Dynamic origins of fermionic D-terms

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The D-term is defined through matrix elements of the energy-momentum tensor, similarly to mass and spin, yet this important particle property is experimentally not known any fermion. In this work we show that the D-term of a spin $\frac{1}{2}$ fermion is of dynamical origin: it vanishes for a free fermion. This is in pronounced contrast to the bosonic case where already a free spin-0 boson has a non-zero intrinsic D-term. We illustrate in two simple models how interactions generate the D-term of a fermion with an internal structure, the nucleon. All known matter is composed of elementary fermions. This indicates the importance to study this interesting particle property in more detail, which will provide novel insights especially on the structure of the nucleon.

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I. INTRODUCTION

The matrix elements of the energy-momentum tensor (EMT) [1] provide most basic information: the mass and spin of a particle. They also define the D-term [2], which is far less known. The most direct way to probe EMT matrix elements would be scattering off gravitons, which is impractical. However, information on the EMT form factors can be accessed through generalized parton distribution functions (GPDs) which enter the description of certain hard exclusive reactions [3–14]. The second Mellin moments of unpolarized GPDs yield EMT form factors. This provides not only the key to access information about nucleon's spin decomposition [4], but also to its mechanical properties [15]. The D-term determines the behavior of unpolarized GPDs in the asymptotic limit of renormalization scale $\mu \to \infty$ [8]. Aspects of the relation of the D-term to GPDs were also investigated in [16].

Similarly to electric form factors providing information on the electric charge distribution [17], the EMT form factors offer insights on the energy density, orbital angular momentum density, and the distribution of internal forces encoded in the stress tensor and directly related to the D-term [15]. The EMT densities allow us to gain insights on the particle stability, and may have interesting practical applications [18]. For a recent review we refer to [19].

The nucleon D-term has been studied in models, lattice QCD, and dispersion relations [20–33]. D-terms have also been investigated in spin-0 [34–40] and in higher-spin [41,42] systems. In all cases the D-terms were found negative. The nucleon D-term was also studied in chiral perturbation theory which cannot predict its value [43]. The fixed poles in virtual Compton amplitudes discussed in the pre-QCD era [44] might be related to the D-term [45].

With the D-term experimentally unknown, theoretical predictions are of importance. A particularly interesting question is: what is the D-term of a free particle? The purpose of this work is to address this question for fermions. To illustrate how instructive it is to investigate this question, one may recall that the free Dirac equation predicts the anomalous magnetic moment $g = 2$ of a charged point-like fermion, which is derived by coupling the free theory to a weak classical magnetic background field. In principle, the same is implicitly done by defining the EMT through coupling the free theory to a classical background gravitational field which for Dirac fields yields the symmetric "Belifante improved" EMT. Interactions alter the value $g = 2$; little for electrons and muons in QED, far more for protons and neutrons in QCD. But in any case, the free theory provides a valuable benchmark to which we can compare results from theoretical approaches and eventually experiment.

In an accompanying work, this question was studied for the bosonic case: free spin-0 bosons have an intrinsic D-term $D = -1$. This prediction pertains to free pointlike bosons, although interacting theories of extended bosons can be constructed where this value is preserved. In general, however, interactions affect the value of D [40].

In this work we will show that free noninteracting fermions have no intrinsic D-term. This means that, in contrast to bosons, fermionic D-terms are generated by dynamics which is an unexpected and highly interesting feature. We will illustrate in two simple models how interactions can generate the D-term of a fermion.

The outline of this work is as follows. After introducing the notation in Sec. II, we will compute the EMT form factors for a free spin $\frac{1}{2}$ particle in Sec. III and show that the D-term of a noninteracting fermion vanishes, which has implicitly already been stated in literature as we became aware after completing this part of our work. In Sec. IV we provide a heuristic argument based on the 3D density formalism to explain why the D-term must be zero for a free pointlike particle for consistency reasons. In Secs. V and VI we use two models of the nucleon to demonstrate how interactions generate a nonzero value for the D-term. We use the bag model, where the interaction is provided by the bag boundary which confines the otherwise free and noninteracting fermion(s). We also use the chiral quark soliton model where the nucleon is described as a solitonic bound state in a strongly interacting theory of quarks, antiquarks and Goldstone bosons. Finally, in Sec. VII we summarize our findings and present the conclusions.

