Strong decays of the higher isovector scalar mesons

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Under the assignment of $a_0(1450)$ as the ground isovector scalar meson, the strong decays of $a_0(1950)$ and $a_0(2020)$ are evaluated in the $^3P_0$ model. Our calculations suggest that $a_0(1950)$ and $a_0(2020)$ can be regarded as the same resonance referring to $a_0(3^3P_0)$. The masses and strong decays of $a_0(2^3P_0)$ and $a_0(4^3P_0)$ are also predicted, which can be useful in the search for radially excited scalar mesons in the future.

I. INTRODUCTION

In the framework of quantum chromodynamics (QCD), apart from the ordinary $q\bar{q}$ states, other exotic states such as glueballs, hybrids, and tetraquarks are permitted to exist in meson spectra. To identify these exotic states, one needs to distinguish them from the background of ordinary $q\bar{q}$ states, which requires one to understand well the conventional $q\bar{q}$ meson spectroscopy both theoretically and experimentally.

Experimentally, $a_0(980)$, $a_1(1260)$, $b_1(1235)$, and $a_2(1320)$, respectively, as the lowest-lying $J^{PC} = 0^{++}$, 1^{++}, 1^{+-}, and 2^{++} states, have been well established [1]. In contrast to $a_1(1260)$, $b_1(1235)$, and $a_2(1320)$, which can be well accommodated in the ordinary $q\bar{q}$ picture, $a_0(980)$, together with its multiplet partners σ, κ, and $f_0(980)$, does not fit well into the predictions of the quark model. For example, the observed mass ordering of these lowest scalar states is $m_\sigma < m_\kappa < m_{a_0,f_0}$ [1], while in the conventional quark model, by a naive counting of the quark mass, the mass ordering of the scalar $q\bar{q}$ nonet should be $m_\sigma \sim m_{a_0} < m_\kappa < m_{f_0}$. These scalar states below 1 GeV are generally believed not to be $q\bar{q}$ states [2–7].

At present, above the $a_0(980)$ mass, three higher isovector scalar states, $a_0(1450)$, $a_0(2020)$, and $a_0(1950)$, have been reported experimentally. $a_0(1450)$ was observed in $p\bar{p}$ annihilation experiments [8,9], $D^\pm \to K^+K^-\pi^\pm$ [10], and $D^0 \to K^0_S K^\pm \pi^\mp$ [11]. $a_0(2020)$ with two alternative solutions of similar masses and widths was found by

the Crystal Barrel Collaboration in the partial wave analysis of the data on $\bar{p}p \to \pi^0\eta$ and $\pi^0\eta'$ [12], and $a_0(1950)$ was observed by the *BABAR* Collaboration in the processes $\gamma\gamma \to K^0_s K^\pm \pi^\mp$ and $\gamma\gamma \to K^+K^-\pi^0$ [13]. The masses and widths of the three isovector scalar states are listed in Table I. The lattice QCD calculations support that the lowest isovector scalar $q\bar{q}$ state corresponds to $a_0(1450)$ rather than $a_0(980)$ [14–16]. It is widely accepted that $a_0(1450)$ is the isovector member of the 1^3P_0 $q\bar{q}$ nonet [1]. The natures of $a_0(2020)$ and $a_0(1950)$ are unclear. To be able to understand the nature of a newly observed state, it is natural and necessary to exhaust the possible $q\bar{q}$ description before restoring to more exotic assignments. Therefore, with the assignment of $a_0(1450)$ as the ground $q\bar{q}$ state, one naturally asks whether the higher isovector scalar states, $a_0(2020)$ and $a_0(1950)$, can be identified as the radial excitations of $a_0(1450)$.

Theoretical efforts to identify the quark model assignments for $a_0(2020)$ and $a_0(1950)$ have been carried out. It is suggested that $a_0(1950)/a_0(2020)$ can be assigned as the 2^3P_0 state based on the extended linear sigma model in Ref. [17], where $a_0(2020)$ is considered earlier evidence for $a_0(1950)$. In addition, $a_0(1950)/a_0(2020)$ is assigned as the 3^3P_0 state based on the relativistic quark model in Ref. [18], where the predicted $a_0(3^3P_0)$ mass is about 1993 MeV, in agreement with both the $a_0(1950)$ and $a_0(2020)$ masses within errors. Obviously, further studies on the quark model assignments for $a_0(1950)$ and $a_0(2020)$

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TABLE I. The masses and widths of the higher isovector scalar states (in MeV).

