Vacuum polarization in Siklos spacetimes

Morteza Mohseni[*]

Physics Department, Payame Noor University, 19395-3697 Tehran, Iran

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We study the effect of one-loop vacuum polarization on photon propagation in Siklos spacetimes in the geometric optics limit. We show that for photons with a general polarization in the transverse plane, the quantum correction vanishes in spacetimes with $H_{xy} = 0$. For photons polarized along a transverse axis, subluminal and superluminal solutions are admitted for certain subclasses of Siklos spacetimes. We investigate the results in the Kaigorodov and Defrise spacetimes and obtain explicit expressions for the phase velocities. In Kaigorodov spacetime with $H \sim x^3$, photons polarized along the x axis are subluminal in regions where H is positive and superluminal in regions where H is negative, while photons polarized along the y axis are superluminal in $H > 0$ regions and subluminal in $H < 0$ regions. In the Defrise spacetime, $H \sim x^{-2}$, x-polarized and y-polarized photons are superluminal for $H < 0$, and subluminal for $H > 0$. We comment on motion in other Siklos spacetimes.

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I. INTRODUCTION

The interplay between gravitational and quantum effects has been a topic of huge interest in recent decades, but in spite of massive efforts, there is no theory fully reconciling these effects yet. Still, one can study quantum effects in gravitational systems in certain contexts. Within this framework, [1] have investigated the QED contribution to the photon effective action from one-loop vacuum polarization in curved backgrounds. The calculation showed that the photon propagation gets altered due to the quantum corrections, and superluminal or subluminal motion could be possible, at least in principle. They also applied the formulation to gravitational wave spacetime (in the weak-field approximation) and also to the Schwarzschild spacetime, and the superluminal-subluminal motion and also the birefringences were shown explicitly. Their theory has then been examined in several spacetimes; namely in Reissner-Nordstrum spacetime in [2], in Kerr spacetime in [3], in dilaton black hole spacetimes in [4], in the static and rotating topological back hole backgrounds in [5], and more recently, in [6], in which by considering the Kerr-de Sitter and static de Sitter cosmic string spacetimes, the effect of a positive cosmological constant was studied. Other aspects of this theory have been discussed in the literature, including the generalization to a high frequency limit and the discussion of various kinds of velocity in [7], the issue of causality violation in [8,9], and the problem of superluminality in [10]. A review of the subject may be found in [11]. Also, it has been shown in [12] that including the quantum terms (but with arbitrary coefficients) in the electromagnetic Lagrangian breaks the conformal invariance of the action, and this could be responsible in producing sizable magnetic fields during inflation.

In the present work, we investigate photon propagation in Siklos spacetimes. These spacetimes, first introduced in [13], may be considered as exact gravitational waves propagating in an anti–de Sitter universe [14,15]. A particular Siklos spacetime, the Kaigorodov spacetime [16], has been of interest particularly in the context of AdS/CFT correspondence [17,18]. Some other aspects of the Kaigorodov spacetime have been investigated in the literature, see, e.g. [19] and the references therein. Although the current observations favor a positive cosmological constant, models with negative cosmological constant are still of interest and appear in different contexts including BTZ black holes [20], and string theory and supersymmetry [21]. Plane gravitational and electromagnetic fields in spaces with a cosmological constant have also been studied in [22].

The paper is organized in the following order. We begin with a brief review of the electromagnetic field equations in a general curved spacetime in the limit of geometric optics and also collect the relevant equations for the one-loop vacuum polarization. Then, in Sec. III, we solve the equations of vacuum polarization for photons propagating in a general Siklos spacetime. We obtain expressions for k^2 and show that solutions with positive, zero, or negative values are admitted. We also compute the phase velocities. In Sec. IV, we consider the particular case of propagation in a background Kaigorodov spacetime and show that depending on the photon polarization and the spacetime region on which photons are moving, both subluminal or superluminal photon propagation are possible. In Sec. V, we take the contribution of photons to the background spacetime into account, which

[*]m-mohseni@pnu.ac.ir

is achieved by considering the Defrise spacetime. In Sec. VI, we consider propagation in other Siklos spacetimes. We conclude the paper with a summary of the results. We use the natural units with the Lorentz-Heaviside units for electromagnetic fields, the metric signature $(-+++)$, and the following convention for the Riemann tensor $R^{\mu}{}_{\nu\rho\sigma} = \partial_\rho \Gamma^{\mu}_{\nu\sigma} + \Gamma^{\mu}_{\rho\kappa}\Gamma^{\kappa}_{\nu\sigma} - \{\rho \leftrightarrow \sigma\}$.

