Torsion in gauge theory

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The potential conflict between torsion and gauge symmetry in the Riemann-Cartan curved spacetime was noted by Kibble in his 1961 pioneering paper and has since been discussed by many authors. Kibble suggested that, to preserve gauge symmetry, one should forgo the covariant derivative in favor of the ordinary derivative in the definition of the field strength $F_{\mu\nu}$ for massless gauge theories, while for massive vector fields, covariant derivatives should be adopted. This view was further emphasized by Hehl *et al.* in their influential 1976 review paper. We address the question of whether this deviation from *normal* procedure by forgoing covariant derivatives in curved spacetime with torsion could give rise to inconsistencies in the theory, such as the quantum renormalizability of a realistic interacting theory. We demonstrate in this paper the one-loop renormalizability of a realistic gauge theory of gauge bosons interacting with Dirac spinors, such as the SU(3) chromodynamics, for the case of a curved Riemann-Cartan spacetime with totally antisymmetric torsion. This affirmative confirmation is one step toward providing justification for the assertion that the flat-space definition of the gauge-field strength should be adopted as the proper definition.

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I. INTRODUCTION

In the formulation of a physical theory in curved spacetime, the *normal* procedure is to replace the ordinary derivative with the corresponding covariant derivative. For a gauge theory in the Riemannian spacetime, because the connection is symmetric, the normal procedure yields a field strength tensor $F_{\mu\nu}$ in the form of its flat-space expression, which is gauge symmetric. But in a Riemann-Cartan spacetime with torsion, the connection being nonsymmetric, this same procedure gives rise to an additional torsion term in the gauge-field strength tensor that violates gauge symmetry. Torsion naturally appears in the Einstein-Cartan-Kibble-Sciama theory of gravitation [1,2]. The potential conflict of torsion with gauge symmetry was already noticed by Kibble [1] in his original paper and has since been discussed by a number of authors [3–12] with various alternatives. Kibble [1] himself took the view that, to preserve gauge symmetry, one should forgo the covariant derivative in favor of the ordinary derivative in the definition of the field strength $F_{\mu\nu}$ for massless gauge theories, while for massive vector fields, covariant derivatives should be adopted. This view was adopted by Hehl *et al.* [3] in their influential 1976 review paper. Since all other alternatives suggested by various authors did not seem to hold up, Kibble's original view has been tacitly accepted without further deliberation, seemingly as consensus by default. The situation is the following. We are facing two alternative choices of $F_{\mu\nu}$, one with the torsion term and the other without. It is uncertain whether the latter alternative, forcing gauge symmetry by deviating from the normal procedure of defining $F_{\mu\nu}$ through covariant derivatives, would cause inconsistency or noncovariant issues in a realistic *quantum* gauge theory, such as the SU(3) quantum chromodynamics, in a curved Riemann-Cartan spacetime, in which all other operations, such as gauge fixing and the ensuing ghost supplementation, follow normal covariant procedures. This uncertainty, at least, needs a clarification. We report in this paper our findings regarding system consistency for the two alternative $F_{\mu\nu}$ cases within the framework of the Kibble-Sciama scheme as well as the renormalizability question. We will first show that the system of field equations, even at the classical level, is inconsistent if the field strength $F_{\mu\nu}$ takes the gauge nonsymmetric form, while it is consistent with the gauge symmetric $F_{\mu\nu}$. This clearly rules out the gauge nonsymmetric version of $F_{\mu\nu}$. We will next demonstrate, using the gauge-invariant background-field method [13–17], in conjunction with the heat-kernel technique and dimensional regularization, that the theory is renormalizable at the one-loop level in the case of the gauge-symmetric field strength $F_{\mu\nu}$, in a Riemann-Cartan spacetime with totally antisymmetric torsion. These findings provide substantiation for the choice of the gauge-symmetric version of $F_{\mu\nu}$ and validate the view of Kibble [1] and Hehl *et al.* [3].

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II. SCIAMA-KIBBLE SCHEME

The genesis of the Kibble-Sciama [1,2] theory can be traced back to the formulation of the Dirac equation in curved spacetime by Weyl [18] and Fock [19]. The vierbein fields $e^a{}_\mu$ were introduced by Weyl and Fock to provide a local coordinate basis for defining the Dirac spinor and the spin connection field $\omega^{ab}{}_\mu$ as the gauge potential for the SO(3,1) group of local Lorentz transformations of the Dirac spinor. Utiyama [20] demonstrated that Einstein's Riemannian theory of gravitation can be regarded as a gauge theory of the SO(3,1) Lorentz group when the corresponding gauge potential, the spin connection $\omega^{ab}{}_\mu$, is identified with the Ricci coefficients of rotation [19] in terms of the vierbein fields $e^a{}_\mu$. Sciama and Kibble [1,2] took the step of treating the spin connection field $\omega^{ab}{}_\mu$, in the spirit of a genuine Lorentz group gauge theory, as an independent dynamic variable to be determined by the theory, instead of being identified with the Ricci coefficients. The coupling of the spin connection to the Dirac spinors, for example, gives rise to torsion.

The metric tensor $g_{\mu\nu}$ in the Kibble-Sciama scheme is defined by

$$g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu, \tag{1}$$

where $\eta_{ab} = (1, -1, -1, -1)$, and the covariant derivatives with respect to both local Lorentz transformations and general coordinate transformations, for generic $\chi_a{}^\lambda$ and $\chi^a{}_\nu$, are defined according to

$$\nabla_\mu \chi_a{}^\lambda = \chi_a{}^\lambda{}_{,\mu} - \omega^b{}_{a\mu}\chi_b{}^\lambda + \Gamma^\lambda{}_{\nu\mu}\chi_a{}^\nu, \tag{2}$$

$$\nabla_\mu \chi^a{}_\nu = \chi^a{}_{\nu,\mu} + \omega^a{}_{b\mu}\chi^b{}_\nu - \Gamma^\lambda{}_{\nu\mu}\chi^a{}_\lambda. \tag{3}$$

