

Dissipative dark matter halos: The steady state solution

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Dissipative dark matter, where dark matter particle properties closely resemble familiar baryonic matter, is considered. Mirror dark matter, which arises from an isomorphic hidden sector, is a specific and theoretically constrained scenario. Other possibilities include models with more generic hidden sectors that contain massless dark photons [unbroken $U(1)$ gauge interactions]. Such dark matter not only features dissipative cooling processes but also is assumed to have nontrivial heating sourced by ordinary supernovae (facilitated by the kinetic mixing interaction). The dynamics of dissipative dark matter halos around rotationally supported galaxies, influenced by heating as well as cooling processes, can be modeled by fluid equations. For a sufficiently isolated galaxy with a stable star formation rate, the dissipative dark matter halos are expected to evolve to a steady state configuration which is in hydrostatic equilibrium and where heating and cooling rates locally balance. Here, we take into account the major cooling and heating processes, and numerically solve for the steady state solution under the assumptions of spherical symmetry, negligible dark magnetic fields, and that supernova sourced energy is transported to the halo via dark radiation. For the parameters considered, and assumptions made, we were unable to find a physically realistic solution for the constrained case of mirror dark matter halos. Halo cooling generally exceeds heating at realistic halo mass densities. This problem can be rectified in more generic dissipative dark matter models, and we discuss a specific example in some detail.

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I. INTRODUCTION

A variety of observations, on both large and small scales, suggests the existence of nonbaryonic dark matter in the Universe. Anisotropies of the cosmic microwave background radiation, large scale structure, gravitational lensing observations, etc., are all consistent with cold or warm dark matter candidates, with $\Omega_{dm}/\Omega_b \approx 5.4$ from the analysis of PLANCK data [1]. On smaller scales, dark matter properties can be probed in several ways, including the abundance and distribution of satellite galaxies, and via the structural properties of galactic dark matter halos.

The structural properties of galactic dark matter halos can be constrained by rotation curve measurements of rotationally supported spiral and irregular galaxies, e.g. [2–5]. These measurements indicate that the dark matter halos around small dwarf irregular and low surface brightness galaxies can dominate the gravitational potential at all measured radii, e.g. [6–8]. The observed (\sim) linear rise of the rotational velocity in the inner region of such galaxies provides evidence that baryons must influence the dark matter structural properties, despite being only a small subcomponent of the mass. In addition, various empirical scaling relations, including the Tully-Fisher

relations [9,10], radial acceleration relation [11,12], and others e.g. [13,14], all support the notion that baryons influence dark matter structural properties.

This article concerns a type of self-interacting dark matter, with dissipative interactions, that has the potential to address these (and other) small scale structural issues. The prototype dissipative dark matter particle physics model assumes the existence of a hidden sector that contains a “dark proton” and a “dark electron,” coupled together via a massless “dark photon” [15]. Such dark matter is dissipative in the sense that it can cool via the emission of dark photons. The most theoretically constrained example of dissipative dark matter arises if the hidden sector is exactly isomorphic to the ordinary sector (see e.g. [16] for a review and detailed bibliography). This kind of dark matter has been called *mirror dark matter*, since the existence of an isomorphic hidden sector restores improper Lorentz symmetries, including space-time parity, as full symmetries of the Lagrangian describing fundamental particle interactions [17].

In dissipative dark matter models, the halo around rotationally supported galaxies takes the form of a dark plasma with long range interactions resulting in collective behavior. This type of halo dark matter can be modeled as a fluid governed by Euler’s equations, and if significant dark magnetic fields are present, by magnetohydrodynamics (MHD) equations. The parameter space of interest is one

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where dissipation plays an important role: in the absence of significant heating the dark matter halo would collapse to a disk on a time scale shorter than the current Hubble time. However, at the present epoch, such galactic dark matter can form an extended halo provided that significant heat source(s) exist. The only viable heating mechanism identified so far is from ordinary Type II supernovae (SN), which can provide heat sources for the dark sector if the kinetic mixing interaction exists [18–20].

Dark matter galaxy halos are dynamical in this picture, influenced by the dissipative cooling as well as supernova sourced heating. For a sufficiently isolated and unperturbed galaxy, the dark halo is expected to evolve until it reaches an approximate steady state configuration, where the halo is in hydrostatic equilibrium, and where heating and cooling rates locally balance. This means that the current structural properties of the dark matter halos around such isolated galaxies can be determined by the galaxy’s current baryonic properties, including the SN abundance and distribution. Knowledge of the past history of the galaxy is not essential in this picture.

In this article we aim to study the steady state solution for dissipative dark matter particle models, including mirror dark matter, as well as in more generic models. Previous work examined this problem, within the mirror dark matter context, considering only optically thin cooling, and neglected to fully deal with capture and line emission [21,22] (some work also looked at the optically thin case in more generic models [15,23–25]). Here, we aim to include all of the important cooling processes, and take into account halo reabsorption of cooling radiation. There are some remaining simplifying assumptions: spherical symmetry and negligible dark magnetic fields. We have also followed earlier work [21,22] and assumed that the SN sourced energy is transported to the halo via dark radiation. The alternative case, where the SN sourced energy is transmitted to the halo via local collisional processes in the SN vicinity, will be discussed in a separate paper.

For the parameters studied, we were unable to find physically realistic solutions for the constrained case of mirror dark matter halos. Halo cooling generally exceeds heating at realistic halo mass densities. This result prompts reexamination of the assumptions made; it also provides motivation to explore more generic dissipative models. One such generic model, which features steady state solutions with realistic halo mass densities, will be studied in some detail.

This article is structured as follows. In Sec. II we provide some background information on dissipative dark matter models, focusing mainly on aspects of these models relevant for understanding galaxy halo properties at the present epoch (the steady state solution of Euler’s equations). In Sec. III we discuss relevant halo properties: the ionization state and local heating and cooling rates. In Sec. IV we describe our method of solution of the steady state

equations, give our results for the dissipative particle physics models studied, and discuss. In Sec. V we conclude.

II. DISSIPATIVE DARK MATTER

The standard model provides a remarkable description of the known elementary particles and their fundamental interactions. There are no suitable dark matter candidates in the standard model, but this can easily be rectified by introducing a hidden sector. That is, a sector of additional particles and forces which couples to ordinary matter predominantly via gravity,

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{dark}} + \mathcal{L}_{\text{mix}}. \quad (1)$$

If the hidden sector features an unbroken $U(1)$ gauge symmetry, then there will be “dark electromagnetic” interactions among the dark sector particles mediated by a massless dark photon. This kind of dark matter can be dissipative in the sense that galactic dark matter halos can cool (in the absence of heating) on a time scale shorter than the current Hubble time.

A. Two-component dissipative dark matter

The prototype dissipative dark matter model consists of a dark sector with two $U(1)'$ charged hidden sector particles: a dark electron (e_d), and a dark proton (p_d). The interactions of these particles are governed by the Lagrangian,

$$\begin{aligned} \mathcal{L}_{\text{dark}} = & -\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \bar{e}_d (iD_\mu \gamma^\mu - m_{e_d}) e_d \\ & + \bar{p}_d (iD_\mu \gamma^\mu - m_{p_d}) p_d + \mathcal{L}_{\text{mix}}. \end{aligned} \quad (2)$$

Here, $F'_{\mu\nu} \equiv \partial_\mu A'_\nu - \partial_\nu A'_\mu$ is the field strength tensor associated with the dark $U(1)'$ gauge interaction, and A'_μ is the relevant gauge field. The dark electron and dark proton are described by the quantum fields e_d , p_d , and the covariant derivative is $D_\mu \equiv \partial_\mu + ig' Q' A'_\mu$ (where g' is the coupling constant associated with this gauge interaction). The Lagrangian [Eq. (2)] possesses dark lepton and dark baryon global $U(1)$ symmetries which ensure that the dark proton and dark electron are absolutely stable; this is analogous to the way global baryon and lepton symmetries ensure the stability of the electron and proton in the standard model.

The dark electron and dark proton are assumed to have $U(1)'$ charges opposite in sign, but their charge ratio $|Q'(p_d)/Q'(e_d)|$ is not necessarily unity; it is a fundamental parameter of the theory. The self-interactions of the dark electron can be defined in terms of the dark fine structure constant, $\alpha_d \equiv [g' Q'(e_d)]^2/4\pi$. A fundamental particle-antiparticle asymmetry is presumed to set the relic abundance of dark electrons and dark protons (that is, the relic abundance of dark antielectrons and dark antiprotons is negligible).

In addition to gravity, the dark sector interacts with the standard model particles via the kinetic mixing interaction involving the dark photon and the standard model hypercharge gauge boson [17,20],

$$\mathcal{L}_{\text{mix}} = \frac{\epsilon'}{2} F^{\mu\nu} F'_{\mu\nu}. \quad (3)$$

Here, $F^{\mu\nu}$ is the standard $U(1)_Y$ field strength tensor. This renormalizable gauge-invariant interaction, characterized by the dimensionless parameter ϵ' , leads to photon–dark photon kinetic mixing, which imbues the dark electron and dark proton with an ordinary electric charge: $-\epsilon e$ and $Z'\epsilon e$, where $Z' \equiv |Q'(p_d)/Q'(e_d)|$ [19]. (Note that the parameter ϵ , which is proportional to ϵ' , is conveniently taken as the fundamental parameter.) The new particle physics is fully described by the five fundamental parameters: $m_{e_d}, m_{p_d}, Z', \alpha_d$, and ϵ .

B. Mirror model

The standard model of particle physics has been very successful in describing the interactions of the known elementary particles. Indeed, the recent discovery of a Higgs-like scalar at the LHC [26,27] is the latest chapter in this remarkable story. An intriguing feature of the standard model is that the weak interaction violates improper Lorentz symmetries, including parity and time reversal. However, if the standard model is extended to include an exact “mirror” copy, that is, a duplicate set of matter particles and gauge bosons, labeled henceforth with a prime ($'$), then the improper Lorentz symmetries can be respected [17].

In terms of a fundamental Lagrangian, the standard model is extended with an exact copy,

$$\mathcal{L} = \mathcal{L}_{\text{SM}}(e, u, d, \gamma, \dots) + \mathcal{L}_{\text{SM}}(e', u', d', \gamma', \dots). \quad (4)$$

So far, no new fundamental parameters have been introduced. The elementary “mirror particles” have the same masses as their corresponding ordinary matter counterparts, and their gauge self-interactions have the same coupling strength as the ordinary matter gauge self-interactions. Since the mirror particles are described by a Lagrangian that is exactly analogous to that of the standard model, there will be an entire set of “mirror elements”: $H', \text{He}', \text{Li}', \text{Be}', \text{B}', \text{C}'$, etc., the properties of which will, of course, be completely analogous to the corresponding ordinary elements: H, He, Li, Be, B, C, etc.

The mirror sector particles are largely decoupled from the standard model particles, sharing only gravity and, possibly, additional interactions. Any additional interactions must respect the gauge and space-time symmetries (including the improper space-time symmetries), conditions that lead to only two possible renormalizable interactions. These are the Higgs–mirror Higgs portal

interaction ($\lambda_H \phi^\dagger \phi \phi'^\dagger \phi'$), and the kinetic mixing interaction of the form Eq. (3), where $F^{\mu\nu}$ and $F'_{\mu\nu}$ are the field strength tensors of the standard model $U(1)_Y$ and mirror $U(1)'_Y$ gauge fields. The effect of kinetic mixing is to embellish the mirror sector particles with a tiny ordinary electric charge: $Q = -\epsilon e$ for the mirror electron, e' , and $Q = \epsilon e$ for mirror proton, p' .

C. Dark matter as mirror matter

Mirror particles can be identified with the nonbaryonic dark matter in the Universe, e.g. [28–30]. On large scales, mirror dark matter closely resembles collisionless cold dark matter, e.g. successfully reproducing the cosmic microwave background (CMB) anisotropy spectrum [31–34]. On smaller scales, though, the effects of the self-interactions, and interactions with baryons via the kinetic mixing interaction, lead to very different phenomenology. Here, we provide a short overview of some relevant aspects of mirror dark matter. A more comprehensive review, including a more detailed bibliography, can be found in [16].

Early Universe cosmology of kinetically mixed mirror dark matter has been studied in a number of papers [35–37]. In the early Universe, during the radiation dominated epoch, the ordinary and mirror particles form two almost decoupled sectors, each described by distinct temperatures, T and T' . Successful big bang nucleosynthesis (BBN) limits the energy density of the dark sector so that a temperature asymmetry is required. An asymmetry is also needed to reproduce the CMB.¹ In fact, in the limit where $T' \ll T$, mirror dark matter behaves like collisionless cold dark matter as far as BBN and CMB are concerned. It is well known that collisionless cold dark matter can fit the measured CMB anisotropy spectrum, with $\Omega_{dm} \simeq 5.4\Omega_b$ obtained from the analysis of PLANCK data [1] (Ω is the usual normalized cosmic energy density parameter). These considerations motivate the effective initial conditions at the BBN epoch,²

$$T' \ll T, \quad \Omega_{b'} \simeq 5.4\Omega_b. \quad (5)$$

In the presence of nonzero kinetic mixing, entropy can be transferred from the ordinary sector to the mirror particles, a process driven mainly by the particle interaction: $\bar{e}e \rightarrow \bar{e}'e'$. This entropy transfer ceases to be important for temperatures below the kinematic threshold, $T \lesssim m_e$, and T'/T asymptotes to [36,37]

¹The origin of the temperature asymmetry between the ordinary and mirror particles is unknown, but may potentially arise in chaotic inflation models [29,38,39].