II. THE VACUUM POLARIZATION

The electromagnetic action

$$S_0 = \frac{-1}{4}\int d^4x\sqrt{-g}F_{\mu\nu}F^{\mu\nu} \tag{1}$$

gives the Maxwell equations for free fields

$$D_\mu F^{\mu\nu} = 0, \tag{2}$$

or, in terms of the field A^μ (subject to the Lorentz condition $D_\mu A^\mu = 0$),

$$D_\nu D^\nu A^\mu = R^{\mu}_{\nu}A^\nu, \tag{3}$$

$R_{\mu\nu}$ being the Ricci tensor. In the limit of geometric optics, the solution to Eq. (3) is given by

$$A^\mu = (a^\mu + i\varepsilon b^\mu + \cdots)\exp\left(\frac{i\varphi}{\varepsilon}\right). \tag{4}$$

Here, it is assumed that the wavelength λ is small compared with the length scale of the spacetime curvature, L_0. In fact, one may take $\varepsilon = O(\frac{\lambda}{L_0})$. See, e.g. [23], for more details and a pedagogical review of the geometric optics limit in curved spacetime.

To the leading order, Eqs. (3) and (4) result in $k_\mu k^\mu = 0$, where $k_\mu = \partial_\mu \varphi$. Thus, the integral curve of k^μ is null. Also, the Lorentz condition implies that the polarizations are transverse

$$k_\mu a^\mu = 0. \tag{5}$$

The next-to-leading order terms give

$$k^\nu D_\nu a^\mu = -\frac{1}{2}a^\mu D_\nu k^\nu, \tag{6}$$

and the Lorentz condition leads to

$$D_\mu a^\mu = k_\mu b^\mu. \tag{7}$$

The one-loop corrected action is given by $S = S_0 + S_1$, where

$$S_1 = \frac{1}{m_e^2}\int d^4x\sqrt{-g}(aRF_{\mu\nu}F^{\mu\nu} + bR_{\mu\nu}F^{\mu}{}_{\kappa}F^{\nu\kappa} + cR_{\mu\nu\kappa\lambda}F^{\mu\nu}F^{\kappa\lambda} + dD_\mu F^{\mu\nu}D_\kappa F^{\kappa}{}_{\nu}) \tag{8}$$

in which $a = \frac{\alpha}{144\pi}$, $b = -\frac{13\alpha}{360\pi}$, $c = \frac{\alpha}{360\pi}$, $d = \frac{e^2}{120\pi^2}$, α being the fine structure constant, and m_e is the electron mass. It should be noted that the above action is in fact a truncated form of the QED effective field theory action. The full action contains other terms including some curvature-independent and some UV divergent ones. Omitting such terms from the action restricts the range of energies over which it is valid. A discussion of this may be found in [10].

Including S_1 (with the last term being neglected) into the action results in the following field equation [24]:

$$D_\mu F^{\mu\nu} = \frac{2}{m_e^2}D_\mu Q^{\mu\nu}, \tag{9}$$

where $Q^{\mu\nu} = 2aRF^{\mu\nu} + b(R^{\mu}_{\rho}F^{\rho\nu} - R^{\nu}_{\rho}F^{\rho\mu}) + 2cR^{\mu\nu\kappa\lambda}F_{\kappa\lambda}$.

In the special case of a maximally symmetric spacetime, where $R_{\mu\nu\kappa\lambda} = P(g_{\mu\kappa}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\kappa})$, this reduces to

$$\left(1 + \frac{7P\alpha}{90\pi m_e^2}\right)D_\mu F^{\mu\nu} = 0, \tag{10}$$

and the vacuum polarization does not affect the propagation. For Ricci-flat spacetimes, we have from Eq. (9)

$$D_\mu F^{\mu\nu} - \frac{\alpha}{90\pi m_e^2}R^{\mu\nu}{}_{\kappa\lambda}D_\mu F^{\kappa\lambda} = 0. \tag{11}$$

