Jeans instability in a universe with dissipation

Gilberto M. Kremer,[1,*] Martín G. Richarte,[1,2,†] and Felipe Teston[1,‡]

[1]*Departamento de Física, Universidade Federal do Paraná,*
Caixa Postal 19044, 81531-990 Curitiba, Brazil
[2]*Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires,*
Ciudad Universitaria 1428, Pabellón I, Buenos Aires, Argentina

The problem of Jeans gravitational instability is investigated for static and expanding universes within the context of the five and thirteen field theories which account for viscous and thermal effects. For the five-field theory a general dispersion relation has been derived with the help of relevant linearized perturbation equations, showing that the shear viscosity parameter alters the propagating modes for large and small wavelengths. The behavior of density and temperature contrasts are analyzed for the hard-sphere model in detail. In the small wavelengths regime, increasing the amount of shear viscosity into the system forces the harmonic perturbations to damp faster, however, in the opposite limit larger values of shear viscosity lead to smaller values of density and temperature contrasts. We also consider the hyperbolic case associated with the thirteen-field theory which involves two related parameters, namely the shear viscosity and the collision frequency, the last one is due to the production terms which appear in the Grad method. The dispersion relation becomes a polynomial in the frequency with two orders higher in relation to the five-field theory, indicating that the effects associated with the shear viscosity and heat flux are nontrivial. The profile of Jeans mass in terms of the temperature and number density is explored by contrasting with several data of molecular clouds. Regarding the dynamical evolution of the density, temperature, stress and heat flux contrasts for a universe dominated by pressureless matter, we obtain also damped harmonic waves for small wavelengths. In the case of large wavelengths, the density and temperature contrasts grow with time (due to the Jeans mechanism) while the stress and heat flux contrasts heavily decay with time. For an expanding universe, the Jeans mass and Jeans length are obtained and their physical consequences are explored.

I. INTRODUCTION

The current view of the universe seems to indicate that it is vastly uniform and homogeneous at large-scale for redshifts larger than $z \simeq 10^3$ [1,2]. However, the existence of inhomogeneities such as galaxies and clusters of galaxies requires a physical mechanism to account them for. One way to address the study of structure formation is by considering the process associated with the aggregation of matter in a form of cosmic substratum (or fluid), which permeate the whole universe. From some initial seeds, possibly generated during a primordial inflationary era, one must explore how gravity forces to grow into lumpy structures on small scale that one observes today, namely a clustered distribution of galaxies or super-cluster of galaxies at low redshifts ($z \ll 1$) [2].

First attempts to examine the existence of instability for a fluid within the context of collapsing astrophysical bumps were initiated by Sir Jeans in 1912 [3,4]. He started his analysis by assuming a static universe and found that the dispersion relation associated with the perturbed contrast density could admit, apart from harmonic perturbations, two kinds of propagating modes: a growing mode and a decaying one. To be more precise, he noticed the existence of a physical cutoff, called Jeans' wavelength, such that perturbations with wavelength shorter than Jeans scale will not grow whereas in the opposite case perturbations may grow exponentially in time. A useful manner to understand the gravitational instability associated with the Jeans mechanism is by using the following reasoning: consider a given mass M enclosed in a spherical volume of radius λ in which there exists a mass density inhomogeneity. This inhomogeneity will grow if the gravity force F_G per unit mass is greater than the opposed pressure force F_P per unit of mass, i.e.,

$$F_G = \frac{GM}{\lambda^2} \propto \frac{G\rho\lambda^3}{\lambda^2} = G\rho\lambda > F_P \propto \frac{p\lambda^2}{\rho\lambda^3} \propto \frac{v_s^2}{\lambda},$$

where one used $v_s^2 \propto (p/\rho)$. G stands for the Newton gravitational constant, ρ represents the density, and λ is a wavelength. From dimensional analysis, one shows that the

*kremer@fisica.ufpr.br
†martin@df.uba.ar
‡felipe@fisica.ufpr.br

Jeans wavelength can be written in terms of the Jeans wave-number as $\lambda_J = 2\pi/k_J = 2\pi v_s/\sqrt{4\pi G\rho}$ being $\lambda = 2\pi/k$; then instability arises if the condition $k < k_J$ holds. Equivalently, the Jeans instability can be stated as follows: the timescale associated with the pressure exerted over a region with matter must be bigger than the timescale needed to start the gravitational collapse of the matter due to its own weight, namely $t_{\text{pressure}} = (\lambda/v_s) > t_{\text{gravity}} = (G\rho)^{-1/2}$.

The situation of an expanding universe is considerably different provided the expansion tries to counterbalance the effect of gravity. As a positive density bump is trying to collapse due to its self-gravity the expansion of the universe is trying to pulling it apart [5]. In fact, there are two elements which can help to prevent the formation of structure, which are the usual pressure effects and the fast background expansion. Therefore, the growth rate of matter in an expanding universe tends to be slower than in one with no expansion, evolving as power law in time in the latter case. The Jeans treatment for exploring the evolution of density contrast is just limited to the linear regime, so as the amplitude of density fluctuations approach to the nonlinear stage all the propagating modes do not evolve independently. Nevertheless, Jeans stability analysis seems to be the first method to understand the inclusion of secondary effects in the agglomeration of matter for the nonexpanding case and expanding scenario within the framework of first-order Newtonian dynamics. Some authors devoted several efforts to explore the Jeans instability mechanism within the context of Newtonian cosmology where the universe is dominated by a pressureless matter. For instance, the effect of including a nonvanishing bulk viscosity along with the analysis of the top-down fragmentation process were taken into account in Ref. [6]. In doing so, they compared one collapsing agglomerate with $M \gg M_J$ and an internal noncollapsing substructure with $M < M_J$, M_J being the Jeans mass [7].

The case of Jeans instability criterion for a compressible fluid including viscosity and heat conduction was analyzed many years ago [8]. At high redshifts associated with the recombination epoch ($z \simeq 1000$) the role played by the shear viscosity would be negligible while the bulk viscosity would be important [8]. In addition, the gravitational instability for a fluid which supports viscoelastic stresses was addressed by Janaki *et al.*, showing that quadratic corrections appear in the dispersion relation due to the presence of both shear viscosity and bulk viscosity [9].

On the other hand, the analysis of Jeans instability from the point of view of kinetic theory was also examined in the literature. In such treatment, Trigger *et al.* obtained the dispersion relation for the Boltzmann-Vlasov operator coupled to Poisson equation. The authors showed that a nonvanishing collisional term led to substantial modifications to the unstable propagating modes [10]. The gravitational instability for a collisionless self-gravitating system composed of one or two components described by the coupled collisionless Boltzmann and Poisson equation were analyzed as a first step to understand the cooperative effects of baryons and dark matter in process of structure formation [11,12]. For instance, a system composed of baryons and dark matter lead to a total Jeans mass which is smaller than the one associated with a single component, pointing out that less amount of mass is needed to ignite the collapsing process. One could expect that for bumps with masses greater the Jeans mass initiate the collapsing process but an overdense region in an expanding universe eventually recollapses and virialises. In the case of a single component with an expanding background it turned out that the "swindle" proposal may be avoided while the Jeans instability is expected to arise in the limit of large wavelengths [11]. Leaving general relativity aside, some authors devoted several efforts to explore the Jeans instability within the framework of $f(R)$ gravity or modified gravity theories [13–17]. In particular, Capozziello *et al.* studied the collapse of self-gravitating system composed of neutral particles which is characterized by a coupled of collisionless Boltzmann and Poisson equations in the case of $F(R)$-gravity. Perturbing a background solution and looking at linearized regime (or in the case of weak field approximation) of the aforesaid set of equations, they obtained a modified dispersion relation along with a new kind of an unstable mode. Such propagating mode turned to grow faster than the standard one obtained in GR [16,17].