²We assume that mirror dark matter comprises all of the dark matter in the Universe. Hybrid dark matter models, where a subdominant dissipative component is mixed with a dominant collisionless component, have been discussed in e.g. [40–42].

$$T'/T \simeq 0.31 \sqrt{\epsilon/10^{-9}}. \quad (6)$$

The nonzero value of T'/T appears to be rather important for the evolution of small scale structure. Prior to mirror hydrogen recombination (at $T' \sim 1$ eV), the growth of mirror dark matter density perturbations is impacted by dark acoustic oscillations and dark photon diffusion. These effects can only be important for density perturbations characterized by length scales less than the sound horizon at that time.

The effect of this dark sector physics is to severely suppress power on small scales. This is somewhat analogous to the situation with warm dark matter, although the physical origin of the suppression involves very different physics. This suppression of power on small scales can provide a simple explanation [43] for the observed deficit of satellite galaxies [44,45], and potentially also a similar (albeit more modest) deficit observed for small field galaxies [46–48]. Matching the relevant scales leads to a rough estimate of the fundamental kinetic mixing parameter: $\epsilon \sim 1\text{--}4 \times 10^{-10}$ [43].

In this picture, very small scale perturbations are exponentially suppressed, so much so, that the smallest observable galaxies could only have formed “top-down”; that is, they arose out of the collapse of larger density perturbations. If one contemplates the evolution of a galaxy mass scale perturbation, then collapse occurs when the mean overdensity of such a perturbation reaches a critical value, $\delta \sim 1$. During the nonlinear collapse process, the dissipative dark matter is envisaged to form a disklike structure. The collapse is not expected to be completely uniform, perturbations at the edge of the dark disk could potentially break off and seed the formation of small satellite galaxies. In such a formation scenario, the satellite galaxies would have a planar and corotating distribution, consistent, perhaps, with the properties of the observed satellites of the Milky Way [49] and Andromeda [50]. Meanwhile, the bulk of the dark disk of the host galaxy might conceivably take the form of a diffuse gas of dark sector particles. It is envisaged that this dark disk gas component would eventually disrupt due to the heating from ordinary supernovae (facilitated by the kinetic mixing interaction, to be discussed), and ultimately expand to form a roughly spherical dark plasma halo.

Dark matter, if dissipative, might arise from a more generic hidden sector as opposed to the rather theoretically constrained case of mirror dark matter. The two-component dissipative model, reviewed in Sec. II. A, is one such scenario. That specific model has been examined in some detail in [15] (see also [43]), where some constraints on the fundamental parameters were derived. Importantly, the picture sketched above readily generalizes to this more generic case, and thus it remains a prime candidate for dark matter that is able to explain structure on large scales and, potentially, also on small scales.

III. GALAXY STRUCTURE

The dark matter halo around rotationally supported galaxies is envisaged to be a dissipative plasma.³ The bulk properties of such a plasma can be modeled as a fluid governed by Euler’s equations of fluid dynamics (and MHD equations if dark magnetic fields play an important role). These fluid equations take the form

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0, \\ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} &= - \left(\nabla \phi + \frac{\nabla P}{\rho} \right), \\ \frac{\partial}{\partial t} \left[\rho \left(\frac{\mathbf{v}^2}{2} + \mathcal{E} \right) \right] \\ &+ \nabla \cdot \left[\rho \left(\frac{\mathbf{v}^2}{2} + \frac{P}{\rho} + \mathcal{E} \right) \mathbf{v} \right] - \rho \mathbf{v} \cdot \nabla \phi = \mathcal{H} - \mathcal{C}. \end{aligned} \quad (7)$$

Here P , ρ , and \mathbf{v} , denote the pressure, mass density, and velocity of the fluid, and ϕ is the gravitational potential. \mathcal{E} is the internal energy per unit mass of the fluid, so that $\rho(\mathbf{v}^2/2 + \mathcal{E})$ is the energy per unit volume. Finally, \mathcal{H} and \mathcal{C} are the local heating and cooling rates per unit volume.

Significant simplifications occur if the system evolves to a steady state configuration; the time derivatives vanish, and assuming there is no steady state velocity flow, the system reduces to just two equations,

$$\begin{aligned} \nabla P &= -\rho \nabla \phi, \\ \mathcal{H} &= \mathcal{C}. \end{aligned} \quad (8)$$

These equations need to be satisfied at every location in the halo. We shall assume that the steady state configuration is the current physical state of rotationally supported galaxies that are sufficiently isolated and have stable star formation rates. Naturally, it would be useful to solve the full system of time-dependent fluid equations to examine the evolutionary history and thereby check the consistency of this picture. This would surely require many details about the galaxy’s properties and history, etc., but nevertheless could be attempted.

The halo density and temperature profiles for which the steady state conditions [Eqs. (8)] are satisfied represent a steady state solution. If this solution is unique, then the current halo properties are dictated in a large measure by the baryonic properties as the halo heating is sourced by ordinary Type II supernovae (to be discussed). This makes the dissipative dynamics highly predictive. Moreover, the tight coupling between the physical properties of the halo and the galactic baryon content can potentially address

³The dark matter halo around elliptical and dwarf spheroidal galaxies is expected to have very different physical properties in this picture. See [15,16] for relevant discussions.

long-standing indications of such a connection, e.g. [11–14,51]. Previous work in this direction [15,21–25] offers some encouragement that such a picture might lead to successful phenomenology.

In this paper we endeavor to solve Eqs. (8) to find steady state solutions for mirror dark matter, as well as for the more generic dissipative dark matter model of Sec. II A. We include all the major cooling processes and take into account halo reabsorption of cooling radiation as the optically thin approximation is not always valid. In fact, the wavelength-dependent finite optical depth will be taken into consideration for all dark radiation sources, i.e. heating as well as cooling. It turns out that the equations are somewhat non-trivial, and we do make the simplifying assumption of spherical symmetry. While there is reason to suppose that an approximately spherically symmetric halo would form at large distances [15], departures from spherical symmetry are anticipated in the inner regions of galaxies. Nevertheless, we expect (as will be discussed) that such departures from spherical symmetry are not of critical importance. Naturally, a more sophisticated analysis, without the spherical symmetry assumption, could be undertaken following essentially the same procedure as developed here.

The baryons contribute to the gravitational potential, and their distribution is certainly not spherical. In spiral galaxies the stellar distribution can be modeled as an azimuthally symmetric disk with surface density [52]

$$\Sigma(r) = m \frac{e^{-r/r_D}}{2\pi r_D^2}, \quad (9)$$

where m is the mass of the disk and r_D is the disk scale length. To have a mathematically consistent description we instead adopt a spherically symmetric distribution for the baryons, with density defined by $\int_0^r \rho_{\text{baryons}}^{\text{stars}}(r') 4\pi r'^2 dr' = \int_0^r \Sigma(r') 2\pi r' dr'$, i.e.

$$\rho_{\text{baryons}}^{\text{stars}}(r) = m_* \frac{e^{-r/r_D}}{4\pi r_D^2 r}. \quad (10)$$

Here, m_* is the stellar mass parametrized in terms of a stellar mass fraction: $m_* = f_s m_{\text{baryons}}$. In addition to stars, there is also a baryonic gas component—which generally features a more spatially extended distribution. We model the gas density, $\rho_{\text{baryons}}^{\text{gas}}(r)$, with an exponential profile of the form Eq. (10), but with $r_D^{\text{gas}} = 3r_D$ and total mass $m_{\text{gas}} = (1 - f_s) m_{\text{baryons}}$.

The first equation in Eqs. (8), the hydrostatic equilibrium condition, relates the dark matter fluid density and temperature. If we assume that all particle species are in local thermodynamic equilibrium at a common temperature T , then the fluid pressure is $P = \rho T / \bar{m}$, where $\bar{m} \equiv \sum n_i m_i / \sum n_i$ is the mean mass of the particle species. (Each of these quantities is, of course, location dependent.)

For a spherically symmetric system, the hydrostatic equilibrium condition reduces to⁴

$$\frac{\partial T}{\partial r} = -\frac{T}{\rho} \frac{\partial \rho}{\partial r} + \frac{T}{\bar{m}} \frac{\partial \bar{m}}{\partial r} - \bar{m} \nabla \phi, \quad (11)$$

where the gravitational acceleration is

$$\nabla \phi = \frac{G_N}{r^2} \int_0^r [\rho(r') + \rho_{\text{baryons}}(r')] 4\pi r'^2 dr'. \quad (12)$$

Here, G_N is Newton’s constant. Notice that we have included only the dark matter fluid density, ρ , and the stellar, gas baryon components.⁵ For a given fluid density $[\rho(r)]$ and composition $[\bar{m}(r)]$, the hydrostatic equilibrium condition can be solved for the temperature profile if a boundary condition is specified. In our numerical work, we take $dT/dr = 0$ at $r = 20r_D$. (The results in the physical region of interest, $r \lesssim 6r_D$, are quite insensitive to the boundary condition and its location, so long as the boundary is sufficiently distant.)

To proceed further, we need to evaluate the heating and cooling rates; also required is the ionization state of the halo. We first evaluate these equations for a mirror dark matter halo; the modifications necessary for the generic dissipative dark matter model will be subsequently indicated. Since the heating and cooling rates depend on the ionization state, and vice versa, an iterative method will then be needed to solve this system of equations; one such method will be discussed in Sec. IV.

A. Ionization state of the halo

In the mirror dark matter scenario, the halo is a multi-component plasma comprising a set of elements with varying degrees of ionization. To simplify the discussion, we shall, on occasion, adopt the notation “electron” for “mirror electron,” “photon” for “mirror photon,” etc. Since the discussion of the mirror dark matter plasma exclusively comprises mirror particles with exactly analogous particle properties to ordinary matter, no confusion need arise. We shall label the mirror elements and their ionization state with the notation: A^k , where $A = \text{H}', \text{He}', \text{C}', \text{O}', \text{Ne}'$, etc., and $k = 0, 1, \dots$, $Z(A)$ represents the number of bound electrons present. [$Z(A)$ is the nuclear charge, i.e. $Z(\text{H}') = 1, Z(\text{He}') = 2, Z(\text{C}') = 6$, etc.]

The ionization state in a local region of interest is determined by the balancing of electron capture against the ionization processes:

⁴Natural units with $\hbar = c = k_B = 1$ are used unless otherwise indicated.

⁵In addition to the diffuse dissipative fluid component, there can also be clumped dark matter objects, “dark stars.” In the analysis of this paper, such a component is presumed to be subdominant and is, for simplicity, neglected.

$$\begin{aligned}
A^k + e' &\rightarrow A^{k+1} + \gamma' && \text{Capture (free-bound transition),} \\
A^k + e' &\rightarrow A^{k-1} + e' + e' && \text{Electron impact ionization,} \\
A^k + \gamma' &\rightarrow A^{k-1} + e' && \text{Photoionization.}
\end{aligned} \tag{13}$$

The cross sections for these processes will be denoted by $\sigma[A^k]_{fb}$, $\sigma[A^k]_I$, and $\sigma[A^k]_{PI}$, respectively. We introduce the notation f_{A^k} for the fraction of A states with k bound electrons present. That is, $n_{A^k} = f_{A^k} n_A$, where n_A is the number density of all A states. All these quantities are, of course, location dependent. At any given location, the rate of change of n_{A^0} is

$$\begin{aligned}
\frac{dn_{A^0}}{dt} &= -n_{A^0} \int \frac{dn_{e'}}{dE_e} \sigma[A^0]_{fb} v_e dE_e \\
&+ n_{A^1} \left[\int \frac{dn_{e'}}{dE_e} \sigma[A^1]_I v_e dE_e + \int \frac{dF}{dE_\gamma} \sigma[A^1]_{PI} dE_\gamma \right].
\end{aligned} \tag{14}$$

Here, dF/dE_γ is the mirror photon flux at the location of interest, $dn_{e'}/dE_e$ is the local mirror electron energy distribution, and $v_e = \sqrt{2E_e/m_e}$ is the mirror electron velocity. In the steady state limit, $dn_{A^0}/dt \rightarrow 0$, and one finds

$$f_{A^1} = \frac{\int \frac{dn_{e'}}{dE_e} \sigma[A^0]_{fb} v_e dE_e}{\int \frac{dn_{e'}}{dE_e} \sigma[A^1]_I v_e dE_e + \int \frac{dF}{dE_\gamma} \sigma[A^1]_{PI} dE_\gamma} f_{A^0}. \tag{15}$$

More generally, using $dn_{A^k}/dt = 0$, one can deduce

$$f_{A^{k+1}} = \frac{\int \frac{dn_{e'}}{dE_e} \sigma[A^k]_{fb} v_e dE_e}{\int \frac{dn_{e'}}{dE_e} \sigma[A^{k+1}]_I v_e dE_e + \int \frac{dF}{dE_\gamma} \sigma[A^{k+1}]_{PI} dE_\gamma} f_{A^k}. \tag{16}$$

This equation, together with $\sum_k f_{A^k} = 1$, determines the ionization state at a given location in terms of the mirror electron distribution, the mirror photon flux, and the relevant cross sections. We now discuss each of these three quantities in turn.