Equation (9) can be expanded into the following form:

$$\left(1 - \frac{4aR}{m_e^2}\right)D_\mu F^{\mu\nu} - \frac{2b}{m_e^2}(R^{\mu}_{\sigma}D_\mu F^{\sigma\nu} - R^{\nu}_{\sigma}D_\mu F^{\sigma\mu}) - \frac{4c}{m_e^2}R^{\mu\nu}{}_{\sigma\tau}D_\mu F^{\sigma\tau} + \frac{2b+8c}{m_e^2}F^{\sigma\mu}D_\mu R^{\nu}_{\sigma} - \frac{4a+b}{m_e^2}F^{\mu\nu}D_\mu R = 0, \tag{12}$$

which by using Eqs. (4) and (5), to $O(\frac{1}{\varepsilon^2})$ gives

$$\left(-\left(1 - \frac{4aR}{m_e^2}\right)k_\mu k^\mu + \frac{2b}{m_e^2}R^{\mu}_{\sigma}k_\mu k^\sigma\right)a^\nu - \frac{2b}{m_e^2}R^{\mu}_{\sigma}k_\mu a^\sigma k^\nu + \frac{2b}{m_e^2}k_\mu k^\mu a^\sigma R^{\nu}_{\sigma} - \frac{8c}{m_e^2}R^{\mu\nu}{}_{\sigma\tau}k_\mu k^\tau a^\sigma = 0. \tag{13}$$

The general equation, in which higher order terms in the expansion are also included, involve derivative of a^μ and k^μ. To the leading order which Eq. (13) is based on, such terms are absent.

III. THE SIKLOS SPACETIMES

In the chart (u, x, y, v) with u, v being the light cone coordinates, the Siklos spacetimes metric is described by

$$ds^2 = \frac{-3}{\Lambda x^2}(H(u,x,y)du^2 - dudv + dx^2 + dy^2) \quad (14)$$

in which $\Lambda < 0$ is the cosmological constant. Inserting this into the Einstein equation with a cosmological constant, $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$, results in

$$H_{xx} + H_{yy} = \frac{2}{x}H_x, \quad (15)$$

where subscripts represent differentiation.

For photons propagating along the z axis with $k^\mu = (A, 0, 0, B)$, we take $a^\mu = (0, C, F, 0)$ in which C, F are constants. This is consistent with the condition

$$k_\mu a^\mu = 0. \quad (16)$$

Inserting the above data into Eq. (13) results in (for $\mu = x$, y, respectively) the following two equations:

$$\frac{A}{m_e^2 \Lambda x^2}[A\Lambda(CK + Fcx^2 H_{xy}) + C(AH - B)N] = 0 \quad (17)$$

$$\frac{A}{m_e^2 \Lambda x^2}[A\Lambda(FL + Ccx^2 H_{xy}) + F(AH - B)N] = 0, \quad (18)$$

where

$$K \equiv 4c(x^2 H_{xx} - xH_x) + E,$$
$$L \equiv 4c(x^2 H_{yy} - xH_x) + E,$$
$$N \equiv (48a + 12b + 8c)\Lambda - 3m_e^2,$$
$$E \equiv bx^2\left(H_{xx} + H_{yy} - \frac{2}{x}H_x\right).$$

For H satisfying Eq. (15), we have $E = 0$. The above equations admit the trivial solution $A = 0$, which corresponds to $k^2 = 0$.

Now if $H_{xy} = 0$, $C \neq 0$, and $F \neq 0$, Eqs. (17) and (18) are inconsistent unless we take $A = 0$ (except for the particular case where $K = L$ for which A can be nonzero. This particular condition is satisfied for Siklos spacetimes with $H_{xx} = H_{yy}$). However, if we further take either $F = 0$, $C \neq 0$ or $C = 0$, $F \neq 0$, then the above system of equations can be satisfied with $A \neq 0$.