State	Mass	Width	Ref.
$a_0(1450)$	1474 ± 19	265 ± 13	[8,9]
$a_0(2020)$	2025 ± 30	330 ± 75	[12]
	1980^{+12}_{-80}	225^{+120}_{-32}	[12]
$a_0(1950)$	$1931 \pm 14 \pm 22$	$271 \pm 22 \pm 29$	[13]

in other approaches are needed. Also, from Table I, one can see that the resonance parameters of $a_0(2020)$ are close to those of $a_0(1950)$. The observed mass difference between $a_0(1950)$ and $a_0(2020)$ is less than 100 MeV; in such a small mass interval, it would be very difficult to accommodate two radial excitations of $a_0(1450)$ in practically all the quark models. We therefore conclude that if both $a_0(2020)$ and $a_0(1950)$ can be explained as $q\bar{q}$ states, they should correspond to the same resonance. In this work, we shall discuss the possible quark model assignments of $a_0(2020)$ and $a_0(1950)$ by investigating their strong decays in the $^3P_0$ model and check whether $a_0(2020)$ and $a_0(1950)$ can be identified as the same scalar meson.

The organization of this paper is as follows. In Sec. II, we give a brief review of the $^3P_0$ model. In Sec. III, the calculations and discussion are presented, and the summary and conclusion are given in Sec. IV.

II. THE $^3P_0$ MODEL

In this work, we employ the $^3P_0$ model to evaluate the Okubo-Zweig-Iizuka (OZI)-allowed open flavor two-body strong decays of the initial meson. The $^3P_0$ model, also known as the quark-pair creation model, was originally introduced by Micu [19] and further developed by Le Yaouanc *et al.* [20–22]. The $^3P_0$ model has been widely applied to study strong decays of hadrons with considerable success [23–40]. The main assumption of the $^3P_0$ model is that the strong decay occurs through a quark-antiquark pair with the vacuum quantum number. The new produced quark-antiquark pair, together with the $q\bar{q}$ within the initial meson, regroups into two outgoing mesons in all possible quark rearrangement ways.

Following the conventions in Ref. [31], the transition operator T of the decay $A \to BC$ in the $^3P_0$ model is given by

$$T = -3\gamma \sum_m \langle 1m1 - m|00 \rangle \int d^3\boldsymbol{p}_3 d^3\boldsymbol{p}_4 \delta^3(\boldsymbol{p}_3 + \boldsymbol{p}_4)$$
$$\times \mathcal{Y}_1^m\left(\frac{\boldsymbol{p}_3 - \boldsymbol{p}_4}{2}\right)\chi_{1-m}^{34}\phi_0^{34}\omega_0^{34}b_3^\dagger(\boldsymbol{p}_3)d_4^\dagger(\boldsymbol{p}_4), \quad (1)$$

where γ is a dimensionless parameter denoting the probability of the quark-antiquark pair $q_3\bar{q}_4$ with quantum number $J^{PC} = 0^{++}$. $\boldsymbol{p}_3$ and $\boldsymbol{p}_4$ are the momenta of the created quark q_3 and antiquark $\bar{q}_4$, respectively. $\chi_{1,-m}^{34}$, ϕ_0^{34}, and ω_0^{34} are the spin, flavor, and color wave functions of $q_3\bar{q}_4$, respectively. The solid harmonic polynomial $\mathcal{Y}_1^m(\boldsymbol{p}) \equiv |p|^1 Y_1^m(\theta_p, \phi_p)$ reflects the momentum-space distribution of the $q_3\bar{q}_4$.

The partial wave amplitude $\mathcal{M}^{LS}(\boldsymbol{P})$ of the decay $A \to BC$ can be given by [41]

$$\mathcal{M}^{LS}(\boldsymbol{P}) = \sum_{\substack{M_{J_B}, M_{J_C}, \\ M_S, M_L}} \langle LM_L SM_S|J_A M_{J_A}\rangle \langle J_B M_{J_B} J_C M_{J_C}|SM_S\rangle$$
$$\times \int d\Omega Y_{LM_L}^* \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\boldsymbol{P}), \quad (2)$$

where $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\boldsymbol{P})$ is the helicity amplitude and defined as

$$\langle BC|T|A\rangle = \delta^3(\boldsymbol{P}_A - \boldsymbol{P}_B - \boldsymbol{P}_C)\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\boldsymbol{P}). \quad (3)$$

$|A\rangle$, $|B\rangle$, and $|C\rangle$ denote the mock meson states defined in Ref. [42].