The electron distribution will be assumed to be Maxwellian,

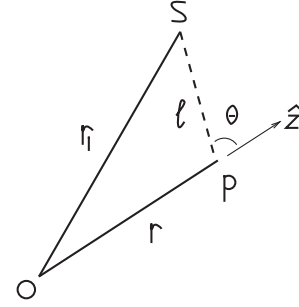
$$\frac{dn_{e'}}{dE_e} = n_{e'} \frac{2}{T} \sqrt{\frac{E_e}{\pi T}} e^{-E_e/T}. \tag{17}$$

This is an important simplification. In general, significant departures from a Maxwellian distribution can occur in the low density plasma environment from a variety of (typically) complex processes. One such process arises due to the halo heating mechanism assumed. As will be discussed in more detail in Sec. III C, the kinetic mixing interaction transforms ordinary supernovae into powerful heat sources.

Supernovae generate energetic mirror photons which are absorbed in the halo via the photoionization process. The ejected mirror electron resulting from photoionization can be very energetic, and it thermalizes primarily by scattering off free and bound mirror electrons in the plasma. The ionization due to such nonthermal scattering off bound mirror electrons is neglected in our analysis, but could be important at low halo temperatures where there are relatively few free mirror electrons.

The mirror photon flux originates from several sources: supernovae, line emission, capture, and bremsstrahlung. In a given volume element, dV , the differential luminosity of photons that arises from all of these sources combined will be denoted as $d\mathcal{L}^S/dVdE_\gamma$. To calculate the flux of photons at a particular location in the halo we must integrate this luminosity over all possible source locations and take into account reabsorption processes.

Within a spherical coordinate system with origin at the galaxy center, O , consider a particular halo location of interest, P , at \mathbf{r} in this coordinate system (see diagram). Some of the photons which arrive at P propagate from a source location, S , at \mathbf{r}_1 . It is



convenient to define a second spherical coordinate system (with coordinates ℓ, θ, ϕ) with origin now at P (and with the z axis in the direction of \mathbf{r}). The photon flux at P (i.e. at the origin of the second spherical coordinate system) can be found by integrating over all source locations. Taking into account the absorption along the photon path from S to P , we have

$$\begin{aligned}
\frac{dF(\mathbf{r})}{dE_\gamma} &= \int \int \frac{1}{E_\gamma} \frac{d\mathcal{L}^S(r_1)}{dVdE_\gamma} \frac{e^{-\tau}}{4\pi\ell^2} 2\pi\ell^2 d\cos\theta d\ell \\
&= \int \int \frac{e^{-\tau}}{2E_\gamma} \frac{d\mathcal{L}^S(r_1)}{dVdE_\gamma} d\cos\theta d\ell,
\end{aligned} \tag{18}$$

where $r_1 = \sqrt{r^2 + \ell^2 + 2r\ell \cos\theta}$ follows from this geometry, and we have set $r_1 = |\mathbf{r}_1|$, $r = |\mathbf{r}|$, etc. The optical depth, $\tau = \tau(\ell, \theta, E_\gamma)$, is given by

$$\tau(\ell, \theta, E_\gamma) = \sum_{A,k} \int_0^\ell \sigma[A^k]_{PI} n_{A^k}(\mathbf{r}, \ell_1, \theta) d\ell_1, \tag{19}$$

where $n_{A^k}(\mathbf{r}, \ell_1, \theta) \equiv n_{A^k}(r')$ with $r' = \sqrt{r^2 + \ell_1^2 + 2r\ell_1 \cos\theta}$. The photoionization cross

section in Eq. (19) is also a function of r, ℓ_1, θ , because it depends on the temperature along the path: $T(r')$. In deriving Eq. (18) azimuthal symmetry has been used to perform the trivial ϕ integration. (Azimuthal symmetry in the coordinate system with origin at the point of interest, P , follows from spherical symmetry in the coordinate system with origin at the Galactic center.)

We now consider the cross sections. In addition to the $\sigma[A^k]_{fb}$, $\sigma[A^k]_I$, and $\sigma[A^k]_{PI}$ cross sections, we will also need the electron excitation cross section, $\sigma[A^k]_{nl'n'l'}$, and the flux of bremsstrahlung photons. For the electron excitation process, we made extensive use of the cross sections calculated by Group T-4 of the Los Alamos National Laboratory (LANL), publicly available via their online web interface [53]. The LANL code is based on the method of Mann [54], and calculated using the first order many body theory. The LANL code also makes use of the Hartree-Fock method of Cowan [55], developed at Group T-4 of the Los Alamos National Laboratory.

Our numerical work used the LANL cross sections for the electron excitation process, with a total of around ~ 300 of the most important $nl \rightarrow n'l'$ transitions considered. For the generic dark matter model, these cross sections will need to be scaled to investigate parameters m_{e_d}, α_d that are different from m_e, α . To understand the appropriate scaling with respect to these parameters an analytic form for the electron excitation cross section is also useful. For hydrogenlike ions, Van Regemorter [56] calculated the cross section for electron impact excitation $nl \rightarrow n'l'$ (with excitation energy $E_{nl'n'l'}$) in the Bethe approximation,

$$\sigma[A^k]_{nl'n'l'} = \pi a_0^2 \frac{8\pi f_{nl'n'l'}}{\sqrt{3}} \left[\frac{Ry}{E_{nl'n'l'}} \right]^2 \frac{G(x)}{x} \Theta(E_e - E_{nl'n'l'}), \quad (20)$$

where $\Theta(y)$ is the Heaviside step function. Also, $a_0 \equiv 1/(m_e \alpha)$, $f_{nl'n'l'}$ is the absorption oscillator strength, $Ry \equiv \alpha^2 m_e / 2$, $x \equiv E_e / E_{nl'n'l'}$, and $G(x)$ is the effective Gaunt factor of order unity.

For electron impact ionization ($A^k + e' \rightarrow A^{k-1} + e' + e'$) we have adopted the Lotz formula [57],

$$\sigma[A^k]_I = \sum_i \frac{0.62 \alpha^2 \pi}{E_e I_i} \ln \left(\frac{E_e}{I_i} \right) \Theta(E_e - I_i), \quad (21)$$

where I_i denotes the ionization energies of the k bound electrons ($i = 1, \dots, k$) in the A^k ion. The set of ionization energies, I_i , for each A^k ion were acquired from the LANL web interface [53].

For photoionization, also called bound-free transition ($A^k + \gamma' \rightarrow A^{k-1} + e'$), we used the Karzas and Latter result [58,59],

$$\sigma[A^k]_{PI} = \sum_i \frac{32\pi \alpha I_i^2 g_{bf}}{3\sqrt{3} m_e E_\gamma^3 n_i} \Theta(E_\gamma - I_i). \quad (22)$$

Here, the index $[i]$ represents the state of the electron prior to its ejection from the atom, n_i is its principal quantum number, and I_i is the ionization energy. The sum runs over all bound electrons [i.e. from $i = 1, \dots, k$]. Near threshold, the Gaunt factor is unity to within 20%, and we set $g_{bf} = 1$ in the numerical work.

For electron capture, also known as free-bound transition ($A^k + e' \rightarrow A^{k+1} + \gamma'$), we used the modified Kramers formula [60],

$$\sigma[A^k]_{fb} = \sum_i \frac{16\pi \alpha I_i^2 w_i g_{fb}}{3\sqrt{3} m_e^2 (E_e + I_i) E_e n_i}, \quad (23)$$

where w_i is the number of unoccupied states in the n_i shell of the ion before recombination. In the numerical work, we considered capture to the valence shell along with the next higher shell ($n_i = n_{\text{valence}}, n_i = n_{\text{valence}} + 1$). Again, we set the Gaunt factor g_{fb} to unity.

The discussion above was relevant for mirror dark matter. The situation with more generic dissipative models is quite analogous. For the two-component model of Sec. II. A, each particle process, capture, excitation, ionization, etc., has an analogue with the obvious replacements: $e' \rightarrow e_d, A^k \rightarrow p_d^k, \gamma' \rightarrow \gamma_d$. The cross sections for these processes are defined in terms of a new set of fundamental parameters: $m_{e_d}, m_{p_d}, \alpha_d, Z', \epsilon$. We need to determine how the relevant cross sections depend on these parameters.

Actually, if $m_{e_d} \ll m_{p_d}$, then to a very good approximation the relevant cross sections depend only on m_{e_d}, α_d, Z' . If Z' is an integer then we can choose the corresponding element A with $Z' = Z(A)$ (e.g. for $Z' = 6$ we take $A = C$). With the element A chosen, the mirror dark matter A^k cross sections can be scaled to take into account values of m_{e_d}, α_d different from m_e, α . The result is the scaling

$$\begin{aligned} \{\sigma[A^k]_{nl'n'l'}, E_e\} &\rightarrow \left\{ \sigma[A^k]_{nl'n'l'} \frac{\alpha^2 m_e^2}{\alpha_d^2 m_{e_d}^2}, E_e \frac{\alpha_d^2 m_{e_d}}{\alpha^2 m_e} \right\}, \\ \{\sigma[A^k]_I, E_e\} &\rightarrow \left\{ \sigma[A^k]_I \frac{\alpha^2 m_e^2}{\alpha_d^2 m_{e_d}^2}, E_e \frac{\alpha_d^2 m_{e_d}}{\alpha^2 m_e} \right\}, \\ \{\sigma[A^k]_{PI}, E_\gamma\} &\rightarrow \left\{ \sigma[A^k]_{PI} \frac{\alpha m_e^2}{\alpha_d m_{e_d}^2}, E_\gamma \frac{\alpha_d m_{e_d}}{\alpha m_e} \right\}, \\ \{\sigma[A^k]_{fb}, E_e\} &\rightarrow \left\{ \sigma[A^k]_{fb} \frac{\alpha_d m_e^2}{\alpha m_{e_d}^2}, E_e \frac{\alpha_d^2 m_{e_d}}{\alpha^2 m_e} \right\}. \end{aligned} \quad (24)$$

The ionization energies also scale,

$$I_i \rightarrow I_i \frac{\alpha_d^2 m_{e_d}}{\alpha^2 m_e}. \quad (25)$$

For the bremsstrahlung process, we only need to know how the differential cooling rate scales with m_{e_d} , α_d , which can be gleaned from the explicit expression for this rate given in the following subsection.

B. Cooling rates

There are three sources of cooling that need to be taken into account: Line emission, capture, and bremsstrahlung. In addition, conduction and convection processes can also contribute to the local cooling/heating rates. These processes could be important if significant temperature gradients exist. It turns out that the halo temperature profile derived from the steady state conditions is close to isothermal, so that neglect of these processes might be justifiable. In any case, conduction/convection processes will not be included in the analysis presented here.

For thermal bremsstrahlung, also called free-free emission, we follow the classical treatment of [59]. The differential rate of energy radiated per unit volume due to electron scattering off ions of charge Z_i , assuming a Maxwellian electron velocity distribution, is

$$\frac{dC_{ff}}{dE_\gamma} = \frac{16\alpha^3}{3m_e} \left(\frac{2\pi}{3m_e T} \right)^{1/2} Z_i^2 n_i n_e e^{-E_\gamma/T} \bar{g}_{ff}. \quad (26)$$

Here, \bar{g}_{ff} is the velocity averaged Gaunt factor, which can be approximated by the simple analytic expression [59]

$$\bar{g}_{ff} = \begin{cases} \frac{\sqrt{3}}{\pi} \ln \left[\frac{4}{\xi} \frac{T}{E_\gamma} \right] & \text{for } E_\gamma < T, \\ \frac{\sqrt{3T}}{\pi E_\gamma} & \text{for } E_\gamma > T. \end{cases} \quad (27)$$

Here $\xi \simeq 1.781$ is Euler's constant. This simple analytic form for the Gaunt factor is known to be valid for $T \gtrsim Z^2 R_y$, where $R_y = 13.6$ eV. For small dwarf galaxies where $T \lesssim Z^2 R_y$, the Gaunt factor is less accurate but still provides a reasonable estimate for our purposes, especially as the bremsstrahlung cooling rate in small galaxies turns out to be much smaller than the other cooling processes.

