# Proposal to look for the anomalous isotopic symmetry breaking in central diffractive production of the $f_1(1285)$ and $a_0^0(980)$ resonances at the LHC

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At very high energies, and in the central region ($x_F \simeq 0$), the double-Pomeron exchange mechanism gives the dominant contribution to the production of hadrons with the positive $C$ parity and isospin $I = 0$. Therefore, the observation of resonances in the states with $I = 1$ will be indicative of their production or decay with the isotopic symmetry breaking. Here, we bear in mind the cases of the anomalous breaking of the isotopic symmetry, i.e., when the cross section of the process breaking the isospin is not of the order of $10^{-4}$ of the cross section of the allowed process but of the order of 1%. The paper draws attention to the reactions $pp \to p(f_1(1285)/f_1(1420))p \to p(\pi^+\pi^-\pi^0)p$ and $pp \to p(K\bar{K})p \to p(a_0^0(980))p \to p(\eta\pi^0)p$, in which a similar situation can be realized, owing to the $K\bar{K}$ loop mechanisms of the breaking of isotopic symmetry. We note that there is no visible background in the $\pi^+\pi^-\pi^0$ and $\eta\pi^0$ channels. Observation of the process $pp \to p(f_1(1285))p \to p(\pi^+\pi^-\pi^0)p$ would be a confirmation of the first results from the VES and BESIII detectors, indicating the very large isospin breaking in the decay $f_1(1285)) \to \pi^+\pi^-\pi^0$.

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## I. INTRODUCTION

The central exclusive production of hadrons, $h$, in the reactions $pp \to p(h)p$ and $p\bar{p} \to p(h)\bar{p}$ at high energies has been studied at the Intersecting Storage Rings and $Sp\bar{p}S$ accelerators at CERN and at the Tevatron collider at Fermilab and is now being investigated in $pp \to p(h)p$ at the LHC (see, for review, Refs. [1–7]). The mass spectra and production cross sections have been measured for a number of hadronic systems $h$ such as $\pi\pi$ [5,6,8–12], $K\bar{K}$ [10,13–15], $\eta\pi^+\pi^-$ [16,17], $K\bar{K}\pi$ [18,19], $4\pi$ [20–22], $\eta\pi^0$ [23], etc. Special attention was paid to the study of resonance contributions.

At very high energies, and in the central region, the double-Pomeron exchange mechanism, $\mathcal{PP}$, gives the dominant contribution to production processes of hadronic resonances with the positive $C$ parity and isospin $I = 0$ (see Fig. 1). The reaction cross sections caused by the double-Pomeron exchange mechanism do not decrease in a power-law manner with increasing energy [1,2,24–27]. Therefore, observation of the well-known resonances in the states $h$ with $I = 1$ will be indicative of their production or decay with the isotopic symmetry breaking. Here, we consider the examples of the reactions $pp \to p(h)p$ in which the anomalous breaking of the isotopic symmetry can occur.



## II. REACTIONS $pp \to p(f_1(1285)/f_1(1420))p \to \to p(\pi^+\pi^-\pi^0)p$

Clear signals from the $f_1(1285)$ and $f_1(1420)$ resonances with $I^G(J^{PC}) = 0^+(1^{++})$ centrally produced in the reactions $pp \to p(f_1(1285)/f_1(1420))p \to p(X^0)p$ have been observed in all their major decay modes $X^0 = \eta\pi\pi$ [16,17], $K\bar{K}\pi$ [4,18,19], and $4\pi$ [20,21]. The experiments

