

Nonthermal production of dark matter from primordial black holes

Rouzbeh Allahverdi,¹ James Dent,² and Jacek Osinski¹

¹*Department of Physics and Astronomy, University of New Mexico, Albuquerque, New Mexico 87131, USA*

²*Department of Physics, Sam Houston State University, Huntsville, Texas 77341, USA*



(Received 13 December 2017; published 8 March 2018)

We present a scenario for nonthermal production of dark matter from evaporation of primordial black holes. A period of very early matter domination leads to formation of black holes with a maximum mass of $\simeq 2 \times 10^8$ g, whose subsequent evaporation prior to big bang nucleosynthesis can produce all of the dark matter in the Universe. We show that the correct relic abundance can be obtained in this way for thermally underproduced dark matter in the 100 GeV–10 TeV mass range. To achieve this, the scalar power spectrum at small scales relevant for black hole formation should be enhanced by a factor of $\mathcal{O}(10^5)$ relative to the scales accessible by the cosmic microwave background experiments.

DOI: [10.1103/PhysRevD.97.055013](https://doi.org/10.1103/PhysRevD.97.055013)

I. INTRODUCTION

Various lines of evidence support the existence of dark matter (DM) in the Universe [1]. Weakly interacting massive particles (WIMPs) are promising candidates for DM in the Universe, and major direct, indirect, and collider searches are currently underway to discover WIMP-like DM and determine its properties. The relic abundance of WIMPs may be nicely explained via the well-explored “WIMP miracle.” This paradigm assumes the Universe was in a radiation-dominated (RD) phase at temperatures about the DM mass m_{DM} . The DM relic abundance in this picture is set when the annihilation rate of DM particles drops below the Hubble expansion rate, a process called “thermal freeze-out,” which typically happens at a temperature $T_f \sim m_{\text{DM}}/20$. The observed DM abundance is obtained if the annihilation rate takes the value $\langle \sigma_{\text{ann}} v \rangle_f = 2\text{--}3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$, nearly independently of the dark matter mass above values of $m_\chi \approx 10$ GeV (and variation with mass roughly up to a factor of two for masses below 10 GeV) [2].

The WIMP miracle has been a staple of DM physics for decades, but is coming under increasing pressure in light of recent experimental data. Notably, Fermi-LAT’s results from observations of dwarf spheroidal galaxies [3] and newly discovered Milky Way satellites [4] rule out the nominal value of the annihilation rate in various channels for DM masses up to ~ 100 GeV. Moreover, a RD phase at the time of freeze-out is an *assumption*, as we currently have no direct observational probe of the early Universe prior to the onset of big bang nucleosynthesis (BBN). A nonstandard thermal history is a generic feature of various early Universe models including some string theory constructions (for a

review, see [5]). The late decay of modulus fields, which can dominate the Universe’s energy density leading to an epoch of early matter domination (EMD), reheats the Universe to temperatures below T_f , thereby rendering the WIMP miracle irrelevant in this framework.

Exploring alternatives to the WIMP miracle is, therefore, well motivated by both theoretical and experimental considerations, and has been gaining increased attention. An attractive scenario is nonthermal production of DM from late decay of moduli that drive an EMD era. This scenario can accommodate both cases with large annihilation rate $\langle \sigma_{\text{ann}} v \rangle_f > 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ (leading to thermal underproduction) [6,7] and small annihilation rate $\langle \sigma_{\text{ann}} v \rangle_f < 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ (resulting in thermal overproduction) [8,9] (for explicit examples in the context of type IIB string compactifications, see [10]).

Also, as WIMP detection remains elusive, alternative models in which DM is not a WIMP-like particle have attracted significant attention in recent years (for example, see [11]). One alternative is that DM, instead of being an elementary particle, is (at least partially) composed of primordial black holes (PBHs) [12,13] formed in the early Universe.¹ PBHs with a mass $\gtrsim 10^{15}$ g would not have evaporated by the present time and may, in principle, constitute a fraction or, perhaps, all of the DM in the Universe. While this possibility has been a subject of study for a long time, the recent discovery of gravitational waves (GW) by the Advanced LIGO group has led to intensified efforts to constrain a

¹For other early works as well as some reviews, see [14–20].

possible PBH population.² A great deal of work has also been devoted to mechanisms for amplifying density perturbations toward the end of inflation to allow production of PBHs [34,62,64,71–79].

In this work, we study a scenario where PBHs are responsible for nonthermal production of DM, including the possibility of producing the entirety of the relic abundance. This scenario invokes a period of very early matter domination (VEMD) that leads to the formation of PBHs within an extended mass range. Evaporation of PBHs in the ensuing RD phase creates DM particles after thermal freeze-out or freeze-in but prior to BBN. Evading tight observational constraints for evaporation after BBN sets an upper bound $\simeq 2 \times 10^8$ g on the maximum mass M_{\max} of PBHs thus formed. We show that the correct DM relic abundance can be obtained within the DM mass range $m_{\text{DM}} = 100$ GeV–10 TeV, provided that the scalar power spectrum at small scales (relevant for PBH formation) is enhanced by a factor of $\mathcal{O}(10^5)$ relative to its value at scales probed by the cosmic microwave background (CMB) experiments. The upper limit on M_{\max} implies that transition from VEMD to RD should occur when the Hubble expansion rate is $H_{\text{reh}} \gtrsim \mathcal{O}(100)$ GeV. The observed DM abundance can be accommodated in cases when thermal freeze-out or freeze-in lead to underproduction of DM. The former case happens when DM annihilation rate is larger than the nominal value for the WIMP miracle, i.e., $\langle \sigma_{\text{ann}} v \rangle_f > 3 \times 10^{-26}$ cm³ s⁻¹. The latter case occurs when DM is extremely weakly coupled to the standard model (SM) particles resulting in a very small annihilation cross section, possibly $\sigma_{\text{ann}} \sim M_{\text{p}}^{-2}$.

We show that the required enhancement of the power spectrum is compatible with the Planck limits on the scalar spectral index and its running within 2σ (for previous work on PBH in the context of a running spectral index, see [80,81]). Such amplification is also attainable, for example, in models where the inflaton undergoes a brief period of ultra slow-roll motion toward the end of inflation. We do

²This includes examination of the LIGO measurement and constraints on the PBH mass distribution (including the possibility of extended mass distributions) and merger rate [21–29], general signatures of exotic compact objects [30], possible PBH progenitors [31,32], future GW searches and expectations [33–38] including those for a GW background [39–45]. A bevy of other constraints (for a recent overview see [19]) on the existence and effects of PBH have been recently reexamined and new constraints proposed including those from dynamical effects in dwarf galaxies [46,47], radio and x-ray sources [48,49], cosmic microwave background (CMB) measurements [50–55], ionization history [56], quasar microlensing [57], neutron star capture [58], lensing of radio bursts [59], near infrared and cosmic infrared background [60], 21 cm measurements [61], current and future pulsar timing arrays [62–64], lensing for intermediate mass PBH [65], future strong lensing tests [66], orbital eccentricity determination [67], spin distribution evaluation [68], spatial clustering [69] along with effects of astrophysical uncertainties on PBH constraints [70].

not present an inflationary model that achieves this, however, as our goal in this work is to discuss the main ingredients for nonthermal DM production via evaporation of PBHs and identify the allowed parameter space. An explicit model that addresses inflation with the desired enhancement of the power spectrum as well as the origin of the VEMD era is the subject of a future work.

