Corpuscular slow-roll inflation

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We show that a corpuscular description of gravity can lead to an inflationary scenario similar to Starobinsky's model without requiring the introduction of the inflaton field. All relevant properties are determined by the number of gravitons in the cosmological condensate or, equivalently, by their Compton length. In particular, the relation between the Hubble parameter H and its time derivative $\dot{H}$ required by cosmic microwave background observations at the end of inflation, as well as the (minimum) initial value of the slow-roll parameter, are naturally obtained from the Compton size of the condensate.

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I. INTRODUCTION

The inflationary scenario in cosmology was introduced by Starobinsky [1] and Guth [2] in the early 1980s in order to explain the homogeneity and flatness of our Universe. The new inflationary scenario was later proposed by Linde [3], and Albrecht and Steinhardt [4], in which the accelerated expansion was driven by a scalar field (the *inflaton*) slowly rolling down a plateau of the potential toward the minimum. If the plateau is sufficiently flat, the process lasts long enough to solve the cosmological problems mentioned above. Moreover, the inflationary model based on the inflaton can be formally mapped into a $f(R)$ (modified) theory of gravity (see, e.g., Ref. [5]). Nowadays, this scenario has become, almost unanimously, accepted as part of the standard model of the cosmos and one case that appears particularly favored by present observations [6,7] is precisely Starobinsky's model [1].

Most models of inflation make use of the semiclassical approximation, in which the (background) metric is classical. However, we are not guaranteed that this approximation is not missing relevant quantum properties of gravity in the early Universe [8]. In this regard, the classical geometry of space-time could as well be conceived as an emerging property of a coherent state describing a large number of gravitons, in close analogy to photons in a laser beam. A peculiar feature of gravity is the attractive graviton-graviton interaction, which allows for their collapse and formation of Bose-Einstein condensates. In Ref. [9], it was conjectured that this picture can reliably describe the physics inside a black hole, which is in turn considered a compact quantum system on the verge of a phase transition. Even when the gravitational regime is strong, the setup is nicely understood as a Newtonian theory of N gravitons, which are loosely confined in a "potential well" that is the size of their Compton wavelength λ and interact with an effective gravitational coupling $\alpha \sim 1/N$. As a result, it is possible to recover the correct post-Newtonian expansion of the gravitational field generated by a static, spherically symmetric source [10] or the renowned Bekenstein-Hawking area law [11] with logarithmic corrections [12] for the Hawking radiation. On the other hand, this framework represents a natural scenario for a cosmological model of inflation [8,13], whose characteristic quantities display quantum properties related to the corpuscular nature of gravity. It will also appear that this description can help to constrain possible modified metric theories of gravity [14], therefore proving to be an interesting benchmark.

In this work we shall show that the corpuscular description of gravity can reproduce the inflationary expansion, purely as a consequence of the graviton self-interaction. Unlike what was considered in Refs. [8,13], the primordial cosmological condensate can give rise to the dynamics of Starobinsky's model [1], without requiring the introduction of an inflaton.

II. CORPUSCULAR COSMOLOGY

Let us start from the assumption that matter and the corpuscular state of gravitons together must reproduce the Friedmann equation of cosmology, which we write as the Hamiltonian constraint

$$\mathcal{H}_{\mathrm{M}} + \mathcal{H}_{\mathrm{G}} = 0, \tag{1}$$

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where $\mathcal{H}_{\rm M}$ is the matter energy and $\mathcal{H}_{\rm G}$ is the analogue quantity for the graviton state. We recall that local (Newton or Einstein) gravity being attractive in general implies that $\mathcal{H}_{\rm G} \le 0$, although this is not true for the graviton self-interaction [10], and might not be true for the cosmological condensate of gravitons as a whole, as we are now going to discuss.

A. Corpuscular de Sitter

In order to obtain the de Sitter space-time in general relativity, one must assume the existence of a cosmological constant term, or vacuum energy density ρ_Λ, so that the Friedmann equation reads[1]

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8}{3}\pi G_{\rm N}\rho_\Lambda. \tag{2}$$

Upon integrating on the volume inside the Hubble radius that solves Eq. (2), that is $L_\Lambda = H_\Lambda^{-1}$, we obtain[2]

$$L_\Lambda \simeq G_{\rm N} L_\Lambda^3 \rho_\Lambda \simeq \ell_{\rm p}\frac{M_\Lambda}{m_{\rm p}}, \tag{3}$$

which looks exactly like the expression of the horizon radius for a black hole of Arnowitt-Deser-Misner mass M_Λ, and is the reason it was conjectured that the de Sitter space-time could likewise be viewed as a condensate of gravitons [8].