The differential rate of energy radiated due to electron capture by an ion, A^k , is

$$\frac{dC_{fb}}{dE_\gamma} = n_{A^k} \frac{dn_{e'}}{dE_e} v_e \sigma[A^k]_{fb} E_e E_\gamma. \quad (28)$$

Note that energy conservation implies $E_\gamma = E_e + I_i$. Assuming that the electron's energy distribution is Maxwellian, we have

$$\frac{dC_{fb}}{dE_\gamma} = 2 \sqrt{\frac{2}{m_e \pi}} \left(\frac{1}{T} \right)^{3/2} e^{-E_e/T} \sigma[A^k]_{fb} E_e E_\gamma. \quad (29)$$

Energy is also radiated from line emission. Electrons can scatter off a bound electron in an ion, A^k , leading to the atomic transition $nl \rightarrow n'l'$ (with excitation energy $E_{nl'n'l'}$). The resulting energy radiated following deexcitation is

$$\frac{dC_{lines}}{dE_\gamma} = \sum n_e n_{A^k} \langle \sigma[A^k]_{nl'n'l'} v_e \rangle \delta(E_\gamma - E_{nl'n'l'}) E_{nl'n'l'}, \quad (30)$$

where the sum runs over the nl quantum numbers that correspond to each of the k bound electrons, and all possible $n'l'$ quantum numbers of the atomic excitations (and also over all A^k ions). For a Maxwellian electron velocity distribution,

$$\langle \sigma[A^k]_{nl'n'l'} v_e \rangle = 2 \sqrt{\frac{2}{m_e \pi}} \left(\frac{1}{T} \right)^{3/2} \times \int_{E_{nl'n'l'}}^{\infty} \sigma[A^k]_{nl'n'l'} e^{-E_e/T} E_e dE_e. \quad (31)$$

The differential rate of radiation energy loss per unit volume at a location P (at position \mathbf{r}) in the halo is the sum of these three contributions,

$$\frac{dC(\mathbf{r})}{dE_\gamma} = \frac{dC_{ff}(\mathbf{r})}{dE_\gamma} + \frac{dC_{fb}(\mathbf{r})}{dE_\gamma} + \frac{dC_{lines}(\mathbf{r})}{dE_\gamma}. \quad (32)$$

These dark photons, together with those originating from Type II supernovae (to be considered in more detail in the following subsection), contribute to the differential source flux, $d\mathcal{L}^S/dVdE_\gamma$, which influences the ionization state of the halo [Eqs. (18) and (16)]. Some of these cooling photons will be reabsorbed and also affect the heating rate, \mathcal{H} (to be discussed shortly).

The cooling rate will depend on the relative abundances of the various mirror elements. Mirror BBN calculations with the initial conditions of Eq. (5) and with $\epsilon \sim 2 \times 10^{-10}$ (as suggested by the observed deficit of satellite galaxies [43]) have concluded that the primordial mirror helium abundance dominates over mirror hydrogen, consistent with general arguments [31], with the helium mass fraction $Y'_p \approx 0.95$ [61] (see also Fig. 3.4 of [16]). Heavier mirror elements are expected to be synthesized in mirror stars at an early epoch; cf. [62]. Unfortunately, the detailed chemical composition of the mirror sector resulting from stellar evolution at the early epoch is rather difficult to surmise: It involves unfamiliar initial conditions, chemical composition, etc., including unknown quantities such as initial mass function. We shall assume for simplicity that the composition of the mirror metal component is the same as the

TABLE I. Solar abundances ($\zeta = 0$) from [63] and modified abundances.

Element	$\log(n/n_{\text{H}})$ (solar/modified)
He	-1.01/0.68
C	-3.44 + ζ
O	-3.07 + ζ
Ne	-3.91 + ζ
Si	-4.45 + ζ
Fe	-4.33 + ζ

solar abundance of the corresponding ordinary elements, but allow for an overall scale factor ζ for the metal component. Naturally, modifications of the relative abundances of the various elements could be looked at it. In Table I the standard solar abundances of the ordinary elements are given, along with modified abundances incorporating the higher primordial mirror helium abundance ($Y'_p \approx 0.95$, suggested by mirror BBN calculations, translates to $\log[n_{\text{He}}/n_{\text{H}}] \simeq 0.68$). The parameter ζ allows adjustment of the mirror metal fraction, and we consider a wide range in our numerical work ($-2.0 < \zeta < 2.0$).

As a check of our code, we have computed the cooling function, $\Lambda_N \equiv \mathcal{C}/(n_e n_i)$ (where $n_i \equiv \sum_A n_A$ is the total number density of mirror ions), for the idealized case of a low density optically thin plasma with ionization dominated by electron impact. In this circumstance the ionization state and cooling function depend only on the local temperature. Adopting solar abundances, but choosing $\zeta = 0.1$ to compensate for the restriction of just five metal components, we found the cooling function shown in Fig. 1. This cooling function compares reasonably well with more accurate results found in the literature, such as the result of Dopita and Sutherland [64]. In the numerical work to follow we use the modified abundances, which

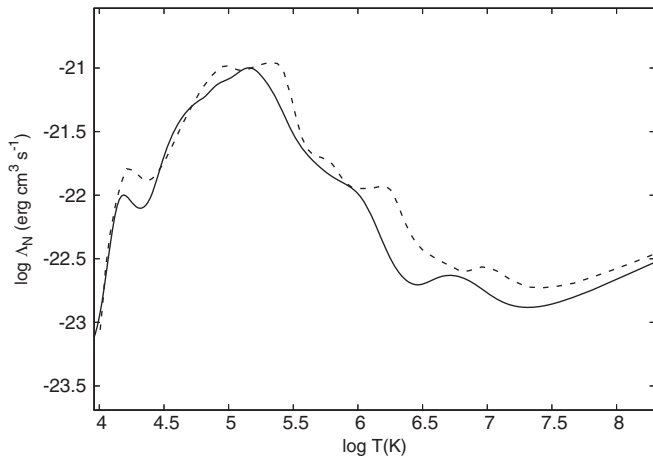


FIG. 1. Optically thin cooling function for solar abundances. The solid line is the result of our code, while the dashed line is the result found by Dopita and Sutherland [64].

take into account the $\log[n_{\text{He}}/n_{\text{H}}] \approx 0.68$ estimated from mirror BBN.

C. Heating rates

The dissipative dark matter halos not only cool but also are heated, with the heat source originating from ordinary Type II supernovae. The assumed mechanism requires a significant fraction of a supernova's core collapse energy to be converted, ultimately, into dark radiation. The kinetic mixing interaction plays an important role as it is responsible for the transfer of energy to the mirror sector. At the particle physics level, kinetic mixing imbues the mirror electron and mirror positron with a tiny ordinary electric charge of magnitude ϵe . This enables particle processes leading to the production of light mirror particles: e' , \bar{e}' , γ' , to readily occur in the hot dense core of ordinary supernovae (e.g. plasmon decay to $e'\bar{e}'$, $e\bar{e} \rightarrow e'\bar{e}'$, $e'\bar{e}' \rightarrow \gamma'\gamma'$, etc.). The light mirror particles interact weakly enough with ordinary matter so that they can escape from the supernova core and also from the collapsing star.⁶

The rate at which the core collapse energy is transferred to light mirror particles can be estimated from [67,68]. This energy loss rate is given by

$$Q_P = \frac{8\zeta_3}{9\pi^3} \epsilon^2 \alpha^2 \left(\mu_e^2 + \frac{\pi^2 T_{\text{SN}}^2}{3} \right) T_{\text{SN}}^3 Q_1, \quad (33)$$

where Q_1 is a factor of order unity, μ_e is the electron chemical potential, and $T_{\text{SN}} \sim 30$ MeV is the temperature of the supernova core. The observation of around a dozen neutrino interactions associated with SN1987A [69,70] suggests that Q_P should not exceed the energy loss rate due to neutrino emission. This indicates a rough upper limit on ϵ of around $\epsilon \lesssim 10^{-9}$ [67].

Supernovae can provide a rather substantial energy source if ϵ does indeed have a value near this upper limit. This energy is initially distributed among the various light mirror particles: e' , \bar{e}' , γ' (potentially also some fraction in ν'). These particles, injected into the region around a supernova, would undergo a variety of complex processes, shocks, etc. In this paper we follow previous work [21,22] and assume that the bulk of this energy is (ultimately) converted into γ' emission. These mirror photons, with total energy up to around half the supernova core collapse energy ($\sim 10^{53}$ erg per supernova), can propagate out into the halo. These photons can be absorbed via the dark photoionization process. The key idea is that such

⁶The interactions of the escaping mirror particles with the baryonic matter, though quite weak, could still transfer a substantial amount of energy to the baryons. It has been speculated [65] that this mechanism might facilitate the explosion of a supernova, as the transfer of energy via the escaping neutrinos may be inadequate, although there is still much debate in the literature [66].

mirror-photon heating, powered by ordinary supernovae, can replace the energy dissipated in the halo due to the various cooling processes.

If ordinary supernovae are the source of the heating of the halo, then we will need to know their rate (R_{SN}) and spatial distribution in a given galaxy. Supernovae are the final evolutionary stage of large stars. Ultraviolet radiation is directly emitted from the photospheres of large stars with $M_* \gtrsim 3m_{\odot}$ (O through later-type B stars). Galactic UV flux measurements, such as those taken by the Galaxy Evolution Explorer (GALEX) satellite [71], can therefore be used to probe the recent star formation rate over a time scale ~ 100 Myr e.g. [72–74]. It follows that a galaxy’s far ultraviolet (FUV) luminosity, L_{FUV} (taken here to be the far UV bandpass of 1350–1750 Å), should provide an estimate of the current supernova rate. That is, we expect the rough scaling: $R_{\text{SN}} \propto L_{\text{FUV}}$. Using $L_{\text{FUV}} \propto 10^{-0.4M_{\text{FUV}}}$, where M_{FUV} is the galaxy’s FUV absolute magnitude, we therefore expect

$$R_{\text{SN}} \approx R_{\text{SN}}^{\text{MW}} \frac{10^{-0.4M_{\text{FUV}}}}{10^{-0.4M_{\text{FUV}}^{\text{MW}}}}. \quad (34)$$

Here, $M_{\text{FUV}}^{\text{MW}}$ is the FUV absolute magnitude for the Milky Way, and $R_{\text{SN}}^{\text{MW}} \sim 10^{-9} \text{ s}^{-1}$ is the Type II supernova rate in the Milky Way. (We take $M_{\text{FUV}}^{\text{MW}} = -18.4$ in the numerical work.)

Supernovae become a powerful source of mirror photons with uncertain spectrum and total energy. We denote the average dark photon luminosity of a single supernova by L_{SN} . The frequency spectrum will be modeled, for simplicity, by a thermal distribution,

$$\frac{dL_{\text{SN}}}{dE_{\gamma}} = \frac{15}{\pi^4 T_{\text{eff}}^4} \frac{E_{\gamma}^3}{e^{E_{\gamma}/T_{\text{eff}}} - 1} L_{\text{SN}}. \quad (35)$$

The relevant effective temperature parameter, T_{eff} , is not known. One would need to be able to model the complex processes in the expanding e' , \bar{e}' , γ' plasma around a supernova. In the absence of such modeling, we consider a wide range of potential T_{eff} values.

A thermal distribution for the supernova sourced spectrum of dark photons is very convenient, but may be a poor representation of the actual spectrum. In fact, this system may have some similarities with gamma ray bursts, which are very complex, and are seldom thermal. Modeling the system with an alternative distribution, e.g. a power law, would appear to be equally valid given this state of ignorance.

Consider now the supernova spatial distribution within a given galaxy. For rotationally supported galaxies, this distribution could be modeled as a Freeman disk (located at $\theta = \pi/2$ in the spherical coordinate system with origin at the Galactic center), so that the differential source luminosity of SN dark photons takes the form

$$\frac{d\mathcal{L}_{\text{SN}}^S(\mathbf{r})}{dV dE_{\gamma}} = \frac{e^{-r/r_D}}{2\pi r_D^2 r} \frac{dL_{\text{SN}}}{dE_{\gamma}} R_{\text{SN}} \delta(\theta - \pi/2). \quad (36)$$

Since we are solving for the steady state configuration assuming spherical symmetry, we shall replace this supernova disk distribution with a spherically symmetric analogue of the form [cf. discussion around Eq. (10)]

$$\frac{d\mathcal{L}_{\text{SN}}^S(r)}{dV dE_{\gamma}} = \frac{e^{-r/r_D}}{4\pi r_D^2 r} \frac{dL_{\text{SN}}}{dE_{\gamma}} R_{\text{SN}}. \quad (37)$$

For the Milky Way, we have a rough upper limit: $R_{\text{SN}}^{\text{MW}} L_{\text{SN}} \lesssim 10^{45} \text{ erg/s}$.

The differential source luminosity, at a given location, is the sum of the radiation cooling function [Eq. (32)] and the SN sourced photons, i.e.

$$\frac{d\mathcal{L}^S(r)}{dV dE_{\gamma}} = \frac{d\mathcal{C}(r)}{dE_{\gamma}} + \frac{d\mathcal{L}_{\text{SN}}^S(r)}{dV dE_{\gamma}}. \quad (38)$$

Recall that the above source luminosity is required to compute the differential flux at a given location [$dF(r)/dE_{\gamma}$] via Eq. (18), and that this flux is needed to compute the ionization state [Eq. (16)]. The flux is also required to calculate the differential rate of radiation absorption (heating rate),

$$\frac{dH(r)}{dE_{\gamma}} = \sum_{A,k} \sigma[A^k]_{PI} n_{A^k} E_{\gamma} \frac{dF(r)}{dE_{\gamma}}. \quad (39)$$

We now have a set of interconnected equations describing the ionization state, cooling and heating rates of a dark plasma. These equations will need to be solved to find the steady state solution for mirror dark matter galaxy halos. These equations, with straightforward modifications (as indicated), are applicable also to the more generic dissipative model of Sec. II. A.

IV. STEADY STATE SOLUTION

A. The numerical method

The system of equations governing the ionization state, the heating and cooling rates, are somewhat nontrivial to solve. Our strategy to solve them is to choose a suitable form for the density profile (defined in terms of several parameters to be determined from the dynamics). The system of equations is then solved iteratively as follows:

- (a) With the chosen density profile, the temperature profile is calculated from the hydrostatic equilibrium condition [Eq. (11)] (in the first iteration the \bar{m} profile is chosen arbitrarily; in the second and subsequent iterations it is input from the previous iteration).
- (b) Using the temperature profile calculated from step (a) the ionization state is computed [Eq. (16)], and a new \bar{m} profile derived (in the first iteration the mirror

photon flux can be neglected; in the second and subsequent iterations the flux is input from the previous iteration).

