

New nilpotent $\mathcal{N} = 2$ superfields

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(Received 25 October 2017; published 8 January 2018; corrected 23 January 2018)

We propose new off-shell models for spontaneously broken local $\mathcal{N} = 2$ supersymmetry, in which the supergravity multiplet couples to nilpotent Goldstino superfields that contain either a gauge one-form or a gauge two-form in addition to spin-1/2 Goldstone fermions and auxiliary fields. In the case of $\mathcal{N} = 2$ Poincaré supersymmetry, we elaborate on the concept of twisted chiral superfields and present a nilpotent $\mathcal{N} = 2$ superfield that underlies the cubic nilpotency conditions given in *J. High Energy Phys.* **08** (2017) 109. in terms of constrained $\mathcal{N} = 1$ superfields.

DOI: [10.1103/PhysRevD.97.026003](https://doi.org/10.1103/PhysRevD.97.026003)

I. INTRODUCTION

In the last three years, there has been much interest in off-shell models for spontaneously broken $\mathcal{N} = 1$ supergravity; see [1–10] and references therein. Recently, we have constructed several off-shell models for spontaneously broken $\mathcal{N} = 2$ supergravity [10], in which the supergravity multiplet couples to the nilpotent Goldstino superfields introduced in [11,12]. The models proposed in [10] make use of both reducible and irreducible Goldstino superfields, following the terminology of [8]. Every irreducible $\mathcal{N} = 2$ Goldstino superfield contains only two independent component fields, the spin-1/2 Goldstone fermions [13,14], while the other component fields are composites constructed from the Goldstini. Reducible Goldstino superfields also contain some independent fields in addition to the Goldstini.

In this paper we propose new models for spontaneously broken $\mathcal{N} = 2$ supergravity in which the Goldstini belong to nilpotent superfields containing either a gauge one-form or a gauge two-form among its independent physical fields. This is achieved by relaxing the constraints obeyed by the Goldstino superfields introduced in [10,15]. The idea can be illustrated by giving two examples. The oldest irreducible Goldstino superfield in four dimensions is the $\mathcal{N} = 1$ chiral scalar superfield ϕ [16,17], $\bar{D}_{\dot{\alpha}}\phi = 0$, which is subject to the constraints [16]

$$\phi^2 = 0, \quad (1.1a)$$

$$f\phi = -\frac{1}{4}\phi\bar{D}^2\bar{\phi}, \quad (1.1b)$$

where f is a real parameter of mass dimension +2, which characterizes the supersymmetry breaking scale. Removing the second constraint, Eq. (1.1b), leads to the reducible Goldstino superfield advocated in [18,19]. Our second example is the irreducible Goldstino superfield introduced in [8]. It is described by a real scalar $\mathcal{N} = 1$ superfield V subject to the nilpotency constraints¹

$$V^2 = 0, \quad (1.2a)$$

$$VD_A D_B V = 0, \quad (1.2b)$$

$$VD_A D_B D_C V = 0, \quad (1.2c)$$

where $D_A = (\partial_a, D_\alpha, \bar{D}^{\dot{\alpha}})$ are the covariant derivatives of $\mathcal{N} = 1$ Minkowski superspace, in conjunction with the nonlinear constraint

$$fV = \frac{1}{16}VD^\alpha\bar{D}^2D_\alpha V. \quad (1.3)$$

If the nonlinear constraint (1.3) is removed, we end up with the reducible Goldstino superfield introduced in [10].

This paper is organized as follows. In Sec. II we couple $\mathcal{N} = 2$ supergravity to a deformed reduced chiral superfield subject to a cubic nilpotency condition. In Sec. III, $\mathcal{N} = 2$ supergravity is coupled to a linear superfield [also known as the $\mathcal{O}(2)$ multiplet] subject to a cubic nilpotency condition. In Sec. IV, we elaborate on the concept of

¹The constraints (1.2a) and (1.3) were introduced for the first time in [20].

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twisted chiral superfields in $\mathcal{N} = 2$ Minkowski superspace and present a nilpotent $\mathcal{N} = 2$ superfield that underlies the cubic nilpotency conditions given in [21] in terms of constrained $\mathcal{N} = 1$ superfields. The reason for restricting our analysis to the super-Poincaré case is that there is no simple definition of twisted chiral superfields on arbitrary $\mathcal{N} = 2$ curved superspace backgrounds. The main body of the paper is accompanied by two technical appendixes. Appendix A reviews the prepotential formulations for the $\mathcal{N} = 2$ reduced chiral and linear multiplets. Appendix B provides a solution to the nilpotency condition (3.2) in the flat case using the harmonic superspace techniques.

II. NILPOTENT CHIRAL SUPERFIELD

In recent papers [15,22], a deformed reduced chiral superfield \mathcal{Z} coupled to $\mathcal{N} = 2$ supergravity was introduced. It is described by the constraints

$$\bar{\mathcal{D}}_{\dot{\alpha}}^i \mathcal{Z} = 0, \quad (2.1a)$$

$$(\mathcal{D}^{ij} + 4S^{ij})\mathcal{Z} - (\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{\mathcal{Z}} = 4iG^{ij}, \quad (2.1b)$$

where we have defined $\mathcal{D}^{ij} = \mathcal{D}^{\alpha(i}\mathcal{D}_{\alpha}^{j)}$ and $\bar{\mathcal{D}}^{ij} = \bar{\mathcal{D}}_{\dot{\alpha}}^{(i}\bar{\mathcal{D}}^{j)\dot{\alpha}}$. Here G^{ij} is a linear multiplet that obeys the constraints (1.2). In addition, G^{ij} is required to be nowhere vanishing, $G^{ij}G_{ij} \neq 0$. As reviewed in Appendix A, G^{ij} is the gauge-invariant field strength of a tensor multiplet. In this paper, we identify G^{ij} with one of the two conformal compensators of the minimal formulation for $\mathcal{N} = 2$ supergravity proposed in [23]. The superfields S^{ij} and \bar{S}^{ij} in (2.1) are special dimension-1 components of the torsion; see [24] for the technical details of the superfield formulation for $\mathcal{N} = 2$ conformal supergravity [25] that we use. The constraints (2.1a) and (2.2) are invariant under the $\mathcal{N} = 2$ super-Weyl transformations [24,25] if \mathcal{Z} is chosen to be a primary superfield of dimension 1.

