Dynamical study of Ω_c^0 in the chiral quark model

Gang Yang and Jialun Ping*

Department of Physics and Jiangsu Key Laboratory for Numerical Simulation of Large Scale Complex Systems, Nanjing Normal University, Nanjing 210023, People's Republic of China

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Recently, the experimental results of the LHCb Collaboration suggested the existence of five new excited states of Ω_c^0: $\Omega_c(3000)^0$, $\Omega_c(3050)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$, and $\Omega_c(3119)^0$; however, the quantum numbers of these new particles are not determined now. To understand the nature of these states, a dynamical calculation of Ω_c^0 both in five-quark configuration with quantum numbers $IJ^P = 0(\frac{1}{2})^-$, $0(\frac{3}{2})^-$, $0(\frac{5}{2})^-$ and in three-quark configuration with positive parity and negative parity was performed in the framework of the chiral quark model with the help of the Gaussian expansion method. The results show the masses both of the $1P$ and the $2S$ states in ssc systems are comparable to experimental data; Besides, $\Xi\bar{D}$, $\Xi_c\bar{K}$, and $\Xi_c^*\bar{K}$ are also possible candidates of these new particles if the parity is negative. The distances between quark pairs suggest a compact structure nature.

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I. INTRODUCTION

Recently, CERN announced an exceptional new discovery that was made by the LHCb Collaboration, which unveiled five new states all at once [1]. Each of the five particles were found to be the excited states of Ω_c^0, a particle with three quarks css in the conventional quark model. These particles are named, according to the standard convention: $\Omega_c(3000)^0$, $\Omega_c(3050)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$, and $\Omega_c(3119)^0$. Just after the announcement, the theoretical interpretations were proposed. Agaev *et al.* interpreted two of these excited charmed baryons [$\Omega_c(3066)^0$ and $\Omega_c(3119)^0$] as the first radial excitation with $(2S, 1/2^+)$ and $(2S, 3/2^+)$, respectively, in QCD sum rules [2]. The same conclusion is proposed by Chen *et al.* [3] in studying the decay properties of P-wave charmed baryons from light-cone QCD sum rules. They also suggest that one of these Ω_c^0 states [$\Omega_c(3000)^0$, $\Omega_c(3050)^0$, or $\Omega_c(3066)^0$] is a $J^P = 1/2^-$ state, and the other two states are with $J^P = 3/2^-$ and $J^P = 5/2^-$. In Ref. [4], Karliner and Rosner suggested that the parity was negative for all of the five states: two $J^P = 1/2^-$ states [$\Omega_c(3000)^0$ and $\Omega_c(3050)^0$], two $J^P = 3/2^-$ states [$\Omega_c(3066)^0$ and $\Omega_c(3090)^0$], and one $J^P = 5/2^-$ state [$\Omega_c(3119)^0$]. These exciting announcements and the theoretical work along with the pentaquarks P_c^+ discovered also by the LHCb Collaboration in 2015 [5] have brought us more understanding of the world of microcosmic particles.

The quantum numbers of these new particles are not determined for the moment, and the explanation of them as the excited states of q^3 baryon may be reasonable. However, the possibility of the multiquark candidates of these excited states cannot be excluded. The ground states of Ω_c have been observed experimentally, $\Omega_c(2695)^0$ with $J^P = \frac{1}{2}^+$ and $\Omega_c(2770)^0$ with $J^P = \frac{3}{2}^+$. The excited energies of the newly reported states with respect to the ground states are 230–424 MeV, which are enough to excite a light quark-antiquark pair from the vacuum. From the masses of the Ξ_c baryon and the K meson, 2468 MeV and 495 MeV, we have the threshold for the Ξ_c-$\bar{K}$ state around 2963 MeV. It is expected that the 5-q components will play a role in these Ω_c's. In Ref. [6], the spectrum of low-lying pentaquark states with strangeness $S = -3$ and negative parity is studied in three kinds of constituent quark models. The results indicate that the lowest energy state Ω^* is around 1.8 GeV, which is about 200 MeV lower than predictions of various quenched three quark models, and the energy cost to excite the ground state of Ω to a five-quark state is less than that to an orbital excitation.

The interest in the pentaquark is revived after the observation of the exotic hadrons, $P_c^+(4380)$ and $P_c^+(4450)$, in the decay of Λ_b^0, $\Lambda_b^0 \to J/\psi K^- p$ by the LHCb Collaboration [5]. Lots of theoretical calculations have been performed to investigate these two exotic states [7–17], even though the $\Theta^+(1540)$ pentaquark was reported by several experimental groups [18–20] in 2003

*Corresponding author.
jlping@njnu.edu.cn

and has been denied by JLab with more higher precision results [21] [LEPS Collaboration still insisted on the existence of pentaquark $\Theta^+(1540)$ [22]]. Besides, it is shown that there should be notable five-quark components in the baryon resonances [23–26]. In addition, the valence–sea quark mixing (Fock space expansion) model ($q^3 + q^3 q\bar{q}$) of the nucleon ground state had been used to explain the mysterious proton spin structure well [27]. Such a sea quark excitation model had also been used to show that the $q^3 q\bar{q}$ excitation is more favorable than the p-wave excitation in the q^3 configuration for $1/2^-$ baryons [28].

The quark model is the most common approach to the multiquark system. With the recent experimental data on hadron states and the development of the quark model, it is expected to perform a serious calculation of hadron states in the framework of the quark model. In the present work, the chiral quark model (ChQM) is employed to study the three-quark and five-quark states corresponding to Ω_c^0. To find the structure of the five-quark states, a general, powerful method of the few-body system, Gaussian expansion method (GEM) [29] is used to do the calculation. The GEM has been successfully applied to many few-body systems, light nuclei, hypernuclei, hadrons, and so on [29]. It suits for both of the compact multiparticle systems and loosely bound moleculelike states. In this approach, the relative orbital motions of the system are expanded by Gaussians with various widths. By taking into account all of the possible couplings for color-flavor-spin degrees of freedom, the structure of the system determined by its dynamics can be found.