If we take $H_{xy} = 0$, with $F = 0$, $C \neq 0$, Eq. (18) is automatically satisfied and from Eq. (17), we get

$$\frac{3A^2 K}{Nx^2} = -\frac{3A}{\Lambda x^2}(AH - B), \quad (19)$$

where $K = c(x^2 H_{xx} - xH_x) + E$. From Eq. (19), one can easily read off the value of $k^2 = \frac{-3A}{\Lambda x^2}(AH - B)$. Thus,

$$k^2 = \frac{3A^2 K}{Nx^2}. \quad (20)$$

The phase velocity of photons can be obtained from $v_p = \frac{\omega}{|\vec{k}|}$. To compute this, we first perform the coordinates transformation

$$du = \frac{1}{\sqrt{h(h+H)}}(hdt - dz), \quad (21)$$

$$dv = \frac{1}{\sqrt{h(h+H)}}(dt + hdz), \quad (22)$$

in which $h \equiv \sqrt{H^2 + 1} - H$. This brings the metric to the following form:

$$ds^2 = \frac{-3}{\Lambda x^2}\left(-hdt^2 + \frac{1}{h}dz^2 + dx^2 + dy^2\right). \quad (23)$$

Thus, we obtain

$$\omega = \sqrt{\frac{-3}{4\Lambda x^2(h+H)}}|hA + B|$$
$$= \sqrt{\frac{-3}{4\Lambda x^2(h+H)}}|A|\left|h + H + \frac{\Lambda K}{N}\right|, \quad (24)$$

and

$$|\vec{k}| = \sqrt{\frac{-3}{4\Lambda x^2 h^2(h+H)}}|A - Bh|$$
$$= \sqrt{\frac{-3}{4\Lambda x^2(h+H)}}|A|\left|h + H - \frac{\Lambda K}{N}\right|. \quad (25)$$

We therefore obtain

$$v_p = \left|\frac{N(h+H) + \Lambda K}{N(h+H) - \Lambda K}\right|. \quad (26)$$

Similarly, if we take $C = 0$, $F \neq 0$, then Eq. (17) is automatically satisfied, and from Eq. (18), we get the same expressions as above but with K replaced by $L = c(x^2 H_{yy} - xH_x) + E$.

In the case, where $H_{xy} \neq 0$, Eqs. (17) and (18) are satisfied with $A \neq 0$ by choosing particular $\frac{F}{C}$ ratios. Here, the solutions are given by the roots of

$$\det\begin{pmatrix} A\Lambda K + N(AH-B) & A\Lambda cx^2 H_{xy} \\ A\Lambda cx^2 H_{xy} & A\Lambda L + N(AH-B) \end{pmatrix} = 0. \tag{27}$$

These are given by $B - AH = \frac{\Lambda A}{N} W_\pm$, where $W_\pm = (1+\frac{2c}{b})E \pm cx^2\sqrt{4(H_{xx}-H_{yy})^2 + H_{xy}^2}$ from which we obtain

$$k^2 = \frac{3A^2}{Nx^2} W_\pm \tag{28}$$

corresponding to $\frac{F}{C} = \frac{W_\pm - K}{cx^2 H_{xy}}$, respectively.

IV. MOTION IN KAIGORODOV SPACETIME

A particular solution of Eq. (15), which in addition satisfies $H_{xy} = 0$, is given by $H = \sigma x^3$ in which σ is a constant. With this choice, the metric (14) reduces to

$$ds^2 = \frac{-3}{\Lambda x^2}(\sigma x^3 du^2 - dudv + dx^2 + dy^2). \tag{29}$$

This describes the Kaigorodov space-time [16] in Siklos horospherical-type coordinates (or more formally, Fefferman-Graham coordinates, see, e.g. [25]). This metric can be obtained from the following one:

$$ds^2 = \pm e^{-lr} dX^2 + e^{2lr}(-dXdT + dY^2) + dr^2 \tag{30}$$

by imposing the coordinate transformation $x = \pm e^{-lr}$, $u = lX$, $v = lT$, $y = lY$, where $l = \sqrt{\frac{-\Lambda}{3}}$ [19]. The minus and plus signs correspond to $x > 0$ and $x < 0$ regions, respectively, and σ is regarded as unity for simplicity. The positive and negative x regions are disjointed, and $x = 0$ represents the null infinity.