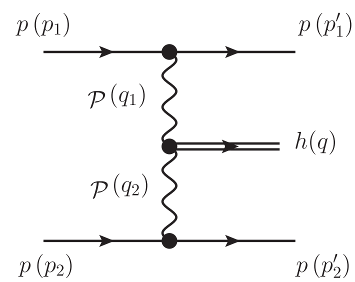


FIG. 1. The central production of a state $h$ by the double-Pomeron exchange mechanism, $\mathcal{PP}$, in the reaction $pp \to p(h)p$. The 4-momenta of the initial and final protons, $\mathcal{P}$ exchanges, and $h$ system are indicated in parentheses; the main kinematic variables in this reaction are $s = (p_1 + p_2)^2$, $s_1 = (p_1' + q)^2$, $s_2 = (p_2' + q)^2$, $M^2 = q^2 = (q_1 + q_2)^2$, $t_1 = q_1^2 \simeq -\vec{q}_{1\perp}^{\,2}$, and $t_2 = q_2^2 \simeq -\vec{q}_{2\perp}^{\,2}$.

were performed at incident proton laboratory momentum of $P^p_{lab} = 85$, 300, 450, and 800 GeV$/c$ or at center-of-mass energies of $\sqrt{s} \approx 12.7$, 23.8, 29, and 40 GeV, respectively. The data on the production cross sections of these resonances are consistent with the $\mathcal{PP}$ exchange mechanism [3,4,16–19,21]. In contrast, there is no evidence for any $0^+(0^{-+})$ contribution in the 1.28 and 1.4 GeV regions [3,4,16–19,21]. In practice, this fact can help one measure more precisely the characteristics of the $f_1(1285)$ and $f_1(1420)$ than is possible in other experiments which see both $0^+(1^{++})$ and $0^+(0^{-+})$ states.

Thus, investigations of the $f_1(1285)$ and $f_1(1420)$ resonances produced in central $pp$ interactions allow one to determine in a single experiment the branching ratios for all their major decay modes. Moreover, we pay attention that the reaction $pp \to p(f_1(1285))p \to p(\pi^+\pi^-\pi^0)p$, due to the isotopic neutrality of the $\mathcal{PP}$ exchange mechanism, gives a unique possibility to investigate the isospin-breaking decay $f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ in a situation free from any visible coherent background in the $\pi^+\pi^-\pi^0$ channel. Because of completely different experimental conditions, such a study would be a good test of the first results from the VES [28] and BESIII [29] detectors, indicating to the strong isospin breaking in this decay.

According to the data from the VES Collaboration [28],

$$\frac{\mathrm{BR}(f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0)}{\mathrm{BR}(f_1(1285) \to \eta\pi^+\pi^-)} = (0.86 \pm 0.16 \pm 0.20)\%. \quad (1)$$

The data from the BESIII Collaboration [29] give

$$\frac{\mathrm{BR}(f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0)}{\mathrm{BR}(f_1(1285) \to \eta\pi^+\pi^-)} = (1.23 \pm 0.55)\%. \quad (2)$$

The $f_1(1285) \to \eta\pi^+\pi^-$ decay channel has the largest branching ratio, $\mathrm{BR}(f_1(1285) \to \eta\pi^+\pi^-) \approx 35\%$ [30], among all other recorded decay channels [3,4,16–21]. Therefore, the ratios mentioned in Eqs. (1) and (2) give a good reason to believe that we really deal with the anomalously large isospin breaking in the transition $f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$. Moreover, the $\pi^+\pi^-$ mass spectrum observed in the decay $f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ represents a narrow resonance structure with a width of 10–20 MeV located near the $K\bar{K}$ thresholds [28,29]. The various $K\bar{K}$ loop mechanisms responsible for the decay $f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ [i.e., the various types of transitions $f_1(1285) \to (K^+K^- + K^0\bar{K}^0)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$] lead to the $\pi^+\pi^-$ mass spectrum of such a type [31–34]. A significant violation of isotopic symmetry in this transitions is a threshold phenomenon. It occurs in the narrow region of the $\pi^+\pi^-$ invariant mass near the $K\bar{K}$ thresholds due to the incomplete compensation between the contributions of the $K^+K^-$ and $K^0\bar{K}^0$ intermediate states caused by the mass difference of the $K^+$ and $K^0$ mesons [31–36]. Certainly, the data on the $f_1(1285) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ decay need to be clarified.