Kibble [1] chose the affine connection

$$\Gamma^\lambda{}_{\mu\nu} = e_a{}^\lambda (e^a{}_{\mu,\nu} + \omega^a{}_{b\nu} e^b{}_\mu) \tag{4}$$

so that it is metric compatible, meaning

$$\nabla_\lambda e^a{}_\mu = 0, \tag{5}$$

$$\nabla_\lambda e_a{}^\mu = 0, \tag{6}$$

and, consequently,

$$\nabla_\lambda g^{\mu\nu} = 0, \tag{7}$$

$$\nabla_\lambda g_{\mu\nu} = 0. \tag{8}$$

In the presence of torsion, which is defined as

$$C^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}, \tag{9}$$

the metric compatibility relations (7) and (8) imply that the connection is of the general form

$$\Gamma^\lambda{}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}(g_{\rho\mu,\nu} + g_{\nu\rho,\mu} - g_{\mu\nu,\rho}) + Y^\lambda{}_{\mu\nu}, \tag{10}$$

where the contortion tensor $Y^\lambda{}_{\mu\nu}$ is given by

$$Y^\lambda{}_{\mu\nu} = \frac{1}{2}(C^\lambda{}_{\mu\nu} + C_{\mu\nu}{}^\lambda + C_{\nu\mu}{}^\lambda). \tag{11}$$

III. SYSTEM OF GLUONS INTERACTING WITH QUARKS IN KIBBLE-SCIAMA SCHEME

For notational convenience of presentation, we shall consider the specific case of the SU(3) chromodynamics, in which the gauge gluons interact with a triplet of massless spinor quarks. Let the gauge field be denoted by $A^{\underline{a}}_\mu$, where the index $\underline{a}$ runs from 1 to 8. It is convenient to adopt the group algebraic notation

$$A_\mu = A^{\underline{a}}_\mu T^{\underline{a}}, \tag{12}$$

where $T^{\underline{a}}$, for concreteness, are the familiar 3×3 $\frac{1}{2}\lambda_{\underline{a}}$ Gell-Mann matrices satisfying the algebra, with the totally antisymmetric $f^{\underline{abc}}$ being the SU(3) group structure constants,

$$[T^{\underline{a}}, T^{\underline{b}}] = i f^{\underline{abc}} T^{\underline{c}}. \tag{13}$$

In flat space, the field strength is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]. \tag{14}$$

In curved space, the natural definition for the field strength is to follow the normal procedure of replacing the partial derivative by the appropriate covariant derivative, like

$$\partial_\mu A_\nu \to \partial_\mu A_\nu - \Gamma^\lambda{}_{\nu\mu} A_\lambda. \tag{15}$$

In the Riemannian space, the connection being symmetric, the connection terms cancel when the replacement (14) is made in (13), leaving the expression for $F_{\mu\nu}$ unchanged. In a Riemann-Cartan space, the connection is nonsymmetric, and the field strength $F_{\mu\nu}$ resulting from the replacement is of the form

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + C^\lambda{}_{\mu\nu} A_\lambda - ig[A_\mu, A_\nu]. \tag{16}$$

The additional torsion term in (16) violates gauge invariance. To preserve gauge symmetry, an alternative is to forgo the torsion term in (16) and adopt the flat-space expression (14) as the definition of the field strength $F_{\mu\nu}$. We now consider the system of SU(3) gauge bosons interacting with spinor quarks in the background of the curved Riemann-Cartan space as described above in the Kibble-Sciama scheme. We first check the consistency of the system of field equations with the two alternative versions of the field strength $F_{\mu\nu}$, Eqs. (14) and (16). For convenience, we shall consider a massless spinor quark, which is denoted by ψ. The action for the system is of the form, with a trace over the color index understood,

$$W = \int d^4xh\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\bar{\psi}i\gamma^a e_a{}^\mu D_\mu\psi - \bar{\psi}\bar{D}_\mu i\gamma^a e_a{}^\mu\psi)\right], \quad (17)$$

where $h = \det e^a{}_\mu$, and

$$D_\mu = \partial_\mu - \frac{i}{4}\sigma_{ab}\omega^{ab}{}_\mu + igA_\mu, \quad (18)$$

$$\bar{D}_\mu = \overleftarrow{\partial}_\mu - igA^{\underline{a}}_\mu T^{\underline{a}} + \frac{i}{4}\sigma_{ab}\omega^{ab}{}_\mu, \quad (19)$$

with $\sigma_{ab} = \frac{i}{2}[\gamma_a, \gamma_b]$ [21]. We note that $F^{\mu\nu} = g^{\mu\lambda}g^{\nu\rho}F_{\lambda\rho}$. For proper normalization of the $F^{\mu\nu}F_{\mu\nu}$ term in the Lagrangian, there should be a factor of $\frac{1}{C_2(R)}$, which is defined by

$$\mathrm{tr}(T^{\underline{a}}T^{\underline{b}}) = C_2(R)\delta^{\underline{ab}}. \quad (20)$$

For convenience, we have omitted this normalization factor, but it will be taken into account when we consider renormalization counterterms. The Lagrangian in the action (17) is invariant under local Lorentz transformations, general coordinate transformations, and local scale transformations, the latter being defined, with the proper scale weights for the various fields, by

$$\begin{aligned} e_a{}^\mu &\to e^{-\Lambda(x)}e_a{}^\mu, \\ e^a{}_\mu &\to e^{\Lambda(x)}e^a{}_\mu, \\ \psi(x) &\to e^{-\frac{3}{2}\Lambda(x)}\psi(x), \\ A_\mu(x) &\to A_\mu(x), \\ \omega^{ab}{}_\mu(x) &\to \omega^{ab}{}_\mu(x). \end{aligned}$$

We note the scale invariance of the Dirac Lagrangian in (17) without the explicit appearance of a Weyl scale gauge field; even if such a gauge field were introduced in the covariant derivative D_μ, it would drop out from the Lagrangian, due to cancellation between the two Hermitian conjugate terms, and would not appear in the ensuing field equation for the Dirac field $\psi(x)$. Regarding the Maxwell-field strength $F_{\mu\nu}$, we consider separately its two alternative versions, namely, Eqs. (14) and (16), respectively.