The rest of this paper is organized as follows. In Sec. II, we briefly review PBH production in the early Universe in both RD and (V)EMD phases (focusing on the latter). In Sec. III, we discuss nonthermal DM production via evaporation of PBHs. We present the main results in Sec. IV. We close the paper by a brief discussion and conclusion in Sec. V. An Appendix includes some calculational details of our results presented in the main body of the paper.

II. FORMATION OF PBHS IN VERY EARLY MATTER DOMINATION

PBHs are postulated to form from density fluctuations in the postinflationary early Universe. For the standard cosmology, the Universe existed in a RD stage after the reheating process that followed inflation, and remained there until matter-radiation equality was reached in a post-BBN and pre-CMB era. However, as mentioned above, it is possible that there existed a period of (V)EMD that ended before the onset of BBN.

It is well known that density fluctuations exhibit remarkably different growth behavior depending on the form of the dominant background energy component, be it radiation or matter. Here we briefly review the formation of PBHs in these two different background scenarios, and will then explore some of the consequences of PBH formation in the context of a VEMD scenario (for some of the works on EMD and PBH see, for example, [14,15,53,82]).

In the case of a RD universe, a density fluctuation of $\mathcal{O}(1)$ would need to overcome the radiative pressure and thus would have a characteristic size on the order of the scale of the horizon. Assuming a Gaussian perturbation profile with root-mean-square amplitude $\delta(M)$, the fractional energy density of the Universe that goes into PBHs with mass M is given by [83]

$$\beta(M) \approx K \delta^{2\gamma}(M) \text{erfc}\left(\frac{\delta_c}{\sqrt{2}\delta(M)}\right), \quad (1)$$

where $\gamma \simeq 0.36$ [84–88], $K \simeq 3.3$ [89], and $\delta_c \simeq 0.45$ [85–87] (see [90] for a smaller value of δ_c). We note that after PBH formation β increases $\propto a(t)$ during RD, where $a(t)$ is the scale factor.

However, the situation is altered if PBH formation takes place during a period of (V)EMD. In that case, one arrives at [14,15]

$$\beta(M) \approx 2 \times 10^{-2} \delta^{13/2}(M), \quad (2)$$

where $\delta(M)$ denotes the amplitude of perturbations for a mode that eventually collapses to form PBH with mass M when it enters the horizon. Such a perturbation enters the horizon when $H \simeq 4\pi M_p^2/M$ (where M_p is the Planck mass). The amplitude then grows according to $\delta \propto a \propto H^{-2/3}$ and black hole formation occurs at

$$H_{\text{form}} \simeq \frac{4\pi M_p^2}{M} \delta^{3/2}(M), \quad (3)$$

when $\delta \sim \mathcal{O}(1)$. It is important to note that subhorizon fluctuations can form black holes due to the absence of pressure in this case. We also note that β remains constant during the (V)EMD era.

The minimum mass of PBHs formed during the VEMD era, M_{min} , depends on the details of the thermal history between the end of inflation (characterized by H_{inf}) and the start of the VMED era (characterized by H_0), namely the window $H_0 \lesssim H \lesssim H_{\text{inf}}$. An absolute lower bound on M can be found by noticing that the minimal inflationary fluctuation wavelength is $\sim H_{\text{inf}}^{-1}$, which implies that

$$M_{\text{min}} \gtrsim \frac{4\pi M_p^2}{H_{\text{inf}}}. \quad (4)$$

The maximum mass M_{max} corresponds to the mode whose amplitude reaches $\mathcal{O}(1)$ at the end of the VEMD epoch, which results in

$$M_{\text{max}} \simeq \frac{4\pi M_p^2}{H_{\text{reh}}} \delta^{3/2}(M_{\text{max}}), \quad (5)$$

where H_{reh} denotes the Hubble rate when the VEMD epoch ends and the Universe enters the RD phase. In order to avoid very tight post-BBN constraints on evaporation of PBHs (for example, see [18]), we require that all PBHs formed during the VEMD era evaporate before BBN. As we shall see, from Eq. (6) in the following section, this results in an upper bound of $M_{\text{max}} \lesssim 2 \times 10^8$ g.

One comment is in order before moving to the next section. Since transition from VEMD to RD is not instantaneous, one should not take the above expression for M_{max} as exact. The spectrum of PBHs formed during VEMD is not suddenly cut at M_{max} . Instead, there is a quick drop in $\beta(M)$ around M_{max} signifying the transition from VEMD to RD. In fact, PBHs with a mass (much) larger than M_{max} may form in the following RD phase from the collapse of fluctuation modes that enter the horizon then. However, the abundance of such PBHs is extremely suppressed due to its exponential dependence on $\delta(M)$ as seen in (1). Therefore, M_{max} provides a good approximation of the mass above which PBH formation during VEMD ceases to be important.

III. NONTHERMAL DM FROM EVAPORATION OF PBHS

Here we study a scenario in which the entire DM relic abundance is due to evaporation of PBHs formed during an epoch of VEMD. In passing, we note that PBHs formed in a RD phase can also produce DM particles via Hawking radiation. However, the exponential dependence of $\beta(M)$ on $\delta(M)$ in this case, see Eq. (1), implies that a parametrically larger $\delta(M)$ and a higher level of tuning are needed in this case in order to obtain the correct DM relic abundance. For this reason, we focus on DM production from PBHs formed in a VEMD phase.

PBHs with mass M evaporate via Hawking radiation [91] and have a lifetime

$$t_{\text{eva}} = \frac{80M^3}{\pi M_p^4}, \quad (6)$$

giving rise to particles with a thermal spectrum at the Hawking temperature,

$$T_H = \frac{M_p^2}{M}. \quad (7)$$

Evaporation of PBHs produces all particles that have a mass below their corresponding Hawking temperature. This implies that DM particles will also be produced as long as $m_{\text{DM}} \ll T_H$ [92]. For $M_{\text{max}} \lesssim 2 \times 10^8$ g, this requires that $T_H \gtrsim 50$ TeV, and hence implying production of particles that are lighter than ~ 50 TeV.³

PBHs with mass $M_{\text{max}} \lesssim 2 \times 10^8$ g evaporate in the RD phase of the Universe at a temperature $T_{\text{BBN}} \approx 1$ MeV. This late process can be responsible for the entire observed DM relic abundance in cases where thermal freeze-out or freeze-in lead to underproduction of DM. Thermal underproduction via freeze-out occurs for WIMPs with a large annihilation rate $\langle \sigma_{\text{ann}} v \rangle_f > 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$.⁴ Thermal underproduction from freeze-in can happen if DM has extremely weak coupling to the SM particles, perhaps even gravitationally suppressed interactions resulting in a very small annihilation cross section $\sigma_{\text{ann}} \sim M_p^{-2}$.

From the conservation of energy, and assuming that there is no other entropy generating process after the transition from VEMD to RD, evaporation of PBHs with mass M results in a DM abundance,

³This includes possible unwanted relics whose late decay may ruin the success of BBN, which leads to constraints on $\beta(M)$ [93,94].