- (c) Using the results from steps (a) and (b), the heating $[\mathcal{H}(r)]$, cooling $[\mathcal{C}(r)]$ rates are evaluated [from Eqs. (39) and (32)], and also the mirror photon flux $dF(r)/dE_\gamma$ [from Eq. (18)].

The above three steps can be repeated until a stable solution for \mathcal{H} and \mathcal{C} emerges (typically requires around 10–20 iterations). If the chosen density profile is such that $\mathcal{H} \simeq \mathcal{C}$ at each location, then this density profile (together with the temperature derived from the hydrostatic equilibrium condition via the above iterative procedure) would represent an approximate steady state solution to the fluid equations.

We have made use of two spherically symmetric dark matter halo density profiles. The first one is the density profile that arises in the idealized case of an isothermal halo in the optically thin limit [24,25]. Under these assumptions, $\mathcal{C}(\mathbf{r}) \propto n(\mathbf{r})^2$ and $\mathcal{H}(\mathbf{r}) \propto n(\mathbf{r})F(\mathbf{r})$ [where $F(\mathbf{r})$ is the flux of dark photons at \mathbf{r}], the steady state condition $\mathcal{C}(\mathbf{r}) = \mathcal{H}(\mathbf{r})$ implies that $n(\mathbf{r}) \propto F(\mathbf{r})$. For a flux originating from a spherical distribution of SN heat sources, this yields a dark matter density profile of the form

$$\rho(r) = \lambda \int \int \frac{d\mathcal{L}_{\text{SN}}^S(r')}{dV} \mathcal{F}(r, r', \theta') 2\pi r'^2 d\cos\theta' dr', \quad (40)$$

where $\mathcal{F}(r, r', \theta') = 1/(4\pi[r^2 + r'^2 - 2rr'\cos\theta'])$. Also, from Eq. (37) we have

$$\frac{d\mathcal{L}_{\text{SN}}^S(r)}{dV} = \frac{\kappa e^{-r/r_D}}{4\pi r_D^2 r}, \quad (41)$$

where $\kappa \equiv R_{\text{SN}} L_{\text{SN}}$. The coefficient λ would be independent of r in the optically thin and isothermal limit. We refer to this one-parameter distribution as the λ -density profile. We have also considered a generic cored profile,

$$\rho(r) = \rho_0 \left[\frac{r_0^2}{r^2 + r_0^2} \right]^\beta. \quad (42)$$

This profile is defined in terms of three independent parameters: ρ_0 , r_0 , β . With either of these profiles we can follow the steps (a)–(c) iterated until a stable solution for \mathcal{H} and \mathcal{C} emerges.

For $\rho(r)$ to be an approximate steady state solution requires $\mathcal{H} \simeq \mathcal{C}$ at every location in the halo. To quantify this, it is useful to introduce the functional Δ ,

$$\Delta \equiv \frac{1}{R_2 - R_1} \int_{R_1}^{R_2} \frac{|\mathcal{H}(r') - \mathcal{C}(r')|}{\mathcal{H}(r') + \mathcal{C}(r')} dr', \quad (43)$$

where we take $R_1 = 0.3r_D$, $R_2 = 10r_D$ in our numerical work. We then minimize Δ with respect to variations in λ for the λ -density profile, and ρ_0 , r_0 , β for the generic cored

profile. If this minimum is sufficiently small, say less than 0.05, then we shall suppose that a candidate steady state solution exists. The value of λ (or ρ_0 , r_0 , β) that minimizes Δ defines the density profile of the candidate solution.

B. Mirror dark matter

Following the iterative procedure outlined above, we have searched for steady state solutions for mirror dark matter halos with realistic asymptotic halo velocity for a Milky Way scale galaxy ($v_{\text{rot}}^{\text{asym}} \approx 200$ km/s). We explored a wide range of the available parameters including the halo metal abundance parameter, $-2.0 \leq \zeta \leq 2.0$, and SN parameters, $T_{\text{eff}} \leq 1000$ keV, $\kappa \leq 10^{46}$ erg/s. We also looked at modifications of the halo metal composition, e.g. Fe'/O' ratio, etc., and different forms for the SN spectrum, e.g. replacing the thermal spectrum, Eq. (35), with a power law. Throughout this parameter space it was found that halo cooling exceeds heating. (However, for a limited parameter space, $\zeta \sim 2.0$, $T_{\text{eff}} \sim 1$ keV, $\kappa \sim 10^{46}$ erg/s, we found that cooling only exceeds heating by a factor of ~ 3 .) That is, we were unable to find a steady state solution for mirror dark matter galactic halos with realistic halo density. Previous more optimistic results of [21,22] were due, in part, to the incomplete treatment of cooling (neglect of line emission and recombination radiation), and, in part, to the incomplete treatment of the ionization state (neglect of the photoionization contribution).

In Fig. 2, we illustrate the problem by showing the integrated heating and cooling rates ($H = \int \mathcal{H} dV$, $C = \int \mathcal{C} dV$) for a Milky Way scale galaxy. We considered the λ -density profile with $\lambda\kappa = 7.3 \times 10^9 [m_\odot/\text{kpc}]$, a parameter choice sufficient to give a realistic asymptotic halo rotation velocity of ~ 220 km/s. With this density profile, the halo temperature and ionization state were determined at each location in the halo from the steady state equations using the iterative numerical method outlined earlier. Figure 2 clearly indicates that the halo cooling always exceeds heating for the range of ζ , T_{SN} , κ parameters examined. With the large values of $\kappa \equiv R_{\text{SN}} L_{\text{SN}}$ considered, there is enough energy available; the problem is that this energy is not readily absorbed in the halo. To illustrate this issue, we show in Fig. 3 the optical depth for a dark photon originating near the Galactic center and escaping the galaxy (again for a Milky Way scale galaxy with halo properties as per Fig. 2). As this figure shows, the optical depth is typically less than unity. Increasing the supernovae rate beyond observational limits does not help as the increased energy in dark photons is compensated by a reduction in the optical depth caused by the increased ionization.

The discouraging results reported here for mirror dark matter might be due to residual simplifications or, possibly, invalid assumptions. The simplifications include treating the mirror ions and electrons as a single-component fluid with a common local temperature $T(\mathbf{r})$, a simplified description of SN sourced dark photon energy spectrum,

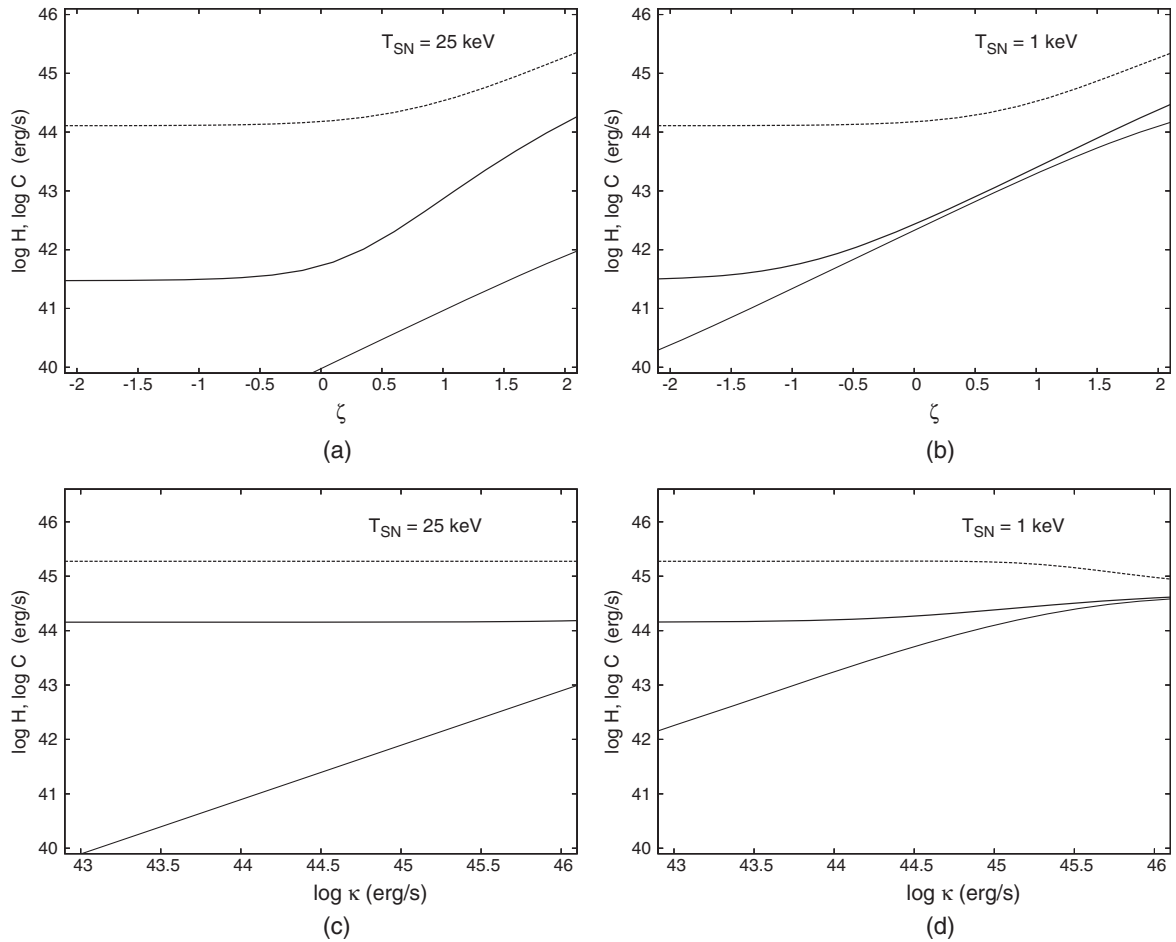


FIG. 2. Halo heating (thick solid line) and cooling rates (dashed line) for a Milky Way scale galaxy. The lower thin solid line is the heating contributed by SN sourced dark photons. The λ -density profile was used with the halo temperature and ionization state numerically determined from steady state equations. (a) and (b) give the rates in terms of the mirror metal abundance parameter, ζ , for $\kappa = 10^{45}$ erg/s and (a) $T_{\text{SN}} = 25$ keV, (b) $T_{\text{SN}} = 1$ keV. While (c) and (d) give the rates in terms of the SN sourced dark photon luminosity, κ [erg/s], for $\zeta = 2$ and (c) $T_{\text{SN}} = 25$ keV, (d) $T_{\text{SN}} = 1$ keV.

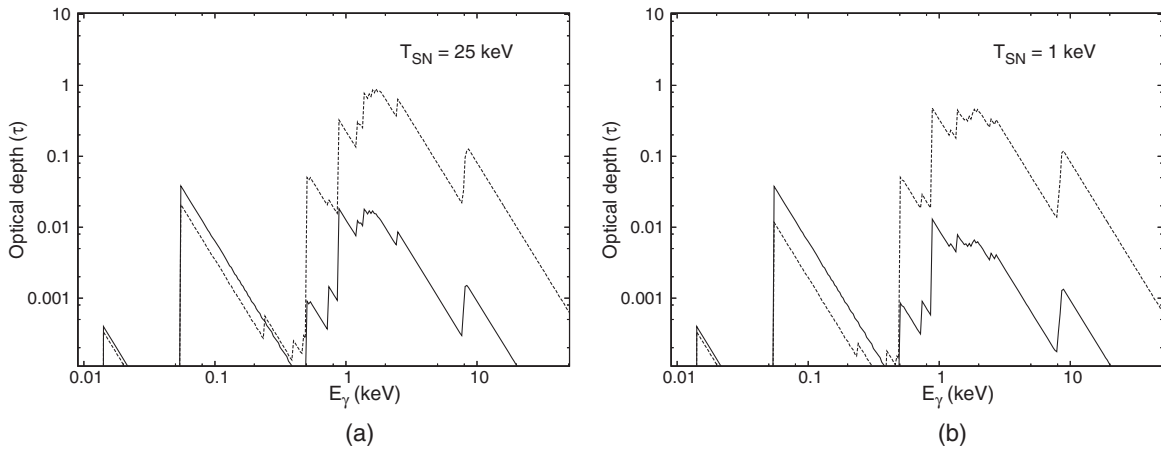


FIG. 3. The optical depth for a dark photon originating near the Galactic center and escaping a Milky Way scale galaxy, with halo properties as per Fig. 2. The solid (dashed) line shows results for $\zeta = 0$ ($\zeta = 2$), with $\kappa = 10^{45}$ erg/s and (a) $T_{\text{SN}} = 25$ keV, (b) $T_{\text{SN}} = 1$ keV.

and a neglect of dark magnetic fields. One of the questionable assumptions involves the mechanism by which the SN sourced energy is transported to the halo. In this paper, we have followed earlier work [21,22] and assumed that this energy is transmitted to the halo via dark radiation. It is possible that the SN sourced energy is instead transported to the halo via local collisional processes in the SN vicinity. A discussion of the implications that follow from adopting this alternative energy source will be given in a separate paper. In any case, the above caveats suggest that no definite conclusion as to the validity, or otherwise, of the mirror dark matter model could be made at this time; in the remaining discussion of this paper we shall focus on alternative dissipative particle models.