IV. CASE (I) GAUGE NONSYMMETRIC $F_{\mu\nu}$

First, we consider the version with the field strength $F_{\mu\nu}$ containing the torsion term, namely,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + C^\lambda{}_{\mu\nu}A_\lambda - ig[A_\mu, A_\nu],$$

which is not gauge invariant. The Euler-Lagrange equation for the Dirac field can be obtained straightforwardly from (17). On account of the relation

$$h^{-1}\partial_\mu h = \Gamma^\lambda{}_{\lambda\mu} = \Gamma^\lambda{}_{\mu\lambda} + C^\lambda{}_{\lambda\mu} \quad (21)$$

and the commutation properties of the Dirac gamma matrices [21], we obtain [22–24] the field equation for the Dirac field ψ,

$$i\gamma^a e^\mu_a\left(D_\mu + \frac{1}{2}C^\lambda{}_{\lambda\mu}\right)\psi = 0, \quad (22)$$

where D_μ is given in (19). We know that the Lagrangian in the action (17) is scale invariant. The Dirac equation (22) is thus expected to be scale invariant. We have, by its construction according to (8), that the connection $\Gamma^\lambda_{\mu\nu}$ has the scale transformation property

$$\Gamma^\lambda{}_{\mu\nu} \to \Gamma^\lambda{}_{\mu\nu} + \delta^\lambda{}_\mu\Lambda_{,\nu}, \quad (23)$$

which implies

$$C^\lambda{}_{\lambda\mu} \to C^\lambda{}_{\lambda\mu} + 3\Lambda_{,\mu}. \quad (24)$$

We denote

$$B_\mu = \frac{1}{3}C^\lambda{}_{\lambda\mu}. \quad (25)$$

It transforms as an effective *Weyl gauge field* for local scale transformations [22–24]:

$$B_\mu \to B_\mu + \Lambda_{,\mu}. \quad (26)$$

The Dirac equation (22) is then expressed as

$$i\gamma^a e^\mu_a\left(D_\mu + \frac{3}{2}B_\mu\right)\psi = 0. \quad (27)$$

So, indeed, the massless Dirac equation written in this form shows explicit scale invariance, with the proper scale weight $\frac{3}{2}$ for the Dirac field ψ. The Euler-Lagrange equation for the gauge field is obtained straightforwardly. It is of the form

$$(\nabla_\nu + 3B_\nu)F^{\mu\nu} = gJ^\mu, \quad (28)$$

where the covariant derivative ∇_ν is defined as in

$$\nabla_\nu F^{\mu\nu} = \partial_\nu F^{\mu\nu} + \Gamma^\mu{}_{\lambda\nu} F^{\lambda\nu} + \Gamma^\nu{}_{\lambda\nu} F^{\mu\lambda} - ig[A_\nu, F^{\mu\nu}] \quad (29)$$

and the current J_μ is given by

$$J^\mu = \bar{\psi}\gamma^a e_a{}^\mu T^{\underline{a}} \psi T^{\underline{a}}. \quad (30)$$

In the presence of torsion, the field equation (28) is not gauge invariant. We would like to check whether current conservation is valid and whether the system of field equations, namely, Eqs. (27) and (28), is mutually consistent. As a consequence of the Dirac equation (27) and its Hermitian conjugate equation for $\bar{\psi}$, it is straightforward to verify that the current J^μ given by Eq. (30) is indeed conserved,

$$(\nabla_\mu + 3B_\mu) J^\mu = 0. \quad (31)$$

Consistency of Eq. (28) with this current conservation equation (31), which follows directly from the Dirac equation (27), requires that

$$(\nabla_\mu + 3B_\mu)(\nabla_\nu + 3B_\nu) F^{\mu\nu} = 0. \quad (32)$$

Making use of the antisymmetry of $F^{\mu\nu}$, it is straightforward, though tedious, to show that

$$(\nabla_\mu + 3B_\mu)(\nabla_\nu + 3B_\nu) F^{\mu\nu} = -R^\mu{}_{\rho\mu\nu} F^{\rho\nu} + \frac{1}{2} C^\mu{}_{\rho\nu} \nabla_\mu F^{\rho\nu} + \frac{3}{2} F^{\mu\nu} (\nabla_\mu B_\nu - \nabla_\nu B_\mu). \quad (33)$$

For the right-hand side of Eq. (33) to vanish, it is necessary, due to its structure, that the second term vanishes. That is, we have to set $C^\mu{}_{\rho\nu} = 0$. This results in B_μ being equal to 0 and $R^\mu{}_{\rho\mu\nu}$ being symmetric in ρ and ν because the connection $\Gamma^\lambda{}_{\mu\nu}$ now reduces to the Christoffel connection. The three terms on the right-hand side of Eq. (29) then all vanish. Consistency of the two field equations of the system (24) and (25) is thus seen to require the vanishing of torsion. The upshot is that the system of field equations is inconsistent for the gauge nonsymmetric version (16) of $F_{\mu\nu}$.

V. CASE (II) GAUGE SYMMETRIC $F_{\mu\nu}$

We next consider the case of gauge symmetric $F_{\mu\nu}$,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu],$$

which is the version with the torsion term removed. With this expression for $F_{\mu\nu}$ in the action (17), the field equation for the Dirac field ψ remains the same as Eq. (27), resulting in the same current conservation equation (31), while the field equation for the Maxwell field (28) is replaced by

$$(\nabla_\nu + 3B_\nu) F^{\mu\nu} - \frac{1}{2} C^\mu{}_{\rho\nu} F^{\rho\nu} = J^\mu. \quad (34)$$

Consistency of Eq. (34) with Eq. (31) requires that $(\nabla_\mu + 3B_\mu)$ operating on the left-hand side of Eq. (34) vanishes. The result of operating on the first term on the left-hand side of Eq. (34) is already found and given by Eq. (33). Operating on the second term yields the contribution

$$-\frac{1}{2} C^\mu{}_{\rho\nu} \nabla_\mu F^{\rho\nu} - \frac{1}{2} \nabla_\mu C^\mu{}_{\rho\nu} F^{\rho\nu} - \frac{3}{2} B_\mu C^\mu{}_{\rho\nu} F^{\rho\nu}. \quad (35)$$

Summing the two contributions given in Eqs. (33) and (35) yields

$$-R^\mu{}_{\rho\mu\nu} F^{\rho\nu} + \frac{1}{2}[3(B_{\nu,\mu} - B_{\mu,\nu}) - \nabla_\mu C^\mu{}_{\rho\nu}] F^{\mu\nu}. \quad (36)$$

