

# Covariant kinetic theory for effective fugacity quasiparticle model and first order transport coefficients for hot QCD matter

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An effective relativistic kinetic theory has been constructed for an interacting system of quarks, antiquarks and gluons within a quasiparticle description of hot QCD medium at finite temperature and baryon chemical potential, where the interactions are encoded in the gluon and quark effective fugacities with nontrivial energy dispersions. The local conservations of stress-energy tensor and number current require the introduction of a mean field term in the transport equation which produces non-vanishing contribution to the first order transport coefficients. Such contribution has been observed to be significant for the temperatures which are closer to the QCD transition temperature, however, induces negligible contributions beyond a few times the transition temperature. As an implication, impact of the mean field contribution on the temperature dependence of the shear viscosity, bulk viscosity and thermal conductivity of a hot QCD medium in the presence of binary, elastic collisions among the constituents, has been investigated. Visible effects have been observed for the temperature regime closer to the QCD transition temperature.

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## I. INTRODUCTION

In view of the fact that heavy-ion experiments at relativistic heavy-ion collider (RHIC) and large hadron collider (LHC) have already realized strongly coupled quark-gluon plasma (QGP) [1–4], interacting hot QCD equations of state (EOSs) computed either within the lattice QCD framework [5–8] or the improved hard thermal loop (HTL) perturbation theory up to three loops [9], might play important roles in modeling the equilibrium/isotropic state of the QGP. On the other hand, effective transport theory approaches beyond hot QCD transition temperature (weak coupling domain) have already shown their usefulness in understanding the bulk and the transport properties of the QGP/hot QCD matter [10–14]. These approaches not only require the microscopic definitions of various thermodynamic quantities for the QGP but also the appropriate momentum distributions as the inputs. To that end, mapping hot QCD equation of state (EOS) effects in a system of effective gluons and quark-antiquarks (quasiparticles) with nontrivial dispersion relations [15–20], has turned out to be a viable approach in developing covariant transport theory. Moreover, the effective kinetic equation is needed to obtain the first and second order dissipative hydrodynamic equations that depict the fluid-dynamic evolution of the QGP medium in addition to the determination of the first and the second order transport coefficients itself.

In this work, we are presenting the foundations of a relativistic kinetic theory of many particle, multicomponent systems, that effectively represent the partonic interactions within the system through a quasiparticle model, *viz.*, effective fugacity quasiparticle model (EQPM)[21–23]. The EQPM has been constructed on the idea of mapping the hot QCD medium effects present in the EOSs of the strongly interacting system, created in the heavy ion collision experiments in terms of quasi-gluons and quasi-quarks/antiquarks with respective temperature dependent effective fugacity parameters. The temperature dependence of the effective fugacities has been determined from the recent  $(2 + 1)$ -flavor lattice data of HotQCD Collaboration [6], realizing the medium as an effective grand canonical system of these quasi-particles. Further, the EQPM at higher temperature (much beyond QCD transition temperature,  $T_c$ ) approaches to the perturbative QCD as far as the effective coupling or the Debye mass are concerned.

The key finding of the present article is to identify the presence of mean field terms that is necessary for the conservation of particle number and energy momentum tensor from a covariant kinetic equation in terms of its appropriate moments. In our analysis, we have observed that, the mean field term turns out to be dependent on the medium modified part of the energy dispersions for the effective gluons and quark-antiquarks. Treating the above mentioned function as the force term in the relativistic transport equation and expressing the thermodynamic quantities in terms of the quasiparticle four-momenta, we conveniently obtain the conservation relations for particle

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current and energy-momentum under the EQPM. Following the conservation relations we can further achieve all the equilibrium thermodynamic laws for a first order hydrodynamic theory. Under this scheme, a complete formalism for estimation of the first order transport coefficients, that quantifies the thermal and viscous dissipations in a strongly interacting medium can be developed consistently, preserving the quasiparticle excitations in the transport theory of the system.

It is to be noted, that the presence of the mean field terms in the effective kinetic theory with quasiparticle models based on the temperature dependent effective masses in the hot QCD medium has long been realized in the context of conservation laws from kinetic theory [24–26] along with an explanation on the fundamental reason for the presence of such mean field terms. As mentioned earlier, this modifies the kinetic theory (microscopic) definition of the energy-momentum tensor so that the hot QCD thermodynamics could exactly be reproduced from the quasiparticle model realizing hot QCD as an effective grand-canonical ensemble of effective gluon and quark-antiquark degrees of freedom. These aspects are crucial while computing transport coefficients for the hot QCD/QGP medium along with deriving hydrodynamic equations from covariant kinetic theory including second and higher order relativistic dissipative hydrodynamic evolution equations [24,25,27–32]. In the context of effective mass quasiparticle model, dissipative hydrodynamics with and without anisotropy has already been constructed and the predictions are tested against the experimental observation [33–37].

Here, in the context of the EQPM, an effective kinetic theory is constructed with appropriate form of the energy-momentum tensor with mean field contribution. The impact of mean field contributions to first order transport coefficients such as shear and bulk viscosities and thermal conductivity of a hot QCD medium with binary, elastic collisions among the effective gluons and quarks-antiquarks has been presented in the current manuscript. The derivation for second and third order dissipative hydrodynamics is beyond the scope of the present work.

The manuscript is organized as follows. Section II deals with the details of the EQPM model, the development of the effective kinetic theory and hydrodynamics under it and its application for estimating the viscous coefficients and thermal conductivity for a QGP system. Section III presents the results, depicting the significance of the mean field term on the temperature dependence of the transport coefficients. The article ends with a conclusion and outlook section, summarizing the relevance and details of the work and with a discussion about the possible open horizons in this direction.

## II. FORMALISM

This section consists of the theoretical set up required to construct a complete, many particle effective theory that

follows the EQPM consistently and hence the estimations of relevant transport parameters.