II. FORM FACTORS OF THE ENERGY-MOMENTUM TENSOR

The energy momentum tensor of a theory described by the Lagrangian $\mathcal{L}$ is defined by coupling the theory to a background gravitational field and varying the action $S_{\rm grav} = \int \mathrm{d}^4x\sqrt{-g}\mathcal{L}$ with respect to the background field,

$$\hat{T}_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_{\rm grav}}{\delta g^{\mu\nu}}, \tag{1}$$

where g denotes the determinant of the metric. The matrix elements of the EMT operator in spin-$\frac{1}{2}$ states are described by three form factors [1]

$$\langle p'|\hat{T}_{\mu\nu}(0)|p\rangle = \bar{u}(p')\left[M_2(t)\frac{P_\mu P_\nu}{m} + J(t)\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2m} + D(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m}\right]u(p), \tag{2}$$

with states and spinors normalized by $\langle p'|p\rangle = 2p^0(2\pi)^3\delta^{(3)}(\mathbf{p}'-\mathbf{p})$ and $\bar{u}(p)u(p) = 2m$ where m denotes the mass. We suppress spin indices for brevity, and define $P = (p+p')/2$, $\Delta = (p'-p)$, $t = \Delta^2$.

The form factors of the EMT in Eq. (2) can be interpreted [15] in analogy to the electromagnetic form factors [17] in the Breit frame characterized by $\Delta^0 = 0$. In this frame one can define the static EMT

$$T_{\mu\nu}(\mathbf{r},\mathbf{s}) = \int\frac{\mathrm{d}^3\mathbf{\Delta}}{(2\pi)^3 2E}\exp(i\Delta\mathbf{r})\langle p',S'|\hat{T}_{\mu\nu}(0)|p,S\rangle \tag{3}$$

with the initial and final polarization vectors of the nucleon S and S' defined such that they are equal to $(0,\mathbf{s})$ in the respective rest-frame, where we introduce the unit vector $\mathbf{s}$ denoting the quantization axis for the nucleon spin. The energy density $T_{00}(\mathbf{r})$ yields the fermion mass according to $\int \mathrm{d}^3r T_{00}(\mathbf{r},\mathbf{s}) = m$, while $\epsilon^{ijk}r_jT^Q_{0k}(\mathbf{r},\mathbf{s})$ corresponds to the distribution of angular momentum inside the fermion. The components of $T_{ik}(\mathbf{r})$ constitute the stress tensor. The form factors $M_2(t)$ and $D(t)$ are related to $T_{\mu\nu}(\mathbf{r},\mathbf{s})$ by

$$M_2(t) - \frac{t}{4m^2}(M_2(t) - 2J(t) + D(t)) = \frac{1}{m}\int \mathrm{d}^3\mathbf{r}e^{-i\mathbf{r}\Delta}T_{00}(\mathbf{r},\mathbf{s}), \tag{4}$$

$$D(t) + \frac{4t}{3}D'(t) + \frac{4t^2}{15}D''(t) = -\frac{2}{5}m\int \mathrm{d}^3\mathbf{r}e^{-i\mathbf{r}\Delta}T_{ij}(\mathbf{r})\left(r^ir^j - \frac{\mathbf{r}^2}{3}\delta^{ij}\right), \tag{5}$$

where the primes denote derivatives with respect to the argument. The explicit expressions relating $\epsilon^{ijk}r_jT^Q_{0k}(\mathbf{r},\mathbf{s})$ to $J(t)$, see [15,46], will not be needed in this work. At zero momentum-transfer the form factors satisfy the constraints

$$M_2(0) = 1, \qquad J(0) = \frac{1}{2},$$
$$D(0) = -\frac{2}{5}m\int \mathrm{d}^3\mathbf{r}T_{ij}(\mathbf{r})\left(r^ir^j - \frac{\mathbf{r}^2}{3}\delta^{ij}\right) \equiv D. \tag{6}$$