In the presence of torsion, the antisymmetric part of $R^\mu{}_{\rho\mu\nu}$ does not vanish, and explicit evaluation gives the result

$$\frac{1}{2}(R^\mu{}_{\rho\mu\nu} - R^\mu{}_{\nu\mu\rho}) = \frac{1}{2}[3(B_{\nu,\rho} - B_{\rho,\nu}) - \nabla_\mu C^\mu{}_{\rho\nu}]. \quad (37)$$

The two contributions from operating $(\nabla_\mu + 3B_\mu)$ on the two right-hand side terms of Eq. (34) miraculously cancel each other out, and the final result is zero. Consistency of the field equations is thus established. The unpleasing $C^\mu{}_{\rho\nu} F^{\rho\nu}$ term in the field equation (34) looks formidable, but it actually helped save consistency. We have thus seen that the system of classical field equations of chromodynamics in the curved Riemann-Cartan space is self-consistent when the gauge-field strength is defined by the gauge-symmetric expression (14), while it is not for the gauge nonsymmetric version (16). The latter version is thus ruled out, even at the classical level. We next check whether the gauge-symmetric version (13) of the interacting gauge theory, chromodynamics, is one-loop renormalizable.

VI. ONE-LOOP RENORMALIZATION BY BACKGROUND-FIELD METHOD

The background-field method [13–17] is ideally suited to the computation of effective interaction in curved spaces. It has been used to study the renormalization property of gauge theories in curved Riemannian spacetime by various authors [25–28], establishing renormalizability at one-loop level and beyond. In the case of Riemann-Cartan spacetime, there do not seem to exist investigations in the literature of the renormalizability question of gauge theories. The question in focus is whether torsion could create complications, a question we would like to study. Based on the background-field method, there is the unified superspace computation [27] of the one-loop renormalization counterterms, treating both gauge bosons and Dirac fermions within the framework of the Schwinger-DeWitt proper-time representation of the propagator functions [13,29]. Rather than using this elegant framework for evaluating the renormalization counterterms, we will instead combine the normal treatment based on the heat-kernel technique with 't Hooft's algorithm [15,26] for extracting one-loop divergences. The one-loop renormalization counterterms arise from four types of loops, the boson gluon loop, the ghost loop, the spinor quark loop, and the mixed gluon-quark loops (quark self-energy loop and gluon-quark vertex loops). For the gluon, ghost, and quark loops, we follow Toms's treatment, which is based on the heat-kernel method (a variant of the Schwinger-DeWitt proper-time method) and dimensional regularization, while for the mixed gluon-quark loops, we make use of the 't Hooft algorithms [15,26]. The divergent part of the one-loop effective action is given by an integral of the coefficient $[a_2]$ of the heat-kernel expansion [13,30]. Its explicit expression is given by DeWitt [13] and Gilkey [31], in the case of Riemannian spacetime. In the case of Riemann-Cartan spacetime, the presence of torsion makes the evaluation of the corresponding $[a_2]$ quite involved, and there does not seem to exist a definitive result for a general torsion. The special case of totally antisymmetric torsion has been carefully studied by Yajima [32]. We will make use of Yajima's result, and we will thus restrict ourselves to the special case of totally antisymmetric torsion, for which the effective Weyl gauge field vanishes, namely, $B_\mu = 0$.

Let us denote the classical background fields by η and $\hat{A}_\mu$, which satisfy the field equations (22) and (28), respectively. We replace in the action (17) the field A_μ by $A_\mu + \hat{A}_\mu$ and ψ by $\psi + \eta$. In the respective sums, A_μ and ψ (and $\bar{\psi}$) are regarded as quantum fields, while $\hat{A}_\mu$ and ξ are regarded as classical fields. We remind ourselves that in the action (17) the gauge-field strength $F_{\mu\nu}$ is defined by the gauge-symmetric expression (14), namely,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu].$$

We will also need to add to the action the gauge-fixing term and the corresponding Faddeev-Popov ghost term [33]. The gauge-fixing term is chosen in accordance to the Landau-DeWitt gauge condition and is given by [25]

$$W_{(GF)} = \int d^4xh\left[-\frac{1}{2}(\hat{\nabla}_\mu A^\mu)^2\right], \tag{38}$$

where

$$\hat{\nabla}_\mu A^\mu = \partial_\mu A^\mu + \Gamma^\mu{}_{\lambda\mu}A^\lambda - ig[\hat{A}_\mu, A^\mu]. \tag{39}$$

The gauge-fixing action (38) brakes gauge symmetry if only the quantum field A^μ undergoes gauge transformation but can be made gauge covariant under suitably combined gauge transformations of both A^μ and $\hat{A}_\mu$. The corresponding Faddeev-Popov ghost term can be obtained by changing the integration "variable" in the path integral and is given by

$$W_{(\mathrm{ghost})} = \int d^4xh\bar{\zeta}(-\hat{\nabla}_\mu\hat{\nabla}^\mu - \hat{\nabla}_\mu A^\mu - A^\mu\hat{\nabla}_\mu)\zeta, \tag{40}$$

where the ghost fields $\bar{\zeta}$ and ζ are Grassmann scalars and carry the same color index as A_μ. The gauge-fixing action $W_{(GF)}$ and Faddeev-Popov ghost action $W_{(\mathrm{ghost}}$ are to be added to the action W, given by Eq. (17), to form the total action. Expand the action in powers of the quantum fields A_μ and ψ. The coefficients of terms linear in quantum fields vanish, as a result of the classical field equations (21) and (28). The terms quadratic in the quantum fields (including the ghost fields) give rise to the quark-loop and gluon-loop contributions. They are also sufficient for evaluating the mixed gluon-quark loops in accordance with the 't Hooft's algorithm. The terms quadratic in quantum fields, up to a total divergence term in the integrand, are exhibited in

$$\begin{aligned} W^{(2)} = \int d^4xh\Big\{\Big(&-\frac{1}{4}\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu} - 2iA_\mu[\hat{F}^{\mu\nu}, A_\nu]\Big) \\ &-\frac{1}{2}(\hat{\nabla}_\mu A^\mu)^2 + \bar{\psi}i\gamma^\mu(\hat{D}_\mu)\psi + \bar{\psi}i\gamma^\mu iA_\mu\eta + \bar{\eta}i\gamma^\mu iA_\mu\psi \\ &-\frac{1}{2}(\hat{\nabla}_\mu A^\mu)^2 + \bar{\zeta}(-\hat{\nabla}_\mu\hat{\nabla}^\mu)\zeta\Big\}, \end{aligned} \tag{41}$$

where $\hat{\nabla}_\mu A^\mu$ is given in Eq. (39) and

$$\tilde{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[\hat{A}_\mu, A_\nu] + ig[\hat{A}_\nu, A_\mu], \tag{42}$$

$$\gamma^\mu = \gamma^a e^\mu_a, \tag{43}$$

$$\hat{D}_\mu = \partial_\mu - \frac{i}{4}\sigma_{ab}\omega^{ab}{}_\mu + ig\hat{A}_\mu. \tag{44}$$