One can roughly describe the corpuscular model by assuming that the (soft virtual) graviton self-interaction gives rise to a condensate of N_Λ gravitons of typical Compton length $\lambda \simeq L_\Lambda$, so that $M_\Lambda = N_\Lambda \ell_{\rm p} m_{\rm p}/L_\Lambda$, and the usual consistency condition

$$M_\Lambda \sim \sqrt{N_\Lambda}\, m_{\rm p} \tag{4}$$

for the graviton condensate immediately follows from Eq. (3). Equivalently, one finds

$$L_\Lambda \sim \sqrt{N_\Lambda}\,\ell_{\rm p}, \tag{5}$$

which shows that for a macroscopic universe one needs $N_\Lambda \gg 1$. Note also that we have $\rho_\Lambda \sim L_\Lambda^{-3} M_\Lambda \sim 1/N_\Lambda$, so that the number of gravitons in the vacuum increases for smaller vacuum energy, and $L_\Lambda \sim M_\Lambda \sim 1/\sqrt{\rho_\Lambda}$. It is important to remark that the above relations do not need to hold for gravitons that are not in the condensate, and therefore one expects deviations to occur if regular matter is added [15], or if the system is driven out of equilibrium.

We can refine the above corpuscular description of the de Sitter space by following the line of reasoning of Ref. [10], where it was shown that the maximal packing condition which yields the scaling relations (5) for a black hole actually follows from the energy balance (1) when matter becomes totally negligible. In the present case, matter is absent *a priori* and $\mathcal{H}_{\rm M} = 0$, so that one is left with

$$\mathcal{H}_{\rm G} \simeq U_{\rm N} + U_{\rm PN} = 0. \tag{6}$$

The negative "Newtonian energy" of the N_Λ gravitons can be obtained from a coherent state description of the condensate [10] in which each graviton has negative binding energy ε_Λ given by the Compton relation, that is,

$$U_{\rm N} \simeq M_\Lambda \phi_{\rm N} = N_\Lambda \varepsilon_\Lambda = -N_\Lambda \frac{\ell_{\rm p} m_{\rm p}}{L_\Lambda}. \tag{7}$$

The positive "post-Newtonian" contribution is then given by the graviton self-interaction term [10]

$$U_{\rm PN} \simeq N_\Lambda \varepsilon_\Lambda \phi_{\rm N} = N_\Lambda^{3/2}\frac{\ell_{\rm p}^2 m_{\rm p}}{L_\Lambda^2}, \tag{8}$$

where we used the Newtonian potential

$$\phi_{\rm N} = -\frac{N_\Lambda \ell_{\rm p} m_{\rm p}}{M_\Lambda L_\Lambda} = -\sqrt{N_\Lambda}\frac{\ell_{\rm p}}{L_\Lambda}, \tag{9}$$

as follows from Eq. (7) and the scaling relation (4).

B. Metric de Sitter

Before we consider explicit ways of perturbing the de Sitter solution, let us try to reinterpret our results in terms of a metric theory. We have just seen that the de Sitter universe is, in a sense, a solution of our Hamiltonian constraint and we also know that the de Sitter metric is an exact solution of a modified theory of gravity [14]

$$S = \frac{1}{16\pi G_{\rm N}}\int {\rm d}^4x\sqrt{-g}f(R), \tag{10}$$

with [16,17]

$$f(R) = \gamma \ell_{\rm p}^2 R^2, \tag{11}$$

where γ is a dimensionless constant. We recall that the equation of motion following from Eq. (10) for a spatially flat Friedmann-Lemaître-Robertson-Walker metric,

$${\rm d}s^2 = -{\rm d}t^2 + a^2(t)({\rm d}r^2 + r^2 {\rm d}\Omega^2), \tag{12}$$

is given by [5,16]

$$6f'(R)H^2 = Rf'(R) - f(R) - 6H\dot R f''(R), \tag{13}$$

[1]We shall use units with $c = 1$ and the Newton constant $G_{\rm N} = \ell_{\rm p}/m_{\rm p}$, where $\ell_{\rm p}$ and $m_{\rm p}$ are the Planck length and mass, respectively, and $\hbar = \ell_{\rm p} m_{\rm p}$.