The aim of this work is to explore the Jeans instability for a static and expanding backgrounds from the phenomenological point of view of a five-field and a thirteen-field theory, so it can be considered as a complementary analysis to the full kinetic theory approach mentioned above. First, we are going to consider a proper fluid-description in terms of a five-field model which is characterized by several fields such as mass density, velocity field, temperature, gravitational potential along with an equation of state to close the coupled Poisson-balance system of equations. In the thirteen-field theory, we are going to deal with a hyperbolic system of equations which include mass density, velocity, pressure tensor, and heat flux vector. For an ideal gas these balance equations can be obtained from the Grad method of kinetic theory [18] or from the so-called extended thermodynamic theory (see [19]). The main idea of such approach is to analyze the evolution of plane waves and obtained the modified dispersion relation for static universe. Then, we are going to solve numerically the dynamical set of equations for inspecting the evolution of contrast density, contrast temperature, contrast heat flux, and contrast shear tensor with time when the expanding universe is dominated by dustlike matter. In doing so, we will explore the impact and effect introduced by the shear viscosity parameter in the evolution of contrast quantities such as density, temperature, heat flux, and shear's stress. In the phenomenological side of our model, we will examine two different kinds of constraints. We will explore for static configurations how the Jeans mass varies with the temperature and number density and compare such outcomes with several data of molecular clouds. In addition,

we will study the dynamical Jeans mass in terms of the redshift.

The outline of this paper is as follows. Section II A is dedicated to analyze the Jeans instability for static universe along with the computation of the dispersion relation (and growing mode) within the five-field theory while in Sec. II B several constraints on the static Jeans mass are obtained in the case of stellar molecular clouds. Section II C is devoted to evaluate the evolution of contrast quantities (namely, mass density, temperature) in terms of the cosmic time. In Sec. II D the dynamical Jeans mass is calculated to estimate the clustering properties of the model. In Sec. III A, the thirteen-field theory is presented, the dispersion relation in the presence of shear viscosity is calculated. In Sec. III B, the dynamical evolution of thirteen fields coupled to the Poisson equation is explored along with the physical behavior of such quantities with the shear viscosity. In Sec. IV, the conclusions are stated.

II. JEANS INSTABILITY FROM A FIVE-FIELD THEORY

The aim of ordinary thermodynamics of fluids is the determination of five fields of mass density ρ, velocity v_i and temperature T in space-time (t, x_i). The equations for the determination of these fields are based on the balance equations of mass, momentum and energy which in the presence of a gravitational potential ϕ read (see, e.g., [18,19])

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_i}{\partial x_i} = 0, \qquad \frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} + \frac{1}{\rho} \frac{\partial p_{ij}}{\partial x_j} + \frac{\partial \phi}{\partial x_i} = 0,$$
$$\frac{\partial \varepsilon}{\partial t} + v_i \frac{\partial \varepsilon}{\partial x_i} + \frac{1}{\rho} \frac{\partial q_i}{\partial x_i} + \frac{p_{ij}}{\rho} \frac{\partial v_i}{\partial x_j} = 0. \quad (1)$$

In the above equations p_{ij} denotes the pressure tensor, ε stands for the specific internal energy, and q_i is the heat flux vector.

The gravitational potential is connected with the mass density through Poisson's equation, namely

$$\nabla^2 \phi = 4\pi G \rho, \qquad (2)$$

where G is the gravitational constant. In order to close the system of equations (1) and (2) constitutive equations for p_{ij}, ε, q_i in terms of the basic fields ρ, v_i, T must be specified. Here we are interested in studying an ideal gas where velocity and temperature gradients are present. According to the thermodynamic theory the constitutive equations for the pressure tensor and heat flux vector are governed by the so-called Navier-Stokes and Fourier laws and their expressions read

$$p_{ij} = p \delta_{ij} - \mu_v \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \frac{\partial v_r}{\partial x_r} \delta_{ij} \right), \qquad (3)$$

$$q_i = -\lambda_c \frac{\partial T}{\partial x_i}, \qquad (4)$$

respectively. Above $p = \rho k T / m$ is the pressure with k denoting the Boltzmann constant and m the mass of a gas particle, μ_v is the shear viscosity and λ_c the thermal conductivity coefficients, respectively. According to the kinetic theory for an ideal nonrelativistic gas [18] the bulk viscosity vanishes and the coefficients of shear viscosity and thermal conductivity are related by the following expression:

$$\frac{\lambda_c}{\mu_v} = \frac{15k}{4m}. \qquad (5)$$

Furthermore, for an ideal gas the specific internal energy is given by $\varepsilon = 3kT/2m$.

In the next subsections, we shall investigate the Jeans instability for a static and expanding Universe by taking into account the above equations.

A. Static Universe

Here we search for plane wave solutions of (1) and (2) supplemented by the constitutive equations (4). At equilibrium the density, temperature and gravitational potential have constant values, namely ρ_0, T_0, ϕ_0 while the velocity vanishes. The equilibrium solution is subjected to perturbations characterized by a Fourier expansion with frequency ω, wavenumber vector k and small amplitudes $\{\delta\rho, \delta v, \delta T, \delta\phi\}$ in relation to unperturbed variables:

$$\rho = \rho_0 + \delta\rho\, e^{i(\mathsf{k}x - \omega t)}, \qquad v_x = \delta v_x e^{i(\mathsf{k}x - \omega t)}, \qquad (6)$$

$$T = T_0 + \delta T e^{i(\mathsf{k}x - \omega t)}, \qquad \phi = \phi_0 + \delta\phi\, e^{i(\mathsf{k}x - \omega t)}. \qquad (7)$$

For simplicity, we have assumed that $\bar{k} = \mathsf{k}\hat{x}$. Further, it is import to remark the fact that the equilibrium values for the fields satisfy the equations (1) but not Poisson equation (2). This inconsistency is removed by considering Jeans "swindle" approximation which states that the Poisson equation is valid only for the perturbations.

Replacing the Fourier expansion modes (6) and (7) together with the constitutive equations (4) into the balance (1) and Poisson (2) equations, one obtains a system of algebraic equations for the amplitudes. This system of algebraic equations has a solution if the determinant associated with the coefficients of the perturbations vanishes and consequently one gets the following dispersion relation

$$\omega \left(\omega + i\frac{4\mu_v}{3\rho_0}\mathsf{k}^2 \right) \left(\omega + i\frac{5\mu_v}{2\rho_0}\mathsf{k}^2 \right) - \frac{2k}{3m} T_0 \omega \mathsf{k}^2$$
$$- \left(\omega + i\frac{5\mu_v}{2\rho_0}\mathsf{k}^2 \right) \left(\frac{k}{m} T_0 \mathsf{k}^2 - 4\pi G \rho_0 \right) = 0, \quad (8)$$

where μ_v stands for the viscosity coefficient which is a function of the temperature [20]. Note that one has to use the relationship between the coefficients of thermal conductivity and shear viscosity (5) in order to recast (8) in its current form.

Now we shall analyze the dispersion relation (8) and for that end we introduce the adiabatic speed of sound v_s, the Jeans wave number k_J and the dimensionless parameters associated with the shear viscosity coefficient μ_*, frequency ω_*, and wave number k_*:

$$v_s = \sqrt{\frac{5k}{3m}T_0}, \qquad k_J = \frac{\sqrt{4\pi G\rho_0}}{v_s}, \qquad (9)$$

$$\mu_* = \frac{\mu_v k_J}{\rho_0 v_s}, \qquad k_* = \frac{k}{k_J} = \frac{\lambda_J}{\lambda}, \qquad \omega_* = \frac{\omega}{v_s k_J}, \qquad (10)$$

where $\lambda_J = 2\pi/k_J$ is the Jeans wavelength.

