Quantum interaction between two gravitationally polarizable objects in the presence of boundaries

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We investigate, in the framework of the linearized quantum gravity and the leading-order perturbation theory, the quantum correction to the classical Newtonian interaction between a pair of gravitationally polarizable objects in the presence of both Neumann and Dirichlet boundaries. We obtain general results for the interaction potential and find that the presence of a boundary always strengthens in the leading order the interaction as compared with the case in absence of boundaries. But different boundaries yield a different degree of strengthening. In the limit when one partner of the pair is placed very close to the Neumann boundary, the interaction potential is larger when the pair is parallel with the boundary than when it is perpendicular to it, which is just opposite to the case when the boundary is Dirichlet where the latter is larger than the former. In addition, we find that the pair-boundary separation dependence of the higher-order correction term is determined by the orientation of the pair with respect to boundary, with the parallel case giving a quadratic behavior and the perpendicular case a linear one.

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I. INTRODUCTION

The classical Newtonian theory of gravity tells us that the interaction potential of two massive objects behaves as r^{-1} with r being the separation between them. This interaction is expected to be modified if gravity is quantized. However, a complete study of quantum corrections to the classical Newtonian interactions requires a full theory of quantum gravity which is elusive at the present. Even though, quantum gravity effects at the low energies can however be analyzed by treating the general relativity as an effective field theory or in the framework of linearized quantum gravity. For example, by summing one-loop Feynman diagrams with off-shell gravitons, it has been found that the monopole-monopole interaction provides a quantum correction, which behaves as r^{-3}, to the Newtonian force [1].

A direct consequence of quantization of gravity is the appearance of quantum vacuum fluctuations of gravitational fields, i.e., fluctuations of spacetime itself. These fluctuations are expected to induce instantaneous quadrupole moments in gravitationally polarizable objects. As a result, the induced quadrupole-quadrupole interaction produces a quantum correction to the classical Newtonian interaction, which has been studied in different contexts [2–4]. The quantum potential between gravitational quadrupoles is found to behave as r^{-11} and r^{-10} in the far and near regimes respectively. Recently, the quadruple-quadruple interaction was extended to include the contribution of fluctuations of thermal gravitons at finite temperature [5]. In the high-temperature limit, the potential behaves like T/r^{-10}; thus, the thermal fluctuations of gravitons produce a dominant contribution, while in the low-temperature limit, the zero-point fluctuations dominate the interaction and the thermal fluctuations only generate a small correction.

It is well known that field modes will be changed when boundaries are present [6–8], which leads to modifications of vacuum fluctuations. Changes in vacuum fluctuations can produce observable effects. The Casimir-Polder potential [9] between two neutral atoms near a perfectly conducting plate is an example of such effects that arise from the changes of vacuum modes of electromagnetic fields [10–12]. In the case of gravitation, one also finds that interesting effects appear when boundaries are present; for example, light cone fluctuations are modified [13–16], which leads to flight time fluctuations of a probe light signal from its source to a detector [17].

In this paper, we shall examine the impact of plane boundaries on the induced quadrupole-quadrupole interaction between a pair of gravitationally polarizable objects in vacuum. Our approach is based upon the leading-order perturbation theory in the framework of linearized quantum gravity [13], which has been used to investigate quantum gravitational corrections in [4,5]. Throughout this paper, the latin indices run from 0 to 3, while the greek letter is from 1 to 3. The Einstein convention is assumed for repeated indices and $\hbar = c = k_B = 1$ is set. Here, c is

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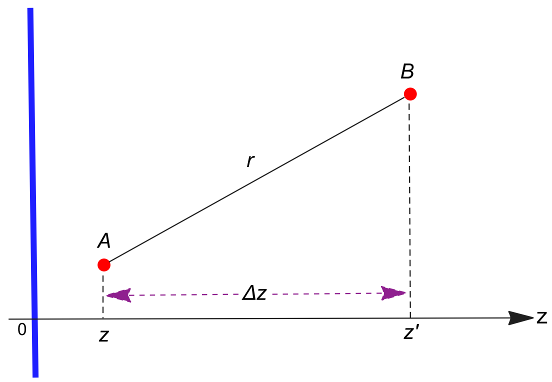

FIG. 1. The system consists of objects A and B in a flat spacetime with a plane boundary at $z = 0$.

the light speed, $\hbar$ is the reduced Planck constant, and k_B is the Boltzmann constant.