Information on the reaction $pp \to p(f_1(1285))p \to p(\pi^+\pi^-\pi^0)p$ could probably be extracted from the data collected by the CERN Omega Spectrometer and Collider Detector at Fermilab. However, for this, enthusiasts are needed, since these facilities have long been closed. At present, the reaction $pp \to p(f_1(1285))p \to p(\pi^+\pi^-\pi^0)p$ could be measured, for example, using the CMS detector at the LHC (it is interesting also to study the related reaction $pp \to pf_1(1420)p \to p(\pi^+\pi^-\pi^0)p$ [32,37]). Recently, the CMS Collaboration has presented the data on the central exclusive and semiexclusive $\pi^+\pi^-$ production in $pp$ collisions at $\sqrt{s} = 7$ TeV [6]. With such a huge total energy, the energy values $\sqrt{s_1}$ and $\sqrt{s_2}$ for the subprocesses $p(p_1)\mathcal{P}(q_2) \to p(p_1')h(q)$ and $p(p_2)\mathcal{P}(q_1) \to p(p_2')h(q)$ (see Fig. 1) are also very large. In fact, they fall into the region in which the contributions of the secondary Regge trajectories, $\mathcal{R}$, can be neglected in comparison with the contribution of the $\mathcal{P}$ exchange. If we put $s_1 \approx s_2$, $M \approx 1$ GeV, and $\sqrt{s} = 7$ TeV and use the relation $s_1 s_2 \approx M^2 s$ (valid for the processes in the central region [24–26]), we find that $\sqrt{s_1} \approx \sqrt{s_2} \approx 84$ GeV. Thus, the dominance of the $\mathcal{PP}$ exchange mechanism appears to be a good approximation at the LHC energies. Note that in the above-mentioned experiments carried out at CERN and Tevatrov (at fixed target) the values of $\sqrt{s_1} \approx \sqrt{s_2}$ were approximately equal to ≈3.6, 4.9, 5.4, and 6.3 GeV. Therefore, in a number of cases, when interpreting the results, it was necessary to take into account, along with the $\mathcal{PP}$ exchange mechanism, the mechanisms involving the secondary Regge trajectories, $\mathcal{R}$, i.e., the $\mathcal{RP}$ and $\mathcal{RR}$ exchanges.

The $f_1(1285))$ resonance production with its subsequent decay to $\pi^+\pi^-\pi^0$ can be also studied in central $pp$, $pA$, $\pi^-p$, and $\pi^-A$ interactions at the Serpukhov acceleration in Protvino.

## III. REACTION $pp \to p(a_0^0(980))p \to p(\eta\pi^0)p$

The central production of the $a_0^0(980)$ resonance in the reaction $pp \to p(a_0^0(980))p \to p(\eta\pi^0)p$ at the LCH energies has to be dominated by the mechanism shown in Fig. 2. The incomplete compensation between the $K^+K^-$ and $K^0\bar{K}^0$ intermediate state produced in $\mathcal{PP}$ collisions leads to the isospin-breaking $a_0^0(980)$ production amplitude, which does not decrease with increasing energy. In so doing, the $\eta\pi^0$ mass spectrum has to be a narrow resonance peak (with a width of 10–20 MeV) located near the $K\bar{K}$ thresholds [31–33] (see, for example, Fig. 4 below). Judging from the existing data, the transition amplitudes

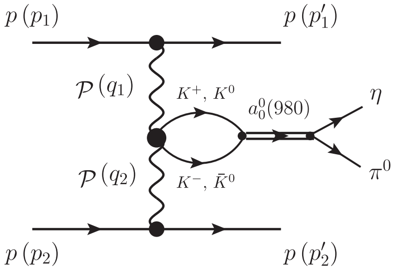


FIG. 2. The $K\bar{K}$ loop mechanism of the $a_0^0(980)$ production in the central region via $\mathcal{PP}$ exchange.