In terms of the dimensionless quantities (10) Eq. (8) can be rewritten as

$$\omega_*^3 + i\frac{23}{6}\mu_* k_*^2 \omega_*^2 + \left(1 - k_*^2 - \frac{10}{3}\mu_*^2 k_*^4\right)\omega_*$$
$$+ i\frac{5}{2}\mu_* k_*^2\left(1 - \frac{3}{5}k_*^2\right) = 0. \qquad (11)$$

At this point, one must look for an important consistency check which corresponds to the case of a nonviscous and nonthermal conducting gas or equivalently $\mu_* = 0$. In that limit (11) leads to

$$\omega_* = \pm\sqrt{\frac{\lambda_J^2}{\lambda^2} - 1}, \qquad (12)$$

which is the well-known Jean's solution. For small wavelengths $\lambda_J > \lambda$, one finds that ω_* remains a real quantity. Due to the factor $\exp(-i\omega t)$ this perturbations will propagate as harmonic waves in time. For big wavelengths $\lambda_J < \lambda$, ω_* becomes a pure imaginary quantity then the perturbations will grow or decay in time, depending on the $\pm$ sign. Further, the one which grows in time is connected with Jeans instability. Now it is essential to compare the dispersion relations in the case $\mu^* = 0$ and $\mu_* \neq 0$. We check that dimensionless dispersion relation in the presence of dissipative effects is smaller than the dispersion relation with $\mu^* = 0$ [cf. Fig. (1)]. This is physically equivalent to state that the time needed to collapse a mass configuration is larger when the dissipative effects are included.

For large wavelengths $\lambda_J/\lambda < 1$, we can obtain from (11) three different values associated with the dimensionless frequencies that read

$$\omega_{*1} = -i\left[\frac{5\mu_*}{2}\left(\frac{\lambda_J}{\lambda}\right)^2 + \cdots\right], \qquad (13)$$
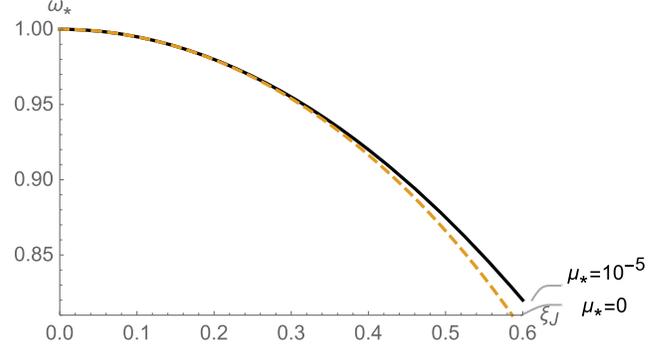
FIG. 1. Behavior of the dimensionless dispersion relations in terms of $\xi_J = \lambda_J/\lambda$ for two different cases, namely, $\mu^* = 0$ and $\mu_* = 10^{-5}$.

$$\omega_{*2} = -i\left[1 - \frac{3 - 4\mu_*}{6}\left(\frac{\lambda_J}{\lambda}\right)^2 - \cdots\right], \qquad (14)$$

$$\omega_{*3} = i\left[1 - \frac{3 + 4\mu_*}{6}\left(\frac{\lambda_J}{\lambda}\right)^2 - \cdots\right]. \qquad (15)$$

These dimensionless frequencies are pure imaginary. In particular, ω_{*1} and ω_{*2} result in decaying perturbations in time while ω_{*3} represents a growing mode. This last refers to Jeans instability which shows a dependence on dissipative phenomena through the dimensionless shear viscosity coefficient μ_*. For small wavelengths $\lambda_J/\lambda > 1$, we obtain two complex conjugate solutions for the dimensionless frequencies which refer to harmonic waves in time and a decaying mode.

B. Stellar structure formation

In this subsection, we are going to explore the impact of viscosity within the context of stellar structure formation as a useful application of the previous analysis. Galaxies have different components such as stars and the interstellar medium. In particular, the interstellar medium contains clouds of cold and ionized gas and dust, like giant molecular clouds, HII regions, Bok globules, etc. These clouds are usually stable, with pressure (due to a finite temperature) balancing self-gravity. In general, the physical processes of a collapsing clouds are well described in the fluid limit, however, it can not take into account some effects due to the particle nature of the interstellar medium that could much be larger if an alternative gravity theory is selected [16]. In order to compare the standard Jeans mass with the case of nonzero viscosity, we must begin by obtaining the Jeans masses in both cases. In the standard case with $\mu_* = 0$, the Jeans mass can be visualized as the minimum mass for an overdensity to collapse. It is defined as the mass contained within a sphere of diameter λ_J, so it reads $M_J^s = (4\pi/3)(\lambda_J/2)^3\rho_0$, where the mass density is ρ_0. We assume that the main component of these clouds is

hydrogen (H_2), so the density is given by $\rho_0 = \mu m_p n$, n being the number density, m_p stands for the proton mass and $\mu = 2.06$ is the mean molecular weight for Hydrogen. For an ideal gas, the squared thermal velocity is proportional to the temperature of the cloud, $v_s^2 \propto kT/m_{H_2}$, so with the help of (10) the Jeans mass can be recast as

$$M_J^s = \frac{\pi^4}{6(\mu m_p)^2 n^{\frac{1}{2}}} \left(\frac{5}{12\pi} \frac{kT}{G} \right)^{\frac{3}{2}}. \tag{16}$$

Equation (16) can be written in solar units as $M_J^s \simeq 3.31 \, T^{\frac{3}{2}} n^{-\frac{1}{2}} M_\odot$, where $T[\mathrm{K}]$ is the numerical value of temperature in Kelvin units and $n[\mathrm{cm}^{-3}]$ the number density in centimeters. At this point some comments are in order. When we are dealing with the structure formation at the astrophysical level there are several mechanisms which we must take into account. For instance, we must include the fragmentation/starvation process, the radiation pressure, stellar winds and so on. Here we are studying an unstable media which collapse gravitationally and how they produce objects with a characteristic mass comparable to the Jeans mass (see [21]). Given the fact that the great majority of stars form via the collapse of cold, gravitationally-unstable, molecular gas, and the subsequent accretion of cold gas onto the protostellar seeds that the collapse produce, it is quite natural to estimate the Jeans mass:

$$M_J^s = 0.5 \left(\frac{n}{10^4 \, \mathrm{cm}^{-3}} \right)^{\frac{1}{2}} \left(\frac{T}{10 \, \mathrm{K}} \right)^{\frac{3}{2}} M_\odot. \tag{17}$$

Equation (17) shows the typical values of the temperature and the number density associated with star-forming regions. The general mechanism of how a gas does fragment into numerous small stars rather than forming a single large one is a really complex process. Nevertheless we can mention the standard fragmentation mechanism. As can be seen from (17), the critical mass is only a function of temperature and density. Increasing density and decreasing temperature lead to lower Jeans masses which, in turn, will lead to fragmentation into smaller cores. For a given initial mass, a molecular cloud (or other astrophysical object) may be formed in spiral density waves and other density perturbations (e.g., caused by the expansion of a supernova remnant or superbubble). What exactly happens during the collapse depends very much on the temperature evolution of the cloud. Initially, the cooling processes (due to molecular and dust radiation) are very efficient. If the cooling time scale is much shorter than free fall time, the collapse is approximately isothermal. As M_J^s decreases, inhomogeneities with mass larger than the actual M_J^s will collapse by themselves with their local free fall time. This fragmentation process will continue as long as the local cooling time is shorter than the local free fall time, producing increasingly smaller collapsing subunits.