II. BASIC EQUATIONS

The system, which is shown in Fig. 1, consists of two gravitationally polarizable objects (A and B) in a bath of fluctuating quantum vacuum gravitational fields with a plane boundary at $z = 0$. For simplicity, we assume A and B to be described by two-level harmonic oscillators with their Hamiltonians being $H_{A(B)} = E^0_{A(B)}|0_{A(B)}\rangle\langle 0_{A(B)}| + E^1_{A(B)}|1_{A(B)}\rangle\langle 1_{A(B)}|$. For this system, the total Hamiltonian can be written as

$$H = H_F + H_A + H_B + H_{AF} + H_{BF}, \tag{1}$$

where H_F is the Hamiltonian of gravitational fields and

$$H_{A(B)F} = -\frac{1}{2} Q^{ij}_{A(B)} E_{ij} \tag{2}$$

represents the interactions between the objects and gravitational fields. Here Q_{ij} is the object's quadrupole moment induced by the gravitational vacuum fluctuations and the gravito-electric tensor E_{ij} is defined as $E_{ij} = R_{0i0j}$ by analogy of linearized Einstein field equation with the Maxwell equations [18], where $R_{\mu\nu\alpha\beta}$ is the Riemann tensor defined in terms of the metric tensor. A fluctuating metric tensor can be expanded in a flat background spacetime as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $h_{\mu\nu}$ being the linearized perturbations which can be quantized as [13]

$$h_{ij}(\mathbf{x}, t) = \sum_{\mathbf{k},\lambda} [a_\lambda(\omega, \mathbf{x}) f^\lambda_{ij,\mathbf{k}} + \text{H.c.}], \tag{3}$$

TABLE I. Ten intermediate states contributing to the two-objects potential.

Case	$\|\mathrm{I}\rangle$	$\|\mathrm{II}\rangle$	$\|\mathrm{III}\rangle$
(1)	$\|1_A, 0_B\rangle\|1^{(1)}\rangle$	$\|0_A, 0_B\rangle\|1^{(2)}, 1^{(3)}\rangle$	$\|0_A, 1_B\rangle\|1^{(4)}\rangle$
(2)	$\|1_A, 0_B\rangle\|1^{(1)}\rangle$	$\|1_A, 1_B\rangle\|\{0\}\rangle$	$\|0_A, 1_B\rangle\|1^{(2)}\rangle$
(3)	$\|1_A, 0_B\rangle\|1^{(1)}\rangle$	$\|1_A, 1_B\rangle\|\{0\}\rangle$	$\|1_A, 0_B\rangle\|1^{(2)}\rangle$
(4)	$\|1_A, 0_B\rangle\|1^{(1)}\rangle$	$\|1_A, 1_B\rangle\|1^{(2)}, 1^{(3)}\rangle$	$\|0_A, 1_B\rangle\|1^{(4)}\rangle$
(5)	$\|1_A, 0_B\rangle\|1^{(1)}\rangle$	$\|1_A, 1_B\rangle\|1^{(2)}, 1^{(3)}\rangle$	$\|1_A, 0_B\rangle\|1^{(4)}\rangle$
(6)	$\|0_A, 1_B\rangle\|1^{(1)}\rangle$	$\|0_A, 0_B\rangle\|1^{(2)}, 1^{(3)}\rangle$	$\|1_A, 0_B\rangle\|1^{(4)}\rangle$
(7)	$\|0_A, 1_B\rangle\|1^{(1)}\rangle$	$\|1_A, 1_B\rangle\|\{0\}\rangle$	$\|1_A, 0_B\rangle\|1^{(2)}\rangle$
(8)	$\|0_A, 1_B\rangle\|1^{(1)}\rangle$	$\|1_A, 1_B\rangle\|\{0\}\rangle$	$\|0_A, 1_B\rangle\|1^{(2)}\rangle$
(9)	$\|0_A, 1_B\rangle\|1^{(1)}\rangle$	$\|1_A, 1_B\rangle\|1^{(2)}, 1^{(3)}\rangle$	$\|1_A, 0_B\rangle\|1^{(4)}\rangle$
(10)	$\|0_A, 1_B\rangle\|1^{(1)}\rangle$	$\|1_A, 1_B\rangle\|1^{(2)}, 1^{(3)}\rangle$	$\|0_A, 1_B\rangle\|1^{(4)}\rangle$

where H.c. denotes the Hermitian conjugate, $\mathbf{k} = \{k_1, k_2, k_3\}$, $\mathbf{x} = \{x, y, z\}$, $a_\lambda(\omega, \mathbf{x})$ is the gravitational field operator, which defines the vacuum $a_\lambda(\omega, \mathbf{x})|\{0\}\rangle = 0$, $\omega = \sqrt{k_1^2 + k_2^2 + k_3^2}$, λ labels the polarization states, and $f^\lambda_{ij,\mathbf{k}}(\mathbf{x}, t)$ is the field mode. Substituting the metric tensor into the Riemann tensor gives

$$E_{ij} = \frac{1}{2} \ddot{h}_{ij}, \tag{4}$$

where a dot denotes a derivative with respect to time t.