The form factors $M_2(t)$ and $J(t)$ are constrained at $t = 0$ because the total energy of the fermion is equal to its mass and its spin is 1/2, see [47] for a recent rigorous discussion in an axiomatic approach. But the value of $D \equiv D(0)$ is *a priori* unknown, and must be determined from experiments. The physical interpretation of the D-term is the following. $D(t)$ is connected to the distribution of pressure and shear forces experienced by the partons in the nucleon [15]: $T_{ij}(\mathbf{r})$ can be decomposed as

$$T_{ij}(\mathbf{r}) = s(r)\left(\frac{r_ir_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(r)\delta_{ij}. \tag{7}$$

Hereby $p(r)$ describes the distribution of the "pressure" inside the hadron, while $s(r)$ is related to the distribution of the "shear forces." Both functions are related to each other due to the conservation of the EMT [15]. Another important

consequence of the EMT conservation is the von Laue condition [48]

$$\int_0^\infty \mathrm{d}r r^2 p(r) = 0, \tag{8}$$

which is a necessary (but not sufficient) condition for stability. Further worthwhile noticing properties which follow from the conservation of the EMT are discussed in Ref. [25].

III. EMT FORM FACTORS FOR A FREE DIRAC PARTICLE

The simplest case is the theory of a free spin $\frac{1}{2}$ fermion described by the Lagrangian

$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi. \tag{9}$$

For a free spin $\frac{1}{2}$ particle Eq. (1) yields the EMT operator given by

$$\hat{T}_{\mu\nu}(x) = \frac{1}{4}\bar{\Psi}(x)(i\gamma_\mu\vec{\partial}_\mu + i\gamma_\nu\vec{\partial}_\mu - i\gamma_\mu\overleftarrow{\partial}_\mu - i\gamma_\nu\overleftarrow{\partial}_\mu)\Psi(x), \tag{10}$$

where the arrows indicate on which fields the derivatives act. Evaluating the matrix elements yields

$$\langle p'|\hat{T}_{\mu\nu}(x)|p\rangle = \frac{1}{4}\bar{u}(p')[\gamma_\mu p_\nu + p_\mu\gamma_\nu + \gamma_\mu p'_\nu + p'_\mu\gamma_\nu]u(p)e^{i(p'-p)x}. \tag{11}$$

Exploring the Gordon identity we can rewrite this result as

$$\langle p'|\hat{T}_{\mu\nu}(x)|p\rangle = \bar{u}(p')\left[\frac{P_\mu P_\nu}{m} + \frac{1}{2}\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{2m}\right]u(p)e^{i(p'-p)x}, \tag{12}$$

from which we read off the predictions of the free Dirac theory for the EMT form factors, namely

$$M_2(t) = 1, \qquad J(t) = \frac{1}{2}, \qquad D(t) = 0. \tag{13}$$

Several comments are in order. The form factors are constant functions of t as expected for a free point-like particle, and we consistently recover the general constraints for $M_2(t)$ and $J(t)$ at $t = 0$ in Eq. (6). The value of the D-term is therefore the only nontrivial result from this exercise: it is remarkable it vanishes for a free pointlike fermion [49].

It is important to remark that the vanishing of the D-term in the free case was implicitly known in literature, see e.g. [50] where quantum corrections to the metric were studied. Although a quantum gravity theory is not yet known, the leading quantum corrections can be computed from the known low energy structure of the theory [51]. These calculations are challenging [52–54]. But the "tree level" results for EMT form factors were obtained unambiguously already in [50]. Our free field calculation, Eq. (13), agrees with Ref. [50]. The loop corrections to the Reissner-Nordström and Kerr-Newman metrics [52–54] show how (QED, gravity) interactions generate quantum long-range contributions to the stress tensor. A consistent description of the D-term requires, however, the full picture of the stress tensor including short-distance contributions which cancel exactly the long-distance ones in Eq. (8). The results of these works therefore do not allow us to gain insights on how much these corrections contribute to the D-terms of elementary (and charged) fermions.

