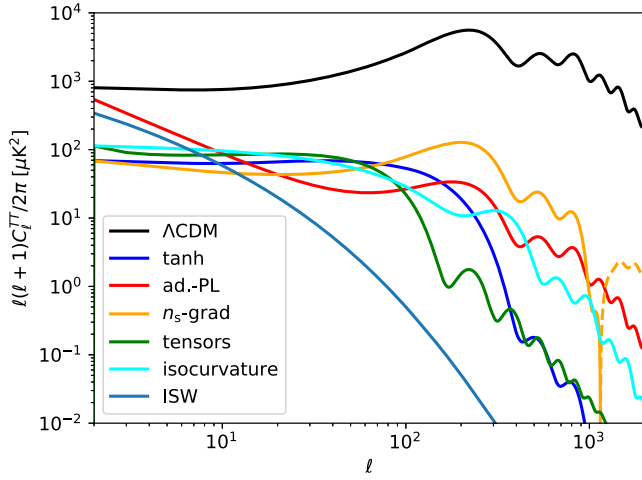


FIG. 1. Λ CDM temperature spectrum compared to the best-fit asymmetry spectra, C_ℓ^0 , for the various models. The best fits correspond roughly to a 5–10% asymmetry for $\ell \lesssim 100$, as expected, with the exception of the ISW modulation, whose maximum amplitude (and shape) is fixed by Λ CDM.

isocurvature and tensor modulation, the joint constraints are inconsistent with the level of modulation preferred by the asymmetry data. In other words, the addition of the independent isotropy data has substantially reduced the “signal” seen in the asymmetry data.

Note that in Fig. 2 we have assumed that the isocurvature and tensor contributions are maximally modulated, via Eq. (1). This allows us to directly compare the asymmetry and isotropic posteriors, but is also a conservative choice, because for less than full modulation the corresponding r and α values preferred by the asymmetry constraints would necessarily be larger with larger uncertainties. This would increase the tension we find between asymmetry and isotropic constraints and increase the dominance of the isotropic data in the joint constraints.

In order to express the above graphical results quantitatively, and determine which models are viable for explaining the original asymmetry signal, we will consider two quantities for each model. The first is the probability, $P_{>3\sigma}$, that the data allow a modulation amplitude A that is at least 3 times larger than the cosmic variance σ_X . Note that the choice of the value 3 is arbitrary; however, if $P_{>3\sigma}$ is small then the model cannot source significant modulation and can be ruled out, even if $P_{>3\sigma}$ being large is an insufficient condition to prefer a modulation model over Λ CDM. The second quantity we use is the maximum-likelihood amplitude of modulation compared to the cosmic-variance value, A/σ_X . For both quantities σ_X is calculated for asymmetry only [via Eq. (14)].

We present these quantities for the various model and data combinations in Table II. For the asymmetry data, both quantities are large (except for the ISW model), which simply tells us that the models can produce the considerable asymmetry present in the data. However, in all cases the