Using the leading-order perturbation theory, we find that the interaction potential between two objects, which is just the shift of the ground-state energy, arises from fourth-order perturbations [9,11,19] and can be expressed as

$$\begin{aligned} U_{AB}(\mathbf{x}_A, \mathbf{x}_B) &= -{\sum_{\mathrm{I,II,III}}}' \frac{\langle 0|\hat{H}_{AF} + \hat{H}_{BF}|\mathrm{I}\rangle\langle \mathrm{I}|\hat{H}_{AF} + \hat{H}_{BF}|\mathrm{II}\rangle}{(E_\mathrm{I} - E_0)(E_\mathrm{II} - E_0)} \\ &\quad \times \frac{\langle \mathrm{II}|\hat{H}_{AF} + \hat{H}_{BF}|\mathrm{III}\rangle\langle \mathrm{III}|\hat{H}_{AF} + \hat{H}_{BF}|0\rangle}{(E_\mathrm{III} - E_0)}, \end{aligned} \tag{5}$$

where $|0\rangle = |0_A\rangle|0_B\rangle|\{0\}\rangle$ is the ground state of the whole system, which is omitted in the summation as indicated by a prime, and the summation includes position and frequency integrals. $|\mathrm{I}\rangle$, $|\mathrm{II}\rangle$, and $|\mathrm{III}\rangle$ are the intermediate states. In Ref. [4] it has been shown that there are ten possible combinations of intermediate states, which are listed in Table I. Summing up all of them, we obtain that the interaction potential for isotropically polarizable objects can be expressed as

$$\begin{aligned} U_{AB}(\mathbf{x}_A, \mathbf{x}_B) = &-\frac{1}{4(\omega_A + \omega_B)} \int_0^\infty d\omega \int_0^\infty d\omega' \frac{\tilde{\alpha}_A \tilde{\alpha}_B (\omega_A + \omega_B + \omega)}{(\omega_A + \omega)(\omega_B + \omega)} \left(\frac{1}{\omega + \omega'} - \frac{1}{\omega - \omega'}\right) \\ &\times G_{ijkl}(\omega, \mathbf{x}_A, \mathbf{x}_B) G_{ijkl}(\omega', \mathbf{x}_A, \mathbf{x}_B), \end{aligned} \tag{6}$$

where $\omega_{A(B)} = (\omega^1_{A(B)} - \omega^0_{A(B)})$ with $\omega^1_{A(B)} = E^1_{A(B)}$ and $\omega^0_{A(B)} = E^0_{A(B)}$ represents the transition frequency of the object, $\tilde{\alpha}_{A(B)} \equiv \tilde{Q}^{ij}_{A(B)} \tilde{Q}^{*ij}_{A(B)} = |\tilde{Q}^{ij}_{A(B)}|^2$ with $\tilde{Q}^{ij}_{A(B)} = \langle 0_{A(B)} | Q^{ij}_{A(B)} | 1_{A(B)} \rangle$ and $\tilde{Q}^{*ij}_{A(B)} = \langle 1_{A(B)} | Q^{ij}_{A(B)} | 0_{A(B)} \rangle$, and $G_{ijkl}(\omega, \mathbf{x}_A, \mathbf{x}_B)$ is the two-point correlation function of gravitoelectric fields

$$G_{ijkl}(\omega, \mathbf{x}_A, \mathbf{x}_B) = \langle 0 | E_{ij}(\omega, \mathbf{x}_A) E_{kl}(\omega, \mathbf{x}_B) | 0 \rangle. \quad (7)$$

III. NEUMANN BOUNDARY CONDITION

Now we consider what happens to the potential when a Neumann boundary is present. For metric perturbations which satisfy the Neumann boundary condition $\partial_z f^\lambda_{ij,\mathbf{k}}|_{z=0} = 0$, the field mode $f^\lambda_{ij,\mathbf{k}}$ can be expressed as

$$f^\lambda_{ij,\mathbf{k}}(\mathbf{x}, t) = \sqrt{\frac{8\pi G}{2\omega(2\pi)^3}} \times [e_{ij}(\mathbf{k}, \lambda) e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)} + e_{ij}(\mathbf{k}^-, \lambda) e^{i(\mathbf{k}^-\cdot\mathbf{x} - \omega t)}] \quad (8)$$

in the transverse tracefree (TT) gauge with $e_{ij}(\mathbf{k}, \lambda)$ being polarization tensors. Here

$$\mathbf{k}^- = \{k_1, k_2, -k_3\},$$

and G is the Newton's gravitational constant.