[2]Factors of order one will often be omitted from now on.

where primes denote derivatives with respect to R and dots derivatives with respect to the cosmic time t. In particular, from Eq. (11), one obtains

$$12RH^2 = R^2 - 12H\dot{R}, \tag{14}$$

and, for de Sitter with $a(t) = e^{\sqrt{\Lambda/3}t}$, one has

$$R = 6\left(H^2 + \frac{\ddot{a}}{a}\right) = 6\left(H^2 + \frac{\Lambda}{3}\right), \tag{15}$$

and

$$\dot{R} = 6\left(2H\dot{H} + \frac{\dddot{a}\,a - \ddot{a}\,\dot{a}}{a^2}\right) = 12H\dot{H}. \tag{16}$$

By inserting the above expressions into Eq. (14), we simply obtain

$$H^2 = \frac{\Lambda}{3} - \frac{4H^2\dot{H}}{H^2 + \Lambda/3}, \tag{17}$$

which is solved by

$$H_\Lambda^2 = \Lambda/3, \tag{18}$$

and $R = 4\Lambda$ as expected.

Upon comparing with the corpuscular description, we can therefore say that, up to a common numerical factor,

$$U_{\rm N} \simeq -L_\Lambda^3 H_\Lambda^2 = -L_\Lambda, \tag{19}$$

and

$$U_{\rm PN} \simeq L_\Lambda^3(\Lambda/3) = L_\Lambda, \tag{20}$$

where we recall that $U_{\rm N}$ and $U_{\rm PN}$ follow from integrating over the Hubble volume. This will be our starting point to build a connection between the corpuscular model and Starobinsky's inflation [1].

III. SLOW-ROLL INFLATION

The "post-Newtonian" analysis of the graviton condensate has shown that one can have an eternally inflating universe without the need for vacuum (matter) energy. Of course, one next needs a source that drives the universe out of inflation. Unlike the analysis in Ref. [13], this contribution may just be a small perturbation with respect to the post-Newtonian term (8) which breaks the balance with the Newtonian term (7).

A. Starobinsky model

By means of a conformal transformation given by [14]

$$\tilde{g}_{\mu\nu} = f'(R) g_{\mu\nu}, \tag{21}$$

we can rewrite the action (10) in the Einstein frame as

$$\tilde{S} = \int d^4x\sqrt{-\tilde{g}}\left[\frac{\tilde{R}}{16\pi G_{\rm N}} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right], \tag{22}$$

where the Ricci scalar of the original metric now appears as a new scalar field

$$\varphi \equiv \sqrt{\frac{3}{16\pi G_{\rm N}}}\ln f'(R) \tag{23}$$

with the potential

$$V(\varphi) \equiv \frac{f'(R(\varphi))R(\varphi) - f(R(\varphi))}{16\pi G_{\rm N} f'(R(\varphi))^2}. \tag{24}$$

This shows that, unless f' is a constant, the original metric $g_{\mu\nu}$ contains two massless degrees of freedom, corresponding to the helicity-2 gravitons of the metric $\tilde{g}_{\mu\nu}$, and a spin-0 degree of freedom φ associated with the trace of its Ricci tensor (see Ref. [17] for more details).

In particular, for

$$f(R) = \alpha R + \gamma\ell_{\rm p}^2 R^2, \tag{25}$$

one finds

$$\varphi = \sqrt{\frac{3m_{\rm p}}{16\pi\ell_{\rm p}}}\ln(\alpha + 2\gamma\ell_{\rm p}^2 R), \tag{26}$$

from which we also deduce that

$$R(\varphi) = \frac{\exp\left(\sqrt{\frac{16\pi\ell_{\rm p}}{3m_{\rm p}}}\varphi\right) - \alpha}{2\gamma\ell_{\rm p}^2}, \tag{27}$$

and then

$$V(\varphi;\alpha,\gamma) = \frac{m_{\rm p}}{64\pi\ell_{\rm p}^3\gamma}\left[1 - \alpha\exp\left(-\sqrt{\frac{16\pi\ell_{\rm p}}{3m_{\rm p}}}\varphi\right)\right]^2,$$

which is precisely Starobinsky's potential for the inflaton [1] (see Fig. 1).