In the super-Poincaré case, a chiral superfield obeying the constraint (2.1b) with a *constant* $\text{SU}(2)$ triplet G^{ij} appeared in the framework of partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking [26–28].

In our previous paper [15], \mathcal{Z} was subject to the quadratic nilpotency condition

$$\mathcal{Z}^2 = 0. \quad (2.2)$$

The constraints (2.1) and (2.2) imply that, for certain $\mathcal{N} = 2$ supergravity backgrounds, the degrees of freedom described by the $\mathcal{N} = 2$ chiral superfield \mathcal{Z} are in one-to-one correspondence with those of an Abelian $\mathcal{N} = 1$ vector multiplet. The specific feature of such $\mathcal{N} = 2$ supergravity backgrounds is that they possess an $\mathcal{N} = 1$ subspace $\mathcal{M}^{4|4}$ of the full $\mathcal{N} = 2$ curved superspace $\mathcal{M}^{4|8}$. This property is not universal. In particular, there exist maximally $\mathcal{N} = 2$ supersymmetric backgrounds with no admissible truncation

to $\mathcal{N} = 1$ [29]. As shown in [15], the superfield constrained by (2.1) and (2.2) is suitable for the description of partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ rigid supersymmetry breaking in every maximally supersymmetric spacetime \mathcal{M}^4 , which is the bosonic body of an $\mathcal{N} = 1$ superspace $\mathcal{M}^{4|4}$ described by the following algebra of $\mathcal{N} = 1$ covariant derivatives,²

$$\{\mathcal{D}_{\alpha}, \mathcal{D}_{\beta}\} = 0, \quad \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} = 0, \quad \{\mathcal{D}_{\alpha}, \bar{\mathcal{D}}_{\dot{\beta}}\} = -2i\mathcal{D}_{\alpha\dot{\beta}}, \quad (2.3a)$$

$$[\mathcal{D}_{\alpha}, \mathcal{D}_{\beta\dot{\beta}}] = i\varepsilon_{\alpha\beta}G^{\gamma}_{\dot{\beta}}\mathcal{D}_{\gamma}, \quad [\bar{\mathcal{D}}_{\dot{\alpha}}, \mathcal{D}_{\beta\dot{\beta}}] = -i\varepsilon_{\dot{\alpha}\dot{\beta}}G_{\beta}^{\gamma}\bar{\mathcal{D}}_{\dot{\gamma}}, \quad (2.3b)$$

$$[\mathcal{D}_{\alpha\dot{\alpha}}, \mathcal{D}_{\beta\dot{\beta}}] = -i\varepsilon_{\dot{\alpha}\dot{\beta}}G_{\beta}^{\gamma}\mathcal{D}_{\alpha\dot{\gamma}} + i\varepsilon_{\alpha\beta}G^{\gamma}_{\dot{\beta}}\mathcal{D}_{\gamma\dot{\alpha}}, \quad (2.3c)$$

where the real four-vector G_b is covariantly constant,

$$\mathcal{D}_{\alpha}G_b = 0, \quad G_b = \bar{G}_b. \quad (2.3d)$$

Since $G^2 = G^b G_b$ is constant, the geometry (2.3) describes three different superspaces, for $G_b \neq 0$, which correspond to the choices $G^2 < 0$, $G^2 > 0$, and $G^2 = 0$, respectively. The Lorentzian manifolds \mathcal{M}^4 supported by these superspaces are $\mathbb{R} \times S^3$, $\text{AdS}_3 \times S^1$ or its covering $\text{AdS}_3 \times \mathbb{R}$, and a pp-wave spacetime, respectively.

We constructed in [15] the Maxwell-Goldstone multiplet actions for partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking for all of them. In each of these cases, the action coincides with a unique curved-superspace extension of the $\mathcal{N} = 1$ supersymmetric Born-Infeld action [31–33], which is singled out by the requirement of $\text{U}(1)$ duality invariance [34–36]. In the super-Poincaré case, $G_b = 0$, the approach developed in [15] provided a simple $\mathcal{N} = 2$ superfield derivation of the Bagger-Galperin action for partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking [33], which differs in some technical details from the original derivation given by Roček and Tseytlin [37].

If one is interested in $\mathcal{N} = 2 \rightarrow \mathcal{N} = 0$ breaking of local supersymmetry, the nilpotency condition (2.2) should be replaced with a weaker constraint

$$\mathcal{Z}^3 = 0. \quad (2.4)$$

In the super-Poincaré case, such a constraint has recently been considered in [21]. As was demonstrated in [21], for a certain range of parameters, in Minkowski superspace the superfield \mathcal{Z} constrained by (2.1) and (2.4) contains the following independent fields: two Goldstini, a gauge one-form, and a real, nowhere vanishing, $\text{SU}(2)$ triplet of auxiliary fields $D^{ij} = D^{ji}$, with $D^{ij}D_{ij} \neq 0$. We now present a dynamical system describing $\mathcal{N} = 2$ supergravity coupled to \mathcal{Z} .

²These backgrounds are maximally supersymmetric solutions of pure R^2 supergravity [30].

The action for our supergravity-matter theory involves two contributions,

$$S = S_{\text{SUGRA}} + S_{\mathcal{Z}}, \quad (2.5)$$

where S_{SUGRA} denotes the pure supergravity action and $S_{\mathcal{Z}}$ corresponds to the Goldstino superfield. We make use of the minimal formulation for $\mathcal{N} = 2$ supergravity with vector and tensor compensators [23]. In the superspace setting, the supergravity action can be written in the form [38] (derived using the projective-superspace formulation [39] for this theory)

$$\begin{aligned} S_{\text{SUGRA}} &= \frac{1}{\kappa^2} \int d^4x d^4\theta \mathcal{E} \left\{ \Psi \mathbb{W} - \frac{1}{4} W^2 + m \Psi W \right\} + \text{c.c.} \\ &= \frac{1}{\kappa^2} \int d^4x d^4\theta \mathcal{E} \left\{ \Psi \mathbb{W} - \frac{1}{4} W^2 \right\} + \text{c.c.} \\ &\quad + \frac{m}{\kappa^2} \int d^4x d^4\theta d^4\bar{\theta} E G^{ij} V_{ij}, \end{aligned} \quad (2.6)$$

where κ is the gravitational constant and m the cosmological parameter. Here E and \mathcal{E} denote the full superspace and chiral densities, respectively. The covariantly chiral scalar Ψ and the real $\text{SU}(2)$ triplet V_{ij} are the prepotentials of the tensor and vector multiplets, respectively; see Appendix A for the technical details. The supergravity action involves the composite

$$\mathbb{W} := -\frac{G}{8} (\bar{D}_{ij} + 4\bar{S}_{ij}) \left(\frac{G^{ij}}{G^2} \right), \quad (2.7)$$

which proves to be a reduced chiral superfield. The superfield (2.7) is one of the simplest applications of the powerful approach to generate composite reduced chiral multiplets, which was presented in [38]. Another application is given in the next section.