The structure of the paper is as follows. In Sec. II the quark model, wave functions, and calculation method are presented. The results for the three-quark configuration are also given in Sec. II. Section III is devoted to the calculated results and discussions. A brief summary is given in the last section.

II. MODEL AND WAVE FUNCTION

The chiral quark model has achieved a success in describing both the hadron spectra and the hadron-hadron interaction. In this model, the constituent quark and antiquark interact with each other through the Goldstone boson exchange and the effective one-gluon exchange (OGE), in addition to the phenomenological color confinement. Besides, the scalar nonet (the extension of chiral partner σ meson) exchange is also introduced. The details of the model can be found in Ref. [30]. So the Hamiltonian in the present calculation takes the form

$$H = \sum_{i=1}^{n}\left(m_i + \frac{p_i^2}{2m_i}\right) - T_{CM} + \sum_{j>i=1}^{n}[V_{CON}(\mathbf{r}_{ij}) + V_{OGE}(\mathbf{r}_{ij}) + V_\chi(\mathbf{r}_{ij}) + V_s(\mathbf{r}_{ij})], \tag{1}$$

$$V_{CON}(\mathbf{r}_{ij}) = \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c[-a_c(1 - e^{-\mu_c r_{ij}}) + \Delta], \tag{2}$$

$$V_{OGE}(\mathbf{r}_{ij}) = \frac{1}{4}\alpha_s \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c\left[\frac{1}{r_{ij}} - \frac{1}{6m_i m_j}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij} r_0^2(\mu)}\right], \qquad r_0(\mu) = \hat{r}_0/\mu, \tag{3}$$

$$V_\chi(\mathbf{r}_{ij}) = v_\pi(\mathbf{r}_{ij})\sum_{a=1}^{3}\boldsymbol{\lambda}_i^a \cdot \boldsymbol{\lambda}_j^a + v_K(\mathbf{r}_{ij})\sum_{a=4}^{7}\boldsymbol{\lambda}_i^a \cdot \boldsymbol{\lambda}_j^a + v_\eta(\mathbf{r}_{ij})[\boldsymbol{\lambda}_i^8 \cdot \boldsymbol{\lambda}_j^8 \cos\theta_P - \boldsymbol{\lambda}_i^0 \cdot \boldsymbol{\lambda}_j^0 \sin\theta_P],$$

$$v_\chi(\mathbf{r}_{ij}) = \frac{g_{ch}^2}{4\pi}\frac{m_\chi^2}{12 m_i m_j}\frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2} m_\chi \left[Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} Y(\Lambda_\chi r_{ij})\right]\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, \qquad \chi = \pi, K, \eta, \tag{4}$$

$$V_s(\mathbf{r}_{ij}) = v_\sigma(\mathbf{r}_{ij})(\boldsymbol{\lambda}_i^0 \cdot \boldsymbol{\lambda}_j^0) + v_{a_0}(\mathbf{r}_{ij})\sum_{a=1}^{3}\boldsymbol{\lambda}_i^a \cdot \boldsymbol{\lambda}_j^a + v_\kappa(\mathbf{r}_{ij})\sum_{a=4}^{7}\boldsymbol{\lambda}_i^a \cdot \boldsymbol{\lambda}_j^a + v_{f_0}(\mathbf{r}_{ij})(\boldsymbol{\lambda}_i^8 \cdot \boldsymbol{\lambda}_j^8),$$

$$v_s(\mathbf{r}_{ij}) = -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_s^2}{\Lambda_s^2 - m_s^2} m_s \left[Y(m_s r_{ij}) - \frac{\Lambda_s}{m_s} Y(\Lambda_s r_{ij})\right], \qquad s = \sigma, a_0, \kappa, f_0. \tag{5}$$

All the symbols take their usual meanings. μ is the reduced mass of two interacting quarks. To simplify the calculation, only the central parts of the interactions are employed in the present work to consider the ground state of the multiquark system. The model parameters related to the Goldstone-boson exchange are taken from Ref. [30], which are fixed by the meson spectrum and nucleon-nucleon interaction; other parameters are fixed by fitting the masses of the ground state baryons. The values of model parameters are listed in Table I, and the calculated masses of baryons and mesons in the ground state are shown in Table II. To check the effectiveness of the model for the excited states, the masses of $2S$ and $1P$ states of nucleon and Δ are also calculated and listed in Table II. From the tables, we can see that the baryon spectrum can be described well, but the meson spectrum is not described well as the baryon spectrum. Especially the masses of $2S$ and $1P$ states of nucleon and Δ are in agreement with the experimental data, 1440 MeV and 1520 MeV for nucleon

TABLE I. Quark model parameters. The masses of mesons take their experimental values. $m_\pi = 0.7\ \mathrm{fm}^{-1}$, $m_K = 2.51\ \mathrm{fm}^{-1}$, and $m_\eta = 2.77\ \mathrm{fm}^{-1}$.

		Set I	Set II
	$m_u = m_d$ (MeV)	313	313
Quark mass	m_s (MeV)	555	555
	m_c (MeV)	1752	1752
	$\Lambda_\pi = \Lambda_\sigma$ (fm^{-1})	4.20	4.20
Goldstone boson	$\Lambda_K = \Lambda_\eta$ (fm^{-1})	5.20	5.20
	$\theta_P(^\circ)$	−15	−15
	$g^2_{ch}/(4\pi)$	0.54	0.54
SU(3)	m_s (fm^{-1})	4.97	4.97
Scalar nonet	Λ_s (fm^{-1})	5.20	5.20
$s = \sigma, a_0, \kappa, f_0$	m_σ (fm^{-1})	3.42	3.42
	a_c (MeV)	180	184.08
Confinement	μ_c (fm^{-1}	0.645	0.634
	Δ (MeV)	55.5	40.249
OGE	α_s	0.69	$\alpha_0 = 1.293$ $\Lambda_0 = 1.5585\ \mathrm{fm}^{-1}$ $\mu_0 = 621.5$ MeV
	$\hat{r}_0$ (MeV fm)	28.170	43.882

and 1600 MeV and 1620 MeV for Δ. Two sets of parameters are given: the fixed quark-gluon coupling constant is used in set I, and set II has the running coupling constants which are given as

$$\alpha_s = \frac{\alpha_0}{\ln((\mu^2 + \mu_0^2)/\Lambda_0^2)}. \tag{6}$$

It is worthwhile to mention that the above quark-quark interaction is assumed to be universal according to the "Casimir scaling" [31]; it can be applied to the multiquark system directly. The possible multibody interaction in the multiquark system is not considered, although it may give different spectra of multiquark states [32].