Due to different choices of the pair-production vertex, phase space convention, and employed meson space wave function, various $^3P_0$ models exist in the literature. In this work, we employ the simplest vertex as introduced originally by Micu, who assumes a spatially constant pair-production strength γ [19], relativistic phase space, and simple harmonic oscillator (SHO) wave functions. With the relativistic phase space, the decay width $\Gamma(A \to BC)$ can be expressed in terms of the partial wave amplitude,

$$\Gamma(A \to BC) = \frac{\pi|\boldsymbol{P}|}{4M_A^2}\sum_{LS}|\mathcal{M}^{LS}(\boldsymbol{P})|^2, \quad (4)$$

where $|\boldsymbol{P}| = \frac{\sqrt{[M_A^2 - (M_B + M_C)^2][M_A^2 - (M_B - M_C)^2]}}{2M_A}$, and M_A, M_B, and M_C are the masses of the mesons A, B, and C, respectively. The explicit expressions for $\mathcal{M}^{LS}(\boldsymbol{P})$ can be found in Refs. [31–33].

Under the SHO approximation, the meson space wave function in the momentum space is

$$\psi_{nLM_L}(\boldsymbol{p}) = R_{nL}^{\text{SHO}}(p)Y_{LM_L}(\Omega_p), \quad (5)$$

where the radial wave function is given by

$$R_{nL}^{\text{SHO}}(p) = \frac{(-1)^n(-i)^L}{\beta^{3/2}}\sqrt{\frac{2n!}{\Gamma(n + L + 3/2)}}$$
$$\times \left(\frac{p}{\beta}\right)^L e^{-(p^2/2\beta^2)} L_n^{L+(1/2)}\left(\frac{p^2}{\beta^2}\right). \quad (6)$$

Here β is the SHO wave function scale parameter, and $L_n^{L+(1/2)}(\frac{p^2}{\beta^2})$ is an associated Laguerre polynomial.

III. CALCULATION AND RESULTS

In our calculations, the model parameters include the light nonstrange quark pair creation strength γ, the SHO wave function scale β, and the constituent quark masses. β is set to be $\beta_A = \beta_B = \beta_C = 0.4$ GeV, the typical values used to evaluate the light meson decays, as in Refs. [25,26,31–35,39,43–45], and the constituent quark masses are taken to be $m_u = m_d = 330$ MeV and $m_s = 550$ MeV, as in Refs. [27,31–35,39]. We take $\gamma = 7.1$ by fitting to the total width of $a_0(1450)$ as the 1^3P_0 state. The strange quark pair creation strength $\gamma_{s\bar{s}}$ can be related by $\gamma_{s\bar{s}} = \gamma\frac{m_u}{m_s}$ [46]. The meson flavor wave functions follow the conventions of Refs. [26,47] except for

TABLE II. Decay widths of $a_0(1450)$ as the 1^3P_0 state (in MeV). The initial state mass is set to be 1474 MeV.

| Channel	Mode	$\Gamma_i(1^3P_0)$
$0^+ \rightarrow 0^-0^-$	$\pi\eta$	72.77
	$\pi\eta'$	144.47
	$\pi\eta(1295)$	0.86
	$K\bar{K}$	34.40
$0^+ \rightarrow 0^-1^+$	$\pi b_1(1235)$	9.96
	$\pi f_1(1285)$	3.98
Total width		266.45
Experiment [1]		265 ± 13

$f_1(1285) = -0.28n\bar{n} + 0.96s\bar{s}$, $f_1(1420) = -0.96n\bar{n} - 0.28s\bar{s}$, as in Ref. [48], and $\eta(1295) = (n\bar{n} - s\bar{s})/\sqrt{2}$, $\eta(1475) = (n\bar{n} + s\bar{s})/\sqrt{2}$, as in Ref. [49], where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$. Masses of the final state mesons are taken from [1].

The decay widths of $a_0(1450)$ as the 1^3P_0 state are listed in Table II. The dominant decay modes of the 1^3P_0 isovector state are $\pi\eta$, $\pi\eta'$, and $K\bar{K}$, consistent with observations of $a_0(1450)$ [8,9,50].