VII. FERMION LOOP

The term in $W^{(2)}$ that gives rise to the pure fermion loop is quadratic in the quantum fermion fields, namely,

$$\int d^4xh\bar{\psi}i\gamma^\mu\hat{D}_\mu\psi.$$

The effective action due to the fermion loop is given by [25,30]

$$\Gamma_{\mathrm{fermion-loop}} = -i\ln \mathrm{Det}(i\gamma^\mu\hat{D}_\mu). \tag{45}$$

To make use of the heat-kernel technique, while the heat equation is of *second order* in the differential operator, we need to reformulate $\Gamma_{\mathrm{fermion-loop}}$ so that the differential operator in the determinant is of second order. This can be accomplished by replacing in Eq. (45) the linear differential operator $i\gamma^\mu \hat{D}_\mu$ by its square $(i\gamma^\mu \hat{D}_\mu)^2$ and multiplying an overall factor of $\frac{1}{2}$, namely,

$$\Gamma_{\text{fermion-loop}} = -i\frac{1}{2}\ln \mathrm{Det}[(i\gamma^\mu \hat{D}_\mu)^2]. \tag{46}$$

The square in the determinant can be expressed as [22]

$$-\left(g^{\mu\nu}\hat{D}_\mu \hat{D}_\nu - \frac{i}{2}\sigma^{\mu\nu}C^\lambda_{\mu\nu}\hat{D}_\lambda + Z\right), \tag{47}$$

where

$$Z = \frac{1}{4}R + \frac{i}{8}\gamma_5 \tilde{R} + \frac{i}{2}\sigma^{\mu\nu}(R_{\mu\nu} + i\hat{F}_{\mu\nu}), \tag{48}$$

with

$$R = R^{\mu\nu}{}_{\mu\nu},$$
$$\tilde{R} = \varepsilon^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho},$$
$$R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}.$$

We note that when torsion vanishes $\tilde{R} = 0$, $R_{\mu\nu} = R_{\nu\mu}$, and Z in Eq. (48) reduces to $\frac{1}{4}R$, a well-recognized result for Riemannian spacetime. The linear derivative term in Eq. (47), which is proportional to the torsion tensor, can be absorbed into the quadratic derivative term by redefining the covariant derivative

$$\hat{\hat{D}}_\mu = \hat{D}_\mu - \frac{i}{4}\sigma^{\lambda\rho}C_{\lambda\rho\mu}. \tag{49}$$

The fermion-loop effective action (46) thus becomes

$$\Gamma_{\text{fermion-loop}} = -i\frac{1}{2}\ln \mathrm{Det}[g^{\mu\nu}\hat{\hat{D}}_\mu \hat{\hat{D}}_\nu + X], \tag{50}$$

with

$$X = Z - D_\mu Q^\mu - Q_\mu Q^\mu, \tag{51}$$

where

$$Q_\mu = -\frac{i}{4}\sigma^{\lambda\rho}C_{\lambda\rho\mu}. \tag{52}$$

The divergent part of the fermion-loop effective action (48) is of the form [30]

$$\mathrm{Div}\Gamma_{\text{fermion-loop}} = \frac{1}{\epsilon}\int d^4x h \mathrm{tr}[a_2](x), \tag{53}$$

where $\epsilon = (4\pi)^2(n-4)$ and the corresponding kernel for $[a_2]$ is $g^{\mu\nu}\hat{\hat{D}}_\mu\hat{\hat{D}}_\nu + X$ in Eq. (50). For Riemann-Cartan spacetime and in the case of the totally antisymmetric torsion tensor, the $[a_2]$ corresponding to the differential operator in Eq. (47) has been obtained by Yajima [32]. It is given by, as adopted with our metric,

$$[a_2] = \frac{1}{12}\tilde{W}^{\mu\nu}\tilde{W}_{\mu\nu} + \frac{1}{180}(R^{(o)\mu\nu\lambda\rho}R^{(o)}_{\mu\nu\lambda\rho} - R^{(o)\mu\nu}R^{(o)}_{\mu\nu}) - \frac{1}{6}\hat{\hat{D}}_\mu\hat{\hat{D}}^\mu\left(\frac{1}{5}R^{(o)} - X\right) + \frac{1}{2}\left(\frac{1}{6}R^{(o)} - X\right)^2, \tag{54}$$

where $R^{(o)}_{\mu\nu\lambda\rho}$, etc., are the Riemannian curvature tensors and $\tilde{W}_{\mu\nu}$ is defined [32] according to

$$[\hat{\hat{D}}_\mu, \hat{\hat{D}}_\nu]\psi = (\tilde{W}_{\mu\nu} + C^\lambda{}_{\mu\nu}\hat{\hat{D}}_\lambda)\psi. \tag{55}$$