TABLE III. The calculated masses of Ω_c (ssc) in the ground states and the low-lying excited states (unit: MeV).

nL	$\Omega_c(2695)$	$\Omega_c(2765)$
	Set I	
1S	2675	2748
2S	3074	3112
1P	3005	3017
	Set II	
1S	2748	2818
2S	3167	3201
1P	3090	3103

In Table III, the masses of $\Omega_c(ssc)$ in low-lying excited states (2S, 1P) are listed, we can see that the masses both of the P-wave states and the first radial excitation ones of Ω_c could give a comparable results to the corresponding experimental data. The parameters of set II are used to check the dependence of the results on the model parameters. Firstly, the results show that the masses of P-wave Ω_c are in the energy interval from 3.0 GeV to 3.1 GeV which is announced by the LHCb collaboration. Hence, according to the results from set I in Table III, two of the excited states of Ω_c ($\Omega_c(3000)^0$ and $\Omega_c(3050)^0$) could be explained as the negative parity states of three quark system ssc. Secondly, the masses of 2S states of Ω_c are in the range, 3074–3201 MeV, it is possible two of the five reported states ($\Omega_c(3090)^0$ and $\Omega_c(3119)^0$) can also be identified as the 2S states of Ω_c if their parity is fixed to be positive.

If the parity of these states are found to be negative, the five-quark configuration cannot be ignored; at least the five-quark components will mix with the P-wave ssc states. In the following, we use set I parameters to study the five-quark states.

The wave functions for the system are constructed just as in Ref. [7]. Here only the wave functions of each degree of freedom for the five-quark system and parts of the sub-clusters of three-quark and quark-antiquark are listed. One needs to notice that there are many different ways to

TABLE II. Masses of ground states of baryon and meson in ChQM (unit: MeV).

	$N(939)$	$\Delta(1232)$	$\Omega(1672)$	$\Lambda(1116)$	$\Sigma(1189)$	$\Xi(1315)$
Set I	936	1208	1643	1154	1173	1362
Set II	939	1231	1671	1187	1209	1408
	$\Sigma^*(1383)$	$\Xi^*(1532)$	$\Omega_c(2695)$	$\Omega_c(2765)$	$\Xi_c(2467)$	$\Xi_c^*(2645)$
Set I	1342	1488	2675	2748	2541	2603
Set II	1393	1539	2748	2818	2629	2727
	$N^*(1440)$	$\Delta^*(1520)$	$N^*(1520)$	$\Delta^*(1620)$		
Set I	1389	1522	1601	1668		
Set II	1448	1588	1683	1758		
	$\pi(140)$	$\rho(775)$	$\eta(548)$	$\omega(782)$	$K(495)$	$K^*(892)$
Set I	93	800	611	705	326	965
	$\eta'(958)$	$\phi(1019)$	$D^0(1865)$	$D^*(2007)$		
Set I	914	1056	1842	2043		

construct the wave functions of the system. However, it makes no difference by choosing any one configuration if all the possible couplings are considered.

For the Ω_c^0 with quark content $ssc q\bar{q}$, $q = u$, d, s in the flavor SU(3) case, there are two types of separation: one is $(qss)(\bar{q}c)$ and the other is $(ssc)(\bar{q}q)$. The flavor wave functions for the subclusters constructed are shown below,

$$\begin{aligned}
&B^1_{00} = ssc, \qquad B^2_{00} = sss,\\
&B^1_{\frac{1}{2},\frac{1}{2}} = \frac{1}{\sqrt{6}}(sus + uss - 2ssu),\\
&B^1_{\frac{1}{2},-\frac{1}{2}} = \frac{1}{\sqrt{6}}(sds + dss - 2ssd),\\
&B^2_{\frac{1}{2},\frac{1}{2}} = \frac{1}{\sqrt{2}}(us - su)s,\\
&B^2_{\frac{1}{2},-\frac{1}{2}} = \frac{1}{\sqrt{2}}(ds - sd)s,\\
&B^3_{\frac{1}{2},\frac{1}{2}} = \frac{1}{\sqrt{3}}(ssu + sus + uss),\\
&B^3_{\frac{1}{2},-\frac{1}{2}} = \frac{1}{\sqrt{3}}(ssd + sds + dss),\\
&B^4_{\frac{1}{2},\frac{1}{2}} = \frac{1}{\sqrt{2}}(us + su)c, \qquad (7)
\end{aligned}$$

$$\begin{aligned}
&B^4_{\frac{1}{2},-\frac{1}{2}} = \frac{1}{\sqrt{2}}(ds + sd)c,\\
&B^5_{\frac{1}{2},\frac{1}{2}} = \frac{1}{\sqrt{2}}(us - su)c, \qquad B^5_{\frac{1}{2},-\frac{1}{2}} = \frac{1}{\sqrt{2}}(ds - sd)c,\\
&M^1_{\frac{1}{2},\frac{1}{2}} = \bar{d}c, \quad M^1_{\frac{1}{2},-\frac{1}{2}} = -\bar{u}c, \quad M^2_{\frac{1}{2},\frac{1}{2}} = \bar{d}s, \quad M^2_{\frac{1}{2},-\frac{1}{2}} = -\bar{u}s,\\
&M^1_{00} = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d), \quad M^2_{00} = \bar{s}s, \quad M^3_{00} = \bar{s}c. \qquad (8)
\end{aligned}$$