C. Two-component dissipative model

The mirror model is rather unique in that the fundamental interaction cross sections have no free parameters: they are all identical to those of the corresponding ordinary particle processes. Naturally, it is also worthwhile to look at more generic dissipative models, the simplest such model is the two-component model of [15], reviewed in Sec. II. A. In that model the dark halo consists of just two matter components, the dark electron and dark proton [with dark charge ratio $Z' \equiv |Q'(p_d)/Q'(e_d)|$]. That model has five fundamental parameters: m_{e_d} , m_{p_d} , Z' , α_d , and ϵ . The considered parameter space is somewhat restricted, $m_{e_d} \ll m_{p_d}$ and $Z' \geq 1$, so that atoms could potentially form with a p_d nucleus surrounded by one or more dark electrons.

For such a dissipative dark matter model to have the potential of being realistic, galactic halos should not be fully ionized. This is required so that the halo can absorb the supernova sourced dark photons via the photoionization process. This restriction leads to the rough criterion $T_{\text{halo}} \lesssim I_1$, where I_1 is the binding energy of the innermost (K shell) dark electron. This condition is most restrictive for the largest galaxies and can be used to estimate an upper bound on m_{p_d} . The halo also needs to have a non-negligible degree of ionization, even for the smallest galaxies. Under the assumption that the ionization is due primarily to dark electron scattering, this criterion leads to a lower bound on m_{p_d} . (It might be possible to weaken this lower bound given the photoionization contribution, and further study could be done to clarify this issue.) These conditions, derived in Eq. (91) of [15], imply that $Z' \geq 3$ and that m_{p_d} lies in the range

$$\begin{aligned} & \left(\frac{Z'}{10}\right) \left(\frac{\alpha_d}{10^{-2}}\right)^2 \left(\frac{m_{e_d}}{\text{MeV}}\right) \\ & \lesssim \frac{m_{p_d}}{\text{GeV}} \\ & \lesssim 100 \left(\frac{Z'}{10}\right)^3 \left(\frac{\alpha_d}{10^{-2}}\right)^2 \left(\frac{m_{e_d}}{\text{MeV}}\right) g(\alpha_d, Z'), \end{aligned} \quad (44)$$

where $g(\alpha_d, Z') \equiv \max(\alpha_d^3 Z'^4, 1)$.

For a given choice of SN parameters (we take $T_{\text{eff}} = 25$ keV, $\kappa = 10^{45}$ erg/s for definiteness), we have searched for m_{p_d} , m_{e_d} , and α_d values which give realistic asymptotic rotational velocity for a Milky Way scale galaxy (we fixed $Z' = 6$ for definiteness). There is a significant parameter space where this occurs, and we shall focus here on a specific example,

$$m_{p_d} = 100m_p, \quad m_{e_d} = 8m_e, \quad \alpha_d = 4\alpha, \quad Z' = 6. \quad (45)$$

For the particular dissipative dark matter model defined by these parameters, we have undertaken a search for steady state solutions for a representative range of galaxies.

Consider first a Milky Way scale galaxy with baryonic parameters: $m_{\text{baryons}} = 10^{11}m_\odot$, $f_s = 0.8$, $r_D = 3.95$ kpc, $M_{\text{FUV}} = -18.4$. The system of equations describing the ionization state, as well as the heating and cooling rates, were solved iteratively with the λ -density profile [Eq. (40)]. An approximate steady state solution was identified for $\lambda\kappa = 7.3 \times 10^9 [m_\odot/\text{kpc}]$, with $\Delta_{\text{min}} \simeq 0.05$. To better represent this putative solution, we replaced the coefficient, λ , in Eq. (40), with the radial expansion,

$$\lambda \rightarrow \lambda \left[1 + \sum_{n=1}^N a_n \left(\frac{r}{r_D}\right)^n + b_n \left(\frac{r_D}{r}\right)^n \right]. \quad (46)$$

In our numerical work we considered only $N = 1$ as this was sufficient to significantly reduce the Δ_{min} value of the approximate steady state solution. In Fig. 4 we show some physical properties of the solution found. Evidently, the halo's temperature profile is nearly isothermal, and the density profile is close to quasi-isothermal (to be discussed in more detail shortly).

The above procedure can be repeated for other model galaxies. One need only input the baryonic parameters, m_{baryons} , f_s , r_D , M_{FUV} , and the halo properties can be computed from the steady state condition. We have examined large stellar dominated galaxies (putative spiral galaxies) with baryon masses: $10^{9.5}$, 10^{10} , $10^{10.5}$, $10^{11}m_\odot$. The stellar mass component, with mass fraction set to $f_s = 0.80$, was assumed to be distributed as in Eq. (10), with baryonic scale length values (r_D) typical of high surface brightness spirals [taken from Eq. (8) of [75]]. The remaining baryon fraction ($1 - f_s$), the gas component, was modeled with a more extended distribution ($r_D^{\text{gas}} = 3r_D$, as in e.g. [76]). We also looked at two small gas rich galaxies (putative dwarf irregular galaxies) with baryon masses $5 \times 10^8 m_\odot$ and $10^8 m_\odot$ and with stellar mass fraction $f_s = 0.20$. The FUV absolute magnitude values for this parameter set were chosen consistently with the measured GALEX luminosities [71] of THINGS [5] and LITTLE THINGS [8] galaxies. For these six examples we have numerically solved the system of equations iteratively with the generalized λ -density profile of Eqs. (40) and (46). For all these examples approximate steady state solutions were found with $\Delta_{\text{min}} \approx 0.01$ – 0.04 .

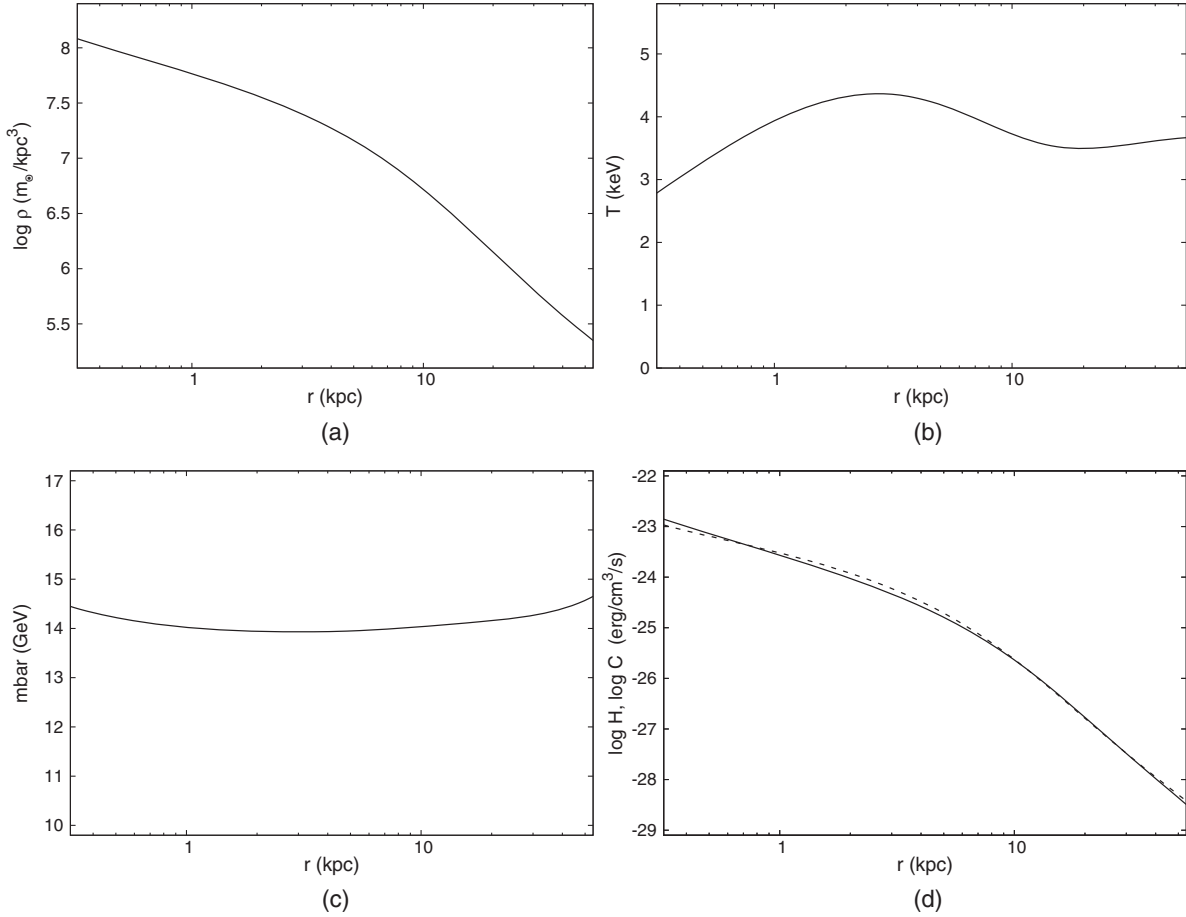


FIG. 4. Properties of the steady state solution obtained for a Milky Way scale galaxy ($m_{\text{baryons}} = 10^{11} m_{\odot}$, $f_s = 0.8$, $r_D = 3.95$ kpc, $M_{\text{FUV}} = -18.4$). Shown are (a) the halo density, (b) the halo temperature, (c) the mean mass, \bar{m} , and (d) the heating, cooling rates [\mathcal{H} , \mathcal{C}] (solid, dashed lines).

For ease of notation, these approximate steady state solutions will hereafter be referred to as *steady state solutions*. The galaxy baryonic parameters chosen are summarized in Table II.

The one-parameter λ -density profile, Eq. (40), represents a reasonable first order approximation to all of the steady state solutions found. It is not surprising then that much of what dissipative dark matter models predict can be understood from the properties of that profile. The λ -density profile has the asymptotic behavior

$$\rho(r) = \frac{\lambda\kappa}{4\pi r^2} \quad \text{for } r \gg r_D, \quad (47)$$

and rises logarithmically for $r \lesssim r_D$. Over the finite range $0.3 < r/r_D < 10$, the λ -density profile is numerically equivalent, to a good approximation (within \sim few percent), to the density,

$$\rho(r) = \rho_0 \left[\frac{r_0^2}{r^2 + r_0^2} \right] \left[1 + \ln \left(\frac{r^2 + r_0^2}{r^2} \right) \right] \quad (48)$$

with $r_0 = 1.75 r_D$ and $\rho_0 = 0.029 \lambda \kappa / r_D^2$. This profile resembles the quasi-isothermal profile often adopted in the literature to fit rotation curves, e.g. [5,77]. There are two important differences. First, it has a logarithmically increasing density profile in the inner region, and second, it is constrained as r_0 is not a free parameter but set by the baryonic disk scale length. The logarithmically increasing inner density profile is expected to be observationally (virtually) indistinguishable from a truly flat profile, while the scaling of the core radius, $r_0 \sim r_D$, is a noted feature derived from observations [13].

TABLE II. Baryonic properties (baryon mass, baryonic scale length, FUV absolute magnitude, and stellar mass fraction) for the six “canonical” model galaxies considered.

$m_{\text{baryons}} (m_{\odot})$	r_D (kpc)	M_{FUV}	f_s
10^{11}	3.95	-18.4	0.8
$10^{10.5}$	2.70	-17.9	0.8
10^{10}	2.00	-17.4	0.8
$10^{9.5}$	1.60	-16.9	0.8
$5 \cdot 10^8$	0.60	-15.0	0.2
10^8	0.40	-13.4	0.2

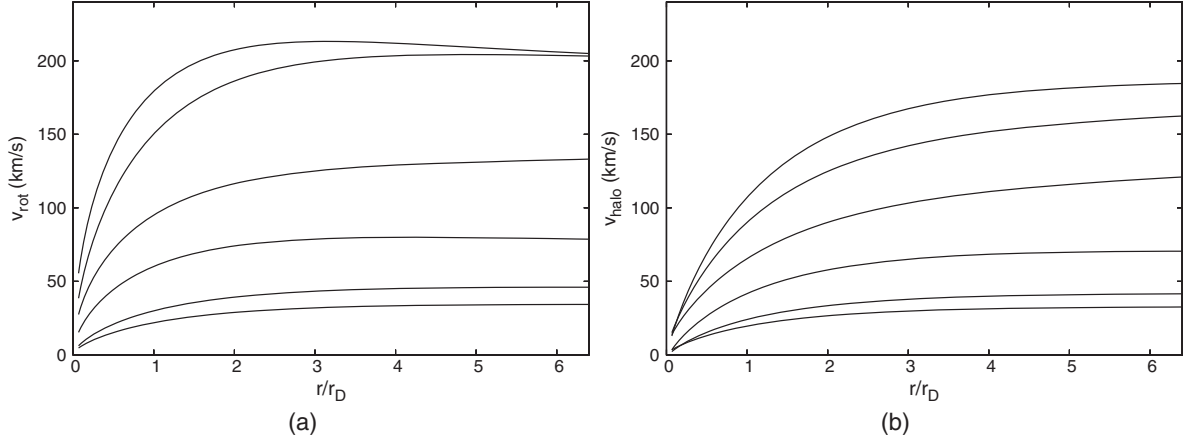


FIG. 5. (a) The rotation curves (halo + baryons) derived from the computed steady state solutions. The baryon mass ranges from $m_{\text{baryons}} = 10^8 m_{\odot}$ (bottom curve) to $m_{\text{baryons}} = 10^{11} m_{\odot}$ (top curve). See Table II for other baryonic parameters chosen. (b) The corresponding halo rotation curves (halo contribution only).