From Eqs. (3), (4), (7), and (8), one finds that the two-point correlation function of E_{ij} has the form

$$\begin{aligned} G_{ijkl}(r, \bar{r}, \Delta t) &= \frac{1}{4} \langle 0 | \ddot{h}_{ij}(\mathbf{x}, t) \ddot{h}_{kl}(\mathbf{x}', t') | 0 \rangle \\ &= \frac{G}{8\pi^2} \int d^3\mathbf{k}\, \omega^3 e^{i\omega\Delta t} \sum_\lambda [e_{ij}(\mathbf{k}, \lambda) e_{kl}(\mathbf{k}, \lambda) e^{i\mathbf{k}\cdot\mathbf{r}} + e_{ij}(\mathbf{k}, \lambda) e_{kl}(\mathbf{k}^-, \lambda) e^{i\mathbf{k}\cdot\bar{\mathbf{r}}} \\ &\quad + e_{ij}(\mathbf{k}^-, \lambda) e_{kl}(\mathbf{k}, \lambda) e^{i\mathbf{k}^-\cdot\bar{\mathbf{r}}} + e_{ij}(\mathbf{k}^-, \lambda) e_{kl}(\mathbf{k}^-, \lambda) e^{i\mathbf{k}^-\cdot\mathbf{r}}]. \end{aligned} \quad (9)$$

Here $r = |\mathbf{r}|$, $\bar{r} = |\bar{\mathbf{r}}|$, and

$$\mathbf{r} = \{x - x', y - y', z - z'\}, \qquad \bar{\mathbf{r}} = \{x - x', y - y', z + z'\}. \quad (10)$$

In the TT gauge, the summation of polarization tensors gives [13]

$$\begin{aligned} \sum_\lambda e_{ij}(\mathbf{k}, \lambda) e_{kl}(\mathbf{k}', \lambda) &= \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl} + \hat{k}_i \hat{k}_j \hat{k}'_k \hat{k}'_l + \hat{k}_i \hat{k}_j \delta_{kl} \\ &\quad + \hat{k}'_k \hat{k}'_l \delta_{ij} - \hat{k}_i \hat{k}'_l \delta_{jk} - \hat{k}_i \hat{k}'_k \delta_{jl} - \hat{k}_j \hat{k}'_l \delta_{ik} - \hat{k}_j \hat{k}'_k \delta_{il}, \end{aligned} \quad (11)$$

where

$$\hat{k}_i = \frac{k_i}{\omega}. \quad (12)$$

From this summation of polarization tensors, we can obtain the two following relations:

$$\begin{aligned} \sum_\lambda e_{ij}(\mathbf{k}, \lambda) e_{kl}(\mathbf{k}, \lambda) e^{i\mathbf{k}\cdot\mathbf{r}} &= \frac{1}{\omega^4} [(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl})\nabla^4 + (\partial_i\partial_j\delta_{kl} + \partial_k\partial_l\delta_{ij} \\ &\quad - \partial_i\partial_l\delta_{jk} - \partial_i\partial_k\delta_{jl} - \partial_j\partial_l\delta_{ik} - \partial_j\partial_k\delta_{il})\nabla^2 + \partial_i\partial_j\partial_k\partial_l] e^{i\mathbf{k}\cdot\mathbf{r}} \\ &\equiv \frac{1}{\omega^4} \hat{g}^r_{ijkl} e^{i\mathbf{k}\cdot\mathbf{r}}, \end{aligned} \quad (13)$$