The flavor wave functions for the five-quark system with isospin $I = 0$ are obtained by the following couplings:

$$\begin{aligned}
&\chi^f_1 = \sqrt{\frac{1}{2}}(B^1_{\frac{1}{2},\frac{1}{2}} M^1_{\frac{1}{2},-\frac{1}{2}} - B^1_{\frac{1}{2},-\frac{1}{2}} M^1_{\frac{1}{2},\frac{1}{2}}),\\
&\chi^f_2 = \sqrt{\frac{1}{2}}(B^2_{\frac{1}{2},\frac{1}{2}} M^1_{\frac{1}{2},-\frac{1}{2}} - B^2_{\frac{1}{2},-\frac{1}{2}} M^1_{\frac{1}{2},\frac{1}{2}}),\\
&\chi^f_3 = \sqrt{\frac{1}{2}}(B^3_{\frac{1}{2},\frac{1}{2}} M^1_{\frac{1}{2},-\frac{1}{2}} - B^3_{\frac{1}{2},-\frac{1}{2}} M^1_{\frac{1}{2},\frac{1}{2}}),\\
&\chi^f_4 = \sqrt{\frac{1}{2}}(B^4_{\frac{1}{2},\frac{1}{2}} M^2_{\frac{1}{2},-\frac{1}{2}} - B^4_{\frac{1}{2},-\frac{1}{2}} M^2_{\frac{1}{2},\frac{1}{2}}),\\
&\chi^f_5 = \sqrt{\frac{1}{2}}(B^5_{\frac{1}{2},\frac{1}{2}} M^2_{\frac{1}{2},-\frac{1}{2}} - B^5_{\frac{1}{2},-\frac{1}{2}} M^2_{\frac{1}{2},\frac{1}{2}}),\\
&\chi^f_6 = B^1_{00} M^1_{00}, \qquad \chi^f_7 = B^1_{00} M^2_{00},\\
&\chi^f_8 = B^2_{00} M^3_{00}. \qquad (9)
\end{aligned}$$

In a similar way, the spin and color wave functions for the five-quark system can be constructed, which are the same as the expressions in Ref. [7]. Here we only give the expressions of the five-quark system; the wave functions for the subclusters can be found in Ref. [7],

$$\begin{aligned}
&\chi^{\sigma 1}_{\frac{1}{2},\frac{1}{2}}(5) = \sqrt{\frac{1}{6}}\chi^{\sigma}_{\frac{3}{2},-\frac{1}{2}}(3)\chi^{\sigma}_{11} - \sqrt{\frac{1}{3}}\chi^{\sigma}_{\frac{3}{2},\frac{1}{2}}(3)\chi^{\sigma}_{10} + \sqrt{\frac{1}{2}}\chi^{\sigma}_{\frac{3}{2},\frac{3}{2}}(3)\chi^{\sigma}_{1-1},\\
&\chi^{\sigma 2}_{\frac{1}{2},\frac{1}{2}}(5) = \sqrt{\frac{1}{3}}\chi^{\sigma 1}_{\frac{1}{2},\frac{1}{2}}(3)\chi^{\sigma}_{10} - \sqrt{\frac{2}{3}}\chi^{\sigma 1}_{\frac{1}{2},-\frac{1}{2}}(3)\chi^{\sigma}_{11},\\
&\chi^{\sigma 3}_{\frac{1}{2},\frac{1}{2}}(5) = \sqrt{\frac{1}{3}}\chi^{\sigma 2}_{\frac{1}{2},\frac{1}{2}}(3)\chi^{\sigma}_{10} - \sqrt{\frac{2}{3}}\chi^{\sigma 2}_{\frac{1}{2},-\frac{1}{2}}(3)\chi^{\sigma}_{11},\\
&\chi^{\sigma 4}_{\frac{1}{2},\frac{1}{2}}(5) = \chi^{\sigma 1}_{\frac{1}{2},\frac{1}{2}}(3)\chi^{\sigma}_{00}, \qquad \chi^{\sigma 5}_{\frac{1}{2},\frac{1}{2}}(5) = \chi^{\sigma 2}_{\frac{1}{2},\frac{1}{2}}(3)\chi^{\sigma}_{00},\\
&\chi^{\sigma 1}_{\frac{3}{2},\frac{3}{2}}(5) = \sqrt{\frac{3}{5}}\chi^{\sigma}_{\frac{3}{2},\frac{3}{2}}(3)\chi^{\sigma}_{10} - \sqrt{\frac{2}{5}}\chi^{\sigma}_{\frac{3}{2},\frac{1}{2}}(3)\chi^{\sigma}_{11},\\
&\chi^{\sigma 2}_{\frac{3}{2},\frac{3}{2}}(5) = \chi^{\sigma}_{\frac{3}{2},\frac{3}{2}}(3)\chi^{\sigma}_{00}, \qquad \chi^{\sigma 3}_{\frac{3}{2},\frac{3}{2}}(5) = \chi^{\sigma 1}_{\frac{1}{2},\frac{1}{2}}(3)\chi^{\sigma}_{11},\\
&\chi^{\sigma 4}_{\frac{3}{2},\frac{3}{2}}(5) = \chi^{\sigma 2}_{\frac{1}{2},\frac{1}{2}}(3)\chi^{\sigma}_{11}, \qquad \chi^{\sigma 1}_{\frac{5}{2},\frac{5}{2}}(5) = \chi^{\sigma}_{\frac{3}{2},\frac{3}{2}}(3)\chi^{\sigma}_{11}, \qquad (10)
\end{aligned}$$

$$\begin{aligned}
\chi^c_1 = &\frac{1}{\sqrt{18}}(rgb - rbg + gbr - grb + brg - bgr)\\
&\times (\bar{r}r + \bar{g}g + \bar{b}b), \qquad (11)
\end{aligned}$$

$$\begin{aligned}
\chi^c_k = &\frac{1}{\sqrt{8}}(\chi^k_{3,1}\chi_{2,8} - \chi^k_{3,2}\chi_{2,7} - \chi^k_{3,3}\chi_{2,6} + \chi^k_{3,4}\chi_{2,5}\\
&+ \chi^k_{3,5}\chi_{2,4} - \chi^k_{3,6}\chi_{2,3} - \chi^k_{3,7}\chi_{2,2} + \chi^k_{3,8}\chi_{2,1}), \qquad (12)
\end{aligned}$$

with $k = 2$, 3. For the color part, both the color singlet channels ($k = 1$) and the hidden color channels ($k = 2$, 3) are considered here to have an economic way to describe multiquark system [7].