### A. Effective fugacity quasiparticle model

As mentioned earlier, the EQPM maps the hot QCD medium effects in to medium consist of noninteracting/weakly interacting quasiglons and quarks possessing the following form for their equilibrium momentum distribution functions:

$$f_{g,q}^0 = \frac{z_{g,q} \exp\{-\frac{E_p}{T}\}}{1 \mp z_{g,q} \exp\{-\frac{E_p}{T}\}}. \quad (1)$$

Here,  $T$  is the temperature of the system and  $E_p$  simply denotes the energy of a single bare parton, which for a gluon becomes  $E_p = |\vec{p}|$  and for a quark turns out to be  $E_p = \sqrt{|\vec{p}|^2 + m_q^2}$ , with  $m_q$  as the quark mass. This model can be straightforwardly extended to include finite baryon chemical potential in the quark/antiquark equilibrium distribution function in the following way,

$$f_{q,\bar{q}}^0 = \frac{z_{g,q} \exp\{-\frac{E_p \mp \mu_q}{T}\}}{1 \mp z_{g,q} \exp\{-\frac{E_p \mp \mu_q}{T}\}}. \quad (2)$$

The local equilibrium can be straightforwardly described simply by generalizing Eq. (1) in the comoving frame of the fluid, defined by the hydrodynamic four-velocity  $u^\mu = (1, 0)$  in the local rest frame (LRF) as,

$$f_{g,q} = \frac{z_{g,q} \exp\{-\frac{u^\mu p_\mu}{T}\}}{1 \mp z_{g,q} \exp\{-\frac{u^\mu p_\mu}{T}\}}. \quad (3)$$

Here, we define  $p^\mu = (E_p, \vec{p})$  is the bare four-momenta (without including the effects of interactions) and  $\tilde{p}^\mu = (\omega_p, \vec{p})$  is the quasiparticle four-momenta under the EQPM, corresponding to a parton. The three momenta  $\vec{p}$  is not altered under EQPM, where the single particle energy has been modified via a dispersion relation as follows,

$$\omega_p = E_p + \delta\omega, \quad \delta\omega = T^2 \partial_T \ln z_{g,q}, \quad (4)$$

with  $z_{g,q}$  is the fugacity parameter for gluons and quarks respectively, through which the interactions are being mapped into Eq. (1).  $\delta\omega(T)$  is a pure temperature ( $T(x)$ ) dependent quantity which is again function of four space-time coordinate  $x^\mu \equiv (t, \vec{x})$ . For a massless case with gluons and light quarks, Eq. (4) simply reduces to,

$$\omega_p = |\vec{p}| + \delta\omega. \quad (5)$$

In the light of above discussion the quasiparticle and bare particle four momenta can be related in a local rest frame as follows,

$$\tilde{p}^\mu = p^\mu + \delta\omega u^\mu, \quad (6)$$

which picks up modification of only the energy (zeroth) component of particle 4-momenta through the dispersion relation (4).

### B. Fundamental quantities of effective kinetic theory under EQPM

In order to set up a covariant kinetic theory for a many particle, multicomponent system, under the assumptions of EQPM mentioned above, we first need to define the basic macroscopic quantities that describe the thermodynamic state of the system. We start with the particle 4-flow which manifests the particle number density  $n(x)$  and particle current  $\vec{j}(x)$  as its zeroth and  $i^{\text{th}}$  component. The quasiparticle four flow  $N^\mu(x)$  can be defined in terms of bare momenta as the following,

$$N^\mu(x) = \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3} \frac{p_k^\mu}{E_{p_k}} f_k(x, p_k), \quad (7)$$

that retains the expression of particle number density  $n(x)$  under EQPM as the following,

$$n(x) = N^\mu u_\mu = \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3} f_k(x, p_k). \quad (8)$$

Here  $f_k(x, p_k)$  is the single particle momentum distribution belonging to  $k$ th species, that is a function of space-time coordinate and particle momenta and  $\nu_k$  is the corresponding degeneracy factor. Throughout the analysis, the subscript  $k$  denotes the particle species. Now, it can be shown that  $N^\mu$  can be expressed in terms of dressed momenta  $\tilde{p}$  as follows,

$$N^\mu(x) = \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \tilde{p}_k^\mu f_k(x, \tilde{p}_k) + \delta\omega \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \frac{\langle \tilde{p}_k^\mu \rangle}{|\vec{p}_k|} f_k(x, \tilde{p}_k). \quad (9)$$

Here  $\langle \tilde{p}^\mu \rangle = \Delta^{\mu\nu} \tilde{p}_\nu$  is the irreducible tensor of rank one, with  $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$  as the projection operator. Throughout the analysis the metric  $g^{\mu\nu}$  has taken to be  $g^{\mu\nu} = (1, -1, -1, -1)$ . The identical individual components of  $N^\mu$  from Eqs. (7) and (9) and the unaltered form of  $n$  as obtained from Eq. (8), confirms the expression of  $N^\mu$  as given by Eq. (9) in terms of dressed momenta  $\tilde{p}^\mu$ .

Next, we focus on the energy momentum tensor  $T^{\mu\nu}(x)$  whose different components describes the energy density and momentum flow. The quasiparticle energy-momentum tensor  $T^{\mu\nu}(x)$  can be defined under EQPM in terms of bare momenta as the following,

$$T^{\mu\nu}(x) = \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 E_{p_k}} p_k^\mu p_k^\nu f_k(x, p_k) + \delta\omega u^\mu u^\nu \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3} f_k(x, p_k). \quad (10)$$

Note that Eq. (10) gives the expression of quasiparticle energy density and pressure respectively as,

$$\begin{aligned} \epsilon(x) &= u_\mu u_\nu T^{\mu\nu} \\ &= \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3} \omega_{p_k} f_k(x, p_k), \end{aligned} \quad (11)$$

$$\begin{aligned} P(x) &= -\frac{1}{3} \Delta_{\mu\nu} T^{\mu\nu} \\ &= \frac{1}{3} \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3} |\vec{p}_k| f_k(x, p_k). \end{aligned} \quad (12)$$