⁴DM particles produced from PBH evaporation will not undergo further annihilation if $\langle \sigma_{\text{ann}} v \rangle < 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ (T_f/T_{BBN}), where $T_f \sim m_{\text{DM}}/20$. This is the case for DM masses up to 10 TeV as indicated by the latest Fermi-LAT constraints [3,4].

$$\begin{aligned} \left(\frac{n_{\text{DM}}}{s}\right)_M &\sim \text{Br}_{\text{DM}} \left(\frac{n_{\text{PBH}}(M)}{s}\right) \left(\frac{M}{T_{\text{H}}}\right) \\ &= \text{Br}_{\text{DM}} \left(\frac{n_{\text{PBH}}(M)}{s}\right) \left(\frac{M}{M_{\text{p}}}\right)^2, \end{aligned} \quad (8)$$

where $n_{\text{PBH}}(M)$ and s are the number density of PBHs and the entropy density in the RD phase, respectively, and Br_{DM} denotes the fraction of energy density in PBHs that goes into DM particles. For supersymmetric (SUSY) DM, we have $\text{Br}_{\text{DM}} \sim 1$ in the case that all SUSY particles have a mass below T_{H} . The reason being that all SM particles and their SUSY partners are produced from PBH evaporation in this case with the latter eventually decaying to DM. However, $\text{Br}_{\text{DM}} < 1$ if some of the SUSY particles have a mass above T_{H} . In the case that DM interacts extremely weakly with the SM particles (and their SUSY partners), Br_{DM} can be as small as $\mathcal{O}(10^{-2})$ based on direct production of DM along with all SM degrees of freedom (and their SUSY partners) from PBH evaporation.

The parameter $\beta(M)$ is related to the DM abundance through

$$\beta(M) = \left(\frac{\rho_{\text{PBH}}(M)}{\rho_{\text{tot}}}\right)_{\text{reh}} = \frac{4M}{3T_{\text{reh}}} \left(\frac{n_{\text{PBH}}(M)}{s}\right), \quad (9)$$

and hence

$$\left(\frac{n_{\text{DM}}}{s}\right)_M \sim \text{Br}_{\text{DM}} \beta(M) \left(\frac{3T_{\text{reh}}M}{4M_{\text{p}}^2}\right). \quad (10)$$

The observed DM relic abundance is

$$\left(\frac{n_{\text{DM}}}{s}\right)_{\text{obs}} \simeq 4 \times 10^{-12} \left(\frac{100 \text{ GeV}}{m_{\text{DM}}}\right). \quad (11)$$

Requiring that the contribution from PBHs with mass M does not exceed this value, and after using Eqs. (7), (8), (9), (10), we arrive at the following relation,

$$\begin{aligned} \beta(M) &\simeq 2.3 \times 10^{-26} \text{Br}_{\text{DM}}^{-1} \left(\frac{g_{*,\text{reh}}}{106.75}\right)^{1/4} \left(\frac{10^{11} \text{ g}}{M}\right) \\ &\times \left(\frac{100 \text{ GeV}}{m_{\text{DM}}}\right) \left(\frac{M_{\text{p}}}{H_{\text{reh}}}\right)^{1/2}, \end{aligned} \quad (12)$$

where $g_{*,\text{reh}}$ denotes the number of relativistic degrees of freedom at T_{reh} . We note that the similar expression in [18], which holds in the case of PBH formation in the RD phase, includes formation temperature T_i instead of the reheat temperature T_{reh} .

In reality, all of the PBHs formed within an extended mass range during the VEMD epoch contribute to the DM relic density. However, see Eq. (12), the constraint on $\beta(M)$ becomes weaker for lighter black holes. To be precise, one has to integrate over the whole relevant mass range to find

the total contribution to the DM abundance. As shown in the Appendix, that integral is typically dominated by the heaviest PBHs in the mass range. We, thus, have

$$\begin{aligned} \beta(M_{\text{max}}) &\simeq 10^{-23} \text{Br}_{\text{DM}}^{-1} \left(\frac{g_{*,\text{reh}}}{106.75}\right)^{1/4} \left(\frac{2 \times 10^8 \text{ g}}{M_{\text{max}}}\right) \\ &\times \left(\frac{100 \text{ GeV}}{m_{\text{DM}}}\right) \left(\frac{M_{\text{p}}}{H_{\text{reh}}}\right)^{1/2}, \end{aligned} \quad (13)$$

where M_{max} is normalized to its largest value for evaporation before the onset of BBN. By using Eq. (5), we can cast this expression in terms of M_{max} and $\delta(M_{\text{max}})$

$$\begin{aligned} \beta(M_{\text{max}}) &\simeq 9 \times 10^{-24} \text{Br}_{\text{DM}}^{-1} \left(\frac{g_{*,\text{reh}}}{106.75}\right)^{1/4} \left(\frac{2 \times 10^8 \text{ g}}{M_{\text{max}}}\right)^{1/2} \\ &\times \left(\frac{100 \text{ GeV}}{m_{\text{DM}}}\right) (\delta(M_{\text{max}}))^{-3/4}. \end{aligned} \quad (14)$$

IV. RESULTS

Equating the theoretical prediction for $\beta(M_{\text{max}})$ in Eq. (2) with that satisfying the observational constraint in Eq. (14) singles out the value of $\delta(M_{\text{max}})$ that is required to obtain the correct DM relic abundance for a given value of m_{DM}

$$\begin{aligned} (\delta(M_{\text{max}}))^{29/4} &\simeq 5 \times 10^{-22} \text{Br}_{\text{DM}}^{-1} \left(\frac{g_{*,\text{reh}}}{106.75}\right)^{1/4} \left(\frac{2 \times 10^8 \text{ g}}{M_{\text{max}}}\right)^{1/2} \\ &\times \left(\frac{100 \text{ GeV}}{m_{\text{DM}}}\right). \end{aligned} \quad (15)$$

We can trade out δ for the amplitude of the scalar power spectrum $A_s \equiv 25\delta^2/4$ [95]. Planck has measured a value $A_s = 2.196 \times 10^{-9}$ at the pivot scale of $k_* = 0.05 \text{ Mpc}^{-1}$ [96]. We translate the value of $\delta(M_{\text{max}})$ in Eq. (15) to the enhancement factor in A_s , denoted by f , from k_* to that relevant for forming PBHs with mass M_{max} .

Fig. 1 depicts the $\beta(M_{\text{max}})$ curve from theory and bands representing the observational constraint as a function of f for $M_{\text{max}} = 2 \times 10^8 \text{ g}$. In the left panel, $\text{Br}_{\text{DM}} = 1$ and the band corresponds to $m_{\text{DM}} = 100 \text{ GeV} - 10 \text{ TeV}$ mass range. In the right panel, $m_{\text{DM}} = 1 \text{ TeV}$ and the band corresponds to $\text{Br}_{\text{DM}} = 10^{-2} - 1$ range. The intersection region lies between $f \approx 6 \times 10^4 - 2 \times 10^5$ and $f \approx (1 - 4) \times 10^5$ in the left and right panels respectively.