This potential has a minimum for

$$\varphi = \frac{3m_{\rm p}\ln\alpha}{16\pi\ell_{\rm p}}, \tag{28}$$

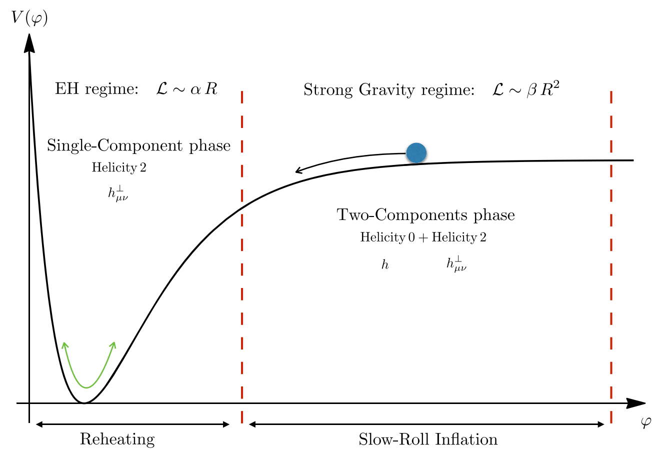

FIG. 1. Starobinsky's potential for the inflaton.

and $\lim_{\varphi\to\infty} V(\varphi;\alpha,\gamma) = \frac{m_p}{64\pi\ell_p\gamma} = \lim_{\alpha\to 0} V(\varphi;\alpha,\gamma)$. For $\alpha = 0$, one again recovers the de Sitter space with $R = 4\Lambda$ and a correspondingly constant scalar field φ. As soon as $\alpha > 0$, this configuration becomes unstable, as can also be inferred from the equation of motion (13), which now reads

$$6\frac{\alpha}{\gamma\ell_p^2}H^2 + 12RH^2 = R^2 - 12H\dot{R}. \tag{29}$$

By assuming that the solution to the above equation is still of the de Sitter form, with a time-dependent Hubble function $H(t) \simeq H_\Lambda = \Lambda/3$, we then obtain

$$\dot{H} \simeq -\frac{\alpha}{\gamma\ell_p^2}, \tag{30}$$

and the Hubble function is then slowly decreasing, as we expected, for $0 < \alpha/\gamma \ll 1$. In particular, the slow-roll parameter is given by

$$\epsilon = -\frac{\dot{H}}{H^2} \sim \frac{m_p}{\ell_p}\left(\frac{V'}{V}\right)^2, \tag{31}$$

and is very small along the plateau of the potential (see Fig. 1). On the other hand, at the end of inflation, when the slow-roll parameter $\epsilon \sim 1$, one infers from the cosmic microwave background data [6] that $\gamma/\alpha \simeq 10^8 \simeq N_\Lambda$, and

$$\dot{H} \simeq -L_\Lambda^{-2}. \tag{32}$$

This is precisely the relation that the corpuscular description naturally yields, which we will now show.

B. Corpuscular model

In an ideal de Sitter universe, gravitons should satisfy the balance condition (6). Let us rewrite the Hamiltonian in Eq. (6) as

$$\mathcal{H}_G^{(2)} \simeq \beta(U_N + U_{PN}), \tag{33}$$

corresponding to the effective metric action (10) with Eq. (11). Note that we introduced the dimensionless parameter $\beta > 0$ of order one, in order to keep track of this contribution. The complete dynamics of our Universe, however, must also include a term corresponding to the Einstein-Hilbert action, that is,

$$\mathcal{H}_G^{(1)} \simeq \alpha U_N, \tag{34}$$

where $\alpha > 0$ can here be viewed as the same parameter of the metric counterpart (25). The full energy balance is therefore given by

$$\mathcal{H}_G^{(1)} + \mathcal{H}_G^{(2)} \simeq (\alpha+\beta)U_N + \beta U_{PN} = 0, \tag{35}$$

and, because of the term proportional to α, we expect that the expressions (19) and (20) for the ideal de Sitter condensate are no longer a solution.