In terms of dressed momenta,  $T^{\mu\nu}$  can be shown to take the form,

$$\begin{aligned} T^{\mu\nu}(x) &= \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \tilde{p}_k^\mu \tilde{p}_k^\nu f_k(x, \tilde{p}_k) \\ &+ \delta\omega \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \frac{\langle \tilde{p}_k^\mu \tilde{p}_k^\nu \rangle}{|\vec{p}_k|} f_k(x, \tilde{p}_k), \end{aligned} \quad (13)$$

with  $\langle \tilde{p}^\mu \tilde{p}^\nu \rangle = \frac{1}{2} \{ \Delta^{\mu\alpha} \Delta^{\nu\beta} + \Delta^{\mu\beta} \Delta^{\nu\alpha} \} \tilde{p}_\alpha \tilde{p}_\beta$  as the irreducible tensor of rank two. Equation (13) readily traces back the expression of  $\epsilon$  and  $P$  as given by Eqs. (11) and (12).

Finally, we provide the microscopic definition of entropy 4-current as,

$$\begin{aligned} S^\mu &= - \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \\ &\times \tilde{p}_k^\mu \{ f_k \ln f_k \mp (1 \pm f_k) \ln(1 \pm f_k) \}. \end{aligned} \quad (14)$$

Contraction of Eq. (14) with  $u^\mu$  gives the entropy density as follows,

$$\begin{aligned} s &= S^\mu u_\mu = - \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3} \\ &\times \{ f_k \ln f_k \mp (1 \pm f_k) \ln(1 \pm f_k) \}. \end{aligned} \quad (15)$$

### C. Conservation laws

We start with the relativistic transport equation of the single quasiparticle distribution function, that can be given by the following covariant equation,

$$\frac{1}{\omega_{p_k}} \tilde{p}_k^\mu \partial_\mu f_k(x, \tilde{p}_k) + \vec{F} \cdot \vec{\nabla}_{p_k} f_k = \sum_{l=1}^N C_{kl} [f_k, f_l],$$

$$[k = 1, \dots, N] \quad (16)$$

with  $\vec{F}$  as the external force and  $C_{kl}$  as the collision integral given by,

$$C_{kl} [f_k, f_l] = \frac{1}{22\omega_{p_k}} \int d\Gamma_{\tilde{p}_l} d\Gamma_{\tilde{p}'_l} d\Gamma_{\tilde{p}'_k} \delta^4(\tilde{p}_k + \tilde{p}_l - \tilde{p}'_k - \tilde{p}'_l)$$

$$\times (2\pi)^4 [f_k(\tilde{p}'_k) f_l(\tilde{p}'_l) \{1 \pm f_k(\tilde{p}_k)\} \{1 \pm f_l(\tilde{p}_l)\}$$

$$- f_k(\tilde{p}_k) f_l(\tilde{p}_l) \{1 \pm f_k(\tilde{p}'_k)\} \{1 \pm f_l(\tilde{p}'_l)\}]$$

$$\times \langle |M_{k+l \rightarrow k+l}|^2 \rangle. \quad (17)$$

The phase space factor is given by the notation  $d\Gamma_{\tilde{p}_i} = \frac{d^3 \tilde{p}_i}{(2\pi)^3 2\omega_i}$ . The overall,  $\frac{1}{2}$  factor appears due to the symmetry in order to compensate for the double counting of final states that occurs by interchanging  $\tilde{p}'_k$  and  $\tilde{p}'_l$ .  $\nu_l$  is the degeneracy of 2nd particle that belongs to  $l$ th species.  $\langle |M_{k+l \rightarrow k+l}|^2 \rangle$  is the QCD scattering amplitudes for  $2 \rightarrow 2$  binary, elastic processes are taken from [38], which are averaged over the spin and color degrees of freedom of the initial states and summed over the final states. However, the inelastic processes like  $q\bar{q} \rightarrow gg$ , have been ignored in the present case, because of the fact that they do not have a forward peak in the differential cross section and thus their contributions will presumably be much smaller compared to the elastic ones.

### 1. Conservation of particle current

Integrating both sides of Eq. (16) over  $\int \frac{d^3 |\tilde{p}_k|}{(2\pi)^3}$  and summing over  $k = [0, N]$  we obtain,

$$\sum_{k=1}^N \int \frac{d^3 |\tilde{p}_k|}{(2\pi)^3} \frac{1}{\omega_{p_k}} \tilde{p}_k^\mu \partial_\mu f_k(x, \tilde{p}_k)$$

$$+ \sum_{k=1}^N \int \frac{d^3 |\tilde{p}_k|}{(2\pi)^3} F^i \frac{\partial f_k}{\partial p_k^i} = 0. \quad (18)$$

The right-hand side of Eq. (18) is zero by the virtue of zeroth moment of summation invariance. Now we define the force term as,

$$F^i = -\partial_\mu \{ \delta\omega u^\mu u^i \}. \quad (19)$$

With this form of  $F^i$ , the integration on the second term of left hand side of Eq. (18) also reduces to zero. Now following the definition of particle 4-flow from Eq. (9) and performing the necessary integrations of Eq. (18), we can achieve the conservation of the particle flow,

$$\partial_\mu N^\mu = 0, \quad (20)$$

where the momentum integration over the second term of  $N^\mu$  from Eq. (9) and over  $\int \frac{d^3 |\tilde{p}_k|}{(2\pi)^3} \partial \{ \frac{\tilde{p}_k^\mu}{\omega_{p_k}} \} f_k$  exactly cancels each other to preserve particle flow conservation.