The action for the Goldstino superfield \mathcal{Z} in (2.5) is

$$\begin{aligned} S_{\mathcal{Z}} &= \int d^4x d^4\theta \mathcal{E} \left\{ \frac{1}{4} \mathcal{Z}^2 + \zeta W \mathcal{Z} + \rho \left(\mathcal{Z} \Psi - \frac{i}{2} \Psi^2 \right) \right\} \\ &\quad + \text{c.c.}, \end{aligned} \quad (2.8)$$

where ζ and ρ are complex and real parameters, respectively. The ρ -term in (2.8) was introduced in [15], where it was shown to be invariant under gauge transformations (A4).

The Goldstino superfield action (2.8) can be generalized to include higher derivative couplings, for instance,

$$I = \int d^4x d^4\theta d^4\bar{\theta} E \left\{ \lambda_1 \frac{\bar{\mathcal{Z}} \mathcal{Z}}{\bar{W} W} + \lambda_2 \left(\frac{\bar{\mathcal{Z}} \mathcal{Z}}{\bar{W} W} \right)^2 \right\}, \quad (2.9)$$

where λ_1 and λ_2 are coupling constants.

III. NILPOTENT LINEAR SUPERFIELD

We now introduce a linear superfield \mathcal{H}^{ij} ,

$$\mathcal{D}_\alpha^{(i} \mathcal{H}^{jk)} = \bar{\mathcal{D}}_\alpha^{(i} \mathcal{H}^{jk)} = 0, \quad (3.1)$$

which is subject to the following cubic nilpotency condition [10]

$$\mathcal{H}^{(i_1 i_2} \mathcal{H}^{i_3 i_4} \mathcal{H}^{i_5 i_6)} = 0. \quad (3.2)$$

This algebraic constraint is one of the several nonlinear constraints, which define the irreducible linear Goldstino superfield \mathbb{H}^{ij} introduced in [10]. As is shown in Appendix B, the cubic constraint (3.2) expresses the $\text{SU}(2)$ triplet of physical scalars, $\mathcal{H}^{ij}|_{\theta=0}$, in terms of the other component fields of \mathcal{H}^{ij} . Thus the field content of \mathcal{H}^{ij} is as follows: two Goldstini, a gauge two-form, and a complex nowhere vanishing auxiliary scalar. As for the Goldstino superfield \mathbb{H}^{ij} , its only independent component fields are the Goldstini, since the additional nonlinear constraints, which \mathbb{H}^{ij} obeys, express the gauge two-form and the auxiliary fields in terms of the Goldstone fermions [10].

To describe the dynamics of $\mathcal{N} = 2$ supergravity coupled to \mathcal{H}^{ij} we choose an action of the form

$$S = S_{\text{SUGRA}} + S_{\mathcal{H}}, \quad (3.3)$$

where the supergravity action is given by (2.6). The action $S_{\mathcal{H}}$ for the Goldstino superfield has, probably, the simplest form within the projective-superspace formulation for $\mathcal{N} = 2$ supergravity [24,40]. Here we refer the reader to [24,40] for the technical details of that formulation, and we simply give the projective superfield Lagrangian corresponding to $S_{\mathcal{H}}$. Using the modern projective-superspace notation [41], the Lagrangian is

$$\mathcal{L}_{\mathcal{H}}^{(2)} = -\frac{1}{2} \frac{\mathcal{H}^{(2)} \mathcal{H}^{(2)}}{G^{(2)}} + \xi V \mathcal{H}^{(2)}, \quad (3.4)$$

with ξ being a coupling constant. Here we have denoted $\mathcal{H}^{(2)} = \mathcal{H}_{ij} v^i v^j$, $G^{(2)} = G_{ij} v^i v^j$, where $v^i \in \mathbb{C}^2 \setminus \{0\}$ denotes the homogeneous coordinates for $\mathbb{C}P^1$. Finally, the superfield $V(v^i)$ in (3.4) is the tropical prepotential for the compensating vector multiplet; in particular, it is a holomorphic homogeneous function of v^i of degree 0.

The action $S_{\mathcal{H}}$ can also be written, in a reasonably compact form, in the conventional curved superspace using the techniques developed in [38]. It is

$$\begin{aligned} S_{\mathcal{H}} &= -\frac{1}{2} \int d^4x d^4\theta \mathcal{E} \Psi \mathbb{W}_2 + \text{c.c.} \\ &\quad + \xi \int d^4x d^4\theta d^4\bar{\theta} E \mathcal{H}^{ij} V_{ij}, \end{aligned} \quad (3.5)$$

where \mathbb{W}_2 denotes the reduced chiral superfield [38]

$$\begin{aligned}\mathbb{W}_2 &= -\frac{G}{16}(\bar{D}_{ij} + 4\bar{S}_{ij})\mathcal{R}_2^{ij}, \\ \mathcal{R}_2^{ij} &= \frac{1}{G^4}\left(\delta_{kl}^{ij} - \frac{1}{2G^2}G^{ij}G_{kl}\right)\mathcal{H}^{(kl}\mathcal{H}^{mn)}G_{mn}.\end{aligned}\quad (3.6)$$

The action (3.5) can be generalized to include higher derivative terms that can be constructed using the techniques developed in [38].