In fact, we are interested in a stage when departures from the de Sitter scalings are small, and we can therefore assume that the potentials now take the slightly more general form

$$U_N \simeq -L^3 H^2 \tag{36}$$

and

$$U_{PN} \simeq L^3 L_\Lambda^{-2}, \tag{37}$$

where $L \sim L_\Lambda$ is the new Hubble radius. Upon inserting this into Eq. (35), we obtain

$$L^3[-(\alpha+\beta)H^2 + \beta L_\Lambda^{-2}] \simeq 0, \tag{38}$$

which is solved by

$$H^2 \simeq \frac{\beta}{\alpha+\beta}\frac{1}{L_\Lambda^2}. \tag{39}$$

Of course, the de Sitter case is properly recovered when $\alpha = 0$, but $\alpha > 0$ implies that $H < H_\Lambda$, as expected. If the system starts with $H = H_\Lambda$, the time derivative $\dot{H}$ must be negative in order to ensure that the constraint (35) holds at all times. This can be explicitly seen by writing

$$H = H_\Lambda + \dot{H}\delta t, \tag{40}$$

where the typical time scale $\delta t \simeq L_\Lambda$ (since gravitons of Compton length L_Λ cannot be sensitive to shorter times). Equation (38) finally yields

$$\dot{H} \simeq -\frac{\alpha}{\alpha+\beta}\frac{H_\Lambda}{\delta t} \simeq -\frac{\alpha}{\alpha+\beta}\frac{1}{L_\Lambda^2}. \tag{41}$$

We can further notice that the slow-roll parameter

$$\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{\alpha}{\alpha+\beta} \tag{42}$$

in the corpuscular model, and one therefore obtains Eq. (32) with the natural choice $\alpha/(\alpha+\beta) \simeq 1$.

Having recovered the prediction of Starobinsky's model at the end of inflation, we can then assume that α and β are proportional to the fraction of gravitons in the condensate whose dynamics is mostly affected by the Hamiltonian $\mathcal{H}_G^{(1)}$ in Eq. (34) and $\mathcal{H}_G^{(2)}$ in Eq. (33), respectively. At the beginning of inflation most of the N_Λ gravitons are in the de Sitter condensate and just interact via the term $\mathcal{H}_G^{(2)} \sim R^2$ (which means $\alpha \ll 1$ and $\beta \simeq 1$), whereas at the end of inflation all of the N_Λ gravitons also interact via the term $\mathcal{H}_G^{(1)} \sim R$, so that $\alpha \sim \beta \sim 1$. In some more detail, gravitons in the condensate generate the effective Hubble expansion parameter $H \sim N_\Lambda^{-1/2} \sim L_\Lambda^{-1}$, but they also scatter and deplete. Their number therefore changes in time according to Eq. (3.23) of Ref. [13], which we can rewrite as

$$-\frac{\dot{H}}{H^2} \simeq \frac{\dot{N}_\Lambda}{N_\Lambda} \simeq \epsilon\left(1 - \frac{1}{\epsilon^{3/2}N_\Lambda}\right), \tag{43}$$

where $\epsilon \sim \alpha$ from Eq. (42); the first term reproduces the background evolution in the slow-roll approximation and the second term is due to the depletion. It is now clear that near the end of inflation, when $\epsilon \sim 1$, the relative effect of depletion becomes of order N_Λ^{-1} and therefore negligibly small. On the other hand, for

$$\epsilon = \epsilon_* \sim N_\Lambda^{-3/2} \sim \left(\frac{\ell_p}{L_\Lambda}\right)^3, \tag{44}$$

one obtains $\dot{N}_\Lambda \simeq 0$, which can be viewed as the closest the corpuscular model can get to the pure de Sitter space (ideally represented by $\epsilon = \alpha = 0$).[3] Equivalently, we deduce that the parameter α will run from the minimum value of order L_Λ^{-3} to the maximum of order one during the inflationary expansion. The minimum value (44) is a peculiar prediction of the corpuscular model for the slow-roll parameter.

C. Physical outcomes

As we have seen, the corpuscular model allows one to recover the background evolution equations of Starobinsky's model with no ambiguous coefficient at the end of inflation, and the minimum value ϵ_* of the slow-roll parameter given in Eq. (44) at the beginning. We therefore expect that the leading-order phenomenology is the same as in Starobinsky's model with initial conditions compatible with Eq. (44). In fact, it was already shown in Ref. [13] that the corpuscular model correctly reproduces the behavior determined by the given background evolution, with corrections of order $N_\Lambda^{-1} \sim L_\Lambda^{-2}$.