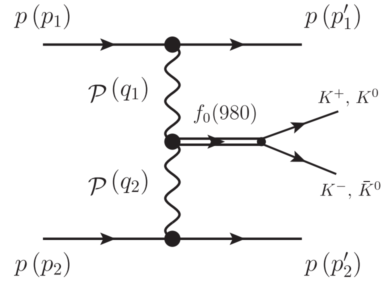


FIG. 3. The central production of the $f_0(980)$ resonance in the $K\bar{K}$ decay channels via $\mathcal{PP}$ exchange.

$\mathcal{PP}\to K\bar{K}$, that generate the process shown in Fig. 2, are dominated by the $f_0(980)$ resonance production [3,4,8–15], $\mathcal{PP}\to f_0(980)\to K\bar{K}$ (see Fig. 3). Really, the $K^+K^-$ and $K^0\bar{K}^0$ mass spectra reveal powerful enhancements near their thresholds [13–15]. The $f_0(980)$ resonance manifests itself also clearly in the $\pi^+\pi^-$ and $\pi^0\pi^0$ mass spectra, but in the form of the sharp dip due to the destructive interference with the large and smooth coherent background [3,5,6,8–12]. Thus, the $a_0^0(980)$ production in the $\eta\pi^0$ channel can predominantly occur via the $a_0^0(980)-f_0(980)$ mixing [31], i.e., owing to the $K\bar{K}$ loop transition $\mathcal{PP}\to f_0(980)\to (K^+K^-+K^0\bar{K}^0)\to a_0^0(980)\to\eta\pi^0$. The corresponding cross section as a function of the $\eta\pi^0$ invariant mass, $M\equiv m$, has the form (see Fig. 4)

$$\begin{aligned}\sigma(\mathcal{PP}\to f_0(980)&\to (K^+K^-+K^0\bar{K}^0)\\ &\to a_0^0(980)\to\eta\pi^0;m)\\ &=|C_{\mathcal{PP}\to f_0}|^2 m\Gamma_{a_0^0\to\eta\pi^0}(m)\\ &\times\left|\frac{\Pi_{a_0^0f_0}(m)}{D_{a_0^0}(m)D_{f_0}(m)-\Pi^2_{a_0^0f_0}(m)}\right|^2,\end{aligned}\tag{3}$$

where $\Gamma_{a_0^0\to\eta\pi^0}(m)$ is the width of the $a_0^0(980)\to\eta\pi^0$ decay, $D_{a_0^0}(m)$ and $D_{f_0}(m)$ are the inverse propagators of the $a_0^0(980)$ and $f_0(980)$, respectively, $\Pi^2_{a_0^0f_0}(m)$ is the $f_0(980)\to(K^+K^-+K^0\bar{K}^0)\to a_0^0(980)$ transition amplitude (all these functions, together with the corresponding values of the resonance parameters, have been written in Ref. [32]), and $C_{\mathcal{PP}\to f_0}$ is the $f_0(980)$ production amplitude.

Note that the $a_0^0(980)$ production cross section $\sigma(\mathcal{PP}\to (K^+K^-+K^0\bar{K}^0)\to a_0^0(980)\to\eta\pi^0;m)$ can be estimated without detailing the $\mathcal{PP}\to(K^+K^-+K^0\bar{K}^0)$ transition mechanism (see Fig. 2), which, in principle, can be caused by not only the $f_0(980)$ resonance contribution (see Fig. 3) but also some nonresonance $K\bar{K}$ production mechanism. To do this, we use the relation valid according to the unitarity condition near the $K\bar{K}$ thresholds (see Refs. [32,33] for details)

$$\begin{aligned}\sigma(\mathcal{PP}\to (K^+K^-+K^0\bar{K}^0)&\to a_0^0(980)\to\eta\pi^0;m)\\ &\approx|\tilde{A}(2m_{K^+})|^2|\rho_{K^+K^-}(m)\\ &\quad-\rho_{K^0\bar{K}^0}(m)|^2\\ &\quad\times\frac{g^2_{a_0^0K^+K^-}}{16\pi}\frac{m\Gamma_{a_0^0\to\eta\pi^0}(m)}{|D_{a_0^0}(m)|^2},\end{aligned}\tag{4}$$