Eventually the density of subunits becomes so large that they become optically thick and the evolution becomes adiabatic. As the density has to increase, the evolution will always reach a point when $M = M_J^s$, when a subunit reaches approximately hydrostatic equilibrium. We assume that a stellar object is born [22]. At this point, we should remark the main reason for studying molecular clouds. As is well known, the primary sites for star formation are molecular clouds. These are thought to form out of the remnants of supernova explosions, outflows and winds from stars, and the gas reservoirs in the interstellar and the intergalactic medium. Besides, we should also stress that from a giant molecular cloud it can form a group of stars with their mass distribution being determined by the fragmentation process. The process depends on the physical and chemical properties of the cloud (ambient pressure, magnetic field, rotation, composition, dust fraction, stellar feedback, etc.); that is, much of the fragmentation process is still under study and is not completely understood.

Our aim is to compare the Jeans mass of the interstellar gas with our without dissipative effects. Then, the study of the ionization feedback, the radiation pressure, and stellar wind in process of fragmentation/starvation are beyond the scope of the present work. In the case of nonvanishing viscosity, we use (10) along with the definition of Jeans mass to exhibit how such quantity scales with the viscosity, it reads

$$M_J^\mu = \frac{\pi^4}{3\mu_*^3} \rho_0^{-2} \mu_v^3 v_s^{-3}. \tag{18}$$

To consider the viscosity effects as subleading ones we must choose μ_* considerably small, typically of order 10^{-6} or 10^{-7} otherwise we can obtain unrealistic scenarios. As is well known, the viscosity depends on the temperature of the gas, so we have to select a model for $\mu_v(T)$. We choose the toy model of Maxwell given by $\mu_v = 3.1 \times 10^{-7} T \, [\mathrm{g\,cm}^{-1}\,\mathrm{s}^{-1}\,\mathrm{K}^{-1}]$, where once again we assumed that the gas is entirely composed of hydrogen (see [20]). From the previous choice, we obtain that the Jeans mass behaves as $M_J^\mu \simeq 3.79 \times 10^{-17} T^{\frac{3}{2}} n^{-2} \mu_*^{-3} M_\odot$. Before embark us in the numerical analysis of the Jeans mass in both scenarios, we begin by noting several points. Eqs. (16)–(18) tell us that Jeans mass scales as $T^{3/2}$ so the $M_J - T$ relation for the interstellar medium is the same in both cases. Nevertheless, the behavior with the numerical density is different in both cases and in addition the viscous case contains the parameter μ_*. It is illustrative to consider different astrophysical systems to compare the Jeans mass in these two cases [16].

(i) *Giant molecular clouds* are composed of gas, dust and different substructures extended over a region of 10 pc. They exhibit a high number density 10^2–10^3 cm^{-3} and a very low-temperature, 10 [K]. Their masses vary between $10^4 \, M_\odot$ and $10^6 \, M_\odot$.

(ii) *Cold neutral medium* is entirely composed of neutral hydrogen and exhibits a low number density 30 cm^{-3} and a very low-temperature, 10–100 [K].

(iii) *Warm neutral medium* is composed of neutral hydrogen HI with a temperature of 10^3–10^4 [K] and a low density $n \simeq 0.6$ cm^{-3}.

(iv) *Warm ionized medium* is a considerably large low-density photo-ionized zone with a temperature of 8×10^3 [K], bigger than the Sun's temperature at its surface. It has a lower density than the warm neutral medium, namely $n \simeq 0.1$ cm^{-3}.

(v) *HII regions* are interesting objects in the galaxy provided they basically emit in the radio and IR part of the visible spectrum, then they are easy to explore observationally. They have an average temperature around 10^4 [K] and a typical density in the range of 0.1–10^4 cm^{-3}. The mass of HII regions varies between 10^2 $M_\odot$ and 10^4 $M_\odot$.

(vi) *Hot intercloud region* is composed of ionized gas HII with high temperatures, 10^5–10^6 [K]. Such region is extended over 20 pc with an extremely low density of 0.004 cm^{-3}.

(vii) *Intracluster medium* exhibits high temperatures, 10^7–10^8 [K], a very low density 10^{-3} cm^{-3} and is extended over a a few Mpc.

(viii) *Fermi bubbles* are extended over 10 kpc with a temperature that varies from 10^8 [K] to 10^9 [K], exhibiting a density of 0.01 cm^{-3}. These configurations are located above and below the galactic plane.

As is expected the Jeans mass for a system with a small viscosity gives always greater mass than the case without viscosity. For instance, the smallest difference corresponds to the case of giant molecular clouds, being $M_J^\mu/M_J^s \simeq 11.45$, while the largest disagreement appears for the intracluster medium provided $M_J^\mu/M_J^s \simeq 3.6 \times 10^5$. The previous result can be easily understood by taking into account that the temperature factor is the same in both cases, but the behavior with the number density is stronger for the viscosity case. Further, we also have to add the μ_*^{-3} term, which accounts for a 10^{21} factor in the Jeans mass with viscosity [cf. Table I]. At this point, two comments are in order regarding the high values associated with the Jeans' mass for HIR, ICM and FB [cf. Table I]. Clearly these astrophysical objects are observed in the galaxy; however, the fragmentation process that leads to such configurations endowed with the small values of viscosity seems to be very unlikely. So, we think that the small viscosity condition breaks down for such objects and we need to consider a scenario where the viscosity is extremely high, for instance, recent numerical analysis showed that FBs are well described in terms of high viscosity values [23]. In addition to that, it is possible that we should change the phenomenological law adopted for $\mu_v(T)$ [23].

TABLE I. Jeans mass measured in solar units for different values of temperatures and number densities.

Object	T [K]	n [cm^{-3}]	$M_J^\mu[M_\odot]$	$M_J^s[M_\odot]$
GMC	10	100	120.15	10.49
CNM	80	30	30 207	433.44
WNM	8000	0.6	7.55×10^{10}	3.06×10^6
WIM	8000	0.1	2.71×10^9	7.51×10^6
HII	10^4	0.1	3.79×10^9	1.04×10^7
HIR	10^4	4×10^{-3}	2.37×10^{15}	5.24×10^{10}
ICM	10^7	10^{-3}	1.20×10^{18}	3.31×10^{12}
FB	10^8	10^{-2}	3.79×10^{17}	3.31×10^{13}

In general, we expect that the system with viscosity leads to less effective system regarding the aggregation of mass and the posterior collapse of a gravitational cloud provided the viscosity and conductive work against the structure formation process. To reinforce such idea, we are going to explore another kinds of molecular clouds which are characterized by really low temperature, $T \leq 10$ K and slightly high number density, that is, $n \leq 10^2$ cm^{-3} [cf. Table II]. In doing so, we will use some of the samples mentioned in Capoziello *et al.* [16] which were extracted from Ref. [24].

Interestingly enough, Table II tells us that the largest difference appears in the case of $T = 6.48$ K and $n = 1.54 \times 10^2$ cm^{-3}, which leads to a ratio $M_J^\mu/M_J^s \simeq 5.99$ while the smallest disagreement gives $M_J^\mu/M_J^s \simeq 1.21$. Notice that the last case is associated with larger values of temperature ($T = 8.87$ K) along with larger value in the number density, $n = 4.99 \times 10^2$ cm^{-3}.

Our analysis was focused on the collapse of overdense region with mass near the Jeans mass which are associated with an interstellar medium (forming molecular clouds or

TABLE II. Temperature, number density and Jeans masses with and without the dissipative effects for some molecular clouds reported in [24]. For the sake of brevity, we shortened the name of the above objects.