Now, noting that for this metric we have $E = 0$, Eq. (20) gives

$$k^2 = -\frac{9\beta A^2 \sigma}{7\Lambda(3+\beta)} x, \tag{31}$$

in which $\beta = \frac{7\alpha\Lambda}{90\pi m_e^2}$. Also, Eq. (26) results in

$$v_p = \left| \frac{(3\sigma x^3 - 7\sqrt{\sigma^2 x^6 + 1})\beta - 21\sqrt{\sigma^2 x^6 + 1}}{(3\sigma x^3 + 7\sqrt{\sigma^2 x^6 + 1})\beta + 21\sqrt{\sigma^2 x^6 + 1}} \right| \tag{32}$$

or, for $\beta \ll 1$,

$$v_p = 1 + \beta \frac{6\sigma x^3}{21\sqrt{\sigma^2 x^6 + 1}}. \tag{33}$$

Similarly, for $C = 0$, $F \neq 0$, we obtain

$$k^2 = \frac{9\beta A^2 \sigma}{7\Lambda(3+\beta)} x, \tag{34}$$

and

$$v_p = \left| \frac{(3\sigma x^3 + 7\sqrt{\sigma^2 x^6 + 1})\beta + 21\sqrt{\sigma^2 x^6 + 1}}{(3\sigma x^3 - 7\sqrt{\sigma^2 x^6 + 1})\beta - 21\sqrt{\sigma^2 x^6 + 1}} \right| \tag{35}$$

or

$$v_p = 1 - \beta \frac{6\sigma x^3}{21\sqrt{\sigma^2 x^6 + 1}}, \tag{36}$$

respectively. In both cases, we have $v_p \to 1$ as $x \to 0$.

Now, noting that $m_e = 5.1 \times 10^5$ eV and $|\Lambda| = 4.6 \times 10^{-66}$ eV^2 (corresponding to the experimental value $+1.19 \times 10^{-52}$ m^{-2}), we have $\beta = -3.2 \times 10^{-81}$. Thus, for the case $F = 0$, Eq. (31) gives $\mathrm{sign}(k^2) = \mathrm{sign}(-\sigma x)$ which, if we assume $\sigma > 0$, corresponds to subluminal photons in $x > 0$ region and superluminal photons in $x < 0$ region. Similarly, for the case $C = 0$, Eq. (34) gives $\mathrm{sign}(k^2) = \mathrm{sign}(\sigma x)$, which shows superluminal photons in the $x > 0$ region and subluminal photons in the $x < 0$ region. These are also confirmed explicitly by Eqs. (33) and (36). One can obtain the reverse situation by choosing $\sigma < 0$.

For the spacetime described by the metric (29), the curvature length scale is of the order of $L_0 \sim (-\Lambda)^{-1/2} \sim 10^{33}$ eV^{-1}, which is very large compared to the Compton wavelength $\lambda_c \sim 10^{-5}$ eV^{-1}. Thus, the requirement $L_0 \gg \lambda_c$ ([1]) for the validity of the one-loop calculations is well satisfied. On the other hand, we should also have $-k^2 < 4m_e^2$ (see, e.g. [26]). Thus, from Eqs. (31) or (34), we obtain $A < \sqrt{|\frac{28\Lambda}{3\beta\sigma x}|} m_e$ or equivalently, $A < \sqrt{\frac{120\pi}{\alpha|\sigma x|}} m_e^2$. This puts an upper bound on the value of A.

V. MOTION IN DEFRISE SPACETIME

In the previous section, we investigated the photon propagation in a particular Siklos spacetime, the Kaigorodov spacetime, in which the source of spacetime curvature is only the cosmological constant. This implies that the contribution of photons to the energy-momentum tensor is neglected. There are other subclasses of Siklos solutions in which such contributions can be accounted for. In particular, the Defrise spacetime [27] is obtained when in addition to the cosmological constant, there is a pure radiation field with the energy-momentum tensor $T_{\mu\nu} = \rho k_\mu k_\nu$, with ρ being a constant [28]. The Defrise metric is described by Eq. (14) by setting $H = -\delta x^{-2}$ in which δ is constant. It can be obtained by taking $\rho = \frac{5\Lambda^2\delta}{18\pi G}$ and $k^\mu = (0, 0, 0, 1)$, corresponding to massless photons propagating in the z direction.