For the orbital wave functions, there are four relative motions for the five-body system. In the present work, the orbital wave functions for each relative motion of the system are determined by the dynamics of the system. The orbital wave functions for this purpose are written as follows:

$$\psi_{LM_L} = [[[\phi_{n_1 l_1}(\boldsymbol{\rho})\phi_{n_2 l_2}(\boldsymbol{\lambda})]_l \phi_{n_3 l_3}(\boldsymbol{r})]_{l'} \phi_{n_4 l_4}(\boldsymbol{R})]_{LM_L}, \qquad (13)$$

where the Jacobi coordinates are defined as

$$\boldsymbol{\rho} = \boldsymbol{x}_1 - \boldsymbol{x}_2, \qquad (14)$$

$$\boldsymbol{\lambda} = \boldsymbol{x}_3 - \frac{m_1\boldsymbol{x}_1 + m_2\boldsymbol{x}_2}{m_1 + m_2}, \qquad (15)$$

$$\boldsymbol{r} = \boldsymbol{x}_4 - \boldsymbol{x}_5, \qquad (16)$$

$$\boldsymbol{R} = \frac{m_1\boldsymbol{x}_1 + m_2\boldsymbol{x}_2 + m_3\boldsymbol{x}_3}{m_1 + m_2 + m_3} - \frac{m_4\boldsymbol{x}_4 + m_5\boldsymbol{x}_5}{m_4 + m_5}, \qquad (17)$$

where $\boldsymbol{x}_i$ is the position of the ith quark. To find the orbital wave functions, the GEM is employed; i.e., every ϕ is expanded by Gaussians with various sizes [29],

$$\phi_{nlm}(\boldsymbol{r}) = \sum_{n=1}^{n_{\max}} c_n N_{nl} r^l e^{-(r/r_n)^2} Y_{lm}(\hat{\boldsymbol{r}}), \tag{18}$$

where N_{nl} is the normalization constant,

$$N_{nl} = \left[\frac{2^{l+2}(2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi}(2l+1)}\right]^{\frac{1}{2}}. \tag{19}$$

The size parameters of Gaussians r_n are taken as the geometric progression numbers

$$r_n = r_1 a^{n-1}. \tag{20}$$

c_n is the variational parameters, which are determined by the dynamics of the system.

Finally, the complete channel wave function for the five-quark system is written as

$$\Psi_{JM,i,j,k,n} = \mathcal{A}[[\psi_L \chi_S^{\sigma_i}(5)]_{JM_J} \chi_j^f \chi_k^c], \tag{21}$$

where $\mathcal{A}$ is the antisymmetry operator of the system. In the flavor SU(3) case, it has six terms for the system with three identical particles, and it can be reduced to three terms, as follows, due to the symmetry between the first two particles that have been considered when constructing the wave functions of the three-quark clusters. For the two types of separations, 1-$(uss)(\bar{u}c) + (dss)(\bar{d}c), (sss)(\bar{s}c)$ and 2-$(ssc)(\bar{u}u + \bar{d}d), (ssc)(\bar{s}s)$, we have the following antisymmetric operators:

$$\mathcal{A}_1 = 1 - (13) - (23), \tag{22}$$

$$\mathcal{A}_2 = 1 - (15) - (25). \tag{23}$$

The eigenenergy of the system is obtained by solving the following eigenequation:

$$H\Psi_{JM} = E\Psi_{JM}, \tag{24}$$

by using the variational principle. The eigenfunctions Ψ_{JM} are the linear combination of the above channel wave functions in Eq. (21).

In evaluating the matrix elements of the Hamiltonian, the calculation is rather complicated, if the orbital angular momenta of relative motions of the system are not all zero. Here a useful method named the infinitesimally shifted Gaussian are used [29]. In this method, the spherical harmonic function is absorbed into the shifted Gaussians,

$$\phi_{nlm}(\boldsymbol{r}) = N_{nl} \lim_{\varepsilon\to 0} \frac{1}{(\nu\varepsilon)^l} \sum_{k=1}^{k_{\max}} C_{lm,k} e^{-\nu_n(\boldsymbol{r}-\varepsilon\boldsymbol{D}_{lm,k})^2},$$

and the calculation becomes easy with no tedious angular-momentum algebra required.

III. RESULTS AND DISCUSSIONS

In the present calculation, we are interested in the low-lying states of the $ussc\bar{u}$, $dssc\bar{d}$ pentaquark system, so all the orbital angular momenta are set to 0. Then the parity of the five-quark system with one antiquark is negative. In this way, the total angular momentum J can take values 1/2, 3/2, and 5/2. The possible channels under the consideration are listed in Tables IV–VI.

First, the single channel calculations are performed. The eigenenergies of each state with different quantum numbers are shown in Tables VII–IX, where the eigenenergies of the states are shown in column 2, along with the theoretical thresholds (the sum of the calculated masses of the corresponding baryon and meson) in column 3 and experimental thresholds in column 5, column 4 gives the binding energies, the difference between the eigenenergies, and the theoretical thresholds, $E_B = E - E_{\rm th}^{\rm Theo}$. The corrected energies of the states (column 6), which are obtained by taking the sum of experimental thresholds and the binding energies, $E' = E_B + E_{\rm th}^{\rm Exp}$. In this way, we hope that the errors come from the three-quark cluster, and the quark-antiquark calculations cancel with the errors that come from the five-quark calculation, at least partly.

Second, the three types of channel coupling calculations are performed. The first is the channel coupling between

TABLE IV. The channels with $IJ^P = 0\frac{1}{2}^-$.