and

$$\begin{aligned} \sum_\lambda e_{ij}(\mathbf{k}, \lambda) e_{kl}(\mathbf{k}^-, \lambda) e^{i\mathbf{k}\cdot\bar{\mathbf{r}}} &= \frac{1}{\omega^4} \sigma_{km}\sigma_{ln} [(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm} - \delta_{ij}\delta_{mn})\nabla^4 + (\partial_i\partial_j\delta_{mn} + \partial_m\partial_n\delta_{ij} \\ &\quad - \partial_i\partial_n\delta_{jm} - \partial_i\partial_m\delta_{jn} - \partial_j\partial_n\delta_{im} - \partial_j\partial_m\delta_{in})\nabla^2 + \partial_i\partial_j\partial_m\partial_n] e^{i\mathbf{k}\cdot\bar{\mathbf{r}}} \\ &\equiv \frac{1}{\omega^4} \sigma_{km}\sigma_{ln} \hat{g}^{\bar{r}}_{ijmn} e^{i\mathbf{k}\cdot\bar{\mathbf{r}}}, \end{aligned} \quad (14)$$

where $\hat{g}^{r}_{ijkl}$ is a differential operator whose definition straightforwardly follows from Eq. (13), $\sigma = \{\{1,0,0\}, \{0,1,0\}, \{0,0,-1\}\}$, $\nabla^2 = \partial_i \partial^i$, and $\partial_i = \partial_{x_i}$. Substituting Eqs. (13) and (14) into Eq. (9) and performing the Fourier transform, one has

$$G_{ijkl}(r,\bar{r},\omega) = \frac{G}{4\pi^2}\int d\Omega\omega[\hat{g}^{r}_{ijkl}e^{i\omega r\cos\theta} + \sigma_{km}\sigma_{ln}\hat{g}^{\bar{r}}_{ijmn}e^{i\omega\bar{r}\cos\theta}] \\ = \frac{G}{\pi}\left[\hat{g}^{r}_{ijkl}\frac{\sin(\omega r)}{r} + \sigma_{km}\sigma_{ln}\hat{g}^{\bar{r}}_{ijmn}\frac{\sin(\omega\bar{r})}{\bar{r}}\right], \tag{15}$$

where Ω is the solid angle, and the relation

$$\int d\Omega e^{i\mathbf{k}\cdot\mathbf{r}} = \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi e^{i\omega r\cos\theta} = 4\pi\frac{\sin(\omega r)}{\omega r} \tag{16}$$

has been used. Substituting Eq. (15) into Eq. (6) gives

$$U_{AB}(r,\bar{r}) = -\frac{G^2}{4\pi^2(\omega_A+\omega_B)}\int_0^\infty d\omega\int_0^\infty d\omega'\frac{\tilde{\alpha}_A\tilde{\alpha}_B(\omega_A+\omega_B+\omega)}{(\omega_A+\omega)(\omega_B+\omega)}\left(\frac{1}{\omega+\omega'} - \frac{1}{\omega-\omega'}\right) \\ \times\left(\hat{g}^{r}_{ijkl}\frac{\sin(\omega r)}{r} + \sigma_{\mathrm{km}}\sigma_{ln}\hat{g}^{\bar{r}}_{ijmn}\frac{\sin(\omega\bar{r})}{\bar{r}}\right) \\ \times\left(\hat{g}^{\tilde{r}}_{ijkl}\frac{\sin(\omega'\tilde{r})}{\tilde{r}} + \sigma_{km'}\sigma_{ln'}\hat{g}^{\tilde{\bar{r}}}_{ijm'n'}\frac{\sin(\omega'\tilde{\bar{r}})}{\tilde{\bar{r}}}\right)\Big|_{\tilde{r}\to r,\tilde{\bar{r}}\to\bar{r}}. \tag{17}$$

Defining $y(r,r')$ to be

$$y(r,r') = \frac{1}{(\omega_A+\omega_B)}\int_0^\infty d\omega\int_0^\infty d\omega'\frac{\tilde{\alpha}_A\tilde{\alpha}_B(\omega_A+\omega_B+\omega)}{(\omega_A+\omega)(\omega_B+\omega)}\left(\frac{1}{\omega+\omega'} + \frac{1}{-\omega+\omega'}\right) \\ \times\frac{\sin(\omega r)}{r}\frac{\sin(\omega' r')}{r'} \\ = \frac{1}{(\omega_A+\omega_B)}\int_0^\infty d\omega\frac{\sin(\omega r)}{r}\int_{-\infty}^\infty d\omega'\frac{\tilde{\alpha}_A\tilde{\alpha}_B(\omega_A+\omega_B+\omega)}{(\omega_A+\omega)(\omega_B+\omega)} \\ \times\left(\frac{1}{\omega+\omega'} + \frac{1}{-\omega+\omega'}\right)\frac{e^{i\omega' r'}}{2ir'} \\ = \frac{\pi}{(\omega_A+\omega_B)}\int_0^\infty d\omega\frac{\tilde{\alpha}_A\tilde{\alpha}_B(\omega_A+\omega_B+\omega)}{(\omega_A+\omega)(\omega_B+\omega)}\frac{\sin(\omega r)\cos(\omega r')}{rr'}, \tag{18}$$