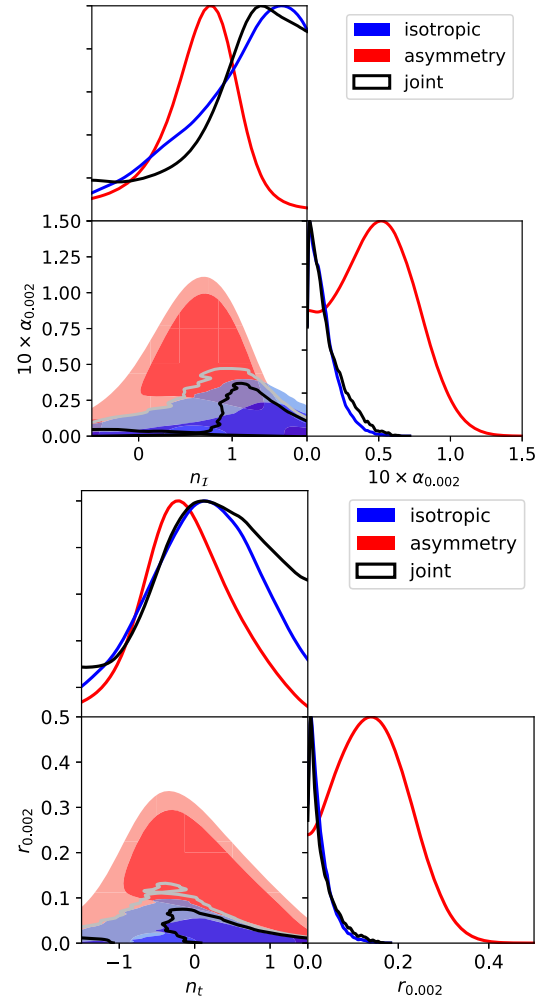


FIG. 2. Posteriors for $\alpha_{0.002}$ or $r_{0.002}$ and tilt of the isocurvature (top panel) and tensor (bottom) models. Contours enclose 68% and 95% of the posteriors. We have conservatively assumed maximal modulation, so that the vertical axes are also a measure of the level of modulation relative to the isotropic Λ CDM spectrum. We can see that the modulation allowed by the asymmetry constraints is reduced substantially when adding the isotropic constraints.

values drop substantially when adding the isotropic data. This implies that even maximally modulated tensor or isocurvature modes cannot source the large asymmetry signal (or can, but with very small probability) due to their respective isotropic constraints (consistent with the result in [32] for tensor modulation). If we attempt to hide the isotropic tensor temperature power by including an anti-

TABLE I. Data sets used for the isotropic constraints. BKP refers to the BICEP2/Keck Array-*Planck* joint analysis [31].

Model	Data set
Isocurvature	<i>Planck</i> TT, TE, EE + lowP
Tensors	<i>Planck</i> TT, TE, EE + lowP + lensing + BKP

TABLE II. Percentage of the posterior for which the amplitude A exceeds $3\sigma_\chi$, i.e., $P_{>3\sigma}$, as well as A/σ_χ for the maximum-likelihood parameters, for different combinations of data. These quantify whether the model can source significant asymmetry given the data, a necessary but not sufficient condition for preferring the model over Λ CDM. The asterisk denotes the addition of a fully anticorrelated isocurvature component.

Model	Asymmetry		Isotropic		Joint	
	$P_{>3\sigma}$ [%]	A/σ_χ	$P_{>3\sigma}$ [%]	A/σ_χ	$P_{>3\sigma}$ [%]	A/σ_χ
tanh	63.1	3.3
ad.-PL	32.4	2.5
n_s -grad	36.3	2.7
ISW	0.0	1.2
n_T free	32.2	3.2	1.5	0.06	0.5	0.03
$n_T = 1$	37.4	3.1	0.33	0.10	1.0	0.10
$n_T = n_s$	39.6	3.1	0.073	0.09	0.24	0.03
n_t free	29.9	3.1	0.003	0.03	0.001	0.02
$n_t = 0$	37.4	3.1	0.000	0.48	0.000	0.63
$n_t = 0^*$	37.4	3.1	0.000	0.31	0.000	0.49
$n_t < 0$	32.1	3.1	0.008	0.48	0.003	0.00

correlated isocurvature mode the conclusions remain the same (see the row marked $n_t = 0^*$ in Table II). This is due to the different shapes of the tensor and anticorrelated isocurvature power spectra, and not, for instance, to the nondetection of primordial B modes in the BICEP2/Keck Array-Planck data. Therefore we expect that, in general, a modulation model for which (like the tensor and isocurvature models) isotropic power is added will be unable to explain the dipolar asymmetry signal. The tanh, ad.-PL, and n_s -grad models are of course unaffected by the isotropic constraint and are thus still viable modulation models as far as CMB temperature is concerned. For the ISW model both $P_{>3\sigma}$ and A/σ_χ are small: even for maximal modulation the standard Λ CDM ISW contribution cannot explain the observed asymmetry. Note that, via Eq. (15), the ratio A/σ_χ is essentially the best-fit χ value, which shows that the tanh model (which has the most free parameters) gives the best fit.