IV. HEURISTIC CONSISTENCY ARGUMENT WHY $D = 0$ FOR A FREE FERMION

The vanishing of the D-term of a free fermion can be made plausible on the basis of a heuristic argument which was already helpful in discussing the EMT densities in the bosonic case [40]. The argument explores the 3D-density framework which strictly speaking requires the particle to be heavy such that relativistic corrections can be neglected. The argument is based on two assumptions: (i) form factors are t-independent constants in the free theory case, and (ii) energy density is formally given by $T_{00}(\vec{r}) = m\delta^{(3)}(\vec{r})$ for a heavy particle [55].

Per assumption (i) we can replace the form factors in Eq. (4) by their values at zero-momentum transfer. Next, we notice that the result in the square brackets in the following equation must be zero to comply with assumption (ii),

$$\frac{1}{m}\int \mathrm{d}^3\mathbf{r}e^{-i\Delta\mathbf{r}}T_{00}(\mathbf{r}) = M_2(0) - \frac{t}{4m^2}\underbrace{[M_2(0) - 2J(0) + D(0)]}_{\overset{!!}{=}0} \overset{!}{=} 1. \tag{14}$$

With the constraints in Eq. (6) it is clear that $M_2(0) - 2J(0) = 0$. From this it then immediately follows that the D-term must vanish for a pointlike particle for consistency reasons.

This is nothing but a heuristic argument. But it is nevertheless helpful to make it plausible why the D-term of a free fermion vanishes. From this argument it is also clear why in the interacting case one may in general encounter a nonzero D-term: when interactions are present form factors can no longer be expected to be t-independent constants, and $D(t)$ in general do not need to be zero. An extended internal structure implies a non-zero D-term along the same lines: now $T_{00}(\vec{r}) \neq m\delta^{(3)}(\vec{r})$ and form factors exhibit a generic t-dependence, e.g., $M_2(t) = 1 + \frac{1}{6}\langle r^2\rangle t + \mathcal{O}(t^2)$ [25].

In Appendix A we include another heuristic argument why the D-term vanishes for elementary fermions but not for elementary bosons, based on a simple analysis of the structure of the Lagrangians.

V. EMERGENCE OF THE D-TERM FROM BAG BOUNDARY FORCES

The bag model describes one or several noninteracting fermions confined inside a "bag" which, in its rest frame, is a spherical region of radius R carrying the energy density $B > 0$. If $N_c = 3$ quarks or a $\bar{q}q$-pair are placed inside the bag in a color-singlet state, this yields the popular model of hadrons with confinement simulated by the bag boundary condition [56]. The Lagrangian of the bag model can be written as [57]

$$\mathcal{L} = (\bar{\psi}(i\partial\!\!\!/ - m)\psi - B)\Theta_V + \frac{1}{2}\bar{\psi}\psi\eta^\mu\partial_\mu\Theta_V, \quad (15)$$

where $\Theta_V = \Theta(R-r)$, $\delta_S = \delta(R-r)$, $\eta^\mu = (0,\vec{e}_r)$, $\vec{e}_r = \vec{x}/r$, $r = |\vec{x}|$ in the bag rest frame. The indices V and S denote respectively the volume and the surface of the bag. The boundary condition for the fields is equivalent to the statement that there is no energy-momentum flow out of the bag, i.e. $\eta_\mu T^{\mu\nu}(t,\vec{r}) = 0$ for $\vec{r} \in S$.

The starting point is as follows. If no bag boundary condition is present, i.e. in the limit $R \to \infty$ in Eq. (15), we deal with the free Lagrangian (9) with an additive constant B which is irrelevant and can be discarded. In such a free theory the D-term is zero, as we have shown in Sec. III.

Next let us discuss what happens if we solve the theory with the bag radius R kept finite. This means we effectively introduce an interaction acting on the otherwise free fermion. We will see that now a nonzero D-term emerges. Below we quote only the main steps needed in our context. The details of this calculation will be reported elsewhere [58].