Index	$\chi_{1/2}^{\sigma_i}$	χ_j^f	χ_k^c	Physical channel
1	$i=1$	$j=3$	$k=1$	$\Xi^*\bar{D}^*$
2	$i=1$	$j=3$	$k=3$	
3	$i=1$	$j=4$	$k=1$	$\Xi_c^*\bar{K}^*$
4	$i=1$	$j=4,5$	$k=2,3$	
5	$i=1$	$j=6$	$k=1$	$\Omega_c^*\omega$
6	$i=1$	$j=6$	$k=3$	
7	$i=2,3$	$j=1,2$	$k=1$	$\Xi\bar{D}^*$
8	$i=2,3$	$j=1,2$	$k=2,3$	
9	$i=2,3$	$j=4,5$	$k=1$	$\Xi_c\bar{K}^*$
10	$i=2,3$	$j=4,5$	$k=2,3$	
11	$i=2$	$j=6$	$k=1$	$\Omega_c\omega$
12	$i=2,3$	$j=6$	$k=2,3$	
13	$i=4,5$	$j=1,2$	$k=1$	$\Xi\bar{D}$
14	$i=4,5$	$j=1,2$	$k=2,3$	
15	$i=4,5$	$j=4,5$	$k=1$	$\Xi_c\bar{K}$
16	$i=4,5$	$j=4,5$	$k=2,3$	
17	$i=4$	$j=6$	$k=1$	$\Omega_c\eta$
18	$i=4,5$	$j=6$	$k=2,3$	

TABLE V. The channels with $IJ^P = 0\frac{3}{2}^-$.

Index	$\chi^{\sigma_i}_{3/2}$	χ^f_j	χ^c_k	Physical channel
1	$i=1$	$j=3$	$k=1$	$\Xi^*\bar{D}^*$
2	$i=1$	$j=3$	$k=3$	
3	$i=1$	$j=4$	$k=1$	$\Xi_c^*\bar{K}^*$
4	$i=1$	$j=4,5$	$k=2,3$	
5	$i=1$	$j=6$	$k=1$	$\Omega_c^*\omega$
6	$i=1$	$j=6$	$k=3$	
7	$i=2$	$j=3$	$k=1$	$\Xi^*\bar{D}$
8	$i=2$	$j=3$	$k=3$	
9	$i=2$	$j=4$	$k=1$	$\Xi_c^*\bar{K}$
10	$i=2$	$j=4,5$	$k=2,3$	
11	$i=2$	$j=6$	$k=1$	$\Omega_c^*\eta$
12	$i=2$	$j=6$	$k=3$	
13	$i=3,4$	$j=1,2$	$k=1$	$\Xi\bar{D}^*$
14	$i=3,4$	$j=1,2$	$k=2,3$	
15	$i=3,4$	$j=4,5$	$k=1$	$\Xi_c\bar{K}^*$
16	$i=3,4$	$j=4,5$	$k=2,3$	
17	$i=3$	$j=6$	$k=1$	$\Omega_c\omega$
18	$i=3,4$	$j=6$	$k=2,3$	

color-singlet and hidden-color channels with the same flavor-spin structures. The second is the coupling among all color-singlet channels with different flavor-spin structures, and the last is the full coupling, including all channels for given J^P. The results are shown in Tables VII–XI. Table XII gives the spatial configurations of the states by calculating the distances between any two quarks or quark and antiquark in the full channel coupling calculation.

In the following we analyze the results in detail:

(a) $J^P = \frac{1}{2}^-$ (Table VII): The single channel calculations show that there are weak attractions for the most channels, the exceptions are $\Omega_c\eta$, $\Omega_c\omega$, $\Omega_c^*\omega$, and $\Xi_c\bar{K}$. The coupling to hidden-color channels helps a little, increasing the attraction a few MeVs and pushing $\Omega_c^*\omega$ and $\Xi_c\bar{K}$ below the corresponding thresholds. So the resonances can be formed. Most of the states have higher masses compared with that of the five new excited states of Ω_c. For $\Xi\bar{D}$, the second lowest state, it has the energy 3156 MeV, which is close to the highest Ω_c, 3119 MeV. The lowest state $\Xi_c\bar{K}$ has the energy 2949 MeV with the help of hidden-color channel coupling, which is a little smaller than the mass of the lowest excited state of Ω_c, 3000 MeV.

TABLE VI. The channels with $IJ^P = 0\frac{5}{2}^-$.

Index	$\chi^{\sigma_i}_{5/2}$	χ^f_j	χ^c_k	Physical channel
1	$i=1$	$j=3$	$k=1$	$\Xi^*\bar{D}^*$
2	$i=1$	$j=3$	$k=3$	
3	$i=1$	$j=4$	$k=1$	$\Xi_c^*\bar{K}^*$
4	$i=1$	$j=4,5$	$k=2,3$	
5	$i=1$	$j=6$	$k=1$	$\Omega_c^*\omega$
6	$i=1$	$j=6$	$k=3$	

TABLE VII. The lowest eigenenergies of the $ssc\bar{u}u + ssc\bar{d}d$ system with $J^P = \frac{1}{2}^-$ (unit: MeV). The percentages of color-singlet (S) and hidden-color (H) channels are also given.