IV. NILPOTENT TWISTED CHIRAL SUPERFIELDS

In this section we restrict our attention to the case of $\mathcal{N} = 2$ Poincaré supersymmetry and introduce new nilpotent superfields on Minkowski superspace $\mathbb{M}^{4|8}$ parametrized by Cartesian coordinates $z^A = (x^a, \theta_i^\alpha, \bar{\theta}_{\dot{\alpha}}^i)$, where $\bar{\theta}^{\dot{\alpha}i}$ is the complex conjugate of θ_i^α , with $i = \underline{1}, \underline{2}$. To start with, we recall some salient features of the so-called projective supermultiplets that live in the generalized $\mathcal{N} = 2$ superspace $\mathbb{M}^{4|8} \times \mathbb{C}P^1$ [42–45]; see [41] for a pedagogical review.³ As usual, the notation $D_A = (\partial_a, D_i^\alpha, \bar{D}_{\dot{\alpha}}^i)$ is used for the superspace covariant derivatives. We denote by ζ the inhomogeneous complex coordinate for $\mathbb{C}P^1$.

An $\mathcal{N} = 2$ superfield $\Xi(z, \zeta)$ of the general form

$$\Xi(z, \zeta) = \sum_{n=-\infty}^{+\infty} \Xi_n(z)\zeta^n \quad (4.1)$$

is called projective if it satisfies the constraints

$$\nabla_\alpha(\zeta)\Xi(\zeta) = 0, \quad \nabla_\alpha(\zeta) = \zeta D_\alpha^1 - D_\alpha^2, \quad (4.2a)$$

$$\bar{\nabla}_{\dot{\alpha}}(\zeta)\Xi(\zeta) = 0, \quad \bar{\nabla}_{\dot{\alpha}}(\zeta) = \bar{D}_{\dot{\alpha}}^1 + \zeta\bar{D}_{\dot{\alpha}}^2. \quad (4.2b)$$

These constraints are equivalent to the following differential conditions,

$$D_\alpha^2\Xi_n = D_\alpha^1\Xi_{n-1}, \quad \bar{D}_{\dot{\alpha}}^2\Xi_n = -\bar{D}_{\dot{\alpha}}^1\Xi_{n+1}, \quad (4.3)$$

which imply

$$(D^{\underline{2}})^2\Xi_n = (D^{\underline{1}})^2\Xi_{n-2}, \quad (\bar{D}_{\underline{2}})^2\Xi_n = (\bar{D}_{\underline{1}})^2\Xi_{n+2}. \quad (4.4)$$

Let us now consider a projective superfield $\Upsilon(\zeta)$ whose Laurent series is bounded below. Without loss of generality, it can be represented by a Taylor series

$$\Upsilon(\zeta) = \sum_{n=0}^{+\infty} \Upsilon_n \zeta^n. \quad (4.5)$$

Then the constraints (4.3) tell us that the lowest component of $\Upsilon(\zeta)$, Υ_0 , satisfies chiral and antichiral constraints

$$\bar{D}_{\underline{1}}^{\dot{\alpha}}\Upsilon_0 = 0, \quad D_{\underline{2}}^{\alpha}\Upsilon_0 = 0, \quad (4.6)$$

while the next-to-lowest component Υ_1 obeys linear constraints

$$(\bar{D}_{\underline{1}})^2\Upsilon_1 = 0, \quad (D^{\underline{2}})^2\Upsilon_1 = 0. \quad (4.7)$$

Making use of the constraints (4.3) and (4.4) also gives

$$\bar{D}_{\underline{2}}^{\dot{\alpha}}\Upsilon_0 = -\bar{D}_{\underline{2}}^{\dot{\alpha}}\Upsilon_1, \quad (4.8a)$$

$$(\bar{D}_{\underline{2}})^2\Upsilon_0 = (\bar{D}_{\underline{1}})^2\Upsilon_2. \quad (4.8b)$$

Constraints of the type (4.6) were considered for the first time thirty-five years ago by Galperin *et al.* [49] in the context of the Fayet-Sohnius hypermultiplet [50,51]. Recently they have been rediscovered, without any reference to [49] and the projective-superspace literature, in [52]. These authors introduced a ring of $\mathcal{N} = 2$ superfields Ω constrained by

$$\bar{D}_{\underline{1}}^{\dot{\alpha}}\Omega = 0, \quad D_{\underline{2}}^{\alpha}\Omega = 0. \quad (4.9)$$

Such superfields were called “chiral antichiral” in [52]. Instead we call them “twisted chiral superfields” by analogy with the two-dimensional terminology introduced in [53]. The most general twisted chiral superfield has the form

$$\Omega(x, \theta_i, \bar{\theta}^j) = e^{-i(\theta_1^{\sigma\alpha}\bar{\theta}^1 - \theta_2^{\sigma\alpha}\bar{\theta}^2)\partial_a}\hat{\Omega}(x, \theta_{\underline{1}}, \bar{\theta}_{\underline{2}}), \quad (4.10)$$

where $\hat{\Omega}(x, \theta_{\underline{1}}^\alpha, \bar{\theta}_{\underline{2}}^{\dot{\alpha}})$ is an arbitrary function of the four Grassmann variables $\theta_{\underline{1}}^\alpha$ and $\bar{\theta}_{\underline{2}}^{\dot{\alpha}}$. We show that every twisted chiral superfield Ω is the lowest component of a projective superfield $\Upsilon(\zeta)$.

Given a projective superfield $\Xi(\zeta)$, the constraints (4.3) imply that the dependence of the component superfields Ξ_n on $\theta_{\underline{2}}^\alpha$ and $\bar{\theta}_{\underline{2}}^{\dot{\alpha}}$ is uniquely determined in terms of their dependence on $\theta_{\underline{1}}^\alpha \equiv \theta^\alpha$ and $\bar{\theta}_{\underline{1}}^{\dot{\alpha}} \equiv \bar{\theta}_{\dot{\alpha}}$. In other words, the projective superfield depends effectively on half the Grassmann variables, which can be chosen to be the spinor coordinates of the $\mathcal{N} = 1$ Minkowski superspace $\mathbb{M}^{4|4}$ parametrized by the coordinates $(x^a, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$. We introduce the spinor covariant derivatives for $\mathbb{M}^{4|4}$, $D_\alpha := D_\alpha^1$ and $\bar{D}_{\dot{\alpha}} := \bar{D}_{\dot{\alpha}}^1$. Associated with every $\mathcal{N} = 2$ superfield U is its $\mathcal{N} = 1$ bar projection $U| := U|_{\theta_2=\bar{\theta}^2=0}$, which is an $\mathcal{N} = 1$