and following an analogy with the electric polarizability of atoms [20] to define the object's ground-state polarizability as

$$\alpha_{A(B)}(\omega) = \lim_{\epsilon\to 0^+}\frac{\tilde{\alpha}_{A(B)}\omega_{A(B)}}{\omega^2_{A(B)} - \omega^2 - i\epsilon\omega}, \tag{19}$$

which satisfies $Q_{ij}(\omega) = \alpha(\omega)E_{ij}(\omega,\mathbf{x})$, one can obtain that

$$y(r,r') = \frac{\pi}{2}\alpha_A(0)\alpha_B(0)\frac{1}{rr'(r+r')}, \tag{20}$$

when $r' \to r$, and when $r \neq r'$

$$y(r,r') = \frac{\pi}{2}\alpha_A(0)\alpha_B(0)\left[\frac{1}{rr'(r+r')} + \frac{1}{rr'(r-r')}\right], \tag{21}$$

where the approximate static polarizability has been assumed. Then, Eq. (17) can be re-expressed as

$$U_{AB}(r,\bar{r}) = -\frac{G^2}{8\pi}\alpha_A(0)\alpha_B(0)\left(\hat{g}^r_{ijkl}\hat{g}^{\tilde{r}}_{ijkl}\frac{1}{r\tilde{r}(r+\tilde{r})} + \sigma_{km}\sigma_{ln}\hat{g}^r_{ijkl}\hat{g}^{\bar{r}}_{ijmn}\frac{1}{r\bar{r}(r+\bar{r})} + \sigma_{km}\sigma_{ln}\hat{g}^{\bar{r}}_{ijmn}\hat{g}^r_{ijkl}\frac{1}{r\bar{r}(\bar{r}+r)} + \hat{g}^{\bar{r}}_{ijkl}\hat{g}^{\tilde{\bar{r}}}_{ijkl}\frac{1}{\bar{r}\,\tilde{\bar{r}}(\bar{r}+\tilde{\bar{r}})}\right)\Big|_{\tilde{r}\to r,\tilde{\bar{r}}\to\bar{r}}. \tag{22}$$

Here $\sigma_{km}\sigma_{lm} = \delta_{kl}$ has been used.

After lengthy calculations, one can arrive at the interaction potential

$$U_{AB}(r,\bar{r}) = -\frac{G^2}{4\pi}\alpha_A(0)\alpha_B(0)\left(\frac{3987}{r^{11}} + \frac{3987}{\bar{r}^{11}} + \frac{144}{r^5\bar{r}^5(r+\bar{r})^9}[A + Br^4\cos 4\theta + 4Cr^2\cos 2\theta + 12Br^2\bar{r}^2\cos 2\theta\cos 2\bar{\theta} + 4\bar{C}\bar{r}^2\cos 2\bar{\theta} + B\bar{r}^4\cos 4\bar{\theta}]\right), \tag{23}$$

where

$$\begin{aligned}
A &= 9(r^8 + 9r^7\bar{r} + 37r^6\bar{r}^2 + 93r^5\bar{r}^3 + 198r^4\bar{r}^4 + 93r^3\bar{r}^5 + 37r^2\bar{r}^6 + 9r\bar{r}^7 + \bar{r}^8),\\
B &= 3r^4 + 27r^3\bar{r} + 83r^2\bar{r}^2 + 27r\bar{r}^3 + 3\bar{r}^4,\\
C &= -3r^6 - 27r^5\bar{r} - 100r^4\bar{r}^2 - 180r^3\bar{r}^3 + 60r^2\bar{r}^4 + 27r\bar{r}^5 + 3\bar{r}^6,\\
\bar{C} &= -3\bar{r}^6 - 27r\bar{r}^5 - 100r^2\bar{r}^4 - 180r^3\bar{r}^3 + 60r^4\bar{r}^2 + 27r^5\bar{r} + 3r^6.
\end{aligned} \tag{24}$$

Here θ and $\bar{\theta}$ are the angles of $\mathbf{r}$ and $\bar{\mathbf{r}}$ with respect to the normal direction of the plane boundary, respectively. The potential includes three terms: the usual r^{-11} interaction potential between two objects in the absence of the plane boundary [2,4], the $\bar{r}^{-11}$ term which is the interaction between the object A, and the image of object B reflected by the plane boundary, and the remaining term depending on both r and $\bar{r}$.