Object	T [K]	n 10^2 [cm^{-3}]	$M_J^\mu[M_\odot]$	$M_J^s[M_\odot]$
GRSMC G053.59	5.97	1.489	25.30	3.97
GRSMC G049.49	6.48	1.54	26.42	4.41
GRSMC G018.89	6.61	1.58	25.86	4.48
GRSMC G030.49	7.05	1.66	25.81	4.82
GRSMC G035.14	7.11	1.89	25.81	4.82
GRSMC G034.24	7.15	2.04	20.16	4.57
GRSMC G019.94	7.17	2.43	17.45	4.41
GRSMC G038.94	7.35	2.61	12.35	4.08
GRSMC G053.14	7.78	2.67	11.11	4.09
GRSMC G023.24	8.57	3.75	11.56	4.40
GRSMC G019.89	8.64	3.87	6.77	4.29
GRSMC G022.04	8.69	4.41	6.44	4.28
GRSMC G018.89	8.79	4.46	4.96	4.04
GRSMC G023.34	8.87	4.99	4.97	4.09

similar compact objects) and how the viscosity effect can reduce the agglomeration of such matter within the extended thermodynamic framework [19]. Any way, as a general rule we can say that a large collapsing cloud has the tendency to fragment. Gas above the critical mass ($M > M_J^s$) will go into a free fall when there is no significant counterpressure. This is likely, at least initially, since molecular clouds are optically thin so that radiation can easily escape, cooling the cloud, and releasing any pressure. The overdensities will therefore contract at different rates. As the molecular cloud collapses, it will break into smaller and smaller pieces in a hierarchical manner, until the fragments reach their Jeans mass. As the density increases, each of these fragments will become increasingly more opaque and are thus less efficient at radiating away the gravitational potential energy. This raises the temperature of the cloud and inhibits further fragmentation. The fragments might merge with one another if the increasing angular momentum did not prevent such aggregation. Then, compact objects with smaller masses can be formed in the fragmentation process provided other important effects such the ionization feedback, the radiation pressure, and stellar wind must be taken into consideration along with their numerical analysis [22]. Interestingly enough, there are several numerical simulations which show that a massive molecular cloud can be fragmented into several pieces with masses lower than the original Jean mass leading to the creation of stars and other objects [25]. Some authors pointed out that the fragmentation process may stop when the mass of the subunit is nearly $M_\odot$ [22]. However, this criterion is not absolute and there is a debate about the possibility of having compact objects with very low-mass. For instance, it was reported with the help of the microlensing technique the detection of objects in the globular cluster M22 with very low-masses (a fraction of the solar mass), namely $M = 0.13^{+0.03}_{-0.02} M_\odot$ [26].

C. Expanding universe

So far we have handled the case of static universe and analyzed the classical Jeans condition for different astrophysical configurations (mostly stellar clouds) when dissipative effects (such as conduction and viscosity) are included. However, the aforesaid analysis cannot be applicable to cosmology provided the expansion of the universe was not considered; such key element is essential for examining the evolution of density inhomogeneities. Then, let us consider a homogeneous and isotropic spatially flat Universe described by the Friedmann-Lamaître-Robertson-Walker metric $ds^2 = (dt)^2 - a(t)^2(dx^2 + dy^2 + dz^2)$, where $a(t)$ denotes the cosmic scale factor. From Einstein's field equation it follows the so-called Friedmann and acceleration equations, which for a pressureless fluid ($p = 0$) read

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho. \qquad (19)$$

The solution of these equations for the mass density ρ and cosmic scale factor a read:

$$\rho = \rho_{i0}\left(\frac{a_{i0}}{a}\right)^3, \qquad a = a_{i0}(6\pi G\rho_{i0}t^2)^{\frac{1}{3}}, \qquad \rho = \frac{1}{6\pi G t^2}. \qquad (20)$$

We will now see what happens to the density perturbation with time. To do so, we write equations of motion for the cosmological fluid. We will consider only one fluid and volumes of the Universe that are small compared with the Hubble distance, $|r| \ll H^{-1}$ (subhorizon scales in a linear regime), so that the Hubble velocities are small, $v \ll c$. For the above background, we take that the unperturbed fields are given by

$$\rho_0 = \rho_{i0}\left(\frac{a_{i0}}{a}\right)^3, \qquad T_0 = T_{i0}\left(\frac{a_{i0}}{a}\right)^2, \qquad (21)$$

$$v_0^i = \frac{\dot{a}}{a}x_i, \qquad \phi_0 = \frac{2\pi}{3}G\rho_0 r^2. \qquad (22)$$

These equations satisfy (1) and (2) with the constitutive equations (4) identically without the necessity of invoking Jeans swindle approximation. Note that here the subscripts 0 denote the zeroth order (i.e., homogeneous) solution, not the necessarily values today. On the other hand, the subscripts $i0$ stand for some reference values that we can choose upon our needs. In dealing with the perturbations some cautions must be taken. For instance, the operator ∇_r in r coordinates must be replaced somehow by a ∇_x operator in x coordinates. To do so, we use that the comoving coordinates and the physical one are related by the law $x^i = r^i/a(t)$, implying that $\nabla_r = a^{-1}\nabla_x$. Besides, the physical time derivative involves the comoving time derivative plus a contribution that accounts for Hubble flow: $\partial_t|_{r=cte} = \partial_t|_{x=cte} - (v_0^i/a)\partial_i$. The physical reason for pointing out the later fact is the slight subtlety of the noncommutation of $\partial_t|_{r=cte}$ and ∂_r, which it will be essential in arriving at the perturbed field equations. In summary, the time and space derivatives defined from t and r were independent in a static spacetime, however, this is not the case anymore for an expanding spacetime.

At this point, we introduce small perturbations by writing $v^i = v_0^i + \delta v^i$, $\rho = \rho_0 + \delta\rho$, $p = p_0 + \delta p$, and $\phi = \phi_0 + \delta\phi$, where the perturbations are assumed to be small compared with the zeroth order quantities. Furthermore, it is customary to define the symbol δ_X as a contrast density of a physical quantity, that is, $\delta X/X_0$. The perturbed continuity equation (1) and perturbed Poisson equation (2) can be recast as

$$\dot{\delta}_\rho + \nabla_i u^i = 0, \qquad (23)$$

$$\nabla^2 \delta\phi = 4\pi G\rho_0 a^2 \delta_\rho, \qquad (24)$$

where $\delta_\rho \equiv \delta\rho/\rho_0$. We have introduced a new perturbed velocity $u^i = \delta v^i/a$, used the relation $\partial_i v_0^j = H\delta_i^j$ along with the fact that at zeroth order in fluctuations (i.e., dropping all perturbations), the usual continuity equation and Poisson equation for unperturbed fields are recovered. Notice that the Laplacian operator refers to derivatives with respect to the comoving coordinates from now on.

Carrying on, we need to calculate the perturbed balance equation (1) and due to the presence of the pressure tensor term, it seems useful to obtain first its perturbed counterpart. The perturbed pressure tensor then is given by

$$p_{ij} = (p_0 + \delta p)\delta_{ij} - \mu_v \left(\partial_j u_i + \partial_i u_j - \frac{2}{3}\delta_{ij}\partial_k u_k \right). \qquad (25)$$

Leaving aside the viscous contribution in (25), we can express the perturbed pressure term associated with the isotropic contribution by using the equation of state of an ideal gas. We immediately see that pressure perturbations come with two contributions: $\delta p = p_0(\delta_\rho + \delta_T)$. We recognize the adiabatic term provided it is proportional to density perturbation, as result it reads $\delta p^{\text{adiab}} \equiv c_s^2 \delta\rho$, where $c_s^2 = (3/5)v_{\text{thermal}}^2$. Nevertheless, there is an additional nonadiabatic term given by $\delta p^{\text{non-adiab}} \equiv (\delta p/\delta T)\delta T = (k\rho_0/m)\delta_T$. As we said before, we are working on subhorizon scales in a linear regime within the context of Newtonian cosmology which means that we only have one gravitational potential. But, the use of the pressure tensor with the viscosity term (25) within the context of a full relativistic perturbation theory would imply that there are two different gravitational potentials [2], namely Ψ and Φ.