We remark that the disentanglement with the definition of $\tilde{W}_{\mu\nu}$ of the torsion term in (55) is crucial in assuring gauge symmetry in the final result. With the definition in Eq. (47), $\tilde{W}_{\mu\nu}$ is computed to be

$$\tilde{W}_{\mu\nu} = -\frac{i}{4}\sigma^{\alpha\beta}\Big[R^{(o)}_{\alpha\beta\mu\nu} + \frac{3}{2}(\bar{\nabla}_\mu C_{\alpha\beta\nu} - \bar{\nabla}_\nu C_{\alpha\beta\mu} - C^\lambda_{\alpha\mu}C_{\beta\lambda\nu} + C_\alpha{}^\lambda{}_\nu C_{\beta\lambda\mu})\Big] + i\hat{F}_{\mu\nu}, \tag{56}$$

where $\bar{\nabla}_\mu$ is the Riemannian covariant derivative. We point out that one important aspect of Eq. (53) is that the torsion term in Eq. (53) is not involved in the definition of $\tilde{W}_{\mu\nu}$. This ensures the clean appearance of the *gauge-invariant* $\hat{F}_{\mu\nu}$ term in $\tilde{W}_{\mu\nu}$ without the involvement of the torsion tensor. The trace over the spinor and quark indices of the quark field ψ of the product $\tilde{W}^{\mu\nu}\tilde{W}_{\mu\nu}$ term appearing in Eq. (54) is given by

$$\frac{1}{12}\mathrm{tr}\tilde{W}^{\mu\nu}\tilde{W}_{\mu\nu} = -\frac{1}{8}\Sigma^{\alpha\beta\mu\nu}\Sigma_{\alpha\beta\mu\nu} - \frac{1}{3}g^2\mathrm{tr}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu}, \tag{57}$$

where

$$\Sigma_{\alpha\beta\mu\nu} = R^{(o)}_{\alpha\beta\mu\nu} + \frac{3}{2}(\bar{\nabla}_\mu C_{\alpha\beta\nu} - \bar{\nabla}_\nu C_{\alpha\beta\mu} - C^\lambda_{\alpha\mu}C_{\beta\lambda\nu} + C^\lambda_{\alpha\nu}C_{\beta\lambda\mu}). \tag{58}$$

As our main interest is in the renormalizabity of gauge theory in the Riemann-Cartan spacetime, we will concentrate on the gauge-field terms in Eq. (53). Explicit evaluation shows that these terms come from the first and last terms on the right-hand side of Eq. (54). The contribution from the first term is contained in Eq. (57). The contribution from the last term is in

$$\mathrm{tr}\frac{1}{2}X^2 = g^2\mathrm{tr}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} + \text{gravitational terms}.$$

The renormalization counterterm for the gluon field due to the fermion loop is the sum of the two contributions and is given by

$$\mathrm{Div}\Gamma_{\text{fermion-loop}} = \frac{1}{\epsilon}\int d^4xh\bigg(\frac{2}{3}g^2\mathrm{tr}\hat{F^{\mu\nu}}\,\hat{F_{\mu\nu}} + \text{gravitational terms}\bigg). \quad (59)$$

VIII. GLUON AND GHOST LOOPS

The relevant terms for the gluon loop in $W^{(2)}$ are

$$W_{\text{gluon-loop}} = \int d^4xh\bigg\{-\frac{1}{4}\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu} - 2iA_\mu[\hat{F}^{\mu\nu}, A_\nu] - \frac{1}{2}(\hat{\nabla}_\mu A^\mu)^2\bigg\}. \quad (60)$$

while that for the ghost loop is

$$W_{\text{ghost-loop}} = \int d^4xh\bar{\zeta}(-\hat{\nabla}_\mu\hat{\nabla}^\mu)\zeta. \quad (61)$$

Up to a total derivative, $W_{\text{gluon-loop}}$ can be expressed in the form

$$\int d^4x\frac{1}{2}hA^\mu(g_{\mu\nu}\hat{\nabla}_\lambda\hat{\nabla}^\lambda - \hat{\nabla}_\nu\hat{\nabla}_\mu + \hat{\nabla}_\mu\hat{\nabla}_\nu + 2C_{\mu\lambda\nu}\hat{\nabla}^\lambda + \hat{\nabla}^\lambda C_{\mu\lambda\nu} - C_{\mu\lambda\rho}C^{\lambda\rho}_\nu + g\hat{F}_{\mu\nu})A^\nu, \quad (62)$$

where $\hat{\nabla}_\mu$ is defined as in

$$\hat{\nabla}_\mu A_\nu = \nabla_\mu A_\nu - i[\hat{A_\mu}, A_\nu]. \quad (63)$$

In Eq. (62), the $A\hat{F}A$ product term is understood to be the product $f^{\underline{abc}}A^{\underline{a}}\hat{F}^{\underline{b}}A^{\underline{c}}$. On account of

$$[\hat{\nabla}_\mu, \hat{\nabla}_\nu]A^\nu = R_{\nu\mu}A^\nu + C^\rho{}_{\mu\nu}\nabla_\rho A^\nu - ig[\hat{F}_{\mu\nu}, A^\nu], \quad (64)$$

we can express Eq. (62) in the form

$$\frac{1}{2}hA^\mu[g_{\mu\nu}\hat{\nabla}_\lambda\hat{\nabla}^\lambda + C_{\mu\lambda\nu}\hat{\nabla}^\lambda + \hat{\nabla}^\lambda C_{\mu\lambda\nu} - C_{\mu\lambda\rho}C^{\lambda\rho}_\nu + R_{\nu\mu} + 2g\hat{F}_{\mu\nu}]A^\nu, \quad (65)$$

The term linear in derivative in Eq. (64), which is brought about by torsion, is to be absorbed into the quadratic derivative term by defining a modified connection

$$\hat{\Gamma}'^\lambda_{\mu\nu} = \hat{\Gamma}^\lambda_{\mu\nu} + \frac{1}{2}C^\lambda_{\mu\nu}, \quad (66)$$

with the corresponding covariant derivative expressed as $\hat{\nabla}'_\mu$. The gluon-loop action (62) can be written as

$$W_{\text{gluon-loop}} = \int d^4x\frac{1}{2}hA^\mu(g_{\mu\nu}\hat{\nabla}'_\lambda\hat{\nabla}'^\lambda + X_{\mu\nu})A^\nu, \quad (67)$$

where

$$X^{ac}_{\mu\nu} = R_{\nu\mu}\delta^{\underline{ac}} + 2gf^{\underline{abc}}\hat{F}^{\underline{b}}_{\mu\nu}. \quad (68)$$

The gluon-loop effective action $\Gamma_{\text{gluon-loop}}$ is then given by

$$\Gamma_{\text{gluon-loop}} = i\frac{1}{2}\ln\mathrm{Det}(g_{\mu\nu}\hat{\nabla}'_\lambda\hat{\nabla}'^\lambda + X'). \quad (69)$$