³The superspace $\mathbb{M}^{4|8} \times \mathbb{C}P^1$ was introduced for the first time by Rosly [46]. The same superspace is at the heart of the harmonic [47,48] and projective [42–44] superspace approaches.

superfield. As we have mentioned, all information about the projective multiplet $\Xi(\zeta)$ is encoded in its bar projection $\Xi(\zeta)|$. In particular, associated with the projective multiplet (4.5) is the following family of $\mathcal{N} = 1$ superfields,

$$\Upsilon(\zeta)| = \phi + \zeta\Gamma + \sum_{n=2}^{+\infty} \Upsilon_n|\zeta^n, \quad \bar{D}^{\dot{\alpha}}\phi = 0, \quad \bar{D}^2\Gamma = 0. \quad (4.11)$$

The explicit structure of the $\mathcal{N} = 1$ superfields $\Upsilon_n|$, with $n = 2, 3, \dots$, depends on the original projective multiplet.

Let us forget for a moment about the projective multiplets and consider a twisted chiral superfield Ω . All information about Ω is encoded in the three $\mathcal{N} = 1$ superfields

$$\phi := \Omega|, \quad \Upsilon^{\dot{\alpha}} := \frac{1}{2}\bar{D}_2^{\dot{\alpha}}\Omega|, \quad \Psi := -\frac{1}{4}(\bar{D}_2)^2\Omega|, \quad (4.12)$$

all of which are chiral,

$$\bar{D}^{\dot{\alpha}}\phi = 0, \quad \bar{D}^{\dot{\alpha}}\Upsilon^{\dot{\beta}} = 0, \quad \bar{D}^{\dot{\alpha}}\Psi = 0, \quad (4.13)$$

by construction. The chirality of $\Upsilon^{\dot{\alpha}}$ implies $\Upsilon^{\dot{\alpha}} = -\frac{1}{4}\bar{D}^2\Lambda^{\dot{\alpha}} = \frac{1}{2}\bar{D}^{\dot{\alpha}}\bar{D}_{\dot{\beta}}\Lambda^{\dot{\beta}} \equiv -\frac{1}{2}\bar{D}^{\dot{\alpha}}\Gamma$. Thus, there exist $\mathcal{N} = 1$ superfields $\Gamma := \Upsilon_1|$ and $U := \Upsilon_2|$, of which Γ obeys the linear constraint $\bar{D}^2\Gamma = 0$, such that

$$\Upsilon^{\dot{\alpha}} = -\frac{1}{2}\bar{D}^{\dot{\alpha}}\Gamma, \quad \Psi = -\frac{1}{4}\bar{D}^2U. \quad (4.14)$$

Thus we have demonstrated that every twisted chiral superfield is the lowest component of a projective superfield. In what follows, we do not indicate explicitly the bar projection.

We now turn to reviewing the structure of supersymmetric actions constructed in terms of the projective multiplets. As is well known, associated with every projective multiplet (4.1) is its smile conjugate

$$\check{\Xi}(\zeta) := \sum_{n=-\infty}^{+\infty} (-1)^n \zeta^n \check{\Xi}_{-n}, \quad (4.15)$$

which is also a projective multiplet. If the theory is formulated in terms of a projective multiplet $\Upsilon(\zeta)$ and its smile conjugate $\check{\Upsilon}(\zeta)$, the dynamics is described with the aid of a Lagrangian $\mathcal{L}(\zeta) \equiv \mathcal{L}(\Upsilon(\zeta), \check{\Upsilon}(\zeta), \zeta)$, which is a projective multiplet. Using this Lagrangian, one can construct a manifestly $\mathcal{N} = 2$ supersymmetric action; see [41] for a pedagogical review. As explained in [41], the manifestly $\mathcal{N} = 2$ supersymmetric action can be recast in two different but equivalent forms,

$$S = \frac{1}{16} \oint \frac{\zeta d\zeta}{2\pi i} \int d^4x (D^{\dot{1}})^2 (\bar{D}_2)^2 \mathcal{L}(\zeta) \\ = \oint \frac{\zeta d\zeta}{2\pi i} \int d^4x d^2\theta_{\dot{1}} d^2\bar{\theta}^2 \mathcal{L}(\zeta) \quad (4.16a)$$

$$= \frac{1}{16} \oint \frac{d\zeta}{2\pi i \zeta} \int d^4x \zeta (D^{\dot{1}})^2 (\bar{D}_1)^2 \mathcal{L}(\zeta) \\ = \oint \frac{d\zeta}{2\pi i \zeta} \int d^4x d^2\theta d^2\bar{\theta} \mathcal{L}(\zeta), \quad (4.16b)$$

of which the latter is used in most applications.

We now consider an important example of applying the action principles (4.16a) and (4.16b). As an extension of the construction given in [54], we choose $\mathcal{L}(\zeta) = -F(\Upsilon(\zeta))\zeta^{-2}$, with $F(z)$ being a holomorphic function of one argument, and consider the action

$$S = - \oint_C \frac{d\zeta}{2\pi i \zeta} \int d^4x d^2\theta d^2\bar{\theta} \frac{F(\Upsilon(\zeta))}{\zeta^2} + \text{c.c.}, \quad (4.17)$$

where C is a contour around the origin. Performing the contour integral gives

$$S = - \int d^4x d^2\theta d^2\bar{\theta} \{ F'(\phi)U + \frac{1}{2}F''(\phi)\Gamma^2 \} + \text{c.c.} \\ = \int d^4x d^2\theta \{ F''(\phi)\Upsilon_{\dot{\alpha}}\Upsilon^{\dot{\alpha}} - F'(\phi)\Psi \} + \text{c.c.} \quad (4.18)$$

On the other hand, making use of (4.16a) leads to the action

$$S = - \int d^4x d^2\theta_{\dot{1}} d^2\bar{\theta}^2 F(\Upsilon_0) + \text{c.c.}, \quad (4.19)$$

which is an example of the twisted chiral supersymmetric action

$$S_{\text{TC}} = \int d^4x d^2\theta_{\dot{1}} d^2\bar{\theta}^2 \mathcal{L}_{\text{TC}}, \quad \bar{D}_1^{\dot{\alpha}}\mathcal{L}_{\text{TC}} = 0, \quad D_{\dot{\alpha}}^2\mathcal{L}_{\text{TC}} = 0. \quad (4.20)$$