The decay widths of $a_0(1950)$ as the 2^3P_0 and 3^3P_0 states are shown in Table III. If $a_0(1950)$ is the 2^3P_0 state, its total width is expected to be about 771 MeV, much larger than the observed $a_0(1950)$ width of $271 \pm 22 \pm 29$ MeV [13]. The possibility of $a_0(1950)$ being the 2^3P_0 state can be ruled out. If $a_0(1950)$ is the 3^3P_0 state, its total width is about 207 MeV, reasonably close to the measurement within errors. The dependence of the total width of $a_0(3^3P_0)$ on the initial state mass is shown in Fig. 1.

TABLE III. Decay widths of $a_0(1950)$ as the 2^3P_0 and 3^3P_0 states (in MeV). The initial state mass is set to be 1931 MeV.

Channel	Mode	$\Gamma_i(2^3P_0)$	$\Gamma_i(3^3P_0)$
$0^+ \rightarrow 0^-0^-$	$\pi\eta$	26.00	5.13
	$\pi\eta'$	2.07	2.52
	$\pi(1300)\eta$	37.75	29.72
	$\pi\eta(1475)$	20.81	13.22
	$\pi\eta(1295)$	7.03	1.97
	$K\bar{K}$	0.42	0.74
$0^+ \rightarrow 0^-1^+$	$\pi b_1(1235)$	339.48	90.06
	$\pi f_1(1285)$	84.42	9.18
	$\pi f_1(1420)$	4.38	0.61
	$KK_1(1270)$	28.31	9.97
	$KK_1(1400)$	2.38	0.72
	$\eta a_1(1260)$	24.12	3.74
$0^+ \rightarrow 1^-1^-$	$\rho\omega$	181.18	36.07
	$K^*\bar{K}^*$	10.94	2.05
$0^+ \rightarrow 0^-2^-$	$\pi\eta_2(1645)$	1.24	1.58
Total width		770.51	207.27
Experiment [13]			$271 \pm 22 \pm 29$
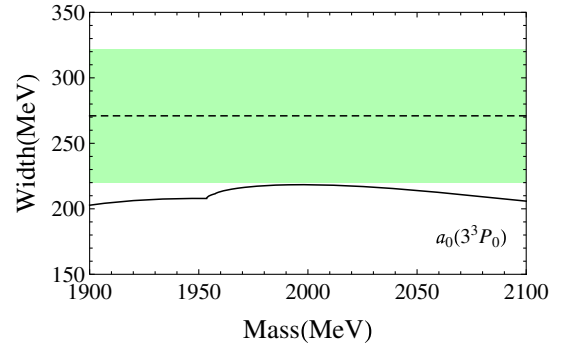
FIG. 1. The dependence of the total width of $a_0(3^3P_0)$ on the initial state mass. The dashed line with a green band denotes the *BABAR* experimental data [13].

Within the $a_0(1950)$ mass errors, the total width does not change too much. The assignment of $a_0(1950)$ as the 4^3P_0 state can also be ruled out because the predicted width for $a_0(4^3P_0)$ with a mass of 1931 MeV is about 37.3 MeV (see also Fig. 3), much smaller than the $a_0(1950)$ width. Therefore, the measured mass and width for $a_0(1950)$ are in favor of it being the 3^3P_0 state.

As shown in Fig. 1, $a_0(3^3P_0)$ with a mass of 2025 MeV is predicted to have a width of about 217 MeV, about 38 MeV smaller than the lower limit of Crystal Barrel's solution I for the $a_0(2020)$ width of 330 ± 75 MeV [12], and the predicted width for $a_0(3^3P_0)$ with a mass of 1980 MeV is about 218 MeV, in agreement with Crystal Barrel's solution II for the $a_0(2020)$ width of 225^{+120}_{-32} MeV [12]. The possibility of $a_0(2020)$ being the 2^3P_0 state can be ruled out because the expected width for $a_0(2^3P_0)$ with a mass of 1980 (2025) MeV is about 895 (995) MeV, much larger than the observed width of $a_0(2020)$, as shown in Table I. The predicted width for $a_0(4^3P_0)$ with a mass of 1980 (2025) MeV is about 36.8 (34.6) MeV (see also Fig. 3), much smaller than the $a_0(2020)$ width, which makes $a_0(2020)$ unlikely to be the 4^3P_0 state. So, the

TABLE IV. Decay widths of $a_0(2^3P_0)$ (in MeV). The initial state mass is set to be 1744 MeV.