### 2. Conservation of energy momentum

Integrating both sides of Eq. (16) over  $\int \frac{d^3 |\tilde{p}_k|}{(2\pi)^3} \tilde{p}_k^\nu$  and summing over  $k$  we now obtain,

$$\sum_{k=1}^N \int \frac{d^3 |\tilde{p}_k|}{(2\pi)^3} \frac{1}{\omega_{p_k}} \tilde{p}_k^\mu \tilde{p}_k^\nu \partial_\mu f_k(x, \tilde{p}_k)$$

$$+ \sum_{k=1}^N \int \frac{d^3 |\tilde{p}_k|}{(2\pi)^3} \tilde{p}_k^\nu F^i \frac{\partial f_k}{\partial p_k^i} = 0 \quad (21)$$

This time the right-hand side of Eq. (21) is zero by the virtue of first moment of summation invariance.

Defining the force term as Eq. (19) and adopting the definition of quasiparticle stress energy tensor from Eq. (13), the space-time derivative over  $T^{\mu\nu}$  can be written as,

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left\{ \delta\omega \sum_{k=1}^N \nu_k \int \frac{d^3 |\tilde{p}_k|}{(2\pi)^3 \omega_{p_k}} \frac{\langle \tilde{p}_k^\mu \tilde{p}_k^\nu \rangle}{|\tilde{p}_k|} f_k(x, \tilde{p}_k) \right\}$$

$$+ \sum_{k=1}^N \nu_k \int \frac{d^3 |\tilde{p}_k|}{(2\pi)^3} \partial_\mu \left\{ \frac{\tilde{p}_k^\mu \tilde{p}_k^\nu}{\omega_{p_k}} \right\} f_k(x, \tilde{p}_k)$$

$$- \nu_k n \partial_\mu \{ \delta\omega u^\mu u^\nu \}, \quad (22)$$

where the integration over the force term simply reduces to a momentum independent quantity  $n \partial_\mu \{ \delta\omega u^\mu u^\nu \}$ . The space-time derivative over  $f_k$  multiplied with  $\delta\omega$  has been ignored considering  $\frac{\delta\omega}{\langle |\tilde{p}| \rangle_T} \ll 1$ , where  $\langle \rangle_T$  stands for the notation of thermal average. It can be shown that the addition of first and the second term on the right-hand side of Eq. (22) exactly cancels the force term to make the right-hand side zero whole together, leading to

$$\partial_\mu T^{\mu\nu} = 0. \quad (23)$$

We must notice that the partial derivative on the first term of left-hand side of Eq. (21) cannot be simply taken outside of the integral, since now the phase space factor  $\int \frac{d^3 |\tilde{p}_k|}{(2\pi)^3 \omega_{p_k}}$  contains a space-time dependent quasiparticle energy  $\omega_{p_k}$  given by Eq. (5). Hence the extraction of the partial derivative outside integral produces an extra term  $\int \frac{d^3 |\tilde{p}_k|}{(2\pi)^3} \partial_\mu \left\{ \frac{\tilde{p}_k^\mu \tilde{p}_k^\nu}{\omega_{p_k}} \right\} f_k(x, \tilde{p}_k)$ . So from this current analysis we can conclude that the difference of this term and the force term generated due to the quasiparticle excitations as a function of dispersion parameter  $\delta\omega$ , produces the exact additional piece of quasi particle energy-momentum tensor over the usual one  $[\int \frac{d^3 |\tilde{p}_k|}{(2\pi)^3 \omega_{p_k}} \tilde{p}_k^\mu \tilde{p}_k^\nu f(x, \tilde{p}_k)]$  from conventional kinetic theory. Equation (23) gives the

desired energy-momentum conservation under the EQPM scheme. Hence Eq. (16) with the force term given in Eq. (19) gives effective kinetic theory description of the interacting partons in a thermal medium under EQPM scheme.

## D. Estimation of transport coefficients

### 1. Solution to relativistic transport equation

Determination of the transport coefficients requires the knowledge of the system away from equilibrium. This could be done by first set-up the relativistic transport equation and look for the appropriate solutions. Here, in order to estimate the transport coefficients, one needs solve the relativistic transport equation (16). For this purpose we employ the Chapman-Enskog (CE) method, which is an iterative technique, where from the known lower order distribution function the unknown next order can be determined by successive approximation. Furthermore, to solve the transport equation Eq. (16) for  $k^{\text{th}}$  species, we need to linearize the collision term on right-hand side by introducing the relaxation time  $\tau_k$  over the deviation part of the next to leading order momentum distribution from the lowest order in following manner,

$$\frac{1}{\omega_{p_k}} \tilde{p}_k^\mu \partial_\mu f_k^0(x, \tilde{p}_k) + F^i \frac{\partial f_k^0}{\partial p_k^i} = -\frac{\delta f_k}{\tau_k} = -\frac{f_k^0(1 \pm f_k^0)\phi_k}{\tau_k}. \quad (24)$$

Clearly,  $f_k^0$  provides the leading order momentum distribution which is the equilibrium distribution function and  $\delta f_k$  accounts for the correction to the next to leading order corresponding to  $k^{\text{th}}$  species. Hence,  $\phi_k$  denotes deviation of momentum distribution from its equilibrium value that quantifies the dissipation in the medium.

Now we retrieve the definition of equilibrium distribution function of quasipartons under the EQPM from Eqs. (1), (2) and (3). In covariant notation with 4-momenta  $\tilde{p}_k^\mu$  the above equations can be written as,

$$f_k^0(x, \tilde{p}_k) = \frac{1}{\exp\left\{\frac{\tilde{p}_k^\mu u_\mu - \mu_k}{T}\right\}}, \quad (25)$$

with  $\mu_k = \mu_{B_k} + \delta\omega + T \ln z_k$ , such that  $\exp\{-\frac{\mu_k}{T}\}$  is the total effective fugacity due to the baryon chemical potential and the quasiparticle excitation effects.