A. Two special cases

Now we analyze the interaction potential in some special circumstances. The first special case is that two objects are placed in parallel with the plane boundary ($z - z' = 0$), which means that $\theta = \frac{\pi}{2}$, $\bar{\theta} = \cos^{-1}\frac{2z}{\bar{r}}$, and $\bar{r} = \sqrt{r^2 + 4z^2}$. When the two-object system is close to the boundary, i.e., when $z \ll r$ ($r \sim \bar{r}$), we find that

$$U_{AB}(r) = -\frac{G^2}{4\pi}\alpha_A(0)\alpha_B(0)\left(\frac{10242}{r^{11}} - 119790\frac{z^2}{r^{13}}\right). \tag{25}$$

It is easy to see that the boundary increases the potential about 2.6 times in the leading order since the coefficient in the case of flat spacetime without boundary is 3987 although the boundary does not change the behavior of r dependence. The boundary also gives a negative higher-order correction term, which is dependent on z^2.

Now we consider that two objects are placed perpendicular to the boundary. Then, one has $\theta = \bar{\theta} = 0$ and $\bar{r} = r + 2z$. In the limit of $z \ll r$, the potential becomes

$$U_{AB}(r) = -\frac{G^2}{4\pi}\alpha_A(0)\alpha_B(0)\left(\frac{9252}{r^{11}} - 101772\frac{z}{r^{12}}\right), \tag{26}$$

which is, in the leading order, about 2.3 times that in the absence of the plane boundary, and is less than that in the parallel case. In addition, we find that the higher-order z-dependent correction term is different from that in the parallel case which relies on z^2.

IV. DIRICHLET BOUNDARY CONDITION

For the Dirichlet boundary condition, the field mode satisfies $f^\lambda_{ij,\mathbf{k}}|_{z=0} = 0$ and thus can be written as

$$f^\lambda_{ij,\mathbf{k}}(\mathbf{x},t) = \sqrt{\frac{8\pi G}{2\omega(2\pi)^3}}\frac{1}{i} \times [e_{ij}(\mathbf{k},\lambda)e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} - e_{ij}(\mathbf{k}^-,\lambda)e^{i(\mathbf{k}^-\cdot\mathbf{x}-\omega t)}] \tag{27}$$

in the TT gauge. From the above equation, one can show that the two-point correlation function defined in (7) becomes

$$\begin{aligned}
G_{ijkl}(r,\bar{r},\omega) &= -\frac{G}{4\pi^2}\int d\Omega\omega[\hat{g}^r_{ijkl}e^{i\omega r\cos\theta} - \sigma_{km}\sigma_{ln}\hat{g}^{\bar{r}}_{ijmn}e^{i\omega\bar{r}\cos\theta}]\\
&= -\frac{G}{\pi}\left[\hat{g}^r_{ijkl}\frac{\sin(\omega r)}{r} - \sigma_{km}\sigma_{ln}\hat{g}^{\bar{r}}_{ijmn}\frac{\sin(\omega\bar{r})}{\bar{r}}\right]
\end{aligned} \tag{28}$$

and then the interaction potential reads

$$U_{AB}(r,\bar{r}) = -\frac{G^2}{8\pi}\alpha_A(0)\alpha_B(0)\left(\hat{g}^{r}_{ijkl}\hat{g}^{\tilde{r}}_{ijkl}\frac{1}{r\tilde{r}(r+\tilde{r})} - \sigma_{km}\sigma_{ln}\hat{g}^{r}_{ijkl}\hat{g}^{\bar{r}}_{ijmn}\frac{1}{r\bar{r}(r+\bar{r})} - \sigma_{km}\sigma_{ln}\hat{g}^{\bar{r}}_{ijmn}\hat{g}^{r}_{ijkl}\frac{1}{r\bar{r}(\bar{r}+r)} + \hat{g}^{\bar{r}}_{ijkl}\hat{g}^{\tilde{\bar{r}}}_{ijkl}\frac{1}{\bar{r}\,\tilde{\bar{r}}(\bar{r}+\tilde{\bar{r}})}\right)\Big|_{\tilde{r}\to r,\tilde{\bar{r}}\to\bar{r}}. \tag{29}$$