Now, we are in position to compare our approach with previous articles reported in the literature [27,28]. First, we are considering not only nonadiabatic contribution in the perturbed pressure but also the explicit contribution of μ_v-viscosity term (25). On the other hand, some authors introduced viscosity for exploring the behavior of viscous dark matter in neo-Newtonian cosmology in a completely different way. They added a bulk viscous (nonadiabatic) term by hand, namely $p \to p + \Pi_v$ [27]. The lack of a microscopy theory for selecting Π_v allowed them to choose a useful parametrization of the bulk viscosity in terms of the density. Of course, such approach led to nonadiabatic perturbation terms quite differently to the one presented in this work. But most importantly, we preferred to rely on a truncated hierarchy of the full Boltzmann equation provided is a well accepted approach within the context of mechanical statistics (for further detail see Appendix). Probably, our line of work is similar to the one employed by Acquaviva *et al.* because they worked within the

framework of Newtonian cosmology and they considered the presence of dissipative effects by including a bulk viscosity term which corrects the standard pressure term of Euler equation. Moreover, they added a transport equation for Π_v based on the Israel and Steward causal theory [28]. This is equivalent to promote Π_v to the rank of another field of the theory which must be determined dynamically.

Coming back to (25), the perturbed balance equation becomes

$$\dot{u}_i + 2Hu_i + \frac{1}{a^2}\left(\partial_i\delta\phi + \frac{\partial_i\delta p}{\rho_0} - \frac{\mu_v}{\rho_0}\delta\mathcal{W}_i \right) = 0, \qquad (26)$$

where the extra term is

$$\delta\mathcal{W}_i = \partial_{jj}u_i + \partial_{ji}u_j - \frac{2}{3}\partial_{ik}u_k. \qquad (27)$$

We derive the general evolution equation for the perturbation in temperature. Our starting point is to consider $\epsilon = \epsilon_0 + \delta\epsilon$, being the thermal energy density $\epsilon = (3k/2m)T$, so it is possible to express $\delta\epsilon = (3k/2m)\delta T$. The evolution equation for $\delta\epsilon$ or δT has the gradient term in the heat flux and the contraction of the pressure tensor with the $\partial_j v^i$ term. For the gradient term, the perturbations are easy to compute, it yields $\rho^{-1}\nabla_i q^i = -\lambda_c a^{-2}(T_0/\rho_0)\partial_{ii}\delta_T$. For the latter case, we need to combine (25) along with the relation $\partial_j v^i = a^{-1}(\partial_j v_{0i} + \partial_j \delta v_i)$. Putting all these facts together leads us

$$\dot{\delta}_T - \frac{5}{2}\frac{\mu(a)}{\rho_0 a^2}\partial_{ii}\delta_T + \frac{2}{3}\partial_i u_i = 0. \qquad (28)$$

To obtain (28), we used that $\dot{T}_0 = -2HT_0$, $\dot{\delta T} \equiv \dot{T}_0\delta_T + T_0\dot{\delta}_T$, and the relation (5).

Combining the time derivative of (23) with the divergence of (26), we find

$$\ddot{\delta}_\rho + 2H\dot{\delta}_\rho - \frac{4\mu_v}{3a^2\rho_0}\nabla^2\dot{\delta}_\rho = \frac{\nabla^2}{a^2}(\delta\phi + \delta p). \qquad (29)$$

Expanding the perturbations in Fourier modes as $\delta_Y = \sum_{\mathsf{q}}\delta_{Y_{(\mathsf{q})}}e^{i\mathsf{q}x}$ the Laplacian transforms as $\nabla^2 \to -\mathsf{q}^2$, which in turn implies that the master equation (29) can be written as

$$\ddot{\delta}_{\rho(\mathsf{q})} + \left(2H + \frac{4\mu_v\mathsf{q}^2}{3a^2\rho_0} \right)\dot{\delta}_{\rho(\mathsf{q})}$$

$$+ \left(\frac{\mathsf{q}^2 p_0}{a^2\rho_0} - 4\pi G\rho_0 \right)\delta_{\rho(\mathsf{q})} = S_{\mathsf{q}}. \qquad (30)$$

Here the source term corresponds to temperature perturbation, $S_{\mathsf{q}} = (\mathsf{q}^2 p_0/a^2\rho_0)\delta_{T(\mathsf{q})}$. Equation (30) tells us that

dissipative effects enters into different ways. The viscosity alters the usual friction term proportional to $2H$ and the nonadiabaticity pressure term appears as a source for the master equation above. We must emphasize that the physical Jeans scale (length or wave-length) is obtained by demanding that the term $\propto \delta_{\rho(\mathsf{q})}$ vanishes. Such scale seems to be sensitive to the equation of state of the gas and consequently it will depend on the thermal velocity of the gas particles.

Before presenting the results of our numerical simulations, we would like to summarize all the master equations expanded in Fourier modes. Using Eqs. (4), (23), (24), and (26), we can arrange the whole five-field system of equations as

$$\dot{\delta}_{\rho(\mathsf{q})} = -i\mathsf{q}u_{(\mathsf{q})}, \tag{31}$$

$$\dot{\delta}_{T(\mathsf{q})} + \frac{5}{2}\frac{\mu_v(a)\mathsf{q}^2}{\rho_0 a^2}\delta_{T(\mathsf{q})} = \frac{2}{3}\dot{\delta}_{\rho(\mathsf{q})}, \tag{32}$$

$$\delta\phi_{(\mathsf{q})} = -\frac{4\pi G\rho_0 a^2}{\mathsf{q}^2}\delta_{\rho(\mathsf{q})}, \tag{33}$$

$$\dot{u}_{(\mathsf{q})} + 2Hu_{(\mathsf{q})} + \frac{4\mathsf{q}^2\mu_v}{3\rho_0 a^2}u_{(\mathsf{q})} + \frac{i\mathsf{q}}{a^2}\delta\phi_{(\mathsf{q})}$$
$$+ \frac{i\mathsf{q}}{a^2}\frac{p_0}{\rho_0}[\delta_{T(\mathsf{q})} + \delta_{\rho(\mathsf{q})}] = 0. \tag{34}$$

For practical purposes, we must distinguish between two kinds of (physical) perturbed fields. On the one hand, we have dynamical perturbed fields denoted by $\delta_{\rho(\mathsf{q})}$ and $\delta_{T(\mathsf{q})}$ and other derived variables which are associated with some constraint equations, in our model they are represented by $u_{(\mathsf{q})}$ and $\delta\phi_{(\mathsf{q})}$. The reason for performing such observation is that $\delta\phi_{(\mathsf{q})}$ can be obtained by simply knowing $\delta_{\rho(\mathsf{q})}$ through Eq. (33). The same goes for $u_{(\mathsf{q})}$ provided it can be obtained from (31) by calculating the time derivative of $\delta_{\rho(\mathsf{q})}$. Taking into account the remaining equations, we obtained the following system of equations for the determination of $\delta_{\rho(\mathsf{q})}$ and $\delta_{T(\mathsf{q})}$:

$$\ddot{\delta}_{\rho(\mathsf{q})} + \left[2\frac{\dot{a}}{a} + \frac{4\mu_v}{3\rho_0}\frac{\mathsf{q}^2}{a^2}\right]\dot{\delta}_{\rho(\mathsf{q})} + \omega_{\mathsf{q}}^2\delta_{\rho(\mathsf{q})} = S_{(\mathsf{q})}, \tag{35}$$

$$\dot{\delta}_{T(\mathsf{q})} + \frac{5\mu_v}{2\rho_0}\frac{\mathsf{q}^2}{a^2}\delta_{T(\mathsf{q})} = \frac{2}{3}\dot{\delta}_{\rho(\mathsf{q})}. \tag{36}$$