Here we need to make an important remark. After defining the local equilibrium distribution function  $f_k^0$ , we can identify  $T$  and  $\exp\{-\frac{\mu_k}{T}\}$ , respectively as the temperature and effective fugacity of the system, only after specifying the following conditions,

$$\sum_{k=1}^N \int \frac{d^3|\tilde{p}_k|}{(2\pi)^3 \omega_{p_k}} \tilde{p}_k^\mu u_\mu \delta f_k = 0, \quad (26)$$

$$\sum_{k=1}^N \int \frac{d^3|\tilde{p}_k|}{(2\pi)^3 \omega_{p_k}} (\tilde{p}_k^\mu u_\mu)^2 \delta f_k = 0, \quad (27)$$

which follows from the fact that contraction of the non-equilibrium part of  $N^\mu$  and  $T^{\mu\nu}$  from Eqs. (9) and (13) with hydrodynamic velocity  $u^\mu$ , gives rise to zero (which is nothing but Landau-Lifshitz condition). In such situation the expression of energy density and pressure from Eqs. (11) and (12) contain only the equilibrium part of the distribution function. So we can conclude within the scheme of first order CE method, the particle number density and energy density can be specified by the equilibrium distribution function alone. Under such circumstances, the equilibrium part of particle 4-flow and energy-momentum tensor can be expressed by the following macroscopic definition,

$$N^\mu = n u^\mu. \quad (28)$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - P \Delta^{\mu\nu}. \quad (29)$$

Following same line of argument, the macroscopic definition of equilibrium entropy density becomes,

$$s = \sum_{k=1}^N \left\{ \frac{\epsilon_k + P_k}{T} - \frac{n_k \mu_k}{T} \right\}. \quad (30)$$

### 2. Equilibrium thermodynamic laws

From the thermodynamic definition of  $N^\mu(x)$  and  $T^{\mu\nu}(x)$  from Eqs. (28) and (29) respectively, and following their conservation laws from Eqs. (20) and (23) respectively, we can achieve the equilibrium thermodynamic laws of macroscopic state variable such as number density ( $n$ ), energy per particle ( $e$ ) and hydrodynamic velocity  $u^\mu(x)$ , as follows,

$$D n_k = -n_k \partial \cdot u, \quad (31)$$

$$\sum_{k=1}^N x_k D e_k = -\left\{ \frac{\sum_{k=1}^N P_k}{\sum_{k=1}^N n_k} \right\} \partial \cdot u, \quad (32)$$

$$D u^\mu = \frac{\nabla^\mu P}{n h}, \quad (33)$$

with  $D = u^\mu \partial_\mu \equiv \frac{\partial}{\partial t}$  as the convective time derivative and  $h$  as enthalpy per particle  $h = e + \frac{P}{n}$  of the system.  $P_k$  is the partial pressure belongs to  $k^{\text{th}}$  species that is related to total pressure as  $P = \sum_{k=1}^N P_k$ .  $x_k = n_k/n$  denotes the particle fraction given by the ratio of particle number of  $n^{\text{th}}$  species to total particle number,  $x_k = n_k/n$ . Following the prescription, the total energy density can be given as,  $\epsilon = \sum_{k=1}^N \epsilon_k = \sum_{k=1}^N e_k n_k$ .

### 3. Linearized solution of the deviation function

Following the definition of equilibrium distribution function from Eq. (25), the second term on the left-hand side of Eq. (24) vanishes for a comoving frame, whereas the first term produces a number of terms containing thermodynamic forces giving rise to a number of transport processes as follows,

$$Q_k X + \langle \tilde{p}_k^\mu \rangle (\omega_{p_k} - h_k) X_{q\mu} - \langle \tilde{p}_k^\mu \tilde{p}_k^\nu \rangle X_{\mu\nu} = -\frac{T\omega_{p_k}}{\tau_k} \phi_k. \quad (34)$$

with  $Q_k = \frac{1}{3} \{ |\vec{p}_k|^2 - 3\omega_{p_k}^2 c_s^2 \}$  where  $c_s$  is the velocity of sound. The thermodynamic forces such as the bulk viscous force, thermal force and shear viscous force are defined respectively as follows,

$$X = \partial \cdot u, \quad (35)$$

$$X_q^\mu = \left\{ \frac{\nabla^\mu T}{T} - \frac{\nabla^\mu P}{nh} \right\}, \quad (36)$$

$$X_{\mu\nu} = \langle \partial_\mu u_\nu \rangle. \quad (37)$$

Since thermodynamic forces are independent, in order to be a solution of Eq. (34), the deviation function  $\phi_k$  must be a linear combination of thermodynamic forces with a number of unknown coefficients,

$$\phi_k = A_k X + B_k^\mu X_{q\mu} - C_k^{\mu\nu} X_{\mu\nu}. \quad (38)$$

The coefficients with proper tensorial ranks can be determined from Eq. (34) itself as the following,

$$A_k = \frac{Q_k}{\left\{ -\frac{T\omega_{p_k}}{\tau_k} \right\}}, \quad (39)$$

$$B_k = \langle \tilde{p}_k^\mu \rangle \frac{(\omega_{p_k} - h_k)}{\left\{ -\frac{T\omega_{p_k}}{\tau_k} \right\}}, \quad (40)$$

$$C_k^{\mu\nu} = \frac{\langle \tilde{p}_k^\mu \tilde{p}_k^\nu \rangle}{\left\{ -\frac{T\omega_{p_k}}{\tau_k} \right\}}. \quad (41)$$

Therefore, it can be observed that through these coefficients which contain the thermal relaxation times of quasipartons, the dynamic interactions of the medium enter in the expression of the deviation function, which are finally inserted in the expressions of transport coefficients.