Its divergent pole term is

$$\mathrm{Div}\Gamma_{\text{gluon-loop}} = -\frac{1}{\epsilon}\int d^4xh\mathrm{tr}[a'_2](x), \quad (70)$$

where $[a'_2]$ is the asymptotic expansion coefficient corresponding to the kernel appearing in Eq. (67), with a structure similar to that for the fermion case, namely, as in Eq. (54). Again, we will concentrate on the corresponding first and last terms in Eq. (54) that give rise to gauge-field terms. The corresponding $W'_{\mu\nu}$ can be obtained from calculating $[\hat{\nabla}'_\mu, \hat{\nabla}'_\nu]A_\lambda$, which can be neatly expressed as

$$[\hat{\nabla}'_\mu, \hat{\nabla}'_\nu]A_\lambda = R'^\rho_{\lambda\mu\nu}A_\rho - i[\hat{F_{\mu\nu}}, A_\lambda] + C'^\rho_{\mu\nu}\hat{\nabla}'_\rho A_\lambda, \quad (71)$$

where $R'^\rho_{\lambda\mu\nu}$ is the curvature tensor formed with $\Gamma'^\lambda_{\mu\nu}$ as the connection. From Eq. (71), we obtain

$$(W'_{\mu\nu})^{\rho\underline{ac}}_\lambda = R'^\rho_{\lambda\mu\nu}\delta^{\underline{ac}} + gf^{\underline{abc}}\hat{F}^{\underline{b}}_{\mu\nu}\delta^\rho_\lambda. \quad (72)$$

We then obtain contribution from the first term,

$$\mathrm{tr}\bigg(\frac{1}{12}W'_{\mu\nu}W'^{\mu\nu}\bigg) = -\frac{1}{3}g^2f^{\underline{abc}}f^{\underline{cda}}\hat{F}^{\underline{b}\mu\nu}\hat{F}^{\underline{d}}_{\mu\nu} + \text{gravitational terms}. \quad (73)$$

Define $C_2(G)$ by

$$f^{\underline{abc}}T^{\underline{a}}T^{\underline{c}} = \frac{i}{2}C_2(G)T^{\underline{b}}. \quad (74)$$

We then have

$$\mathrm{tr}\bigg(\frac{1}{12}W'_{\mu\nu}W'^{\mu\nu}\bigg) = -\frac{1}{3}g^2\frac{C_2(G)}{C_2(R)}\mathrm{tr}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} + \text{gravitational terms}, \quad (75)$$

where $C_2(R)$ is the normalization factor given by Eq. (20). The contribution from the last term can be similarly calculated and is given by

$$\mathrm{tr}\frac{1}{2}X_{\mu\nu}X^{\nu\mu} = 2g^2\frac{C_2(G)}{C_2(R)}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} + \text{gravitational terms}. \tag{76}$$

The divergent pole term of the gluon loop is due to the sum of the contributions in Eqs. (75) and (76) and given by

$$\mathrm{Div}\Gamma_{\text{gluon-loop}} = -\frac{1}{\epsilon}\int d^4xh\left(\frac{5}{3}g^2\frac{C_2(G)}{C_2(R)}\mathrm{tr}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} + \text{gravitational terms}\right). \tag{77}$$

The effective action due to the loop of the *complex* Grassmann ghost field is given by

$$\Gamma_{\text{ghost-loop}} = -2i\ln\mathrm{Det}(-\hat{\nabla}_\mu\hat{\nabla}^\mu). \tag{78}$$

Its divergent part can be similarly calculated, taking into account that the ghost field is a complex Grassmann scalar and there is no spinor index to sum over, and is given by

$$\mathrm{Div}\Gamma_{\text{ghost-loop}} = \frac{1}{\epsilon}\int d^4xh\left(-\frac{1}{6}g^2\frac{C_2(G)}{C_2(R)}\mathrm{tr}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} + \text{gravitational terms}\right). \tag{79}$$

The final result of the divergent parts due to the quark, gluon, and ghost loops is the sum of Eqs. (59), (77), and (79):

$$\mathrm{Div}\Gamma_{\text{loops}} = \frac{1}{\epsilon}\int d^4xh\left(g^2\frac{4C_2(R) - 11C_2(G)}{6C_2(R)}\mathrm{tr}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} + \text{gravitational terms}\right). \tag{80}$$

We recall that we have for convenience omitted the normalization factor of $\frac{1}{C_2(R)}$ for the $F^{\mu\nu}F_{\mu\nu}$ term in the original Lagrangian in Eq. (17). Thus, when we consider renormalization constants, this normalization factor should be similarly omitted, namely, by dropping $C_2(R)$ in the denominator in the above equation. The result for the gauge-field term, we note, is compatible with earlier results [25,26] for the Riemannian spacetime. In our specific case of quantum chromodynamics without additional flavor, $C_2(G) = 3$ and $C_2(F) = \frac{1}{2}$.

IX. MIXED LOOPS

These are the gluon-quark vertex and quark self-energy loops, which contain both internal quark and gluon lines. We will use 't Hooft's algorithm [15,26] to find the renormalization counterterms. Following 't Hooft's procedure, we make the substitutions in the action $W^{(2)}$ in (41)

$$\bar{\psi} \to \bar{\psi},$$
$$\psi \to \gamma^\mu \acute{D}_\mu \xi,$$

where $\acute{D}_\mu = \partial_\mu - \frac{i}{4}\sigma_{ab}\omega_\mu^{ab}$, namely, the covariant derivative D_μ without the gauge-potential term. The fermion part of the Lagrangian in Eq. (41) becomes

$$h[-g^{\mu\nu}\bar{\psi}\acute{D}_\mu\acute{D}_\nu\xi + \frac{i}{2}\sigma^{\mu\nu}C^\lambda_{\mu\nu}\acute{D}_\lambda\xi - \bar{\psi}\acute{Z}\xi + \bar{\psi}i\gamma^\mu ig\hat{A}_\mu i\gamma^\nu\acute{D}_\nu\xi + \bar{\psi}i\gamma^\mu igA_\mu\eta + \bar{\eta}i\gamma^\mu igA_\mu i\gamma^\nu\acute{D}_\nu\xi]. \tag{81}$$