Channel	E	$E_{\rm th}^{\rm Theo}$	E_B	$E_{\rm th}^{\rm Exp}$	E'
1	3526	3531	−5	3539($\Xi^*\bar{D}^*$)	3534
2	4016				
1 + 2	3525		−6		3533
	Percentage(S;H): 99.8%; 0.2%				
3	3566	3568	−2	3537($\Xi_c^*\bar{K}^*$)	3535
4	3616				
3 + 4	3564		−4		3533
	Percentage(S;H): 96.3%; 3.7%				
5	3453	3453	0	3548($\Omega_c^*\omega$)	3548
6	3404				
5 + 6	3402		−51		3497
	Percentage(S;H): 0.2%; 99.8%				
7	3374	3405	−31	3322($\Xi\bar{D}^*$)	3291
8	3672				
7 + 8	3373		−32		3290
	Percentage(S;H): 99.8%; 0.2%				
9	3495	3506	−11	3359($\Xi_c\bar{K}^*$)	3348
10	3613				
9 + 10	3472		−34		3325
	Percentage(S;H): 85.2%; 14.8%				
11	3380	3380	0	3477($\Omega_c\omega$)	3477
12	3608				
11 + 12	3380				
13	3175	3204	−29	3185($\Xi\bar{D}$)	3156
14	3811				
13 + 14	3175		−29		3156
	Percentage(S;H): 100.0%; 0.0%				
15	2867	2867	0	2961($\Xi_c\bar{K}$)	2961
16	3807				
15 + 16	2855		−12		2949
	Percentage(S;H): 96.7%; 3.3%				
17	3286	3286	0	3243($\Omega_c\eta$)	3243
18	3828				
17 + 18	3286				
Mixed (singlet)	2771	2867	−96	2961($\Xi_c\bar{K}$)	2865
Mixed (full)	2675	2867	−192	2961($\Xi_c\bar{K}$)	2769

The situation changes a lot after coupling all the color-singlet channels; the lowest energy we obtained is 2865 MeV. And the full channel-coupling calculation decreases the lowest energy further to 2769 MeV. Table X shows the six lowest eigenenergies in the full-channel calculation. It is not easy to define the corrected energy because of the many channels involved. Just for reference, we define the corrected energy E' as follows:

$$E' = E_{\rm th}^{\rm Exp}(\Xi_c\bar{K}) - E_{\rm th}^{\rm Theo}(\Xi_c\bar{K}) + E.$$

The lowest eigenenergy is smaller than the threshold, and it should be a good resonance. The second lowest

TABLE VIII. The lowest eigenenergies of the $ssc\bar{u}u$+$ssc\bar{d}d$ system with $\frac{3}{2}^-$ (unit: MeV).

Channel	E	$E_{\rm th}^{\rm Theo}$	E_B	$E_{\rm th}^{\rm Exp}$	E'
1	3521	3531	−10	$3539(\Xi^*\bar{D}^*)$	3529
2	4026				
1 + 2	3521		−10		3529
		Percentage(S;H): 100.0%; 0.0%			
3	3565	3568	−3	$3537(\Xi_c^*\bar{K}^*)$	3534
4	3617				
3 + 4	3562		−6		3531
		Percentage(S;H): 94.0%; 6.0%			
5	3453	3453	0	$3548(\Omega_c^*\omega)$	3548
6	3477				
5 + 6	3453				
7	3309	3330	−21	$3397(\Xi^*\bar{D})$	3376
8	4145				
7 + 8	3309		−21		3376
		Percentage(S;H): 100.0%; 0.0%			
9	2929	2929	0	$3139(\Xi_c^*\bar{K})$	3139
10	3782				
9 + 10	2928		−1		3138
		Percentage(S;H): 99.7%; 0.3%			
11	3359	3359	0	$3314(\Omega_c^*\eta)$	3314
12	3763				
11 + 12	3359				
13	3388	3405	−17	$3322(\Xi\bar{D}^*)$	3305
14	3705				
13 + 14	3388		−17		3305
		Percentage(S;H): 100.0%; 0.0%			
15	3506	3506	0	$3359(\Xi_c\bar{K}^*)$	3359
16	3656				
15 + 16	3348		−158		3201
		Percentage(S;H): 57.7%; 42.3%			
17	3380	3380	0	$3477(\Omega_c\omega)$	3477
18	3588				
17 + 18	3380				
Mixed (singlet)	2928	2929	−1	$3139(\Xi_c^*\bar{K})$	3138
Mixed (full)	2857	2929	−72	$3139(\Xi_c^*\bar{K})$	3067

eigen-energy 2867 MeV is just the threshold of $\Xi_c\bar{K}$ and the eigenvectors show that the dominant component is $\Xi_c\bar{K}$, so it is a scattering state of $\Xi_c\bar{K}$. The next three eigenenergies have similar properties; they are all the scattering states of $\Xi_c\bar{K}$. The last eigenenergy of the listed state is 2937 MeV (the corrected energy is 3031), the wave function shows that the main component is $\Xi_c\bar{K}$ (44%), other components with considerable contribution are color-singlet $\Xi_c\bar{K}^*$ (18%), hidden-color $(\Xi_c\bar{K})^8$ (11%), and other components, color-singlet $\Omega_c^*\omega$ and hidden-color $\Omega_c\omega$, $\Omega_c\eta$, $\Omega_c^*\omega$, $\Xi_c\bar{K}^*$, are around 5%. It is possible to have a resonance with energy around 3000 MeV. To assign the state to the excited Ω_c states announced by LHCb, further work is needed. The problem that has to be solved is how to correct the eigenenergies from the full channel-coupling calculation.

TABLE IX. The lowest eigenenergies of the $ssc\bar{u}u$+$ssc\bar{d}d$ system with $\frac{5}{2}^-$ (unit: MeV).

Channel	E	$E_{\rm th}^{\rm Theo}$	E_B	$E_{\rm th}^{\rm Exp}$	E'
1	3508	3531	−23	$3539(\Xi^*\bar{D}^*)$	3516
2	4042				
1 + 2	3507		−24		3515
		Percentage(S;H): 99.8%; 0.2%			
3	3568	3568	0	$3537(\Xi_c^*\bar{K}^*)$	3537
4	3646				
3 + 4	3532		−36		3501
		Percentage(S;H): 80.0%; 20.0%			
5	3453	3453	0	$3548(\Omega_c^*\omega)$	3548
6	3563				
5 + 6	3453				
Mixed (singlet)	3453				
Mixed (full)	3453				

One interesting state is $\Omega_c^*\omega$, the hidden-color channel has lower energy than the colorless one. It is a possible good resonance because of its color structure, although it has a rather high energy, 3497 MeV.