After using Eqs. (2), (5), (13), we find the following expression for H_{reh}

$$\begin{aligned} H_{\text{reh}} &\approx (200 \text{ GeV}) \times \text{Br}_{\text{DM}}^{-6/29} \left(\frac{g_{*,\text{reh}}}{106.75}\right)^{3/58} \left(\frac{100 \text{ GeV}}{m_{\text{DM}}}\right)^{6/29} \\ &\times \left(\frac{2 \times 10^8 \text{ g}}{M_{\text{max}}}\right)^{32/29}. \end{aligned} \quad (16)$$

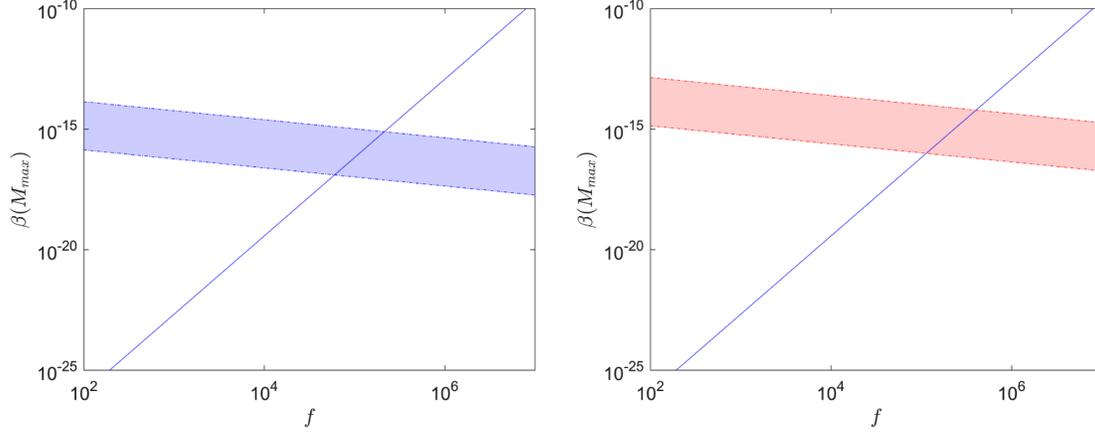


FIG. 1. Left: the solid line corresponds to the theoretical expression for $\beta(M_{\max})$ (2) as a function of the enhancement factor f in the scalar power spectrum at scales relevant for PBH formation. The shaded band shows the observational constraint on $\beta(M_{\max})$ in Eq. (14) for $\text{Br}_{\text{DM}} = 1$ and $m_{\text{DM}} = 100 \text{ GeV} - 10 \text{ TeV}$. Right: same as the left panel, but the shaded band shows the $\beta(M_{\max})$ constraint for $m_{\text{DM}} = 1 \text{ TeV}$ and $\text{Br}_{\text{DM}} = 10^{-2} - 1$ range. We have taken $M_{\max} = 2 \times 10^8 \text{ g}$ for both panels.

This results in $H_{\text{reh}} \approx (200 - 500) \text{ GeV}$ within the DM mass range $m_{\text{DM}} = 100 \text{ GeV} - 10 \text{ TeV}$, for $M_{\max} = 2 \times 10^8 \text{ g}$, with a very mild dependence on Br_{DM} . This corresponds to a very high reheate temperature $T_{\text{reh}} \sim 10^{10} \text{ GeV}$, assuming that the Universe instantly thermalizes, which is why we dubbed the era as ‘‘VEMD.’’ Considering that fluctuations grow as $\delta \propto a \propto H^{-2/3}$ in this epoch, formation of PBHs with mass $M_{\max} = 2 \times 10^8 \text{ g}$ from perturbations whose initial amplitude is enhanced according to Fig. 1 requires that VEMD starts no later than $H_0 \approx 10^6 \text{ GeV}$. This sets an absolute lower bound $H_{\text{inf}} \gtrsim 10^6 \text{ GeV}$, which essentially excludes models of low scale inflation.

We see from Eq. (15) that a larger value of $\delta(M_{\max})$, and hence f , is needed when $M_{\max} < 2 \times 10^8 \text{ g}$. Also, Eq. (16) implies a larger H_{reh} in this case. Therefore, the scenario will be least constrained for $M_{\max} \simeq 2 \times 10^8 \text{ g}$.

One question that arises is whether the large enhancement of the power spectrum that is needed at small scales $f \sim \mathcal{O}(10^5)$ is compatible with Planck limits on n_s and its running at the pivot scale k_* . Following [59], we can write

$$\ln A_s(k) = \ln A_s(k_*) + (n_s - 1) \ln \left(\frac{k}{k_*} \right) + \frac{1}{2} \alpha_s \ln^2 \left(\frac{k}{k_*} \right) + \frac{1}{6} \beta_s \ln^3 \left(\frac{k}{k_*} \right), \quad (17)$$

where $n_s = 0.9655$ at k_* . Choosing $k = k_{\max}$, where k_{\max} denotes the mode that eventually collapses to PBHs with mass M_{\max} , we have

$$\ln f = (n_s - 1) \ln \left(\frac{k_{\max}}{k_*} \right) + \frac{1}{2} \alpha_s \ln^2 \left(\frac{k_{\max}}{k_*} \right) + \frac{1}{6} \beta_s \ln^3 \left(\frac{k_{\max}}{k_*} \right). \quad (18)$$

The question is now whether values of f inferred from Fig. 1 are compatible with constraints from Planck data on

the running parameters α_s and β_s at the pivot scale. As shown in the Appendix,

$$\ln \left(\frac{k_{\max}}{k_*} \right) \approx 47.7 + \frac{1}{4} \ln \delta(M_{\max}). \quad (19)$$

Using this expression, and the relation $f = (\delta(M_{\max})/\delta_*)^2$, we can now check the consistency of Eq. (18) with Planck data. In Fig. 2, we show a band in the $\alpha_s - \beta_s$ plane that corresponds to the intersection region in the left panel of Fig. 1 through Eq. (19). This band is in agreement with Planck constraints on α_s and β_s at the 2σ level.

Another question that naturally arises concerns the mechanism behind a large enhancement of the power

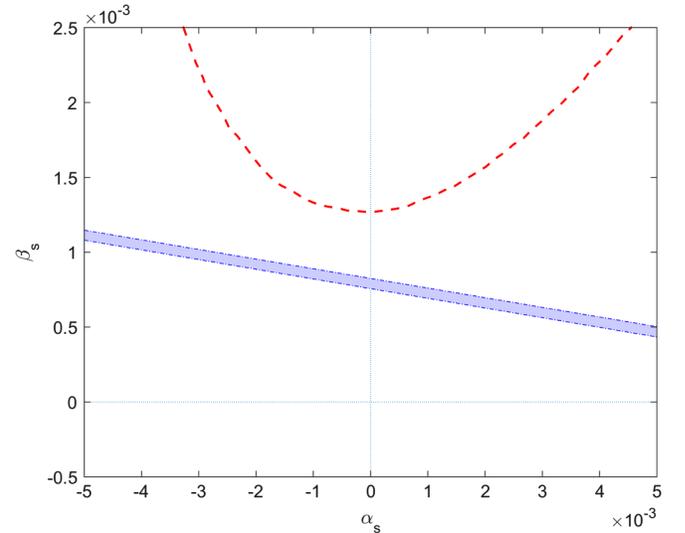


FIG. 2. The shaded band in the $\alpha_s - \beta_s$ plane corresponding to the intersection region in the left panel of Fig. 1. The 68% confidence ellipse from Planck 2015 [95] on the running of the scalar spectral index is outlined by the red dashed line.