The squared frequency is recast as $\omega_{\mathsf{q}}^2 = [kT_0\mathsf{q}^2/ma^2 - 4\pi G\rho_0]$ and the source term yields $S_{(\mathsf{q})} = -(kT_0\mathsf{q}^2/ma^2)\delta_{T(\mathsf{q})}$. The next step is to select a consistent law for the shear viscosity. To do so, we have to recall that according to the kinetic theory of gases the shear viscosity coefficient for an ideal gas is a function only of the

temperature, i.e., $\mu_v \propto T^\alpha$, where the exponent of the temperature may range from $1/2 \leq \alpha \leq 1$; the value $1/2$ is for hard spheres and 1 for soft spheres (Maxwellian molecules) (see, e.g., [18]). Hence, from (22) we may write the shear viscosity coefficient in terms of the cosmic scale factor as

$$\frac{\mu_v}{\mu_{i0}} = \left(\frac{T_0}{T_{i0}}\right)^\alpha = \left(\frac{a_{i0}}{a}\right)^{2\alpha}. \tag{37}$$

We introduce the adiabatic sound speed v_s along with the wavelengths λ_0 and λ_J which are defined by the following expressions:

$$v_s = \sqrt{\frac{5kT_0}{3m}}, \qquad \lambda_0 = \frac{2\pi a_0}{\mathsf{q}}, \qquad \lambda_J = \frac{2\pi v_s}{\sqrt{4\pi G\rho_0}}, \tag{38}$$

Armed with the above definitions, the system of equations (35) and (36) can be rewritten as

$$\tau^2\delta''_{\rho(\mathsf{q})} + \frac{4}{3}\left[1 + \sqrt{\frac{2}{3}}\mu_*\left(\frac{\lambda_J}{\lambda_0}\right)^2\tau^{\frac{5-4\alpha}{3}}\right]\tau\delta'_{\rho(\mathsf{q})}$$
$$- \frac{2}{3}\left[\delta_{\rho(\mathsf{q})} - \frac{3}{5}\left(\frac{\lambda_J}{\lambda_0}\right)^2\frac{\delta_{\rho(\mathsf{q})} + \delta_{T(\mathsf{q})}}{\tau^{\frac{2}{3}}}\right] = 0, \tag{39}$$

$$\tau\left(\delta'_{T(\mathsf{q})} - \frac{2}{3}\delta'_{\rho(\mathsf{q})}\right) + \frac{5}{\sqrt{6}}\mu_*\left(\frac{\lambda_J}{\lambda_0}\right)^2\tau^{\frac{5-4\alpha}{3}}\delta_{T(\mathsf{q})} = 0. \tag{40}$$

Above we have used explicitly that the universe is dominated by matter so the relationships $\dot{a}/a = 2/3t$ and $\rho_0 = 1/(6\pi Gt^2)$ hold. Further, we introduced the dimensionless time $\tau = t\sqrt{6\pi G\rho_{i0}}$ and the prime stands for derivative with respect to τ. We remark that the above system of equations is not easy to tackle for several reasons. We start by recalling that they form a coupled system of equations and it is even worse than it looks provided the Bessel equation (39) is nonhomogeneous. Nevertheless, we are going to obtain the leading contributions due to the viscosity effects by using a series of physical approximations. Further, we will confirm the aforesaid solution by solving the coupled system numerically.

Let us search for a solution of the system of equations (39) and (40). For simplicity, we consider the case where $\alpha = 1/2$, i.e., the case of hard-spheres. First, we introduce an auxiliary contrast density $\delta_{\beta(\mathsf{q})}$ in order to decouple the solution of this system of equations. In doing so, we define

$$\delta_{\beta(\mathsf{q})} = \delta_{T(\mathsf{q})} - \frac{2}{3}\delta_{\rho(\mathsf{q})}, \tag{41}$$

so that (40) can be rewritten as

$$\delta'_{\beta(\mathbf{q})} + \frac{5}{\sqrt{6}}\mu_* \left(\frac{\lambda_J}{\lambda_0}\right)^2 \left(\frac{2}{3}\delta_{\rho(\mathbf{q})} + \delta_{\beta(\mathbf{q})}\right) = 0. \quad (42)$$

The solution of (42) can be splitted as the sum of two contributions as follows:

$$\delta_{\beta(\mathbf{q})} = e^{-\frac{5}{\sqrt{6}}\mu_*\left(\frac{\lambda_J}{\lambda_0}\right)^2\tau} - \frac{2}{3}e^{\frac{5}{\sqrt{6}}\mu_*\left(\frac{\lambda_J}{\lambda_0}\right)^2\tau}$$

$$\times \int^\tau dt \delta_{\rho(\mathbf{q})}(t) \frac{5}{\sqrt{6}}\mu_*\left(\frac{\lambda_J}{\lambda_0}\right)^2 e^{-\frac{5}{\sqrt{6}}\mu_*\left(\frac{\lambda_J}{\lambda_0}\right)^2 t}. \quad (43)$$

From now on, we shall investigate solutions that depart from the case without dissipation ($\mu_* = 0$) by considering the coefficient $\epsilon = \frac{5}{\sqrt{6}}\mu_*\xi_J^2$ as a small quantity. Note that in this case the shear viscosity coefficient for small wavelengths $\xi_J > 1$ should be smaller than that for large wavelengths $\xi_J < 1$. Hence we can approximate the solution (43) as

$$\delta_{\beta(\mathbf{q})} = 1 - \epsilon\tau - \frac{2}{3}\epsilon \int^\tau \delta_{\rho(\mathbf{q})}(t')dt'. \quad (44)$$

Given the physical nature of μ_*, we can consider the density contrast as perturbative series in the small (control) parameter ϵ, namely $\delta_{\rho(\mathbf{q})} = \delta_{1\rho(\mathbf{q})} + \epsilon\delta_{2\rho(\mathbf{q})}$. Replacing the latter ansatz in (39) we obtain two differential equations, one for $\delta_{1\rho(\mathbf{q})}$ and another for $\delta_{2\rho(\mathbf{q})}$:

$$\tau^2\delta''_{1\rho(\mathbf{q})} + \frac{4}{3}\tau\delta'_{1\rho(\mathbf{q})} - \frac{2}{3}(1 - \xi_J^2\tau^{-\frac{2}{3}})\delta_{1\rho(\mathbf{q})} = -\frac{2}{5}\xi_J^2\tau^{-\frac{2}{3}},$$
$$(45)$$

$$\tau^2\delta''_{2\rho(\mathbf{q})} + \frac{4}{3}\tau\delta'_{2\rho(\mathbf{q})} - \frac{2}{3}[1 - \xi_J^2\tau^{-\frac{2}{3}}]\delta_{2\rho(\mathbf{q})}$$

$$- \frac{2}{5}\xi_J^2\tau^{-\frac{2}{3}}\left(\tau + \frac{2}{3}\int^\tau \delta_{1\rho(\mathbf{q})}(t')dt'\right)$$

$$+ \frac{8}{15}\tau^2\delta'_{1\rho(\mathbf{q})} = 0. \quad (46)$$

The homogeneous solution of (45) is given in terms of Bessel functions of first kind $J_{\pm\frac{5}{2}}(\frac{\sqrt{6}\xi_J}{\tau^{\frac{1}{3}}})$ and its general solution reads

$$\delta_{1\rho(\mathbf{q})} = t^{-\frac{1}{6}}\left[C_1 J_{+\frac{5}{2}}\left(\frac{\sqrt{6}\xi_J}{\tau^{\frac{1}{3}}}\right) + C_2 J_{-\frac{5}{2}}\left(\frac{\sqrt{6}\xi_J}{\tau^{\frac{1}{3}}}\right)\right]$$

$$- \frac{3}{5}[1 + \xi_J\tau^{\frac{2}{3}}]. \quad (47)$$

We now calculate the leading terms of (47) under the under the assumption that $\xi_J < 1$, so we are looking at the regime where the Jeans instability has already begun. In that situation, (47) exhibits to contributions, namely $\delta_{1\rho(\mathbf{q})} \simeq \delta_{1g}\tau^{2/3} + \delta_{1d}\tau^{-1}$ being δ_{1g} and δ_{1d} two constants. In order