The equations of motion of the theory (15) are $(i\partial\!\!\!/ - m)\psi = 0$ for $r < R$, while at the surface $\vec{x} \in S$ the linear boundary condition $in\!\!\!/\psi = \psi$ and the nonlinear boundary condition $\frac{1}{2}\eta_\mu\partial^\mu(\bar{\psi}\psi) = -B$ hold. The ground state solution has positive parity and is given by the wavefunction

$$\psi(t,\vec{x}) = e^{-i\omega t/R}\frac{A}{\sqrt{4\pi}}\begin{pmatrix}\alpha_+ j_0(\omega r/R)\chi_s \\ \alpha_- j_1(\omega r/R)i\vec{\sigma}\vec{e}_r\chi_s\end{pmatrix}, \quad (16)$$

where $\alpha_\pm = \sqrt{1 \pm mR/\Omega}$ and $\Omega = \sqrt{\omega^2 + m^2R^2}$, ω denotes the lowest solution of the equation $\omega = (1 - mR - \Omega)\tan\omega$, σ^i are Pauli matrices, χ_s are two-component spinors. The normalization $\int \mathrm{d}^3x\psi^\dagger(\vec{x},t)\psi(\vec{x},t) = 1$ fixes the constant A. If N_c fermions are placed in the bag the D-term is given by

$$D = \frac{1}{3}MN_c\frac{A^2R^4}{\omega^4}\alpha_+\alpha_-\left(-\frac{\omega^3}{3} + \frac{5}{4}(\omega - \sin\omega\,\cos\omega) - \frac{\omega}{2}\sin^2\omega\right). \quad (17)$$

where $M = N_c\Omega/R + \frac{4}{3}\pi BR^3$ is the mass of the system. One can show that always $D < 0$ in this model [58]. For $N_c = 3$ colors and assuming the fermions to be massless quarks (in which case $\omega = 2.04\ldots$) one obtains $D = -1.145$ in agreement with the numerical bag model calculation of nucleon GPDs and EMT form factors from Ref. [20].

As an application of Eq. (17) it is insightful to consider the limit $mR \to \infty$ where $\omega \to \pi$, and the D-term becomes

$$D = N_c^2\frac{(-4\pi^2 + 15)}{45}. \quad (18)$$

This result can be interpreted in two ways.

For the first interpretation we may assume that m is fixed and R becomes much larger [59] than the Compton wavelength of the particle, $R \gg 1/m$. This means the "interaction" decreases, as the confined particle(s) can occupy an increasing volume with the boundary being "moved" further and further away. However, no matter how far away we move the boundary [59]: *some* interaction remains, and generates a nonzero D-term.

For the second interpretation we may assume a fixed R and $m \to \infty$. This is known as the nonrelativistic limit, in which $\alpha_- \to 0$ and the lower component of the spinor in (16) vanishes. The D-term in Eq. (17) is proportional to α_- which vanishes, and to the mass of the system which behaves as $M \to N_c m$ for $m \to \infty$. The product $M\alpha_-$ is finite in the limit $m \to \infty$. As a result the D-term assumes a finite value as quoted in Eq. (18). This result demonstrates that also nonrelativistic systems have a D-term, i.e. this

property is not a relativistic effect. For a detailed discussion of the D-term in the bag model we refer to [58].

One virtue of the bag model is its transparency, which we explored here to learn insightful lessons about the D-term. One caveat is that it does not comply with chiral symmetry which is incorporated in the model discussed next.

VI. CHIRAL INTERACTIONS AND THE D-TERM OF NUCLEON

The spontaneous breaking of chiral symmetry is the dominant feature of strong interactions in the nonperturbative low-energy regime. A theoretically consistent and phenomenologically successful model of baryons based on chiral symmetry breaking is the chiral quark-soliton model [60] defined in the SU(2) flavor-sector by [61,62]

$$\mathcal{L}_{\rm eff} = \bar{\Psi}(i\partial\!\!\!/ - MU^{\gamma_5})\Psi, \quad U^{\gamma_5} = \exp(i\gamma_5\tau^a\pi^a/F_\pi) \tag{19}$$

where $F_\pi = 93$ MeV denotes the pion decay constant. Besides the emergence of Goldstone bosons, another consequence of chiral symmetry breaking is the dynamically generated "constituent" quark mass $M \simeq 350$ MeV. The effective theory (19) was derived from the instanton model of the QCD vacuum [63,64] which provides a microscopic picture of the dynamical breaking of chiral symmetry, see [65] for reviews.