spectrum for modes around k_{\max} . It is known in models of hybrid inflation that large density perturbations, which could lead to formation of PBHs, can be obtained toward the end of inflation [97]. This is also possible to achieve via multiple phases of inflation in single field models [79], or by a brief period of ultra slow-roll motion toward the end of inflation [78,98–102]. It has been shown [100] that an amplification of the power spectrum up to a factor of 10^7 in 10 e -folds (or more) can be obtained from ultra slow-roll inflation near an inflection point. This fits well with the requirement in our scenario, namely, an enhancement factor $f \sim \mathcal{O}(10^5)$ within a few e -folds including the mode k_{\max} . While an explicit model to achieve this is beyond the scope of this paper, it is assuring that the desirable enhancement in the power spectrum is both compatible with the Planck limits and achievable in models of single field inflation.

V. DISCUSSION AND CONCLUSION

We now turn to a discussion of the possible origin and consequences of a VEMD phase, formation of PBHs in the subsequent RD phase, and possible issues related to very light PBHs. Any implementation of our scenario within a specific model must be aware of these issues along with possible ways of addressing them.

A. Origin and consequences of VEMD

An era of VEMD can arise from oscillations of a very heavy modulus field that dominates the energy density of the Universe soon after the end of inflation. It may also start right at the end of inflation when the Universe is dominated by inflaton oscillations that eventually decay slowly via perturbative channels (for example, when the inflaton has gravitationally suppressed couplings to the visible sector fields). It may also be possible that initial stages of inflaton decay occur via nonperturbative effects [103–105] (for reviews, see [106,107]), and the zero-mode quanta of the inflaton (or other scalar fields produced during this process) come to dominate the Universe at some point.

One might worry about dangerous consequences of high reheat temperatures $T_{\text{reh}} \sim 10^{10}$ GeV at the end of the VEMD epoch (see Eq. (16) and the discussion below it). Notably, a concern arises regarding thermal overproduction of gravitinos that decay after BBN [108–110]. This can be avoided if the gravitino mass is $m_{3/2} \gtrsim 50$ TeV so that gravitinos decay before the onset of BBN. Thermal gravitino production will be totally irrelevant if $m_{3/2} \gg 10^{10}$ GeV, which can happen in some string theory constructions (for example, see [111]). Gravitino production can also be suppressed if the Universe has the same equation of state as radiation but thermalization is delayed and full thermal equilibrium is not established at the end of the VEMD [112].

B. PBH formation in the RD phase

The relation for H_{reh} in Eq. (16) ensures that PBHs whose mass is larger than M_{\max} are practically not produced in the VEMD epoch. However, enhancement of the power spectrum for modes around k_{\max} may result in the formation of heavier PBHs in the ensuing RD phase. This can happen from the collapse of those modes that enter the horizon when $H \lesssim H_{\text{reh}}$ (recall that in a RD universe pressure dominates over gravity for subhorizon modes thus preventing their collapse). We do not expect formation of PBHs with exceedingly large masses in the RD phase as they would correspond to modes with $k \ll k_{\max}$, for which $\delta(M) \ll \delta(M_{\max})$ due to the rapid fall off in the power spectrum far away from k_{\max} . In fact, because of the exponential dependence of $\beta(M)$ on $\delta(M)$ in the RD phase, even moderate suppression of the power spectrum at $k < k_{\max}$ can yield a substantial decrease in $\beta(M)$ in accordance with the most stringent observational limits. The abundance of PBHs that may form during RD follows from Eq. (1) by setting $\delta = \delta(M_{\max})$. For values of $\delta(M_{\max})$ corresponding to the intersection regions in Fig. 1, we have checked that $\beta(M) \ll 10^{-30}$ for $M \gtrsim 2 \times 10^8$ g. This easily satisfies even the tightest observational constraints on the abundance of PBHs over the entire mass range that evaporate after BBN [18].

C. Effects of light PBHs

Very light PBHs with mass $M \ll M_{\max}$ may form in the VEMD phase. As mentioned above, and shown in the Appendix, the contribution of such black holes to the DM relic abundance is typically negligible. However, they can act as a site for bubble nucleation in a first order phase transition and seed vacuum decay. This effect can be relevant for the electroweak vacuum that becomes unstable for a certain range of the top quark mass [113]. It has been shown that PBHs that have a mass in the $(10^5\text{--}10^9)M_{\text{p}}$ range can seed decay of the Higgs vacuum as the decay rate dominates over the Hawking evaporation [114,115].

The simplest possibility to avoid this effect is to have an inflationary scale that corresponds to $H_{\text{inf}} \lesssim 10^{-9}M_{\text{p}}$. Since inflation generates density perturbations with physical wave numbers $k < H_{\text{inf}}$, the condition $H_{\text{inf}} \lesssim 10^{-9}M_{\text{p}}$ ensures the absence of fluctuation modes that could collapse to form dangerous PBHs with mass $M \lesssim 10^9M_{\text{p}}$. Another possibility is to have a situation where $\delta(M) \ll \delta(M_{\max})$ for $M \lesssim 10^9M_{\text{p}}$ as a result of the rapid fall off in the power spectrum far from the mode k_{\max} . Formation of dangerously light PBHs can then be prevented if the corresponding fluctuations do not grow to become $\mathcal{O}(1)$ by the time when $H \simeq H_{\text{reh}}$, without any restrictions on H_{inf} .

In conclusion, we have demonstrated that evaporation of PBHs can produce the entire DM relic abundance within the DM mass range of 100 GeV–10 TeV in cases with

thermal underproduction. The necessary ingredient for this nonthermal scenario to be viable is to produce a sufficient abundance of PBHs whose mass is below $\simeq 2 \times 10^8$ g, so that they evaporate before BBN. We have found that an epoch of VEMD can accommodate this if the scalar power spectrum at small scales is enhanced by a factor of $\mathcal{O}(10^5)$ relative to its value for the CMB modes. An explicit model that leads to inflation with the desired enhancement of the power spectrum as well as an epoch of VEMD will be the subject of a future investigation.

ACKNOWLEDGMENTS

The work of R. A. and J. O. is supported in part by National Science Foundation Grant No. PHY-1720174. J. B. D. would like to acknowledge support from the Mitchell Institute for Fundamental Physics and Astronomy at Texas A&M University. The authors acknowledge discussions with N. Afshordi and M. M. Sheikh-Jabbari. The authors also thank Andrew Powell, who worked on an earlier stage of this project.