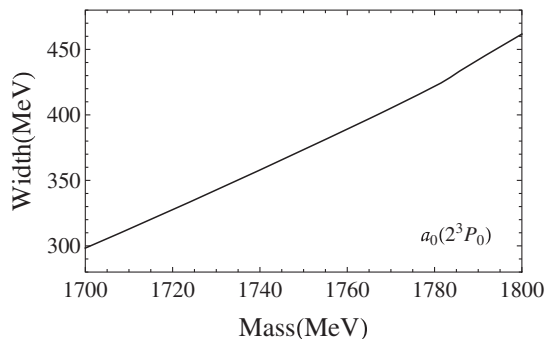| Channel	Mode	$\Gamma_i(2^3P_0)$
$0^+ \rightarrow 0^-0^-$	$\pi\eta$	7.91
	$\pi\eta'$	2.46
	$\pi\eta(1475)$	19.25
	$\pi\eta(1295)$	20.86
	$K\bar{K}$	1.07
$0^+ \rightarrow 0^-1^+$	$\pi b_1(1235)$	213.08
	$\pi f_1(1285)$	38.54
	$\pi f_1(1420)$	1.02
$0^+ \rightarrow 1^-1^-$	$\rho\omega$	59.96
Total width		364.12

FIG. 2. The dependence of the total width of $a_0(2^3P_0)$ on the initial state mass.

measured mass and width for $a_0(2020)$ are consistent with an assignment of the 3^3P_0 state.

The experimental evidence for both $a_0(1950)$ and $a_0(2020)$ turns out to be consistent with the presence of the same resonance corresponding to $a_0(3^3P_0)$. This naturally establishes 1.9 GeV as the approximate mass for the $n\bar{n}$ members of the $3P$ nonets, which could be useful to search for the $n\bar{n}$ members of the $3P$ nonets experimentally. The dominant decay modes of $a_0(3^3P_0)$ are $\pi(1300)\eta$, $\pi\eta(1475)$, $\pi b_1(1235)$, $KK_1(1270)$, and $\rho\omega$.

$a_1(1640)$ and $a_2(1700)$ as the $2P$ radial excitations have been established [25,51], which also fixes the natural mass scale for the $n\bar{n}$ members of the $2P$ multiplets as about 1.7 GeV. One can expect to find $a_0(2^3P_0)$ near 1.7 GeV. At present, no candidate for the isovector scalar state around 1.7 GeV is reported experimentally. An a_0-like pole associated to a resonance with a mass of about 1760 MeV is found by investigating the meson-meson interaction in Refs. [52,53]. The $a_0(2^3P_0)$ mass in the extended linear sigma model is expected to be 1790 ± 35 MeV [17]. Systematic studies on the meson spectra in the relativistic quark models show that the expected $a_0(2^3P_0)$ mass is about 1679–1780 MeV [18,47]. Phenomenologically, it is suggested that the light mesons could be grouped into the following Regge trajectories [54],

$$M_n^2 = M_0^2 + (n-1)\mu^2, \qquad (7)$$

where M_0 is the lowest-lying meson mass, n is the radial quantum number, and μ^2 is the slope parameter of the corresponding trajectory. In the presence of $a_0(1450)$ and $a_0(1950)/a_0(2020)$ being the 1^3P_0 and 3^3P_0 states, respectively, the $a_0(2^3P_0)$ mass can be determined to be about 1744 MeV based on Eq. (7),[1] consistent with the extended linear sigma model prediction [17] and the quark model predictions [18,47].

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[1]We take $M_{a_0(1450)} = 1474$ MeV, $M_{a_0(1950)/a_0(2020)} = (1931 + 2025)/2 = 1978$ MeV, the average value of the $a_0(1950)$ mass reported by the *BABAR* Collaboration [13] and the favored solution for the $a_0(2020)$ mass [12].

TABLE V. Decay widths of $a_0(4^3P_0)$ (in MeV). The initial state mass is set to be 2187 MeV.