Following the same procedure as in the preceding section, we get that in the case of the Dirichlet boundary the interaction potential is

$$U_{AB}(r,\bar{r}) = -\frac{G^2}{4\pi}\alpha_A(0)\alpha_B(0)\left(\frac{3987}{r^{11}} + \frac{3987}{\bar{r}^{11}} - \frac{144}{r^5\bar{r}^5(r+\bar{r})^9}[A + Br^4\cos 4\theta + 4Cr^2\cos 2\theta + 12Br^2\bar{r}^2\cos 2\theta\cos 2\bar{\theta} + 4\bar{C}\bar{r}^2\cos 2\bar{\theta} + B\bar{r}^4\cos 4\bar{\theta}]\right), \tag{30}$$

with A, B, C, and $\bar{C}$ being given in Eq. (24). This result is less than the one obtained in the Neumann boundary since the third term is subtracted in the Dirichlet boundary while it is added in the Neumann boundary, which indicates that different boundary conditions lead to different interaction potentials between two massive objects.

A. Two special cases

For the special case of two objects placed in parallel with the plane boundary, we take the limit of $z \ll r$ and obtain

$$U_{AB}(r) = -\frac{G^2}{4\pi}\alpha_A(0)\alpha_B(0)\left(\frac{5706}{r^{11}} - 55638\frac{z^2}{r^{13}}\right). \tag{31}$$

Thus, a Dirichlet boundary also reinforces the interaction, but it increases only about 1.4 times compared with the case without boundary, which is less than that in the case of a Neumann boundary. Another noteworthy difference is that the higher-order correction term is also less than that in the Neumann boundary case.

If objects A and B are placed in perpendicular to the plane boundary, we obtain

$$U_{AB}(r) = -\frac{G^2}{4\pi}\alpha_A(0)\alpha_B(0)\left(\frac{6696}{r^{11}} - 73656\frac{z}{r^{12}}\right) \tag{32}$$

in the limit of $z \ll r$, which is about 1.7 times that in the absence of the plane boundary and is less than that from the Neumann boundary. Comparing Eqs. (31) and (32) reveals that the leading term in the potential is larger when the pair of the objects is perpendicularly placed than when it is in parallel with the boundary, which is different from the Neumann boundary case where the former is less than the latter. Similar to the Neumann boundary case, the z dependence of the higher-order correction term in the present case is also different from that of the parallel case.

V. CONCLUSION

In this paper, we have investigated the quantum correction to the classical Newtonian force between a pair of polarizable objects in the presence of plane boundaries in the framework of the linearized quantum gravity and the leading-order perturbation theory. Two kinds of boundary conditions, i.e., Neumann and Dirichlet, are imposed. The general results are given in Eqs. (23) and (30). In both cases, the potentials consist of three terms, i.e., the usual r^{-11}-dependent interaction potential between two objects in the absence of the plane boundary where r is the separation of the two objects, the $\bar{r}^{-11}$ term which is the interaction between the object A and the image of object B reflected by the plane boundary where $\bar{r}$ is the distance between the object A and the image of object B, and the term depending on both r and $\bar{r}$. Different boundary conditions in general lead to different interaction potentials, with the Neumann boundary yielding a larger interaction than the Dirichlet boundary.

When one partner of the pair is placed very close to the boundary ($z \ll r$), where z is the distance between the boundary and the closer partner, we find, for both special cases, i.e., the pair is in parallel with or perpendicular to the plane boundary, that the boundary strengthens the interaction potential as compared with the case in the absence of a boundary. In the Neumann boundary case, the potential in the parallel case is larger than that of the perpendicular case, which is just opposite to the Dirichlet boundary case where the latter is larger than the former. In addition, we find that the sign of the higher-order correction term is negative and the pair-boundary separation dependence of the correction is determined by the orientation of the object pair, with the parallel case and the perpendicular case giving a quadratic and a linear correction, respectively.

Finally, let us briefly comment on the issue of how to realize the boundary conditions considered in this paper in some specific physical setups. It is well known that ordinary materials can hardly reflect nor absorb gravitational waves [21], and thus the reflection coefficient for gravitational waves will be extremely small. However, recently, there have been interesting speculations that quantum matter such as superconducting films might behave like highly reflective mirrors that realize the Dirichlet boundary condition for gravitational waves, since the incident gravitational waves may be reflected effectively due to the so-called Heisenberg-Coulomb effect [22]. As for the Neumann boundary condition, we do not know of any specific physical setup that can realize it. So, at present, it only remains as a theoretical curiosity.

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