From the phenomenological point of view, scalar and tensor perturbations should arise from the depletion of the background condensate. An intriguing implication may concern the production of gravitational waves during the inflationary process. In fact, the dimensionless power spectrum of primordial tensor perturbations $\mathcal{P}_T \sim \ell_p^2 H^2 \sim \ell_p^2/L_\Lambda^2$ [18], will receive corrections from Eq. (43), that is,

$$\frac{\Delta\mathcal{P}_T}{\mathcal{P}_T} \simeq \frac{H\delta H}{H^2} \sim \frac{\dot{H}\delta t}{H} \sim -\epsilon\left(1 - \frac{1}{\epsilon^{3/2}N_\Lambda}\right), \tag{45}$$

where we again used $\delta t \sim L_\Lambda \sim H^{-1}$. This correction is negative and proportional to $\dot{N}_\Lambda/N_\Lambda$: it vanishes at the beginning of inflation, when $\epsilon \simeq \epsilon_*$, and $\Delta\mathcal{P}_T + \mathcal{P}_T \simeq 0$ at the end of inflation, when $\epsilon \sim 1$. One might be therefore tempted to relate this feature to the fact that the condensate cools down, as the universe inflates, and the depletion of helicity-2 modes (almost) stops at the end of inflation. On the other hand, we should remark that the correction (45) is very small at the beginning of inflation, when $\epsilon \simeq \epsilon_*$, and it should not affect the standard phenomenological picture in a drastic way. For instance, the tensor-to-scalar ratio could be estimated from Eq. (5.36) of Ref. [8], and further analyzed as Eq. (4.2) in Ref. [13], where it was again shown that results are very close to the ones obtained from the standard approach to cosmological perturbations. A more quantitative analysis of the expected small corrections is left for future developments.

Similar conclusions should hold for the reheating at the end of inflation, where we again remark that the depletion in Eq. (43) leads to order $1/N_\Lambda$ corrections for the background evolution with respect to Starobinsky's model when $\epsilon \sim 1$. It is however known that the behavior of the reheating phase depends strongly on the specific particle content of the theory [19]. Even in simple models, like chaotic inflation, one finds collective (and therefore nonperturbative) effects, such as parametric resonance and Bose enhancement. The equations of motion of the fields can be rewritten, under certain approximations, as Mathieu's differential equations, and the widths of the resonance bands then depend on the parameters of the theory. For the corpuscular model, the precise setup of such a framework has yet to be properly derived, and a detailed analysis is again left for future developments.

[3]A similar argument was already employed in Ref. [13] to estimate the number of e-foldings.

IV. DISCUSSION AND CONCLUSIONS

We started from the simple corpuscular description of the de Sitter universe viewed as a condensate of N_Λ self-interacting (scalar) gravitons of Compton wavelength equal to $L_\Lambda \simeq H_\Lambda^{-1}$, as first proposed in Ref. [8]. We then noticed that a refined, and yet equivalent description can be obtained from the Hamiltonian constraint with "Newtonian" and "post-Newtonian" energy terms [10]. Since the de Sitter metric is an exact solution of the modified $f(R) \simeq R^2$ theory of gravity, we inferred that it should also be possible to reinterpret this quantum state in terms of an effective metric theory of this form. Moreover, this theory is equivalent to the usual Einstein-Hilbert gravity with the addition of a scalar field (replacing the trace of the Ricci scalar), and therefore contains one more degree of freedom (of helicity 0) than the Einstein theory (which contains two helicity-2 modes). In the pure de Sitter space, we hence expect that all degrees of freedom are in an equilibrium state solely characterized by the length scale L_Λ.

Of course, any realistic model of inflation requires a departure from (eternal) de Sitter, which can be achieved by adding the Einstein-Hilbert term R to the previous $f(R) \simeq R^2$ theory in the metric description. Since the corpuscular description of the pure Einstein gravity is just given by the "Newtonian" term, this is equivalent to introducing such an extra term that pushes the de Sitter gravitons off equilibrium. We have seen that this mechanism is compatible with the Hamiltonian constraint and, indeed, it appears that the length scale L_Λ naturally fixes the size of $\dot{H}$ at the end of inflation to the one required by experimental data, as well as the value (44) of the slow-roll parameter at the beginning of inflation. To summarize, the corpuscular model of inflation contains one scale L_Λ from which the main dynamical features of the inflationary background can be extracted.

We conclude by mentioning that it was recently shown in Ref. [15] how the same corpuscular description of the quantum state of the Universe can also explain the observed galaxy rotation curves without the need for dark matter. A more detailed quantitative analysis of both inflation and dark matter phenomenology are part of future developments.

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