APPENDIX DETAILED CALCULATIONS FOR THE NUMBER OF e -FOLDINGS AND AN EXTENDED MASS RANGE

1. Number of relevant e -foldings

For the standard thermal history, the number of e -foldings of inflation between the time when the pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$ left the horizon and the end of inflation is given by [116,117]

$$N_{k_*} \approx 63.5 + \frac{1}{4} \ln \frac{3H_{\text{inf}}^2}{(8\pi M_{\text{p}})^2} + \frac{1}{6} \frac{H_{\text{R}}}{H_{\text{inf}}}. \quad (\text{A1})$$

Here H_{inf} and H_{R} denote the Hubble rate at the end of inflation and when reheating after inflation completes respectively, assuming that the Universe has the same equation of state as a MD phase for $H_{\text{R}} < H < H_{\text{inf}}$. In the presence of an epoch of VEMD for $H_{\text{reh}} < H < H_0$, this relation is modified as follows:

$$N_{k_*} \approx 63.5 + \frac{1}{4} \ln \frac{3H_{\text{inf}}^2}{(8\pi M_{\text{p}})^2} + \frac{1}{6} \frac{H_{\text{R}}}{H_{\text{inf}}} + \frac{1}{6} \frac{H_{\text{reh}}}{H_0}. \quad (\text{A2})$$

The number of e -foldings relevant for the mode k_0 that enters the horizon at $H = H_0$ is given by

$$N_0 = \frac{1}{3} \ln \frac{H_{\text{inf}}}{H_{\text{R}}} + \frac{1}{2} \ln \frac{H_{\text{R}}}{H_0}. \quad (\text{A3})$$

The first and second terms on the right-hand side of the equation take evolution in the MD phase between H_{inf} and H_{R} and the RD phase between H_{R} and H_0 , respectively, into account. The number of e -foldings relevant for the mode k_{max} that eventually collapses to form PBHs with mass M_{max} follows from

$$N_{k_{\text{max}}} - N_0 = \frac{1}{3} \ln \frac{H_0}{H_{\text{max}}}, \quad (\text{A4})$$

where $H_{\text{max}} = 4\pi M_{\text{p}}^2/M_{\text{max}}$ is the Hubble rate when the mode k_{max} enters the horizon.

After using Eqs. (A2), (A3), (A4), and with the help of (5), we find

$$N_{k_*} - N_{k_{\text{max}}} \approx 47.7 + \frac{1}{4} \ln \delta(M_{\text{max}}). \quad (\text{A5})$$

It is interesting to note that H_{inf} does not appear in this expression. However, it implicitly enters as we must have $H_0 \leq H_{\text{R}} \leq H_{\text{inf}}$. For the shaded band shown in Fig. 2, we get $N_{k_*} - N_{k_{\text{max}}} \approx 46.4\text{--}46.5$.

2. Integrating over PBHs in an extended mass range

Here we derive the total contribution from evaporation of PBHs within a mass range $M_{\text{min}} \leq M \leq M_{\text{max}}$ to the DM relic abundance. For simplicity, we assume that $\text{Br}_{\text{DM}} = 1$ in this derivation, but including Br_{DM} in the calculation is straightforward. In general, the number density of regions with mass M in the early Universe can be written as

$$n(M) = \frac{\rho_{\text{tot}}}{M}, \quad (\text{A6})$$

where ρ_{tot} is the total energy density of the Universe. Then the number density of PBHs whose mass is between M and $M + dM$ follows from

$$dn_{\text{PBH}}(M) = \beta(M) dn(M). \quad (\text{A7})$$

Since ρ_{tot} does not depend on M , we have $dn(M)M = -n(M)dM$. This results in

$$dn_{\text{PBH}}(M) = \frac{\beta(M)n(M)}{M} dM = \rho_{\text{tot}} \frac{\beta(M)}{M^2} dM, \quad (\text{A8})$$

and hence

$$\frac{\rho_{\text{PBH}}}{\rho_{\text{tot}}} = \frac{\int_{M_{\text{min}}}^{M_{\text{max}}} M dn_{\text{PBH}}(M)}{\rho_{\text{tot}}} = \int_{M_{\text{min}}}^{M_{\text{max}}} \frac{\beta(M)}{M} dM. \quad (\text{A9})$$

Also, after using the relation $\rho_{\text{tot}}/s = 3T_{\text{reh}}/4$ at the end of the VEMD epoch, we find

$$\frac{dn_{\text{PBH}}(M)}{s} = \frac{3}{4} T_{\text{reh}} \frac{\beta(M)}{M^2} dM. \quad (\text{A10})$$

The corresponding contribution to the DM relic abundance, see Eq. (8), is given by

$$\frac{dn_{\text{DM}}}{s} \sim \frac{dn_{\text{PBH}}(M)}{s} \frac{M^2}{M_{\text{p}}^2}. \quad (\text{A11})$$

After integrating over the entire mass range, we derive the analogue of Eq. (10),

$$\frac{n_{\text{DM}}}{s} \sim \int_{M_{\text{min}}}^{M_{\text{max}}} \frac{3T_{\text{reh}}}{4M_{\text{p}}^2} \beta(M) dM, \quad (\text{A12})$$

which generalizes Eq. (12) to

$$\int_{M_{\text{min}}}^{M_{\text{max}}} \beta(M) \left(\frac{H_{\text{reh}}}{M_{\text{p}}}\right)^{1/2} \left(\frac{106.75}{g_{*,\text{reh}}}\right)^{1/4} \frac{dM}{2 \times 10^8 \text{ g}} \simeq 10^{-23} \left(\frac{100 \text{ GeV}}{m_{\text{DM}}}\right), \quad (\text{A13})$$

where $\beta(M)$ is the theoretical prediction given in Eq. (2).

If $\beta(M)$ varies slowly in the mass range $M_{\text{min}} \leq M \leq M_{\text{max}}$, the above integral is $\propto M_{\text{max}}$ and Eq. (A13) reproduces the bound in (13). The same conclusion holds as long as the minimum of $\beta(M)$ does not happen around M_{max} . We expect this to be the case if the range of modes over which the power spectrum is enhanced includes k_{max} .