In order to solve the strongly coupled theory (19) (the coupling constant of quark and pion field is $M/F_\pi \sim 3.8$) a nonperturbative method based on the limit of a large number of colors N_c is used. In this limit the functional integration over U-fields in Eq. (19) is solved in the saddle-point approximation by evaluating the model expressions at the static solitonic field $U(\vec{x})$ and integrating over the zero-modes of the soliton solution. The spectrum of the Hamiltonian of the effective theory (19), $H = -i\gamma^0\gamma^k\partial_k + M\gamma^0 U^{\gamma_5}(\vec{x})$, contains continua of positive energies $E > M$ and negative energies $E < -M$, and a discrete level with an energy $-M < E_{\rm lev} < M$. The nucleon state is obtained by occupying the discrete level and the states of negative continuum and subtracting the free negative continuum ("vacuum subtraction"). The solitonic field $U(\vec{x})$ is determined from a self-consistent variational procedure which minimizes the soliton energy. In the physical situation the soliton size is $R_{\rm sol} \sim M^{-1}$ [60].

GPDs and EMT form factors including the D-term were studied in this model [21–26]. As a demonstration of the consistency of this effective chiral theory let us mention that in this model the GPDs satisfy polynomiality [22], the Ji sum rule is valid [23], the von Laue condition holds, the model correctly reproduces the leading non-analytic terms of the EMT form factors [25], and agrees with available lattice QCD data [26].

We will now show that the D-term vanishes when one "switches off" the chiral interactions in this model. This can be formally done by replacing $U \to 1$ in Eq. (19) which yields the free theory. One way to practically implement this limit is to consider the formal limit $MR_{\rm sol} \to \infty$. As the soliton size increases the discrete level energy decreases and approaches the negative continuum [60]. Since in this limit the spatial extension of the solitonic field $U(\vec{x})$ grows, its gradients $\nabla U(\vec{x})$ decrease. This allows one to expand model expressions in terms of gradients of the chiral field.

The expression for the D-term valid in such a large soliton expansion was derived in [22] and is given by

$$D = -F_\pi^2 M_N \int \mathrm{d}^3x P_2(\cos\vartheta)\vec{x}^2 \mathrm{tr}_F[\nabla^3 U][\nabla^3 U^\dagger] + \ldots \tag{20}$$

where tr_F is the trace over flavor indices, M_N denotes the nucleon mass, and the dots indicate higher order derivatives. Notice that the expression (20) is quasi model-independent: it is the leading contribution in the chiral expansion of the D-term from which one can derive the leading nonanalytic terms [26]. The second Legendre polynomial reflects that the D-term is related to the traceless part of the stress tensor [15].

After these preparations we can now discuss what happens in the formal limit when we "switch off" the chiral interactions and $U \to 1$. In this limit all gradients vanish in Eq. (20) and we recover that $D = 0$ which is the free field theory prediction obtained in Eq. (13). This shows that the D-term in the chiral quark soliton model is due to the chiral interactions which define and characterize this model.

Let us stress that the above discussion applies only to the formal limit $U \to 1$ which we implemented by means of the large soliton expansion. Only in this limit it is justified to expand model expressions in powers of the derivatives of the chiral field. In the physical situation the soliton size is such that $MR_{\rm sol} \sim 1$ and no such expansion is allowed (though it can be used to derive chiral leading nonanalytic contributions, and it may give useful rough estimates). In order to obtain in the physical situation reliable model predictions for the D-term, and a pressure satisfying the von Laue condition (8), it is necessary to evaluate numerically the full model expression [25].

VII. CONCLUSIONS

The D-term of a free noninteracting fermion vanishes. This is a simple prediction of the free Dirac equation which is, in principle, analog to the prediction $g = 2$ for the anomalous magnetic moment of a charged pointlike fermion. This result is remarkable for several reasons and has interesting implications.