Channel	Mode	$\Gamma_i(4^3P_0)$
$0^+ \to 0^-0^-$	$\pi\eta$	0.00023
	$\pi\eta'$	0.41
	$\pi(1300)\eta$	0.19
	$\pi\eta(1475)$	0.77
	$\pi\eta(1295)$	4.63
	$K\bar{K}$	0.37
	$KK(1460)$	2.74
$0^+ \to 0^-1^+$	$\pi b_1(1235)$	0.0032
	$\pi f_1(1285)$	0.09
	$\pi f_1(1420)$	0.04
	$KK_1(1270)$	4.27
	$KK_1(1400)$	2.50
	$\eta a_1(1260)$	0.34
$0^+ \to 1^-1^-$	$\rho\omega$	4.26
	$K^*\bar{K}^*$	1.49
$0^+ \to 0^-2^-$	$\pi\eta_2(1645)$	13.85
Total width		35.96

The strong decays of $a_0(2^3P_0)$ with a mass of 1744 MeV are presented in Table IV. The total width of $a_0(2^3P_0)$ is expected to be about 364 MeV. The dominant decay modes of $a_0(2^3P_0)$ include $\pi\eta(1475)$, $\pi\eta(1295)$, $\pi b_1(1235)$, $\pi f_1(1285)$, and $\rho\omega$. The dependence of the total width of $a_0(2^3P_0)$ on the initial state mass is shown in Fig. 2. When the initial state mass varies from 1700 to 1800 MeV, the total width of $a_0(2^3P_0)$ varies from about 298 to 460 MeV. With the initial state mass of 1700 MeV, our predicted width of 298 MeV is in agreement with the width of 293 MeV expected by Ref. [25] for $a_0(2^3P_0)$.

$a_0(4^3P_0)$ is predicted to have a mass of about 2187 MeV based on Eq. (7), consistent with 2250 MeV, the expected mass for $a_0(4^3P_0)$ in the quark model [18]. The strong decays of $a_0(4^3P_0)$ with a mass of 2187 MeV are listed in Table V. The dependence of the total width of the 4^3P_0 isovector state on the initial state mass is shown in Fig. 3. A
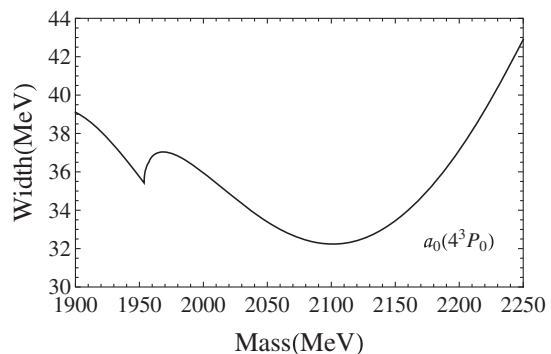

FIG. 3. The dependence of the total width of $a_0(4^3P_0)$ on the initial state mass.

narrow width for $a_0(4^3P_0)$ is predicted. The $\pi\eta(1295)$, $KK_1(1270)$, $\rho\omega$, and $\pi\eta_2(1645)$ channels are the dominant decay modes for $a_0(4^3P_0)$. As we can see in Figs. 1 and 3, the width derivatives are a discontinuity around 1950 MeV, which is because the decay channel $KK(1460)$ is open above this energy.

IV. SUMMARY AND CONCLUSION

Observations of the state $a_0(1950)$ by the *BABAR* Collaboration have enlarged the family of the isovector scalar mesons. In this work, we discuss the possible quark model assignments of $a_0(1950)$ and $a_0(2020)$ by calculating their strong decays in the $^3P_0$ model. We suggest that $a_0(1950)$ and $a_0(2020)$ can be regarded as the same resonance referring to $a_0(3^3P_0)$. The confirmation of $a_0(1950)/a_0(2020)$ as the 3^3P_0 state thereby establishes about 1.9 GeV as a natural mass scale for the $n\bar{n}$ members of the $3P$ nonets.

In the presence of $a_0(1450)$ and $a_0(1950)/a_0(2020)$ being the 1^3P_0 and 3^3P_0 states, respectively, in Regge phenomenology, the masses of $a_0(2^3P_0)$ and $a_0(4^3P_0)$ are predicted to be about 1744 MeV and 2187 MeV, respectively. The predicted masses for $a_0(2^3P_0)$ and $a_0(4^3P_0)$ are consistent with some other theoretical expectations. The total widths of $a_0(2^3P_0)$ and $a_0(4^3P_0)$ are expected to be about 364 MeV and 36 MeV, respectively. Our predictions could be useful to study the higher isovector scalar mesons experimentally.

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