### 4. Decomposition of the energy-momentum tensor

In order to decompose the energy momentum tensor in an equilibrium and an out of equilibrium part, we first define the pressure tensor in the following way,

$$P^{\mu\nu} = \Delta_\sigma^\mu T^{\sigma\tau} \Delta_\tau^\nu. \quad (42)$$

Following the covariant definition of  $T^{\mu\nu}$  under the EQPM from Eq. (13), the pressure tensor yields the form given below,

$$P^{\mu\nu} = \sum_{k=1}^N \nu_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \langle \tilde{p}_k^\mu \tilde{p}_k^\nu \rangle f_k(x, \tilde{p}_k) + \delta\omega \sum_{k=1}^N \nu_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \frac{\langle \tilde{p}_k^\mu \tilde{p}_k^\nu \rangle}{|\vec{p}_k|} f_k(x, \tilde{p}_k). \quad (43)$$

In the LRF,  $P^{\mu\nu}$  is purely spatial,

$$P_{LRF}^{00} = P_{LRF}^{0i} = P_{LRF}^{i0} = 0, \quad (44)$$

$$P_{LRF}^{ij} = \sum_{k=1}^N \nu_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \tilde{p}_k^i \tilde{p}_k^j f_k(x, \tilde{p}_k) + \delta\omega \sum_{k=1}^N \nu_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \frac{\tilde{p}_k^i \tilde{p}_k^j}{|\vec{p}_k|} f_k(x, \tilde{p}_k). \quad (45)$$

Now we decompose  $P^{\mu\nu}$  in a reversible and an irreversible part, that picks up respectively the equilibrium and non-equilibrium components of  $f_k$  in Eq. (43),

$$P^{\mu\nu} = -P\Delta^{\mu\nu} + \Pi^{\mu\nu}. \quad (46)$$

The reversible part is addressed by the equilibrium distribution function  $f_k^0$  as the following,

$$-P\Delta^{\mu\nu} = \sum_{k=1}^N \nu_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \langle \tilde{p}_k^\mu \tilde{p}_k^\nu \rangle f_k^0(x, \tilde{p}_k) + \delta\omega \sum_{k=1}^N \nu_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \frac{\langle \tilde{p}_k^\mu \tilde{p}_k^\nu \rangle}{|\vec{p}_k|} f_k^0(x, \tilde{p}_k), \quad (47)$$

which on contracting with  $\Delta_{\mu\nu}$  simply leads to Eq. (12), revealing  $P$  as the local hydrostatic pressure.

However, the irreversible part  $\Pi^{\mu\nu}$ , named by viscous pressure tensor, includes the nonequilibrium part of  $f_k$  only, leading to the following expression,

$$\Pi^{\mu\nu} = \sum_{k=1}^N \nu_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \langle \tilde{p}_k^\mu \tilde{p}_k^\nu \rangle \delta f_k + \delta\omega \sum_{k=1}^N \nu_k \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \frac{\langle \tilde{p}_k^\mu \tilde{p}_k^\nu \rangle}{|\vec{p}_k|} \delta f_k. \quad (48)$$

From Eq. (48), we can see that the viscous pressure tensor is orthogonal to hydrodynamic velocity,

$$\Pi^{\mu\nu}u_\mu = 0. \quad (49)$$

The heat flow is defined as the difference of energy flow and the flow of enthalpy carried by the particle,

$$I_q^\mu = u_\nu T^{\nu\sigma} \Delta_\sigma^\mu - h N^\sigma \Delta_\sigma^\mu. \quad (50)$$

Putting expressions of  $T^{\nu\sigma}$  and  $N^\sigma$  from Eqs. (13) and (9), respectively, it can be shown that in the LRF the heat flow is purely spatial as well,

$$I_q^0 = 0, \quad (51)$$

$$I_q^i = T_{LRF}^{i0} - N_{LRF}^i. \quad (52)$$

From Eq. (50), it is evident that  $I_q^\mu$  is also orthogonal to  $u^\mu$ ,

$$I_q^\mu u_\mu = 0. \quad (53)$$

From Eq. (50), it is also observed that heat flow only retains the nonequilibrium part of  $f_k$ , while the equilibrium  $f_k^0$  produces zero contraction in heat flow, leading to the following expression,

$$\begin{aligned} I_q^\mu &= u_\nu \Delta_\sigma^\mu \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \tilde{p}_k^\nu \tilde{p}_k^\sigma \delta f_k \\ &\quad - h \Delta_\sigma^\mu \left\{ \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \tilde{p}_k^\sigma \delta f_k \right. \\ &\quad \left. + \delta\omega \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \frac{\langle \tilde{p}_k^\sigma \rangle}{|\vec{p}_k|} \delta f_k \right\}. \quad (54) \end{aligned}$$

It is worth noting that in viscous pressure the additional term due to dispersion parameter  $\delta\omega$  is contributed from  $T^{\mu\nu}$ , where as in heat flow it comes from  $N^\mu$ .

### 5. Shear and bulk viscous coefficients

In both the terms of Eq. (48),  $\langle \tilde{p}^\mu \tilde{p}^\nu \rangle$  can be decomposed in a traceless and a remaining part, giving rise to  $\Pi^{\mu\nu}$  a shear and a bulk part respectively.

Following this argument the shear viscous tensor comes out to be,

$$\begin{aligned} \bar{\Pi}^{\mu\nu} &= \Pi^{\mu\nu} - \Pi \Delta^{\mu\nu}, \\ &= \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \langle \tilde{p}_k^\mu \tilde{p}_k^\nu \rangle f_k^0 (1 \pm f_k^0) \phi_k \\ &\quad + \delta\omega \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \frac{\langle \tilde{p}_k^\mu \tilde{p}_k^\nu \rangle}{|\vec{p}_k|} f_k^0 (1 \pm f_k^0) \phi_k. \quad (55) \end{aligned}$$

Here  $\langle \langle \rangle \rangle$  has been used to denote the traceless irreducible tensor,  $\langle \langle A^\mu B^\nu \rangle \rangle = \{ \frac{1}{2} \Delta_\alpha^\mu \Delta_\beta^\nu + \frac{1}{2} \Delta_\beta^\mu \Delta_\alpha^\nu - \frac{1}{3} \Delta_{\alpha\beta} \Delta^{\mu\nu} \} A^\alpha B^\beta$ . Consequently, the bulk viscous part takes the following form,

$$\begin{aligned} \Pi &= \sum_{k=1}^N \frac{\nu_k}{3} \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \Delta_{\mu\nu} \tilde{p}_k^\mu \tilde{p}_k^\nu f_k^0 (1 \pm f_k^0) \phi_k \\ &\quad + \delta\omega \sum_{k=1}^N \frac{\nu_k}{3} \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \frac{1}{|\vec{p}_k|} \Delta_{\mu\nu} \tilde{p}_k^\mu \tilde{p}_k^\nu f_k^0 (1 \pm f_k^0) \phi_k. \quad (56) \end{aligned}$$