Observe that the existence of a dark matter core, with a core radius $r_0 \sim r_D$, has a clear geometrical origin in this dynamics. The halo evolves toward a steady state configuration, which is strongly influenced by the distribution of supernovae, as these represent the primary source of halo heating. This heat source is cored given the exponential distribution of the Freeman disk and the associated scale length, r_D .

Consider now the rotation curves. The rotational velocity follows directly from Newton's law,

$$\frac{v_{\text{rot}}^2}{r} = \frac{G_N}{r^2} \int_0^r [\rho(r') + \rho_{\text{baryons}}(r')] 4\pi r'^2 dr'. \quad (49)$$

We are also interested in the dark halo contribution to the rotational velocity, for which we use the notation, v_{halo} ,

$$\frac{v_{\text{halo}}^2}{r} = \frac{G_N}{r^2} \int_0^r \rho(r') 4\pi r'^2 dr'. \quad (50)$$

In Fig. 5 we plot the rotational velocity and halo rotational velocity derived from the steady state solutions found.

The rotation curves show an approximate linear rise in the inner region, turning over to a roughly flat asymptotic profile, with the transition radius occurring at $r \sim r_D$ in each case. As already mentioned, these properties can be understood from simple geometrical considerations as the halo mass density is closely aligned with the distribution of supernova sources. Also, recall that these three properties are all well discussed features of measured rotation curves.

We have also looked at the effects of varying some of the other baryonic parameters. The baryonic parameters given in Table II are typical of high surface brightness galaxies. In addition, there exist numerous low surface brightness galaxies within which the stars and gas have a more

extended spatial distribution. To explore these kinds of galaxies, we have computed the steady state solutions for the six galaxies of Table II, but with disk scale length increased by a factor of $\times 2.5$ (i.e. $r_D \rightarrow 2.5r_D$, $r_D^{\text{gas}} \rightarrow 2.5r_D^{\text{gas}}$, with m_{baryons} , M_{FUV} , f_s unchanged). In Fig. 6 we give the halo rotation curves derived from the steady state solutions for the original (Table II) parameter set, as well as the $r_D \rightarrow 2.5r_D$ variation. As the figure shows, there is only a very minor effect on the halo contribution to the rotation curve for the three smallest galaxies studied, while there are noticeable, although still modest, effects for the larger galaxies. That is, the halo rotation velocity is, at least approximately, a function of the dimensionless variable r/r_D (rather than r and r_D separately). This feature is consistent with observations, with the galaxies NGC2403 and UGC128 providing a well studied illustration [78] (see also [51] for a recent discussion).

The near invariance of the halo rotational velocity with respect to the transformation, $r \rightarrow \Lambda r$, $r_D \rightarrow \Lambda r_D$, is not unexpected. Recall the simple analytic argument that motivated the λ -density profile [Eq. (40)]: for an optically thin isothermal halo, $\mathcal{H}(r) = \mathcal{C}(r)$ implies a halo density proportional to the flux of SN sourced dark photons, i.e. $n(r) \propto F(r)$. Since a photon flux scales as $\sim 1/r^2$, this suggests that the halo density will scale as $\rho(r) \rightarrow \rho(\Lambda r)/\Lambda^2$, when $r \rightarrow \Lambda r$, $r_D \rightarrow \Lambda r_D$. It follows directly from Newton's law, Eq. (50), that such a scaling implies a scale invariant rotational velocity: $v_{\text{halo}}(r) \rightarrow v_{\text{halo}}(\Lambda r)$. In fact, the scaling, $\rho(r) \rightarrow \rho(\Lambda r)/\Lambda^2$, when $r \rightarrow \Lambda r$, $r_D \rightarrow \Lambda r_D$, is an exact property of the λ -density profile (for fixed λ), which arises even when the assumption of spherical symmetry is dropped [25]. Of course, scale invariance can only be approximate, rather than exact, due to the effects of halo reabsorption (finite optical depth) and departures from isothermality.

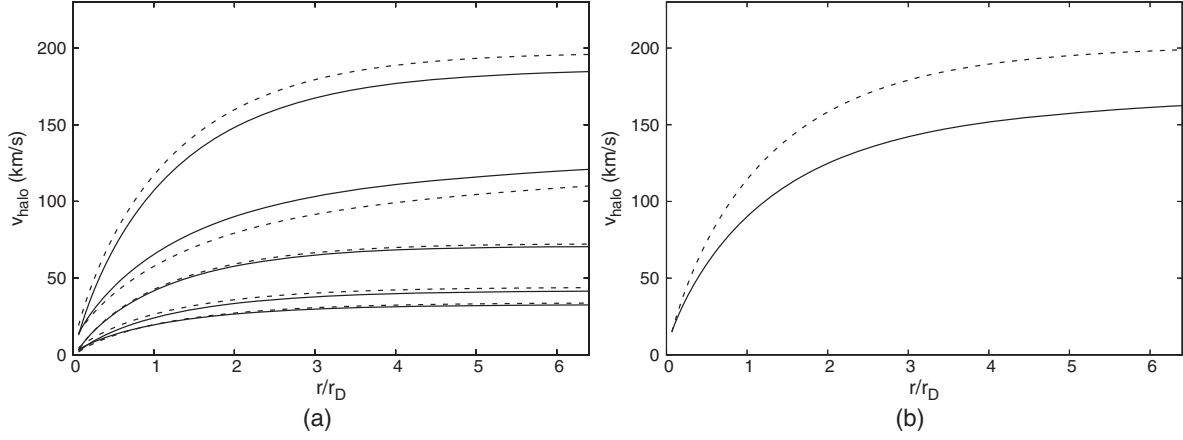


FIG. 6. Halo rotation curves derived from the steady state solutions for the galaxy set of Table II (solid lines), and with a factor of $\times 2.5$ increase in the baryonic disk scale length (dashed lines). (a) The results are given for the five smallest galaxies, with $m_{\text{baryons}} = 10^8 m_{\odot}$ (bottom curve) to $m_{\text{baryons}} = 10^{10.5} m_{\odot}$ (top curve), while (b) is for $m_{\text{baryons}} = 10^{11} m_{\odot}$.

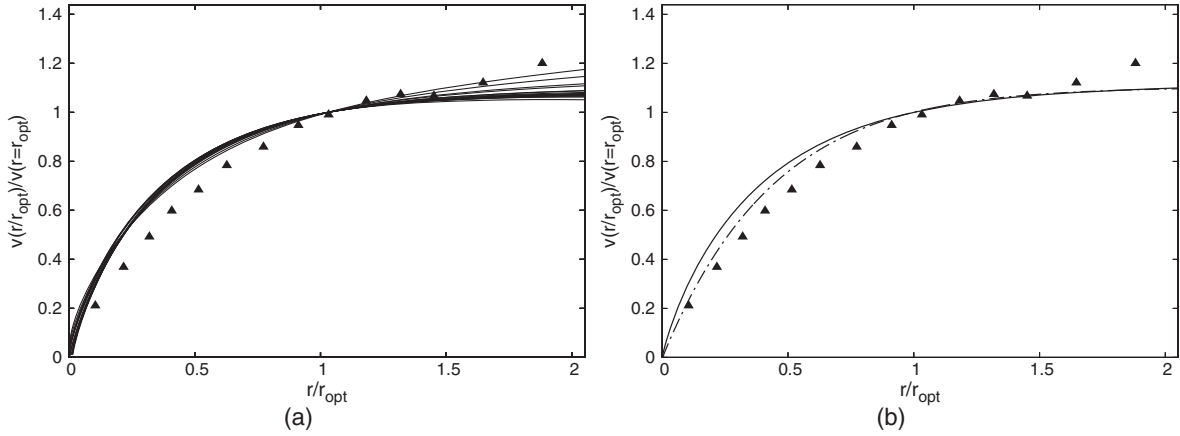


FIG. 7. (a) Normalized halo rotational velocity, $v_{\text{halo}}(r)/v_{\text{halo}}(r_{\text{opt}})$, for all 18 modeled galaxies calculated from the steady state solution. (b) Comparison of the spherically symmetric λ -density profile [Eq. (40)] (solid line) with the corresponding profile for a disk geometry [Eq. (51)] (dash-dotted line). The triangles in the figures are the synthetic rotation curve obtained from dwarf disk galaxies [76].

Consider now the effect of varying the luminosity while keeping the other baryonic parameters fixed.⁷ Specifically, we have computed the steady state solutions for more luminous galaxies, $M_{\text{FUV}} \rightarrow M_{\text{FUV}} - 0.8$ (i.e. a factor of $10^{0.32} \approx 2.1$ increase in luminosity), with the other baryonic parameters (m_{baryons} , f_s , r_D) unchanged from their canonical (Table II) values. This set of baryonic parameters, together with the canonical parameter choice, and those with $r_D \rightarrow 2.5r_D$, provide a total of 18 distinct galaxy parameters. While changing the FUV luminosity will strongly influence the normalization of the halo rotational velocity, let us first look at the effect (if any) on the shape of the rotation curve. Figure 7

⁷The effect on the halo rotational velocity of varying the stellar mass fraction, f_s , with the other parameters kept fixed, was found to be negligible.

shows the derived normalized halo rotational velocity, $v_{\text{halo}}(r)/v_{\text{halo}}(r_{\text{opt}})$, for all 18 modeled galaxies. (Here $r_{\text{opt}} \approx 3.2r_D$ is the optical radius.) Evidently, the halo rotation curves that follow from the solution of the steady state conditions, Eq. (8), have a near universal profile, a notable feature consistent with observations, e.g. [51,76]. This result was anticipated given the major influence of supernova sourced heating on the halo density distribution [24,25]. The shape reflects the geometry of the heating sources.

The steady state solutions discussed here correspond to galaxies simplified with spherical symmetry. Indeed, the supernova source distribution has been artificially modified to make it spherically symmetric. In actuality, the spatial distribution of these sources is far from spherical; a disk geometry would be much closer to realistic. Nevertheless, we anticipate that only modest changes

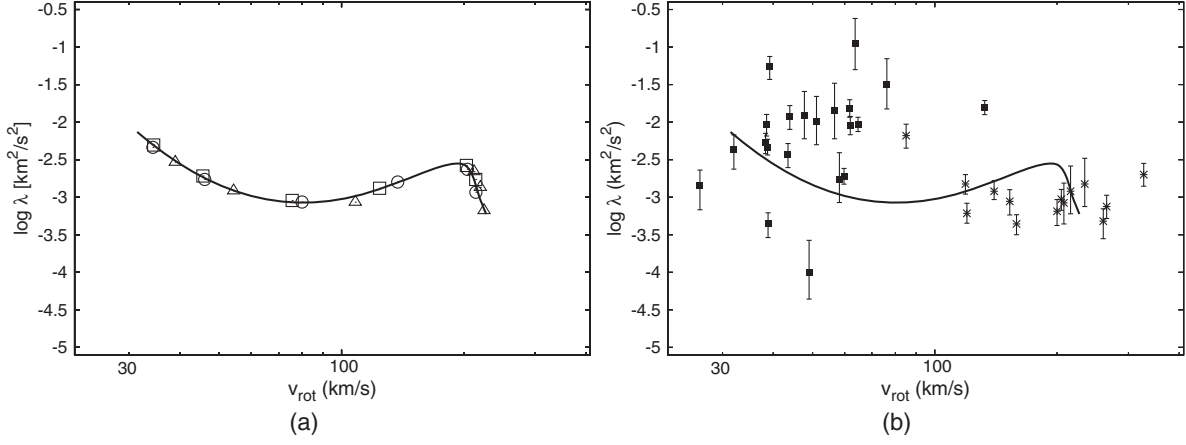


FIG. 8. The quantity $\tilde{\lambda}$ [Eq. (52)] versus $v_{\text{rot}}^{\text{max}}$ computed from the steady state solutions found. (a) The results are given for the 18 modeled galaxies along with an extrapolation (circles for the six canonical baryonic parameters of Table II, squares for the $r_D \rightarrow 2.5r_D$ parameter variation, and triangles for the $M_{\text{FUV}} \rightarrow M_{\text{FUV}} - 0.8$ variation). (b) The extrapolated results together with the $\tilde{\lambda}$ values from THINGS spirals (stars) [5] and LITTLE THINGS dwarfs (squares) [8].

would arise if the system of equations were solved without the spherically symmetric simplification. In fact, the azimuthally symmetric disk analogue of the spherically symmetric λ -density profile [Eq. (40)] is [24,25]

$$\rho(r, \theta) = \lambda \int d\tilde{\phi} \int d\tilde{r} \tilde{r} \frac{\Sigma_{\text{SN}}(\tilde{r})}{4\pi[r^2 + \tilde{r}^2 - 2r\tilde{r}\sin\theta\cos\tilde{\phi}]}, \quad (51)$$

where Σ_{SN} is the supernova distribution (averaged over a suitable time scale) in the disk. The normalized halo rotational velocity corresponding to this density, shown in Fig. 7(b), closely resembles its spherically symmetric counterpart, and in fact agrees slightly better with the observations. While this appears to be a strong indication, and indeed is very encouraging, it still requires verification that Eq. (51) actually does approximate the steady state solution for galaxy systems with disk geometry.