(b) $J^P = \frac{3}{2}^-$ (Table VIII): We have similar results with that of $J^P = \frac{1}{2}^-$. Four channels, $\Omega_c^*\omega$, $\Xi_c^*\bar{K}$, $\Xi_c\bar{K}^*$, and $\Omega_c\omega$, have no attraction in single channel calculations. and the hidden-color channel coupling induces a very weak attraction for $\Xi_c^*\bar{K}$. But, it introduces a large attraction for $\Xi_c\bar{K}^*$, −158 MeV, a good candidate of color structure resonance to be confirmed.

All color-singlet channel-coupling calculations give a very weak bound state with energy 3138 MeV after correction. The full channel coupling lowered the energy further to 3067 MeV. Table XI shows the four lowest eigenenergies in the full channel coupling calculation. After correction, their energies are below 3.2 GeV. The lowest eigenenergy is 2857 MeV (the corrected energy is 3067), which is smaller than the threshold of $\Xi_c^*\bar{K}$, and it should be a resonance, a good candidate of reported Ω_c's.

TABLE X. The eigenenergies of full channel-coupling calculation below 3.1 GeV with $IJ^P = 0\frac{1}{2}^-$ (unit: MeV).

Index	1	2	3	4	5	6
E	2675	2867	2873	2882	2901	2937
E'	2769	2961	2967	2976	2995	3031

TABLE XI. The eigenenergies of full channel-coupling calculation below 3.2 GeV with $IJ^P = 0\frac{3}{2}^-$ (unit: MeV).

Index	1	2	3	4
E	2857	2931	2940	2956
E'	3067	3141	3150	3166

TABLE XII. Distances between quarks: q is for u, d quark, and Q is for c quark (unit: fm).

J^P	Channel	r_{qq}	r_{qQ}	$r_{q\bar{q}}$	$r_{Q\bar{q}}$
$\frac{1}{2}^-$	$\Omega_c^0(2769)$	1.3	1.1	1.4	1.2
$\frac{3}{2}^-$	$\Omega_c^0(3067)$	1.4	1.1	1.2	1.4

The second lowest eigenenergy 2931 MeV is just 2 MeV above the threshold of $\Xi_c^*\bar{K}$, and the eigenvectors show that the dominant component is $\Xi_c^*\bar{K}$, so it is a scattering state of $\Xi_c^*\bar{K}$. The next three eigenenergies have the similar properties; they are all the scattering states of $\Xi_c^*\bar{K}$.

(c) $J^P = \frac{5}{2}^-$ (Table IX): Only one channel, $\Xi^*\bar{D}^*$, is attractive in the single channel calculation. Coupling to the hidden-color channels, an additional channel, $\Xi_c^*\bar{K}^*$, is induced out an attraction. Channel couplings, color-singlet and full, do not produce any bound state. The D-wave Ξ-$\bar{D}$ and/or Ξ_c-$\bar{K}$ scattering phase shift calculation is needed to check that the resonances, $\Xi^*\bar{D}^*$ and $\Xi_c^*\bar{K}^*$, can survive after the coupling.

Table XII gives the distances between quarks for two states, $\Omega_c^0(2769)$ and $\Omega_c^0(3067)$. All the quark pairs have similar distances and all are smaller than 1.5 fm. So these two states are compact ones.

IV. SUMMARY

In the framework of the chiral quark model, the three-quark system with quark content ssc and the five-quark systems with quark contents $sscu\bar{u}$ and $sscd\bar{d}$ are investigated by means of the Gaussian expansion method. For the three-quark system, the calculation shows that both the $1P$ states and the first radial excitation states ($2S$) of Ω_c have the energy around 3.0–3.1 GeV, which are possible candidates of the excited states of Ω_c reported by LHCb Collaboration. In particular, $\Omega_c(3000)^0$ and $\Omega_c(3050)^0$ could be the $1P$ states of ssc configuration, and $2S$ states for $\Omega_c(3090)^0$ and $\Omega_c(3119)^0$. For the five-quark system, the calculation shows that there are several resonance states for $I(J^P) = 0(\frac{1}{2}^-)$, $0(\frac{3}{2}^-)$ below 3.2 GeV. $\Xi\bar{D}$, $\Xi_c\bar{K}$, and $\Xi_c^*\bar{K}$ are possible candidates of the newly announced excited states of Ω_c^0 by LHCb Collaboration if the parities of these states are negative. In the present calculation, the masses of the lowest states with quantum numbers $IJ^P = 0\frac{1}{2}^-$ and $IJ^P = 0\frac{3}{2}^-$ are 2769 MeV and 3067 MeV, respectively; the distances between quark pairs suggest these two states are compact states or pentaquark structures. It manifests the effects of hidden-color channels. So it is interesting to identify the states experimentally. In this work, in fact, we cannot identify the excited states of Ω_c^0 reported by LHCb Collaboration with the pentaquarks we calculated because the information of these states is not enough. We want to stress that the P-wave q^3 baryon will mix strongly with the S-wave pentaquark. The unquenched quark model, including the higher Fock components, study of Ω_c is needed to clarify the situation.

In the present calculation, the internal structures of the subclusters are not fixed and the structure of a five-quark system is determined by the dynamics of the system, because all the possible couplings are included except the high orbital angular momentum. The further work of considering the high orbital angular momenta along with the spin-orbit and tensor interactions is expected.

Pentaquark involves two subclusters, q^3 and $q\bar{q}$. If the two subclusters are colorless, they are corresponding to baryon and meson. To describe baryon and meson simultaneously in a quark model with one set of parameters is still difficult. It is the main source of uncertainty of the model calculation of the pentaquark. Besides in the present work, only the central potential of interactions is considered, and this is an approximation in dealing with angular excitation without spin-orbit and tensor potentials. Moreover, higher Fock components with the same quantum numbers can couple with the conventional three-quark baryons [26,33], and the unquenched effects are not taken into account in the present calculation, which should be considered in the future investigation. The unquenched quark model may be a solution for the unified description of baryon and meson, since the $q\bar{q}$ cluster is always involved.

Multiquark states are ideal places to develop the quark model. Because the model approach is a phenomenological one, its development depends on the accumulated experimental data. We hope that the model description of the multiquark states will be improved with the accumulation of the experimental data on the multiquark state.

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