In this spacetime, we have $E \neq 0$, and with the choice $F = 0$, $C \neq 0$, we have from Eq. (20)

$$k^2 = \frac{54\beta A^2 \delta}{7(3+\beta)\Lambda x^4}, \tag{37}$$

and Eq. (26) results in

$$v_p = \left| \frac{(18\delta + 7\sqrt{x^4+\delta^2})\beta + 21\sqrt{x^4+\delta^2}}{(18\delta - 7\sqrt{x^4+\delta^2})\beta - 21\sqrt{x^4+\delta^2}} \right| \tag{38}$$

or

$$v_p = 1 - \beta \frac{12\delta}{7\sqrt{x^4+\delta^2}} \tag{39}$$

showing subluminal motion for $\delta < 0$ and superluminal motion for $\delta > 0$. Similarly, with the choice $C = 0$, $F \neq 0$, we obtain

$$k^2 = \frac{36\beta A^2 \delta}{7(3+\beta)\Lambda x^4}, \tag{40}$$

and

$$v_p = \left| \frac{(12\delta + 7\sqrt{x^4+\delta^2})\beta + 21\sqrt{x^4+\delta^2}}{(12\delta - 7\sqrt{x^4+\delta^2})\beta - 21\sqrt{x^4+\delta^2}} \right| \tag{41}$$

or

$$v_p = 1 - \beta \frac{8\delta}{7\sqrt{x^4+\delta^2}}, \tag{42}$$

which shows again subluminal motion for $\delta < 0$ and superluminal motion for $\delta > 0$. In both cases, we have $v_p \to 1$ as $x \to \infty$.

VI. MOTION IN OTHER SIKLOS SPACETIMES

It has been shown in [13] that Eq. (15) admits a general solution of the following form:

$$H(u,x,y) = x^2 \frac{\partial}{\partial x}\left(\frac{f(\zeta,u) + \bar{f}(\bar{\zeta},u)}{x}\right), \tag{43}$$

where f is an arbitrary function and $\zeta = x + iy$. Thus, for example, choosing $f(\zeta,u) = \frac{1}{4}\zeta^3$ reproduces the Kaigorodov metric. It is possible to apply the formulation given in Sec. III to various subsets of the above general wave profile. The procedure is straightforward, but the results depend on the explicit form of the wave profile. As an interesting example, for a generalized Kaigorodov metric with $H(u,x,y) = w(u)x^3$ in which $w(u)$ is arbitrary, Eq. (20) reduces to

$$k^2 = \pm \frac{9A^2\beta}{7\Lambda(3+\beta)} x w(u), \tag{44}$$

and the behavior depends on $w(u)$. On the other hand, there are other solutions to Eq. (15), such as $H(u,x,y) = w(u)x^3 + s(u)y$ in which $s(u)$ is arbitrary, which give the same results as Eq. (44). Another profile of this kind is $H(u,x,y) = q(u)(x^2+y^2)$, where $q(u)$ is also arbitrary. Interestingly, the later (up to a conformal transformation) is also the wave profile of an exact gravitational wave produced by a light wave in an otherwise empty spacetime [29], (see also [30]). It is also possible to generalize the Defrise metric by $H(u,x,u) = j(u)x^{-2}$, $j(u)$ being arbitrary, which would result in different behavior compared to the ones discussed in Sec. V.

An example of Siklos spacetimes with $H_{xy} \neq 0$ is $H(u,x,y) = x^3 + \frac{1}{3}y^3 + x^2 y$. For this spacetime, Eq. (28) gives

$$k^2 = \frac{\mp 3\sqrt{37}\beta A^2}{14\Lambda(\beta+3)} x \tag{45}$$

representing both superluminal or subluminal propagation.

VII. CONCLUSIONS

We studied the effect of one-loop correction of photon vacuum polarization on photon propagation in Siklos spacetimes in the geometric optics limit. In Siklos spacetimes with $H_{xy} = 0$, for photons with nonzero polarization in both the x, y directions, the quantum correction vanishes. In Kaigorodov spacetime, $H \sim x^3$, we showed that in addition to usual massless photons, there exists a solution for which photons polarized along the x axis are superluminal in the $H < 0$ regions and subluminal in the $H > 0$ regions, while photons polarized along the y axis are subluminal in the $H < 0$ regions and superluminal in the $H > 0$ regions. Thus, the phenomenon of birefringence is shown to exhibit in Kaigorodov and some other subclasses of Siklos spacetimes. The deviation from the standard speed of light are tiny, of the order of $\frac{\alpha\Lambda}{m_e^2}$. In Defrise spacetime, $H \sim x^{-2}$, photons polarized either along the x axis or the y axis are superluminal in $H < 0$ regions and subluminal in $H > 0$ regions. For the class of Siklos spacetimes with off diagonal terms in the wave profile, $H_{xy} \neq 0$, superluminal/subluminal propagation is possible with an arbitrary polarization in the transverse plane.

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