-
- [1] G. Bertone, D. Hooper, and J. Silk, *Phys. Rep.* **405**, 279 (2005).
- [2] G. Steigman, B. Dasgupta, and J. F. Beacom, *Phys. Rev. D* **86**, 023506 (2012).
- [3] M. Ackermann *et al.* (Fermi-LAT Collaboration), *Phys. Rev. Lett.* **115**, 231301 (2015).
- [4] A. Albert *et al.* (Fermi-LAT and DES Collaborations), *Astrophys. J.* **834**, 110 (2016).
- [5] G. Kane, S. K., and S. Watson, *Int. J. Mod. Phys. D* **24**, 1530022 (2015).
- [6] T. Moroi and L. Randall, *Nucl. Phys.* **B570**, 455 (2000).
- [7] B. S. Acharya, G. Kane, S. Watson, and P. Kumar, *Phys. Rev. D* **80**, 083529 (2009).
- [8] G. B. Gelmini and P. Gondolo, *Phys. Rev. D* **74**, 023510 (2006).
- [9] R. Allahverdi, B. Dutta, and K. Sinha, *Phys. Rev. D* **83**, 083502 (2011).
- [10] R. Allahverdi, M. Cicoli, B. Dutta, and K. Sinha, *Phys. Rev. D* **88**, 095015 (2013).
- [11] M. Battaglieri *et al.*, arXiv:1707.04591.
- [12] B. J. Carr and S. W. Hawking, *Mon. Not. R. Astron. Soc.* **168**, 399 (1974).
- [13] B. J. Carr, *Astrophys. J.* **201**, 1 (1975).
- [14] M. Khlopov, B. A. Malomed, and I. B. Zeldovich, *Mon. Not. R. Astron. Soc.* **215**, 575 (1985).
- [15] A. G. Polnarev and M. Yu. Khlopov, *Usp. Fiz. Nauk* **145**, 369 (1985) [*Sov. Phys. Usp.* **28**, 213 (1985)].
- [16] B. J. Carr, in Proceedings of 59th Yamada Conference on Inflating Horizon of Particle Astrophysics and Cosmology Tokyo, Japan, 2005 [arXiv:astro-ph/0511743].
- [17] M. Yu. Khlopov, *Res. Astron. Astrophys.* **10**, 495 (2010).
- [18] B. J. Carr, K. Kohri, Y. Sendouda, and J. Yokoyama, *Phys. Rev. D* **81**, 104019 (2010).
- [19] B. Carr, F. Kuhnel, and M. Sandstad, *Phys. Rev. D* **94**, 083504 (2016).
- [20] J. Garcia-Bellido, *J. Phys. Conf. Ser.* **840**, 012032 (2017).
- [21] S. Bird, I. Cholis, J. B. Munoz, Y. Ali-Haïmoud, M. Kamionkowski, E. D. Kovetz, A. Raccanelli, and A. G. Riess, *Phys. Rev. Lett.* **116**, 201301 (2016).
- [22] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, *Phys. Rev. Lett.* **117**, 061101 (2016).
- [23] S. Clesse and J. Garcia-Bellido, *Phys. Dark Universe* **15**, 142 (2017).
- [24] E. D. Kovetz, I. Cholis, P. C. Breysse, and M. Kamionkowski, *Phys. Rev. D* **95**, 103010 (2017).
- [25] A. M. Green, *Phys. Rev. D* **94**, 063530 (2016).
- [26] F. Kuhnel and K. Freese, *Phys. Rev. D* **95**, 083508 (2017).
- [27] B. Carr, M. Raidal, T. Tenkanen, V. Vaskonen, and H. Veermäe, *Phys. Rev. D* **96**, 023514 (2017).
- [28] N. Bellomo, J. L. Bernal, A. Raccanelli, and L. Verde, *J. Cosmol. Astropart. Phys.* **01** (2018) 004.
- [29] K. Inomata, M. Kawasaki, K. Mukaida, and T. T. Yanagida, arXiv:1711.06129.
- [30] G. F. Giudice, M. McCullough, and A. Urbano, *J. Cosmol. Astropart. Phys.* **10** (2016) 001.
- [31] A. Raccanelli, E. D. Kovetz, S. Bird, I. Cholis, and J. B. Munoz, *Phys. Rev. D* **94**, 023516 (2016).
- [32] K. Postnov and A. Kuranov, in *IAU Symp.* **12**, 118 (2016); *IAU Symp.* **329**, 118 (2017).
- [33] K. Kyutoku and N. Seto, *Mon. Not. R. Astron. Soc.* **462**, 2177 (2016).
- [34] N. Bartolo *et al.*, *J. Cosmol. Astropart. Phys.* **12** (2016) 026.
- [35] A. Sesana, *J. Phys. Conf. Ser.* **840**, 012018 (2017).
- [36] M. Coleman Miller, *Gen. Relativ. Gravit.* **48**, 95 (2016).
- [37] T. Nakamura *et al.*, *Prog. Theor. Exp. Phys.* **2016**, 093E01 (2016).
- [38] H. Davoudiasl and P. P. Giardino, *Phys. Lett. B* **768**, 198 (2017).
- [39] I. Cholis, *J. Cosmol. Astropart. Phys.* **06** (2017) 037.
- [40] S. Clesse and J. Garcia-Bellido, *Phys. Dark Universe* **18**, 105 (2017).
- [41] S. Wang, Y.-F. Wang, Q.-G. Huang, and T. G. F. Li, arXiv:1610.08725.
- [42] T. Nakama, J. Silk, and M. Kamionkowski, *Phys. Rev. D* **95**, 043511 (2017).
- [43] K. Crocker, T. Prestegard, V. Mandic, T. Regimbau, K. Olive, and E. Vangioni, *Phys. Rev. D* **95**, 063015 (2017).
- [44] E. D. Kovetz, *Phys. Rev. Lett.* **119**, 131301 (2017).
- [45] J. Garcia-Bellido, M. Peloso, and C. Unal, *J. Cosmol. Astropart. Phys.* **09** (2017) 013.
- [46] T. D. Brandt, *Astrophys. J.* **824**, L31 (2016).
- [47] S. M. Koushiappas and A. Loeb, *Phys. Rev. Lett.* **119**, 041102 (2017).