Applying 't Hooft's algorithm to this fermion Lagrangian and the gluon part of the Lagrangian as given in Eq. (67), we have, in 't Hooft's notation,

$$(\alpha)^{\underline{a}}_\lambda = i\gamma_\lambda igT^{\underline{a}}\eta,$$
$$\bar{\beta}^{\lambda\underline{b}} = \bar{\eta}i\gamma^\lambda iigT^{\underline{b}},$$
$$(N^\mu)^{\rho\underline{ac}}_\lambda = g^{\mu\nu}(\hat{\Gamma}^\rho_{\lambda\nu}\delta^{\underline{ac}} + gf^{\underline{abc}}\hat{A}^{\underline{b}}_\nu g^\rho_\lambda),$$

where $\gamma^\mu = e^\mu_{\underline{a}}\gamma^{\underline{a}}$. According to the algorithm, the renormalization counterterms due to the mixed loops are the sum of the following four terms:

$$\frac{2}{\epsilon}\frac{1}{2}\bar{\beta}\gamma^\mu\partial_\mu\alpha, \tag{82}$$

$$\frac{2}{\epsilon}\frac{1}{2}\left[-\bar{\beta}\gamma^\mu\left(-\frac{i}{4}\sigma_{ab}\omega_\mu^{ab} - \frac{i}{4}\sigma^{\lambda\rho}C_{\lambda\rho\mu}\right)\right]\alpha, \tag{83}$$

$$\frac{2}{\epsilon}\frac{1}{2}N_\mu\frac{1}{2}\bar{\beta}\gamma^\mu\alpha, \tag{84}$$

$$\frac{2}{\epsilon}\frac{1}{2}\bar{\beta}\gamma^\mu\frac{1}{2}i\gamma^\nu\hat{A}_\nu\gamma_\mu i\alpha. \tag{85}$$

With the help of the relations

$$\gamma_a\omega_\mu^{ba}e_b{}^\nu = \frac{i}{4}\omega_{bc\mu}[\sigma^{bc},\gamma^\nu], \tag{86}$$

$$\partial_\mu\gamma^\nu = \left[\frac{i}{4}\sigma_{ab}\omega_\mu^{ab},\gamma^\nu\right] - \Gamma^\nu{}_{\lambda\mu}\gamma^\lambda, \tag{87}$$

$$\partial_\mu\gamma_\nu = \left[\frac{i}{4}\sigma_{ab}\omega_\mu^{ab},\gamma_\nu\right] + \Gamma^\lambda{}_{\nu\mu}\gamma_\lambda, \tag{88}$$

the sum of the four terms is given by

$$\frac{2}{\epsilon}[\bar{\eta}g^2(T^{\underline{a}}T^{\underline{a}})i\gamma^\mu(\partial_\mu - \frac{i}{4}\sigma_{ab}\omega_\mu^{ab})\eta - \bar{\eta}g^3(f^{\underline{abc}}T^{\underline{a}}T^{\underline{c}})\hat{A}^{\underline{b}}_\nu i\gamma^\nu\eta + \bar{\eta}g^3(T^{\underline{a}}T^{\underline{b}}T^{\underline{a}})i\hat{A}^{\underline{b}}_\mu i\gamma^\mu\eta]. \tag{89}$$

In addition to $C_2(R)$ defined by Eq. (20) and $C_2(G)$ defined by Eq. (74), we further define $C_2(F)$ by [34]

$$T^{\underline{a}}T^{\underline{a}} = C_2(F)I. \tag{90}$$

We note that $C_2(F)$ and $C_2(R)$ are related. In our specific case here, $C_2(F) = \frac{8}{3}C_2(R) = \frac{4}{3}$. It can be easily shown that

$$f^{\underline{abc}}T^{\underline{a}}T^{\underline{c}} = \frac{i}{2}C_2(G)T^{\underline{b}},$$
$$T^{\underline{a}}T^{\underline{b}}T^{\underline{a}} = -\frac{1}{2}C_2(G)T^{\underline{b}} + C^2(F)T^{\underline{b}}.$$

The sum (89) becomes

$$\frac{2}{\epsilon}C_2(F)g^2\bar{\eta}i\gamma^\mu\left(\partial_\mu - \frac{i}{4}\sigma_{ab}\omega_\mu^{ab} + ig\hat{A}_\mu\right)\eta. \tag{91}$$

This is the final result for the renormalization counterterms due to the gluon-quark vertex and quark self-energy loops. It is also compatible with the earlier result [25,26] for the Riemannian spacetime.

X. CONCLUSIONS

We have in this paper deliberated the compatibility of torsion with gauge symmetry in a realistic interacting gauge theory, namely, quantum chromodynamics of gluons interacting with quarks in Riemann-Cartan spacetime. We have demonstrated that the system of classical field equations is consistent with the choice of the gauge-invariant definition of $F_{\mu\nu}$, which is the flat-space expression, while inconsistent with the choice of the gauge-noninvariant version, which is the one with covariant derivatives replacing the ordinary derivatives in the flat-space expression. To further substantiate the choice of the gauge-invariant version of $F_{\mu\nu}$, we have investigated the quantum renormalizability at one-loop level to make sure that torsion does not somehow get entangled with gauge symmetry at a level beyond the classical. Since the heat-kernel technique is an essential method in our treatment, we restrict ourselves to the special case of totally antisymmetric torsion, as the general case is much more complicated and there is lack of reliable research on the corresponding heat kernel. We would like to note that the results of Yajima *et al.* [32] on the heat kernel in the presence of torsion are essential for our results.

With regard to the renormalization counterterms for the gluon field and the quark field, our one-loop results are contained in Eqs. (80) and (91). It is seen that the counterterms are in the same gauge-invariant forms as the original terms in the Lagrangian. Except for the gravitational counterterms, which we have omitted, the pattern of counterterms for the gluon and quark fields in the present case of Riemann-Cartan spacetime is exactly the same as in the previously studied case of Riemannian spacetime [25,26]. We will hence not repeat here defining the renormalized constants. The conclusion, of course, is that the theory, with the choice of the gauge-invariant version of $F_{\mu\nu}$, is renormalizable and gauge symmetry preserved at the one-loop level.

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