where $\rho_{K\bar{K}}(m)=\sqrt{1-4m_K^2/m^2}$ at $m>2m_K$ and $\rho_{K\bar{K}}(m)=i\sqrt{4m_K^2/m^2-1}$ at $0<m<2m_K$. The resulting shape of the cross section is very similar to the solid curve in Fig. 4. The value of $|\tilde{A}(2m_{K^+})|^2$ should be determined from the data on the $K^+K^-$ production cross section near the threshold

$$\sigma(\mathcal{PP}\to K^+K^-;m)=\rho_{K^+K^-}(m)|\tilde{A}(m)|^2.\tag{5}$$

For $m$ between the $K^+K^-$ and $K^0\bar{K}^0$ thresholds, we get by an order of magnitude [32]

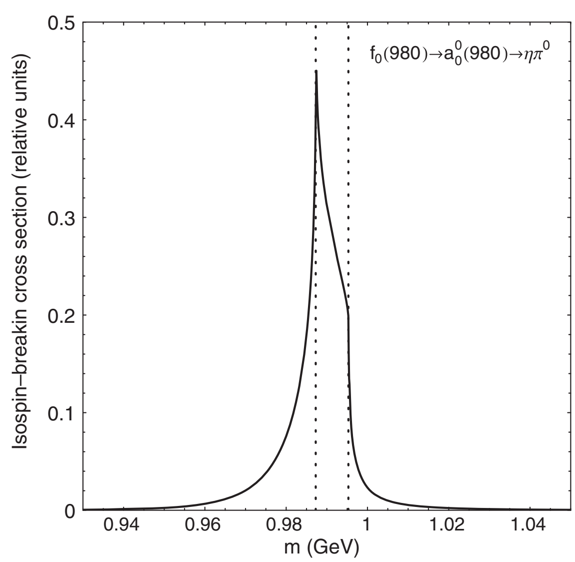


FIG. 4. The solid curve shows the isospin-breaking $\eta\pi^0$ production cross section $\sigma(\mathcal{PP}\to f_0(980)\to(K^+K^-+K^0\bar{K}^0)\to a_0^0(980)\to\eta\pi^0;m)$ caused by the $a_0^0(980)-f_0(980)$ mixing and calculated with the use of Eq. (3). The dotted vertical lines show the locations of the $K^+K^-$ and $K^0\bar{K}^0$ thresholds.

$$\sigma(\mathcal{P}\mathcal{P} \to (K^+K^- + K^0\bar{K}^0) \to a_0^0(980) \to \eta\pi^0; m) \approx |\tilde{A}(2m_{K^+})|^2 \times 0.05. \quad (6)$$

The comparison of this estimate with the data on $\sigma(\mathcal{P}\mathcal{P} \to a_0^0(980) \to \eta\pi^0; m)$ permits one to verify their consistency with the data on $\sigma(\mathcal{P}\mathcal{P} \to K^+K^-; m)$ and with the idea of the $K\bar{K}$ loop breaking of isotopic invariance caused by the mass difference of $K^+$ and $K^0$ mesons. Note that a similar way of the checking the consistency between the data on the decays $f_1(1285) \to \pi^+\pi^-\pi^0$ and $f_1(1285) \to K\bar{K}\pi$ has been discussed in Refs. [32,33]. Detailed formulas connecting $\sigma(\mathcal{P}\mathcal{P} \to h; m)$ with the experimentally measured cross section of the reaction $pp \to p(h)p$ can be found, for example, in Refs. [24–26].