Putting the expression of  $\phi_k$  from Eq. (38) and comparing with the macroscopic definition of  $\Pi^{\mu\nu}$  as follows,

$$\Pi^{\mu\nu} = 2\eta \langle \partial^\mu u^\nu \rangle + \zeta \Delta^{\mu\nu} \partial \cdot u, \quad (57)$$

we obtain following expressions of shear and bulk viscosity respectively,

$$\begin{aligned} \eta &= \sum_{k=1}^N \frac{\nu_k \tau_k}{15T} \int \frac{d^3|\vec{p}_k|}{(2\pi)^3} \frac{|\vec{p}_k|^4}{\omega_{p_k}^2} f_k^0 (1 \pm f_k^0) \\ &\quad + \delta\omega \sum_{k=1}^N \frac{\nu_k \tau_k}{15T} \int \frac{d^3|\vec{p}_k|}{(2\pi)^3} \frac{|\vec{p}_k|^3}{\omega_{p_k}^2} f_k^0 (1 \pm f_k^0), \quad (58) \end{aligned}$$

$$\begin{aligned} \zeta &= \sum_{k=1}^N \frac{\nu_k \tau_k}{9T} \int \frac{d^3|\vec{p}_k|}{(2\pi)^3} \frac{1}{\omega_{p_k}^2} \{ |\vec{p}_k|^2 - 3\omega_{p_k}^2 c_s^2 \}^2 f_k^0 (1 \pm f_k^0) \\ &\quad + \delta\omega \sum_{k=1}^N \frac{\nu_k \tau_k}{9T} \int \frac{d^3|\vec{p}_k|}{(2\pi)^3} \frac{1}{\omega_{p_k}^2} \frac{1}{|\vec{p}_k|} \{ |\vec{p}_k|^2 - 3\omega_{p_k}^2 c_s^2 \}^2 \\ &\quad \times f_k^0 (1 \pm f_k^0). \quad (59) \end{aligned}$$

Clearly the second term on the right-hand side of Eqs. (58) and (59) is the additional term due to quasiparticle excitations, over the first term which comes from the usual kinetic theory of bare particles.

### 6. Thermal conductivity

Since we observe only the nonequilibrium part of the heat-flow is relevant, we can obtain its analytical expression for a multicomponent system, from Eq. (54) after contracting with projection operator and hydrodynamic velocity,

$$\begin{aligned} \delta I^\mu &= \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} (\omega_{p_k} - h_k) \langle \tilde{p}_k^\mu \rangle f_k^0 (1 \pm f_k^0) \phi_k \\ &\quad - \delta\omega \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p}_k|}{(2\pi)^3 \omega_{p_k}} \frac{h_k}{|\vec{p}_k|} \langle \tilde{p}_k^\mu \rangle f_k^0 (1 \pm f_k^0) \phi_k. \quad (60) \end{aligned}$$

Putting the expression of  $\phi_k$  from Eq. (38) and comparing with the macroscopic definition of heat flow,

$$\delta I^\mu = \lambda T X_q^\mu, \quad (61)$$

we obtain the expression of thermal conductivity as follows,

$$\begin{aligned} \lambda = & \sum_{k=1}^N \frac{\nu_k \tau_k}{3T^2} \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3} \frac{(\omega_{p_k} - h_k)^2}{\omega_{p_k}^2} |\vec{p}_k|^2 f_k^0 (1 \pm f_k^0) \\ & - \delta\omega \sum_{k=1}^N \frac{\nu_k \tau_k}{3T^2} \int \frac{d^3 |\vec{p}_k|}{(2\pi)^3} \frac{h_k (\omega_{p_k} - h_k)}{\omega_{p_k}^2} |\vec{p}_k| f_k^0 (1 \pm f_k^0). \end{aligned} \quad (62)$$

The term proportional to  $\delta\omega$  in Eq. (62) is the extra term here over the usual definition of  $\lambda$  from kinetic theory, which renders the quasiparticle excitation in the analytical expression of thermal conductivity.

### 7. Dynamical inputs

The relaxation times that are inverse of the collision frequencies, provide the dynamical interaction measures in the expressions of the above mentioned transport coefficients. The relaxation times, corresponding to the quasi-gluons and quasi-quarks/antiquarks, namely  $\tau_g$  and  $\tau_{q/\bar{q}}$  respectively, have been taken from [39], with updated lattice EOSs [6]. The interaction cross sections therein include the binary, elastic scatterings between quarks and gluons, considering the fusion processes not to be able to contribute forward peak in the differential cross section, leading to insignificant contribution with respect to elastic ones. The effective coupling for an interacting QCD medium, has been introduced following the EQPM prescription of charge renormalization [23], at finite temperature and baryon chemical potential. Throughout the analysis, the quark chemical potential has been taken to be  $\mu_q = 100$  MeV.