- [48] D. Gaggero, G. Bertone, F. Calore, R. M. T. Connors, M. Lovell, S. Markoff, and E. Storm, *Phys. Rev. Lett.* **118**, 241101 (2017).
- [49] Y. Inoue and A. Kusenko, *J. Cosmol. Astropart. Phys.* **10** (2017) 034.
- [50] L. Chen, Q.-G. Huang, and K. Wang, *J. Cosmol. Astropart. Phys.* **12** (2016) 044.
- [51] Y. Ali-Haimoud and M. Kamionkowski, *Phys. Rev. D* **95**, 043534 (2017).
- [52] S. Clark, B. Dutta, Y. Gao, L. E. Strigari, and S. Watson, *Phys. Rev. D* **95**, 083006 (2017).
- [53] J. Georg and S. Watson, *J. High Energy Phys.* **09** (2017) 138; **09** (2017) 138.
- [54] V. Poulin, P. D. Serpico, F. Calore, S. Clesse, and K. Kohri, *Phys. Rev. D* **96**, 083524 (2017).
- [55] T. Nakama, B. Carr, and J. Silk, [arXiv:1710.06945](https://arxiv.org/abs/1710.06945).
- [56] B. Horowitz, [arXiv:1612.07264](https://arxiv.org/abs/1612.07264).
- [57] E. Mediavilla, J. Jimenez-Vicente, J. A. Munoz, H. Vives-Arias, and J. Calderon-Infante, *Astrophys. J.* **836**, L18 (2017).
- [58] F. Capela, M. Pshirkov, and P. Tinyakov, *Phys. Rev. D* **87**, 123524 (2013).
- [59] J. B. Munoz, E. D. Kovetz, L. Dai, and M. Kamionkowski, *Phys. Rev. Lett.* **117**, 091301 (2016).
- [60] A. Kashlinsky, *Astrophys. J.* **823**, L25 (2016).
- [61] J.-O. Gong and N. Kitajima, *J. Cosmol. Astropart. Phys.* **08** (2017) 017.
- [62] K. Inomata, M. Kawasaki, K. Mukaida, Y. Tada, and T. T. Yanagida, *Phys. Rev. D* **95**, 123510 (2017).
- [63] K. Schutz and A. Liu, *Phys. Rev. D* **95**, 023002 (2017).
- [64] N. Orlofsky, A. Pierce, and J. D. Wells, *Phys. Rev. D* **95**, 063518 (2017).
- [65] G. Chapline and P. H. Frampton, *J. Cosmol. Astropart. Phys.* **11** (2016) 042.
- [66] L. Dai, T. Venumadhav, and K. Sigurdson, *Phys. Rev. D* **95**, 044011 (2017).
- [67] I. Cholis, E. D. Kovetz, Y. Ali-Haimoud, S. Bird, M. Kamionkowski, J. B. Munoz, and A. Raccanelli, *Phys. Rev. D* **94**, 084013 (2016).
- [68] T. Chiba and S. Yokoyama, *Prog. Theor. Exp. Phys.* (2017) 083E01.
- [69] J. Garcia-Bellido and S. Clesse, [arXiv:1710.04694](https://arxiv.org/abs/1710.04694).
- [70] A. M. Green, *Phys. Rev. D* **96**, 043020 (2017).
- [71] M. Kawasaki, K. Mukaida, and T. T. Yanagida, *Phys. Rev. D* **94**, 063509 (2016).
- [72] J. M. Ezquiaga, J. Garcia-Bellido, and E. Ruiz Morales, [arXiv:1705.04861](https://arxiv.org/abs/1705.04861).
- [73] S.-L. Cheng, W. Lee, and K.-W. Ng, *J. High Energy Phys.* **02** (2017) 008.
- [74] M. Kawasaki, A. Kusenko, Y. Tada, and T. T. Yanagida, *Phys. Rev. D* **94**, 083523 (2016).
- [75] J. Garcia-Bellido, M. Peloso, and C. Unal, *J. Cosmol. Astropart. Phys.* **12** (2016) 031.
- [76] V. Domcke, F. Muia, M. Pieroni, and L. T. Witkowski, *J. Cosmol. Astropart. Phys.* **07** (2017) 048.
- [77] K. Inomata, M. Kawasaki, K. Mukaida, Y. Tada, and T. T. Yanagida, *Phys. Rev. D* **96**, 043504 (2017).
- [78] J. Garcia-Bellido and E. Ruiz Morales, *Phys. Dark Universe* **18**, 47 (2017).
- [79] K. Kannike, L. Marzola, M. Raidal, and H. Veermäe, *J. Cosmol. Astropart. Phys.* **09** (2017) 020.
- [80] M. Drees and E. Erfani, *J. Cosmol. Astropart. Phys.* **04** (2011) 005.
- [81] M. Drees and E. Erfani, *J. Cosmol. Astropart. Phys.* **01** (2012) 035.
- [82] J. Georg, G. Sengor, and S. Watson, *Phys. Rev. D* **93**, 123523 (2016).
- [83] B. Carr, F. Kuhnel, and M. Sandstad, *Phys. Rev. D* **94**, 083504 (2016).
- [84] J. C. Niemeyer and K. Jedamzik, *Phys. Rev. D* **59**, 124013 (1999).
- [85] I. Musco, M. J. C., and R. L., *Classical Quantum Gravity* **22**, 1405 (2005).
- [86] I. Musco, M. J. C., and P. A. G., *Classical Quantum Gravity* **26**, 235001 (2009).
- [87] I. Musco and M. J. C., *Classical Quantum Gravity* **30**, 145009 (2013).
- [88] T. Koike, H. T., and A. S., *Phys. Rev. Lett.* **74**, 5170 (1995).
- [89] J. C. Niemeyer and K. Jedamzik, *Phys. Rev. Lett.* **80**, 5481 (1998).
- [90] T. Harada, C. M. Yoo, and K. Kohri, *Phys. Rev. D* **88**, 084051 (2013).
- [91] S. W. Hawking, *Commun. Math. Phys.* **43**, 199 (1975); **46**, 206(E) (1976).
- [92] A. M. Green, *Phys. Rev. D* **60**, 063516 (1999).
- [93] M. Y. Khlopov, A. Barrau, and J. Grain, *Classical Quantum Gravity* **23**, 1875 (2006).
- [94] M. Lemoine, *Phys. Lett. B* **481**, 333 (2000).
- [95] J. B. Munoz *et al.*, *J. Cosmol. Astropart. Phys.* **05** (2017) 032.
- [96] P. A. R. Ade *et al.* (Planck Collaboration), *Astron. Astrophys.* **594**, A20 (2016).
- [97] J. Garcia-Bellido, A. D. Linde, and D. Wands, *Phys. Rev. D* **54**, R25 (1996).
- [98] J. M. Ezquiaga, J. Garcia-Bellido, and E. Ruiz Morales, *Phys. Lett. B* **776**, 345 (2018).
- [99] C. Germani and T. Prokopec, *Phys. Dark Univ.* **18**, 6 (2017).
- [100] H. Motohashi and W. Hu, *Phys. Rev. D* **96**, 063503 (2017).
- [101] Y. Gong, [arXiv:1707.09578](https://arxiv.org/abs/1707.09578).
- [102] G. Balletsteros and M. Taoso, *Phys. Rev. D* **97**, 023501 (2018).
- [103] Y. Shtanov, J. H. Traschen, and R. H. Brandenberger, *Phys. Rev. D* **51**, 5438 (1995).
- [104] L. Kofman, A. D. Linde, and A. A. Starobinsky, *Phys. Rev. Lett.* **73**, 3195 (1994).
- [105] L. Kofman, A. D. Linde, and A. A. Starobinsky, *Phys. Rev. D* **56**, 3258 (1997).
- [106] R. Allahverdi, R. H. Brandenberger, F. Y. Cyr-Racine, and A. Mazumdar, *Annu. Rev. Nucl. Part. Sci.* **60**, 27 (2010).
- [107] M. A. Amin, M. T. Hertzberg, D. I. Kaiser, and J. Karouby, *Int. J. Mod. Phys. D* **24**, 1530003 (2015).
- [108] J. R. Ellis, J. E. Kim, and D. V. Nanopoulos, *Phys. Lett. B* **145**, 181 (1984).
- [109] M. Y. Khlopov and A. D. Linde, *Phys. Lett.* **138B**, 265 (1984).

- [110] M. Bolz, M. Brandenburg, and W. Buchmuller, Nucl. Phys. **B606**, 25 (2001). R. Bousso, J. High Energy Phys. **04** (2001) 035.
- [111] R. Blumenhagen, J. P. Conlon, S. Krippendorff, S. Moster, and F. Quevedo, J. High Energy Phys. **09** (2009) 007.
- [112] R. Allahverdi and A. Mazumdar, J. Cosmol. Astropart. Phys. **10** (2006) 008.
- [113] P. Burda, R. Gregory, and I. G. Moss, Phys. Rev. Lett. **115**, 071303 (2015).
- [114] P. Burda, R. Gregory, and I. G. Moss, J. High Energy Phys. **06** (2016) 25.
- [115] J. R. Espinosa, D. Racco, and A. Riotto, arXiv: 1710.11196.
- [116] A. R. Liddle and S. M. Leach, Phys. Rev. D **68**, 103503 (2003).
- [117] T. Rehagen and G. B. Gelmini, J. Cosmol. Astropart. Phys. **6** (2015) 039.