First, the central production of the $a_0^0(980)$ resonance in the reaction $pp \to p(\eta\pi^0)p$ has been studied by the WA102 Collaboration with the use of the CERN Omega Spectrometer at $\sqrt{s} = 29$ GeV [23,38]. The interpretation of these data has been discussed in Refs. [39–42]. Here, we note the following. In the above experiment, the clear peaks from $a_0^0(980)$ and $a_2^0(1320)$ resonances have been observed in the $\eta\pi^0$ mass spectrum. The fit [23] gave the quite usual widths of these states [30]: $\Gamma(a_0(980)) = 72 \pm 16$ MeV and $\Gamma(a_2(1320)) = 115 \pm 20$ MeV. Such a picture indicates that at the energy $\sqrt{s_1} \approx \sqrt{s_2} \approx \sqrt{4m^2_{a_0^0} s} \approx \sqrt{429^2 \text{ GeV}^4} \approx 5.4$ GeV the secondary Regge exchanges, for which the $\eta\pi^0$ production in the central region is not forbidden by $G$ parity, play an important role. For example, the central $a_0^0(980)$ production can proceed via $\mathcal{R}(\eta)\mathcal{R}(\pi^0) \to a_0^0(980)$, $\mathcal{R}(a_2^0)\mathcal{R}(f_2) \to a_0^0(980)$, and $\mathcal{R}(a_2^0)\mathcal{P} \to a_0^0(980)$ transitions, where the type of the secondary Regge trajectory $\mathcal{R}$ is indicated in parentheses. At the LHC energies, the contributions of the secondary Regge trajectories fall off appreciably, and it is natural to expect that the $a_0^0(980)$ resonance must mainly be produced via the double-Pomeron exchange mechanism (see Fig 2), which essentially violates the isotopic invariance in the $K\bar{K}$ threshold region. The narrowing of the $a_0^0(980)$ peak in the $\eta\pi^0$ channel up to the width of 10–20 MeV (see Fig. 4) will serve as an indicator of the changing central production mechanism of the $a_0^0(980)$ resonance with increasing energy.

## ACKNOWLEDGMENTS

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[1] E. Klempt and A Zaitsev, Phys. Rep. **454**, 1 (2007).
[2] M. G. Albrow, T. D. Coughlin, and J. R. Forshaw, Prog. Part. Nucl. Phys. **65**, 149 (2010).
[3] A. Kirk, Int. J. Mod. Phys. A **29**, 1446001 (2014).
[4] G. Gutierrez and M. A. Reyes, Int. J. Mod. Phys. A **29**, 1446008 (2014).
[5] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. D **91**, 091101 (2015).
[6] V. Khachatryan *et al.* (CMS Collaboration), arXiv:1706.08310.
[7] R. Fiore, L. Jenkovszky, and R. Schicker, arXiv:1711.08353.
[8] T. Akesson *et al.* (AFS Collaboration), Nucl. Phys. **B264**, 154 (1986).
[9] K. L. Au, D. Morgan, and M. R. Pennington, Phys. Rev. D **35**, 1633 (1987).
[10] T. A. Armstrong *et al.* (WA76 Collaboration), Z. Phys. C **51**, 351 (1991).
[11] D. Barberis *et al.* (WA102 Collaboration), Phys. Lett. B **453**, 316 (1999).
[12] D. Barberis *et al.* (WA102 Collaboration), Phys. Lett. B **453**, 325 (1999).
[13] M. A. Reyes *et al.* (E690 Collaboration), Phys. Rev. Lett. **81**, 4079 (1998).
[14] D. Barberis *et al.* (WA102 Collaboration), Phys. Lett. B **453**, 305 (1999).
[15] D. Barberis *et al.* (WA102 Collaboration), Phys. Lett. B **462**, 462 (1999).
[16] T. A. Armstrong *et al.* (WA76 Collaboration), Z. Phys. C **52**, 389 (1991).
[17] D. Barberis *et al.* (WA102 Collaboration), Phys. Lett. B **440**, 225 (1998).
[18] D. Barberis *et al.* (WA102 Collaboration), Phys. Lett. B **413**, 225 (1997).
[19] M. Sosa *et al.* (E690 Collaboration), Phys. Rev. Lett. **83**, 913 (1999).
[20] D. Barberis *et al.* (WA102 Collaboration), Phys. Lett. B **413**, 217 (1997).
[21] D. Barberis *et al.* (WA102 Collaboration), Phys. Lett. B **471**, 440 (2000).
[22] D. Barberis *et al.* (WA102 Collaboration), Phys. Lett. B **474**, 423 (2000).
[23] D. Barberis *et al.* (WA102 Collaboration), Phys. Lett. B **488**, 225 (2000).
[24] S. N. Ganguli and D. P. Roy, Phys. Rep. **67**, 201 (1980).
[25] G. Alberi and G. Goggi, Phys. Rep. **74**, 1 (1981).
[26] K. H. Streng, Phys. Lett. B **166**, 443 (1986).
[27] A. B. Kaidalov, V. A. Khoze, A. D. Martin, and M. G. Ryskin, Eur. Phys. J. C **31**, 387 (2003).
[28] V. Dorofeev *et al.* (VES Collaboration), Eur. Phys. J. A **47**, 68 (2011).