## III. RESULTS AND DISCUSSIONS

In this section, we have depicted the temperature dependence of shear and bulk viscous coefficients over entropy density and thermal conductivity scaled by  $T^2$ , with and without the mean field corrections in Figs. 1, 2 and 3 respectively. As we have already discussed in our previous work [39], the mean field corrections are of second order in gradients, which indeed appear as the mean field force term,  $\partial_\mu \{\delta\omega u^\mu u^i\}$ . This term being a derivative over  $\delta\omega$ , which itself is a temperature gradient of the fugacity parameter  $z_{g/q}$ , turns out to be second order in gradient. Since, at higher temperature regions, mostly over  $T/T_c \sim 2.5$ ,  $z_{g/q}$  is a slowly varying function of  $T$  [22,23], we can see that the mean field effects are almost negligible in the temperature dependence of transport

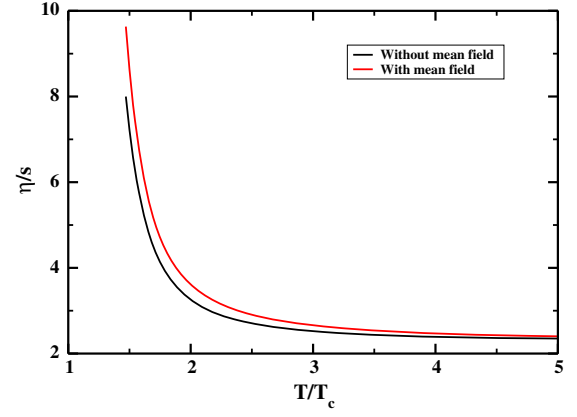


FIG. 1. Shear viscosity over entropy density ratio as a function of  $T/T_c$  with and without mean field correction.

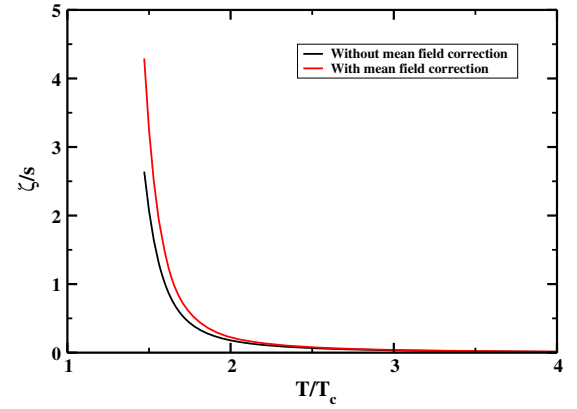


FIG. 2. Bulk viscosity over entropy density ratio as a function of  $T/T_c$  with and without mean field correction.

coefficients over  $T/T_c \sim 3$ . However, below  $T/T_c \sim 3$ , the sharp temperature gradient of  $z_{g/q}$ , makes the mean field term significant, which is consequently reflected in the temperature behavior of transport coefficients. In Figs. 1, 2 and 3, the estimated values of shear viscosity, bulk

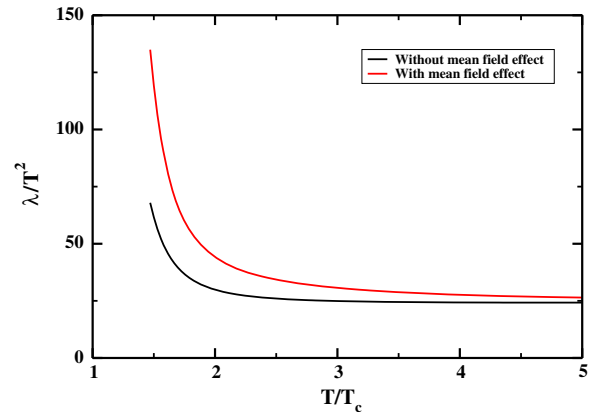


FIG. 3. Thermal conductivity scaled by  $T^2$  as a function of  $T/T_c$  with and without mean field correction.



viscosity and thermal conductivity including mean field corrections, follow the same temperature trend as obtained in the previous estimation [39] without considering it. However, the quantitative difference in the low temperature behavior of shear and bulk viscous coefficients as well as thermal conductivity, with and without mean field corrections, reveals the significance of the mean field term induced by a system's collective behavior, in estimating thermodynamic quantities in those temperature regions. Therefore, although the mean field correction is not affecting the transport coefficients beyond  $T/T_c \sim 3$ , and hence neglecting those terms are justified in explaining the high temperature behavior of the system properties, its inclusion is essential in order to explain the behavior of thermodynamic parameters closer to  $T_c$ .

#### IV. CONCLUSIONS AND OUTLOOK

In conclusion, a covariant kinetic theory is developed for the hot QCD matter/QGP, employing the effective fugacity quasi-particle model (EQPM) for interacting hot QCD equations of state. Since the hot QCD medium effects are encoded in the gluon and quark effective fugacities as well as in the modified part of the dispersion relations, the mean field terms of the current effective kinetic theory also include only these fugacity parameters and their derivatives. The modified energy momentum tensor reproduces the hot QCD thermodynamics exactly, while respecting the thermodynamic consistency condition. The conservation laws are realized in an exact way from the covariant kinetic theory while taking its appropriate moments of the relativistic transport equation.

Interestingly, the mean field contributions induce sizable modifications to the transport coefficients of the hot QCD matter. Be it shear viscosity, bulk viscosity or the thermal conductivity, the respective three momentum integrals involving the out of equilibrium part of the momentum

distribution, get modified with an additive term proportional to  $\delta\omega_{g,q}/|\vec{p}|$ . Recalling that  $\delta\omega_{g,q}$  is the medium modified part of the dispersion relation containing the collective effects of an interacting medium, we can conclude thus this additional term introduces the quasiparticle excitations in the expressions of transport coefficient. The modifications have negligible contribution at higher temperatures ( $\geq 2.5T_c$ ) as far as the first order transport coefficients are concerned, however, in the vicinity of the transition temperature the mean field effects appear to be significant. This observation is in line with our earlier estimates for the transport coefficients such as shear and bulk viscosities, and thermal conductivity, etc. [39].

The work presented in the manuscript is the first step toward developing second and third order dissipative relativistic hydrodynamics from transport theory with the effective fugacity quasiparticle model along with estimating the respective second and third order transport coefficients from the relativistic effective kinetic theory. These aspects will be taken up in immediate near future. Moreover, the electromagnetic responses of the strongly interacting medium in presence of an electric or magnetic field, are scheduled to be explored following the line of work in [40,41], within the scopes of the effective kinetic theory developed in the present work.

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