The prediction of a vanishing D-term from the free Dirac equation should be contrasted with the bosonic case. The free Klein-Gordon equation predicts an intrinsic nonzero D-term already for free and noninteracting bosons. When interactions are introduced in bosonic theories, the value of D is in general affected and, depending on the theory, the

effect can be sizable [40]. However, in the fermionic case interactions do not modify the D-term, but *generate* it. In other words, the D-term of a spin-$\frac{1}{2}$ particle is entirely of dynamical origin.

We have provided an heuristic consistency argument which makes it plausible why the D-term of a free pointlike spin $\frac{1}{2}$ particle should vanish. While not a rigorous derivation, this argument was already successfully applied to explain why a free pointlike boson must have $D = -1$ [40].

We have explored two dynamical models of the nucleon to illustrate how the D-term is generated in interacting systems. In the bag model we have shown how a nonzero D-term emerges when we "switch on" interactions which in this model are formulated in terms of boundary conditions which confine otherwise free fermions. We used also the chiral quark soliton model where we have shown how the D-term vanishes when the strongly coupled chiral interactions in that model are "switched off." These are simple models of the nucleon, but these results solidify our conclusions: in a fermionic system the D-term is generated by dynamics, it arises entirely from interactions.

The calculations of the nucleon D-term in models, lattice QCD, or dispersion relations [20–33] give therefore insights which are completely due to the underlying (effective, model, chiral, QCD) dynamics. With its relation to the internal forces and the stress tensor [15] the D-term emerges therefore as a valuable window to gain new insights on the structure of composite particles, and especially the QCD dynamics inside the nucleon.

In any case, all presently known matter is composed of what we consider elementary fermions, which indicates the importance to study this interesting particle property in more detail. Knowledge of EMT form-factors can be applied to the spectroscopy of the hidden-charm pentaquarks observed at LHCb [18,42]. Also EMT form factors of mesons can be inferred from data [66]. This information may help to discriminate usual from exotic mesons [67]. The recent analysis of KEKB data on $\gamma^*\gamma \to \pi^0\pi^0$ revealed information on EMT form factors and D-term of the π^0 boson [68].

It will be very exciting to learn about the D-term of the nucleon from lattice QCD calculations and experiment and the perspectives are good. The D-term plays an important role for the phenomenological description of hard-exclusive reactions, but cannot yet be extracted model-independently [11,12]. After first, vague and model-dependent glimpses on the nucleon D-term from the HERMES experiment [69] one may expect more quantitative insights from experiments at Jefferson Lab [70–72], COMPASS at CERN [73], and the envisioned future Electron-Ion-Collider [74].

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APPENDIX: WHY CAN THE KLEIN-GORDON EQUATION GIVE $D \neq 0$ BUT DIRAC EQUATION CANNOT?

One may wonder why the Klein-Gordon equation can naturally predict a nonzero D-term, but the Dirac equation cannot. It is instructive to review how this happens. The D-term appears in the decomposition of the matrix elements of the EMT operator (2) with the same structure $\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2$ in the bosonic and fermionic case. In spin-zero case such a structure emerges already from the kinetic term in the Lagrangian $\mathcal{L} = \partial_\mu\Phi^*\partial^\mu\Phi - V$. The kinetic term contains two field derivatives and generates the contribution $\partial^\mu\Phi^*\partial^\nu\Phi + \partial^\mu\Phi\partial^\nu\Phi^*$ to the EMT operator. This is sufficient to generate the needed structure $\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2$ in the EMT matrix elements even in the absence of interactions, when $V = m^2\Phi^*\Phi$ in the free case. Interactions may affect the D-term (and make its value more or less negative, but preserving its sign in all theories studied so far). The main point is, however, that even in the free theory a non-zero D-term arises in the spin-zero case [40] and this can be naturally traced back to the Lagrangian containing 2 derivatives of the fields needed to generate in the EMT a structure proportional to $\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2$. In contrast to this, in the case of free Dirac fields the Lagrangian contains only one derivative, and consequently no D-term can be generated. Let us notice that *if* interactions are present they of course may generate a D-term in the fermionic case, see Secs. V and VI for some illustrations.

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