The $\mathcal{N} = 2$ supersymmetric theory introduced in [54] made use of a short projective multiplet

$$H(\zeta) = H_0 + \zeta H_1 - \zeta^2 \bar{H}_0, \quad \bar{H}_1 = H_1, \quad (4.21)$$

which is known under three different names: (i) real $\mathcal{O}(2)$ multiplet, (ii) linear multiplet, and (iii) tensor multiplet. Its $\mathcal{N} = 1$ components include a chiral scalar $\phi := H_0|$ and a real linear superfield $G := H_1| = \bar{G}$, $D^2G = \bar{D}^2G = 0$. The $\mathcal{N} = 2$ superfield H_0 in $H(\zeta)$ is called a short twisted chiral superfield. Its $\mathcal{N} = 1$ components in (4.12) satisfy

$$\Upsilon^{\dot{\alpha}} = -\frac{1}{2}\bar{D}^{\dot{\alpha}}G, \quad \Psi = \frac{1}{4}\bar{D}^2\bar{\phi}. \quad (4.22)$$

The action (4.18) corresponding to the $\mathcal{O}(2)$ multiplet (4.21) reads [54]

$$S = \int d^4x d^2\theta d^2\bar{\theta} \times \left\{ \bar{\phi} F'(\phi) + \phi \bar{F}'(\bar{\phi}) - \frac{1}{2} (F''(\phi) + \bar{F}''(\bar{\phi})) G^2 \right\}, \quad (4.23)$$

which is a special case of the general models for self-interacting $\mathcal{N} = 2$ tensor multiplets [55]. Dualizing the linear superfield G in (4.23) into a chiral scalar, one ends up with a hyper-Kähler sigma model. The generalization of (4.23) to the case of several $\mathcal{N} = 2$ tensor multiplets, which was given in [54], provides a superspace derivation of the rigid c -map construction [56].

As was shown in [52], there exists a simple deformation of the short twisted chiral superfield that can be used to derive the tensor Goldstone multiplet for partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking [57] from $\mathcal{N} = 2$ superfields. Such a framework is actually closely related to the earlier work of [37,58]. To describe partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ breaking of supersymmetry, the authors of [37,58] deformed the real $\mathcal{O}(2)$ multiplet $H(\zeta)$ to a complex $\mathcal{O}(2)$ multiplet $\mathbf{H}(\zeta)$ given by

$$\begin{aligned} \mathbf{H}(\zeta) &= \mathbf{H}_0 + \zeta \mathbf{H}_1 + \zeta^2 \mathbf{H}_2 \\ &:= \hat{H}(\zeta) + m((\bar{\theta}^{\dot{2}})^2 - \zeta(\bar{\theta}^{\dot{1}}\bar{\theta}^{\dot{2}}) + \zeta^2(\bar{\theta}^{\dot{1}})^2). \end{aligned} \quad (4.24)$$

Here $\hat{H}(\zeta)$ has the functional form (4.21) and obeys the analyticity conditions

$$\nabla_{\alpha}(\zeta)\hat{H}(\zeta) = 0, \quad \bar{\nabla}_{\dot{\alpha}}(\zeta)\hat{H}(\zeta), \quad (4.25)$$

but it does not transform as an $\mathcal{N} = 2$ superfield, unlike $\mathbf{H}(\zeta)$. The deformed short twisted chiral multiplet \mathbf{H}_0 has the properties

$$\begin{aligned} \mathbf{H}_0| &= \phi, \quad \Upsilon^{\dot{\alpha}} := \frac{1}{2} \bar{D}_2^{\dot{\alpha}} \mathbf{H}_0| = -\frac{1}{2} \bar{D}_2^{\dot{\alpha}} G, \\ \Psi &:= -\frac{1}{4} (\bar{D}_2)^2 \mathbf{H}_0 = \frac{1}{4} \bar{D}_2^2 \bar{\phi} + m, \end{aligned} \quad (4.26)$$

which coincide with those of the deformed chiral-antichiral multiplet considered in [52]. The mass parameter m (4.26) plays a role similar to the deformation parameter of the deformed reduced chiral superfield in the flat superspace limit. The presence of the deformation parameter m modifies the second supersymmetry transformation,

$$\delta\phi = -2\bar{\epsilon}_{\dot{\alpha}}^2 \Upsilon^{\dot{\alpha}}, \quad (4.27a)$$

$$\delta\Upsilon^{\dot{\alpha}} = \frac{1}{2} m \bar{\epsilon}^{\dot{\alpha}2} + \frac{1}{4} \bar{\epsilon}^{\dot{\alpha}2} \bar{D}_2^2 \bar{\phi} - i\epsilon_2^{\alpha} \partial_{\alpha}^{\dot{\alpha}} \phi. \quad (4.27b)$$

The action (4.20) with a Lagrangian $\mathcal{L}_{\text{TC}} = -F(\mathbf{H}_0)$ takes, upon reduction to $\mathcal{N} = 1$ superspace, the following form:

$$S = \int d^4x d^2\theta d^2\bar{\theta} W(\phi) \bar{\phi} + \int d^4x d^2\theta \{ W'(\phi) \Upsilon_{\dot{\alpha}} \Upsilon^{\dot{\alpha}} + m W(\phi) \} + \text{c.c.}, \quad (4.28)$$

where $W(\phi) := F'(\mathbf{H}_0)|$.

To describe $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking, it remains to impose the quadratic nilpotency condition [52]

$$\mathbf{H}_0^2 = 0, \quad (4.29)$$

in agreement with the earlier results of [37,58]. In terms of $\mathcal{N} = 1$ superfields, this constraint is equivalent to

$$\phi^2 = 0, \quad \phi \Upsilon^{\dot{\alpha}} = 0, \quad \left(m + \frac{1}{4} \bar{D}_2^2 \bar{\phi} \right) \phi = \Upsilon_{\dot{\alpha}} \Upsilon^{\dot{\alpha}}, \quad (4.30)$$

which are exactly the Bagger-Galperin constraints [57].