[29] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **92**, 012007 (2015).

[30] C. Patrignani *et al.* (Particle Data Group Collaboration), Chin. Phys. C **40**, 100001 (2016) and 2017 update.

[31] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Phys. Lett. **88B**, 367 (1979).

[32] N. N. Achasov, A. A. Kozhevnikov, and G. N. Shestakov, Phys. Rev. D **93**, 114027 (2016).

[33] N. N. Achasov and G. N. Shestakov, Nucl. Part. Phys. Proc. **287–288**, 89 (2017).

[34] F. Aceti, J. M. Dias, and E. Oset, Eur. Phys. J. A **51**, 48 (2015).

[35] N. N. Achasov, A. A. Kozhevnikov, and G. N. Shestakov, Phys. Rev. D **92**, 036003 (2015).

[36] The usual order of the isotopic symmetry breaking in the process amplitude defined by the mass differences of the particles in the mesonic isotopic multiplets is $\simeq(m_{K^0}-m_{K^+})/m_{K^0}\approx 1/126$. The order of the isotopic symmetry breaking in the process amplitude in the region between the $K^+K^-$ and $K^0\bar{K}^0$ thresholds due to any production mechanism of the $K\bar{K}$ pairs with the definite isospin in the $S$ wave, and without the anomalous Landau thresholds [32,33,35], in particular, due to the $a_0^0(980)-f_0(980)$ mixing [31,33], is $\simeq\sqrt{2(m_{K^0}-m_{K^+})/m_{K^0}}\approx 0.127$.

[37] Note that, due to triangle singularity mechanism, the isotopic symmetry-breaking effect was found by J. J. Wu, X. H. Liu, Q. Zhao, and B. S. Zou, Phys. Rev. Lett. **108**, 081803 (2012); and by X. G. Wu, J. J. Wu, Q. Zhao, and B. S. Zou, Phys. Rev. D **87**, 014023 (2013), to be much more enhanced for energies around 1420 MeV. However, the corresponding calculations were fulfilled for a hypothetic case of the stable intermediate $K^*$ meson. In Ref. [35], we showed that the account of the finite width of the $K^*$ ($\Gamma_{K^*\to K\pi}\approx 50$ MeV) smooths the logarithmic singularities in the amplitude and, in particular, results in the suppression of the calculated decay width $\eta(1405)\to f_0(980)\pi^0\to 3\pi$ by the factor of 6–8 as compared with the case of $\Gamma_{K^*\to K\pi}=0$.

[38] A. E. Sobol, Ph.D. thesis, Protvino-Annecy-le-Vienx, 2001.

[39] F. E. Close and A Kirk, Phys. Lett. B **489**, 24 (2000).

[40] N. N. Achasov and A. V. Kiselev, Phys. Lett. B **534**, 83 (2002).

[41] N. N. Achasov and G. N. Shestakov, Phys. Rev. Lett. **92**, 182001 (2004).

[42] J. J. Wu, Q. Zhao, and B. S. Zou, Phys. Rev. D **75**, 114012 (2007).