For our best-fit parameters, the n_s -grad model induces a modulation amplitude of roughly 1.6% at $k = 1 \text{ Mpc}^{-1}$. On such small scales this model should be vulnerable to constraints from large-scale structure surveys [33–37]. Indeed, this modulation amplitude is close to (or in excess of) the 95% upper limit based on quasar data in [38], and so a rigorous joint analysis may already rule this model out.

In order to determine quantitatively the level of modulation allowed by the full data we look at constraints on the r and α parameters for the isocurvature and tensor models (where we are able to use power spectra to provide tighter constraints). In Table III we show the 95% C.L.s (or upper limits where relevant) for $r_{0.002}$ and $\alpha_{0.002}$ for the different combinations of data. While the general tensor and isocurvature models (where the tilts are free to vary) show no strong detection with the asymmetry constraints alone (in

TABLE III. 95% C.L. (or upper limits) for the parameters $r_{0.002}$ and $\alpha_{0.002}$ for various tensor and isocurvature models and data combinations.

Model	Asymmetry	Isotropic	Joint
n_T free	$\alpha \leq 0.092$	$\alpha \leq 0.031$	$\alpha \leq 0.038$
$n_T = 1$	$0.007 \leq \alpha \leq 0.083$	$\alpha \leq 0.038$	$\alpha \leq 0.044$
$n_T = n_s$	$0.008 \leq \alpha \leq 0.086$	$\alpha \leq 0.038$	$\alpha \leq 0.046$
n_t free	$r \leq 0.28$	$r \leq 0.08$	$r \leq 0.09$
$n_t = 0$	$0.02 \leq r \leq 0.28$	$r \leq 0.07$	$r \leq 0.10$
$n_t = 0^*$	$0.02 \leq r \leq 0.28$	$r \leq 0.08$	$r \leq 0.09$
$n_t \leq 0$	$r \leq 0.30$	$r \leq 0.09$	$r \leq 0.09$

the sense that we can only quote upper limits), we see that the addition of power spectrum data strongly constrains the amount of modulation allowed by the data. For models where the tilt is fixed and not allowed to vary, the modulation signal is more apparent; however, the addition of isotropic constraints removes the signal to a similar degree. Note that the asymmetry constraints in Table III allow much larger values of r than α . This is due simply to the fact that identical *primordial* ratios of tensor- and isocurvature-to-adiabatic scalar fluctuations produce much larger isocurvature *temperature* fluctuations.

VI. DISCUSSION

The models we have examined fall into two general classes. In the first, the total statistically isotropic temperature power was constrained to match that of standard Λ CDM. Therefore the degree of modulation could be varied without spoiling the success of Λ CDM. In the second class, the modulated component contributed extra power to the isotropic spectra. Our main conclusion is that models in this latter class fail to provide sufficient modulation to explain the dipole asymmetry without producing too much large-scale statistically isotropic power. Hence these models, which include modulated tensor and uncorrelated isocurvature, can be ruled out as the source of the large-scale dipolar asymmetry.

Models in the first class, however, can fit the asymmetry while maintaining the success of the Λ CDM isotropic spectra, and hence some cannot yet be ruled out. One exception is a modulated ISW contribution, which cannot source enough asymmetry to explain the signal in temperature. The scalar tilt gradient model produces substantial modulation on small scales, and so is at risk from survey data. The surviving models are the phenomenological adiabatic modulation models. Of course the contrived nature of such models should mean that Λ CDM is still preferred: they essentially add parameters to fit features in the data that may simply be random noise. Unfortunately a Bayesian model selection procedure would not provide an unambiguous Bayes factor for these models, since the modulation model evidence is strongly driven by the parameter prior ranges, which are completely

undetermined. It will only be possible to confirm or refute these models by comparing future observations with their predictions for probes (such as CMB polarization) which are sensitive to independent fluctuation modes from CMB temperature [19].

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