Instead of imposing the constraint (4.29), we now consider a cubic nilpotency condition

$$\mathbf{H}_0^3 = 0. \quad (4.31)$$

Upon reduction to $\mathcal{N} = 1$ superfields, it implies

$$\phi^3 = 0, \quad \phi^2 \Upsilon^{\dot{\alpha}} = 0, \quad \left(m + \frac{1}{4} \bar{D}_2^2 \bar{\phi} \right) \phi^2 = \phi \Upsilon_{\dot{\alpha}} \Upsilon^{\dot{\alpha}}. \quad (4.32)$$

These constraints were introduced in [21]. Our analysis derives them in the full $\mathcal{N} = 2$ superspace in terms of a deformed short twisted chiral Goldstone multiplet. As discussed in [21] the solution of (4.32) mimics the case of the deformed reduced chiral Goldstone multiplet subject to a cubic nilpotent constraint. The solution includes two branches: (i) one that is identical to the $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking case solving (4.30), and (ii) one that completely breaks supersymmetry in general and determines \mathbf{H}_0 in terms of the following physical degrees of freedom: a scalar, two Goldstini, and a gauge two-form [21]. Note that for the first branch to exist it is necessary to have the mass parameter be nonvanishing, $m \neq 0$. This feature distinguishes the present deformed short twisted chiral model from a nilpotent linear multiplet.

In this paper, we did not describe the component structure of the supergravity-matter theories proposed. These theories can be reduced to components using the results of [59].

ACKNOWLEDGMENTS

We are grateful to Joseph Novak for comments on the manuscript. The work of S. M. K. is supported in part by the Australian Research Council, Grant No. DP160103633. The work of G. T.- M. was supported by the Interuniversity Attraction Poles Program initiated by the Belgian Science Policy (P7/37).

APPENDIX A: REDUCED CHIRAL AND LINEAR MULTIPLETS

It is well known that the field strength of an Abelian vector multiplet is a reduced chiral superfield [60]. In curved superspace, it is a covariantly chiral superfield W ,

$$\bar{D}_i^{\dot{\alpha}} W = 0, \quad (\text{A1a})$$

subject to the Bianchi identity [25,60]

$$(\mathcal{D}^{ij} + 4S^{ij})W = (\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{W}. \quad (\text{A1b})$$

We recall that the $\mathcal{N} = 2$ tensor multiplet is described in curved superspace by its gauge-invariant field strength G^{ij} , which is a linear multiplet. The latter is defined to be a real $\text{SU}(2)$ triplet (that is, $G^{ij} = G^{ji}$ and $\bar{G}_{ij} := \overline{G^{ij}} = G_{ij}$) subject to the covariant constraints [61,62]

$$\mathcal{D}_\alpha^i G^{jk} = \bar{\mathcal{D}}_{\dot{\alpha}}^i G^{jk} = 0. \quad (\text{A2})$$

These constraints are solved in terms of a chiral prepotential Ψ [63–66] via

$$G^{ij} = \frac{1}{4}(\mathcal{D}^{ij} + 4S^{ij})\Psi + \frac{1}{4}(\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{\Psi}, \quad \bar{\mathcal{D}}_{\dot{\alpha}}^i \Psi = 0, \quad (\text{A3})$$

which is invariant under Abelian gauge transformations

$$\delta_\Lambda \Psi = i\Lambda, \quad (\text{A4a})$$

with the gauge parameter Λ being a reduced chiral superfield,

$$\bar{\mathcal{D}}_{\dot{\alpha}}^i \Lambda = 0, \quad (\mathcal{D}^{ij} + 4S^{ij})\Lambda - (\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{\Lambda} = 0. \quad (\text{A4b})$$

The constraints on Λ can be solved in terms of the Mezincescu prepotential [67] (see also [63]), $V_{ij} = V_{ji}$, which is an unconstrained real $\text{SU}(2)$ triplet. The curved-superspace solution is [38]

$$\begin{aligned} \mathcal{H}^{++}(\zeta_A, u) &= h^{ij}(x_A)u_i^+ u_j^+ + 2[\theta^{+\alpha}\psi_\alpha^i(x_A) - \bar{\theta}_\alpha^+ \bar{\psi}^{\dot{\alpha}i}(x_A)]u_i^+ + (\theta^+)^2 M(x_A) + (\bar{\theta}^+)^2 \bar{M}(x_A) \\ &\quad + 2i\theta^+ \sigma^m \bar{\theta}^+ [V_m(x_A) + \partial_m h^{ij}(x_A)u_i^+ u_j^-] + 2i[(\bar{\theta}^+)^2 \theta^{+\alpha} \partial_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha}i}(x_A) + (\theta^+)^2 \bar{\theta}_\alpha^+ \partial^{\alpha\dot{\alpha}} \psi_\alpha^i(x_A)]u_i^- \\ &\quad + (\theta^+)^4 \square h^{ij}(x_A)u_i^- u_j^-. \end{aligned} \quad (\text{B3})$$

$$\Lambda = \frac{1}{4}\bar{\Delta}(\mathcal{D}^{ij} + 4S^{ij})V_{ij}. \quad (\text{A5})$$

Here $\bar{\Delta}$ denotes the chiral projection operator [68]

$$\begin{aligned} \bar{\Delta} &= \frac{1}{96}((\bar{\mathcal{D}}^{ij} + 16\bar{S}^{ij})\bar{\mathcal{D}}_{ij} - (\bar{\mathcal{D}}^{\dot{\alpha}\dot{\beta}} - 16\bar{Y}^{\dot{\alpha}\dot{\beta}})\bar{\mathcal{D}}_{\dot{\alpha}\dot{\beta}}) \\ &= \frac{1}{96}(\bar{\mathcal{D}}_{ij}(\bar{\mathcal{D}}^{ij} + 16\bar{S}^{ij}) - \bar{\mathcal{D}}_{\dot{\alpha}\dot{\beta}}(\bar{\mathcal{D}}^{\dot{\alpha}\dot{\beta}} - 16\bar{Y}^{\dot{\alpha}\dot{\beta}})), \end{aligned} \quad (\text{A6})$$

with $\bar{\mathcal{D}}^{\dot{\alpha}\dot{\beta}} := \bar{\mathcal{D}}_k^{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\beta}k}$. Its main properties can be formulated using a super-Weyl inert scalar U . It holds that

$$\bar{\mathcal{D}}_i^{\dot{\alpha}} \bar{\Delta} U = 0, \quad (\text{A7a})$$

$$\delta_\sigma U = 0 \Rightarrow \delta_\sigma \bar{\Delta} U = 2\sigma \bar{\Delta} U, \quad (\text{A7b})$$

$$\int d^4x d^4\theta d^4\bar{\theta} E U = \int d^4x d^4\theta \mathcal{E} \bar{\Delta} U, \quad (\text{A7c})$$

where σ is the real unconstrained parameter of a super-Weyl transformation [24,25]. The detailed derivation of (A7c) is given in [69].