The next item of interest is the scaling of the normalization of the halo velocity. From Eq. (47) we expect, at leading order, an asymptotic halo velocity of $v_{\text{halo}}^{\text{asym}} = \sqrt{G_N \lambda \kappa}$, i.e. $\lambda \propto [v_{\text{halo}}^{\text{asym}}]^2 / R_{\text{SN}}$. To make contact with observable quantities, we again make use of the expected $R_{\text{SN}} \propto L_{\text{FUV}} \propto 10^{-0.4M_{\text{FUV}}}$ scaling. It is convenient then to introduce the quantity $\tilde{\lambda}$,

$$\tilde{\lambda} \equiv \frac{[v_{\text{halo}}^{\text{asym}}]^2}{10^{-0.4M_{\text{FUV}}}}. \quad (52)$$

We have evaluated $\tilde{\lambda}$ from the computed steady state solutions for all 18 galaxy parameters examined [taking $v_{\text{halo}}^{\text{asym}} = v_{\text{halo}}(r = 6.4r_D)$]. The result of this exercise is shown in Fig. 8, where we plot the obtained $\tilde{\lambda}$ values versus the maximum of the rotational velocity, $v_{\text{rot}}^{\text{max}}$. Also shown

in the figure are the values of $\tilde{\lambda}$ for THINGS spirals [5] and LITTLE THINGS dwarfs [8].⁸

Figure 8 indicates that, for the model galaxies studied, $\tilde{\lambda}$ is not exactly constant but has some variation with respect to $v_{\text{rot}}^{\text{max}}$.⁹ The overall normalization appears to be in the right ballpark to be consistent with THINGS spirals and LITTLE THINGS dwarfs, although the $\tilde{\lambda}$ values of the dwarfs show significant scatter. The rotation curve shapes of many of the LITTLE THINGS dwarfs are also quite irregular. This may be an indication that many of these small galaxies are not currently in a steady state configuration. In fact, some of these galaxies are known to have unstable star formation (recent) histories. They are starburst galaxies, undergoing large scale oscillations in star formation rate, with a period of order ~ 100 Myr, e.g. [74,80].¹⁰

The results for the halo rotational velocity normalization (Fig. 8) together with our earlier results for the shape (Fig. 7) can be summarized: The halo rotation curve that follows from the steady state conditions has a characteristic

⁸The raw FUV absolute magnitude values were appropriated from the updated nearby galaxy catalogue [79], and corrected for internal and foreground extinction following [73,74].

⁹The sharp downturn at $v_{\text{rot}}^{\text{max}} \sim 200$ km/s is a threshold effect. The plasma is approaching the state of full ionization. Of course, a change in fundamental parameters, e.g. decreasing m_{p_d} (which lowers the halo temperature), can move this threshold to a higher $v_{\text{rot}}^{\text{max}}$ value.

¹⁰In this picture, the oscillations in the rate of star formation might be strongly influenced by nontrivial dissipative halo dynamics. Large scale radial density oscillations of the plasma halo will induce oscillations in the star formation rate, due to the expansion and compression of the baryonic gas under the oscillating gravitational field strength. Such phenomena appear extremely interesting, but complex, requiring solution of the time dependent fluid equations.

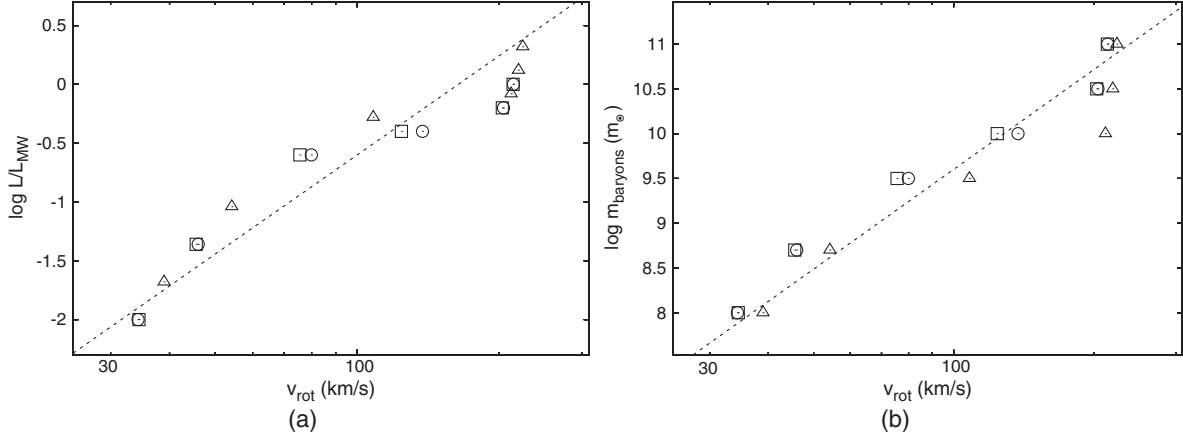


FIG. 9. (a) $L_{\text{FUV}}/L_{\text{FUV}}^{\text{MW}}$ versus $v_{\text{rot}}^{\text{max}}$ and (b) m_{baryons} versus $v_{\text{rot}}^{\text{max}}$, for each of the steady state solutions found. Circles are the results for the canonical galaxy set of Table II, squares for the $r_D \rightarrow 2.5r_D$ parameter variation, and triangles for the $M_{\text{FUV}} \rightarrow M_{\text{FUV}} - 0.8$ variation. The dashed lines are the power laws: (a) $L_{\text{FUV}} \propto [v_{\text{rot}}^{\text{max}}]^{2.8}$ and (b) $m_{\text{baryons}} \propto [v_{\text{rot}}^{\text{max}}]^{3.7}$.

functional form that depends (approximately) only on r/R_D , M_{FUV} . This appears to be consistent with observations—with the existence of such a characteristic functional form discussed for many years, e.g. [3,4]. It has also been argued, though, that the normalization of the rotational velocity might depend more closely on m_{baryons} (rather than luminosity), with $m_{\text{baryons}} \propto [v_{\text{rot}}^{\text{max}}]^\beta$, $\beta \approx 4$ (baryonic Tully-Fisher relation [10]).

In Fig. 9(a) [9(b)] we plot L_{FUV} [m_{baryons}] versus $v_{\text{rot}}^{\text{max}}$ for the modeled galaxies. Evidently, the results are broadly compatible with the empirical scaling relations, but with some scatter. Clearly though, the underlying relation is a predicted scaling of $\tilde{\lambda}$ (Fig. 8), for which there is negligible scatter. (We have checked this further by considering a wider variation of baryonic parameters than those displayed.) That is, our analysis indicates that a $\tilde{\lambda}$ versus $v_{\text{rot}}^{\text{max}}$ correspondence underpins the empirical Tully-Fisher relations, at least in the kind of dissipative model discussed here. Naturally, this

connection could be examined more closely in future studies, noting that the weak variation of $\tilde{\lambda}$ with $v_{\text{rot}}^{\text{max}}$ will surely have some dependence on the fundamental parameters defining the dissipative dark matter model.

Other dark matter galactic scaling relations, e.g. the radial acceleration relation [11,12], can be viewed as a consequence of the predicted universal profile (Fig. 7) with correct normalization (Fig. 8); see discussion in e.g. [25]. Further exploration of such relations is therefore not essential.

Finally, to complete this analysis of the particular dissipative model parameters chosen, we briefly examine the effects of varying the uncertain effective supernova temperature parameter, T_{eff} . [Recall, we have modeled the dark photon frequency spectrum by a thermal distribution with this effective temperature, Eq. (35).] Changing the T_{eff} parameter will modify the level of halo heating and, assuming a steady state solution still exists, will potentially influence the normalization of the halo velocity. However,

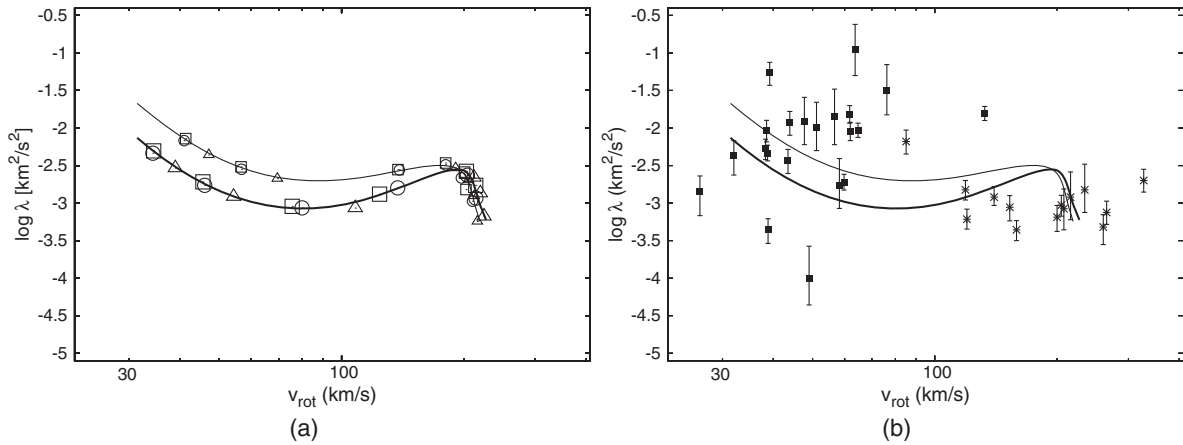


FIG. 10. (a) The effect of varying the effective temperature parameter, T_{eff} [Eq. (35)], on $\tilde{\lambda}$, Eq. (52). Notation as in Fig. 8(a) but with small (large) symbols for $T_{\text{eff}} = 10$ keV ($T_{\text{eff}} = 25$ keV). (b) The extrapolated results for $T_{\text{eff}} = 10$ keV (thin solid line) and $T_{\text{eff}} = 25$ keV (thick solid line). Also shown are the $\tilde{\lambda}$ values from THINGS spirals (stars) [5] and LITTLE THINGS dwarfs (squares) [8].

the shape of the normalized halo rotational velocity remains (approximately) unchanged, and simply reflects the geometry of the SN heat source distribution (as discussed). Figure 10 compares the $\tilde{\lambda}$ [Eq. (52)] values obtained with two choices for the effective temperature parameter, $T_{\text{eff}} = 10$ keV and $T_{\text{eff}} = 25$ keV, for the same 18 baryonic galaxy parameters already discussed.

V. CONCLUSION

We have considered dark matter featuring particle properties that closely resemble familiar baryonic matter. Mirror dark matter, which arises from an isomorphic hidden sector, is a specific and theoretically constrained scenario. Other possibilities include models with more generic hidden sectors that contain massless dark photons [unbroken $U(1)$ gauge interactions]. Such dark matter not only features dissipative cooling processes, but is assumed to have nontrivial heating sourced by ordinary supernovae (facilitated by the kinetic mixing interaction).

The dynamics of dark matter halos around rotationally supported galaxies, influenced by cooling and heating processes, can be modeled by fluid equations. For a sufficiently isolated galaxy with a stable star formation rate, the dissipative dark matter halos are expected to evolve to a steady state configuration which is in hydrostatic equilibrium and where heating and cooling rates locally balance. Here, we have endeavored to take into account the major cooling and heating processes, and we have numerically solved for the steady state solution under the assumptions of spherical symmetry, negligible dark magnetic fields. In addition, we have assumed that supernova sourced energy is transported to the halo via dark radiation. For the parameters considered, and assumptions made, we were unable to find a physically realistic solution for the theoretically constrained case of mirror dark matter halos. Halo cooling generally exceeds heating at realistic halo mass densities.

Naturally, there are a number of possible reasons for this deficiency, and some of the assumptions made could be reexamined, etc. It could also be that mirror dark matter is

not the correct dark matter model. Nature might prefer some other kind of dissipative dark matter, if dark matter is indeed dissipative. To illustrate such a possibility, we examined galaxy structure in the context of more generic dissipative dark matter models. One such model was looked at in some detail which featured steady state solutions with realistic dark matter halos. This analysis confirmed, to some extent, the insight gleaned from simplified analytical considerations, e.g. [15,24].

We conclude this work by summarizing some of the key results from this analysis of the steady state solutions of the particular dissipative model studied. The rotation curves are characterized by three main features:

- (i) Approximate scale invariance of the halo rotation velocity: $v_{\text{halo}}(r) \rightarrow v_{\text{halo}}(\Lambda r)$ under $r \rightarrow \Lambda r, r_D \rightarrow \Lambda r_D$ (Fig. 6).
- (ii) The shape of the normalized halo rotational velocity is close to universal (Fig. 7).
- (iii) The normalization of the halo velocity is characterized by a Tully-Fisher type relation (Fig. 8).

Even though this analysis has focused on a particular dissipative model, the above three features are expected to hold over a significant region of parameter space in generic dissipative models [15,23–25]. Observations appear to be consistent with the above features, including the universal profile for the normalized halo rotational velocity that was here derived in the steady state limit. That observations have these characteristic features has been frequently noted in the literature and is often cited as support for the notion of modified Newtonian dynamics (MOND) [81–83]. The present study, though, reinforces the idea that (approximate) MONDian phenomenology can arise also in a dark matter setting.

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