APPENDIX B: SOLVING THE NILPOTENCY CONDITION (3.2)

In this appendix we show how to solve the nilpotency condition (3.2) in Minkowski superspace. We make use of the harmonic superspace techniques [47,48].

Associated with the linear superfield $\mathcal{H}^{ij}(z)$ constrained by (3.1) is the harmonic superfield $\mathcal{H}^{++}(z, u) := \mathcal{H}^{ij}(z)u_i^+ u_j^+$, which is analytic and short,

$$D_\alpha^+ \mathcal{H}^{++} = 0, \quad \bar{D}_{\dot{\alpha}}^+ \mathcal{H}^{++} = 0, \quad (\text{B1a})$$

$$D^{++} \mathcal{H}^{++} = 0. \quad (\text{B1b})$$

The analyticity constraints mean that \mathcal{H}^{++} lives on the analytic subspace of the harmonic superspace parametrized by $\zeta_A \equiv \{x_A^m, \theta^{+\alpha}, \bar{\theta}_\alpha^+\}$ and u_i^\pm . Here the variables

$$x_A^m = x^m - 2i\theta^{(i} \sigma^m \bar{\theta}^{j)} u_i^+ u_j^-, \quad \theta_\alpha^\pm = u_i^\pm \theta_\alpha^i, \quad \bar{\theta}_\alpha^\pm = u_i^\pm \bar{\theta}_\alpha^i \quad (\text{B2})$$

correspond to the analytic basis of the harmonic superspace.

In the analytic basis, the general expression for \mathcal{H}^{++} was given in [48]. It is

Here $h_{ij} = \overline{h^{ij}}$, $\overline{\psi}_{\dot{\alpha}i} \equiv \overline{\psi^i_{\dot{\alpha}}}$, and V^m is a real conserved vector,

$$\partial_m V^m = 0, \quad (\text{B4})$$

which allows us to interpret V^m as the Hodge dual of the field strength of a gauge two-form. Here one should keep in mind that the operator $D^{++} = u^{+i}\partial/\partial u^{-i}$ in the analytic basis takes the form

$$D_A^{++} = u^{+i} \frac{\partial}{\partial u^{-i}} - 2i\theta^+ \sigma^m \bar{\theta}^+ \frac{\partial}{\partial x_A^m} + \dots, \quad (\text{B5})$$

where the ellipsis denotes two additional terms that do not contribute when acting on analytic superfields.

In the harmonic superspace setting, the nilpotency condition takes the form

$$(\mathcal{H}^{++})^3 = 0. \quad (\text{B6})$$

At the component level, this condition is equivalent to the following equations,

$$0 = (h^{++})^3, \quad (\text{B7a})$$

$$0 = (h^{++})^2 \psi_{\dot{\alpha}}^+, \quad (\text{B7b})$$

$$0 = h^{++} (h^{++} M - 2(\psi^+)^2), \quad (\text{B7c})$$

$$0 = h^{++} (ih^{++} (V_m + \partial_m h^{+-}) + 2\psi^+ \sigma_m \bar{\psi}^+), \quad (\text{B7d})$$

$$0 = h^{++} (M \bar{\psi}_{\dot{\alpha}}^+ + i(V_m + \partial_m h^{+-}) (\psi^+ \sigma^m)_{\dot{\alpha}} + \frac{i}{2} h^{++} (\partial_m \psi^- \sigma^m)_{\dot{\alpha}}) - (\psi^+)^2 \bar{\psi}_{\dot{\alpha}}^+, \quad (\text{B7e})$$

$$\begin{aligned} 0 = h^{++} & \left(M \bar{M} + (V_m + \partial_m h^{+-})^2 - 2i\psi^+ \sigma^m \partial_m \bar{\psi}^- \right. \\ & \left. - 2i\partial_m \psi^- \sigma^m \bar{\psi}^+ + \frac{1}{2} h^{++} \square h^{--} \right) \\ & - (\psi^+)^2 \bar{M} - (\bar{\psi}^+)^2 M - 2i(V_m + \partial_m h^{+-}) \psi^+ \sigma^m \bar{\psi}^+, \end{aligned} \quad (\text{B7f})$$

where we have introduced $h^{\pm\pm} := h^{ij} u_i^{\pm} u_j^{\pm}$ and $\psi_{\dot{\alpha}}^{\pm} := \psi_{\dot{\alpha}}^i u_i^{\pm}$. Equations (B7) are solved by

$$h^{ij} = \frac{\psi^{(i} \psi^{j)} \bar{M} + \bar{\psi}^{(i} \bar{\psi}^{j)} M + 2i\psi^{(i} \sigma_m \bar{\psi}^{j)} V^m}{M \bar{M} + V^n V_n} + \dots, \quad (\text{B8})$$

where the ellipsis denotes all terms with derivatives of the fields. It is assumed that the complex auxiliary field M is nowhere vanishing, $M \neq 0$, and the allowed values of the field strength V^m are restricted by

$$M \bar{M} + V^n V_n \neq 0. \quad (\text{B9})$$

We present the complete solution elsewhere. However it should be pointed out that h^{ij} vanishes if the Goldstini are switched off,

$$\psi_{\dot{\alpha}}^i = 0 \Rightarrow h^{ij} = 0. \quad (\text{B10})$$

Indeed, in the case $\psi_{\dot{\alpha}}^i = 0$ Eq. (B7c) reduces to $h^{++} h^{++} M = 0$. This implies $h^{ij} = 0$ if the components of h^{ij} are ordinary complex numbers, as a consequence of the identity $h^{ij} = iq^{(i} \bar{q}^{j)}$, for some $\text{SU}(2)$ spinor q^i and its conjugate $\bar{q}_i = \overline{q^i}$.

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Correction: The second sentence after Eq. (2.4) contained an error in wording and has been corrected.