to simplify the instability analysis, we have restricted our attention to the growing mode only, thus it is valid to consider $\delta_{1\rho(\mathbf{q})} \simeq \delta_{1g}\tau^{2/3}$. Replacing the latter proposal in (46) and neglecting the term proportional to ξ_J^2, it is possible to put Eq. (46) in a more suitable shape for extracting the growing and decaying modes. Then, Eq. (46) becomes

$$\tau^2\delta''_{2\rho(\mathbf{q})} + \frac{4}{3}\tau\delta'_{2\rho(\mathbf{q})} - \frac{2}{3}\delta_{2\rho(\mathbf{q})} + \frac{16}{45}\delta_{1g}\tau^{\frac{5}{3}} = 0. \quad (48)$$

whose solution is given by

$$\delta_{2\rho(\mathbf{q})} = -\frac{2}{15}\delta_{1g}\tau^{\frac{5}{3}} + \delta_{2g}\tau^{\frac{2}{3}} + \delta_{2d}\tau^{-1}, \quad (49)$$

where δ_{2g} and δ_{2d} are two constants. In other words, the viscosity term generates an additional growing mode which is proportional to $\tau^{\frac{5}{3}}$. Taking into account that the total density contrast is $\delta_{\rho(\mathbf{q})} = \delta_{1\rho} + \epsilon\delta_{2\rho}$, we can conclude that the total density contrast has two growing modes: the standard one given by $\tau^{\frac{2}{3}}$ plus another that appears as a correction in the limit of small viscosity, $\tau^{\frac{5}{3}}$. Of course, the latter term is subleading in the limit of very small viscosity and the standard expression for the density contrast is recovered for a universe dominated by dustlike matter. Having obtained the leading behavior in the contrast density, we are able to determine the leading behavior of some derived quantities. From (31), we observe that $u_{(\mathbf{q})} \propto \delta'_{\rho(\mathbf{q})}$ so the peculiar velocity exhibits also a principal growing mode ($\tau^{2/3}$), a subleading growing mode ($\propto \epsilon\tau^{5/3}$), and a decaying mode ($\propto \tau^{-1}$). Using (33), we arrive at the conclusion that the perturbed gravitational field $\delta\phi_{(\mathbf{q})} \propto a^{-1}\delta_{\rho(\mathbf{q})}$, indicating that the modes include three terms, one is proportional to a constant, another goes like $\tau^{-5/3}$ and the subleading correction scales as $\epsilon\tau^{2/3}$. The latter fact shows us the gravitational potential remains constant in the limit of vanishing viscosity, as one could expect.

As a simple way to confirm our previous analytical results we are going to solve the system of equation (39) and (40) without any approximation. Then, we will numerically determine the conditions for having Jeans instability when the universe is dominated by a pressureless fluid and the viscosity is nonzero. To do so, the coupled system of differential equations (39) and (40) for the density and temperature contrasts were solved numerically for the cases of small ($\xi_J > 1$) and large ($\xi_J < 1$) wavelengths. Without loss of generality, we considered the following initial conditions $\delta_{\rho(\mathbf{q})}(1) = \delta'_{\rho(\mathbf{q})}(1) = \delta_{T(\mathbf{q})}(1) = 10^{-1}$. In Figs. 2 and 3 the time evolution of the density and temperature contrasts are plotted, respectively. In both cases we chosen $\alpha = 1$—which refers to soft particle model—but for other values of $1/2 \leq \alpha \leq 1$ the behavior
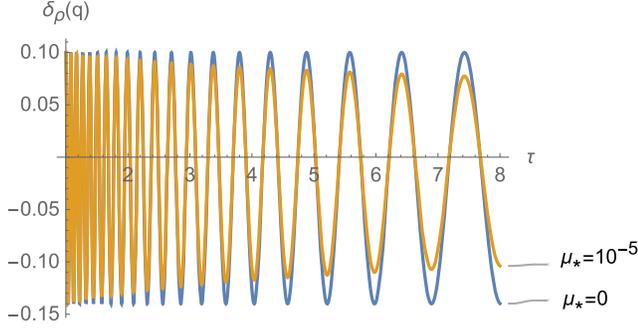
FIG. 2. Time evolution of the density contrast for small wavelengths with $\alpha = 1$ and $\xi_J = 100$.
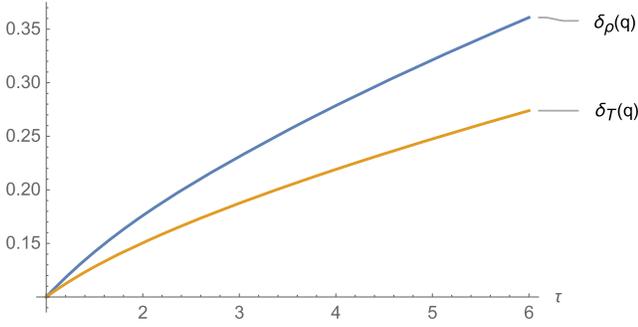
FIG. 3. Time evolution of the density contrast and temperature contrast for large wavelengths with $\alpha = 1$ and $\xi_J = 1/2$.

of the curves seems to be quite similar, so we are not loosing generality. The density and the temperature contrasts shown in Fig. 2 for the case of small wavelengths $\xi_J = 100$ have the same behavior and represent harmonic waves. When the shear viscosity coefficient vanishes the density and temperature contrasts evolve with time without damping, while when the shear viscosity is present the harmonic waves are damped and by increasing the value of this coefficient the damping becomes more accentuated. Figure 3 shows the solutions for large wavelengths $\xi_J = 0.5$ where the density and temperature contrasts grow with time and correspond to Jeans instability. We infer from this figure that when the shear viscosity is present the evolution with time of the density and temperature contrasts are less accentuated than the one without viscosity. This can be understood by noting that in the presence of viscous and thermal effects the dissipation in the energy makes the process of structure formation less effective. Regarding the physical impact of the shear viscosity coefficient, we have noticed that it has a strong influence on the temperature contrast while the density contrast seems to be less sensitive to it.

III. JEANS INSTABILITY FROM A THIRTEEN-FIELD THEORY

In this section, we will examine an enlarged model where the pressure tensor and heat flux vector are endowed with

their own master equations. The physical motivation for extending our previous approach can be explained as follows. The balance equations (1) and (2) with the Navier-Stokes and Fourier (4) constitutive equations lead to a parabolic system of equations which imply infinite speeds for the heat propagation. This was known in the literature as the heat paradox (see [19]). It can be solved by introducing a hyperbolic system of equations and the most simple system of equations is to consider the thirteen fields of mass density ρ, velocity v_i, pressure tensor p_{ij}, and heat flux vector q_i (cf., Appendix). For an ideal gas these balance equations can be obtained from the Grad method of kinetic theory (see, e.g., [18]) or from the so-called extended thermodynamic theory (see [19]). The field equations for the mass density ρ, velocity v_i, pressure tensor p_{ij}, and heat flux vector q_i read

$$\frac{\partial \rho}{\partial t} + v_i \frac{\partial \rho}{\partial x_i} + \rho \frac{\partial v_i}{\partial x_i} = 0, \tag{50}$$

$$\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} + \frac{kT}{m\rho}\frac{\partial \rho}{\partial x_i} + \frac{k}{m}\frac{\partial T}{\partial x_i} + \frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j} + \frac{\partial \phi}{\partial x_i} = 0, \tag{51}$$

$$\frac{3k}{2m}\left(\frac{\partial T}{\partial t} + v_i \frac{\partial T}{\partial x_i}\right) + \frac{1}{\rho}\frac{\partial q_i}{\partial x_i} + \frac{k}{m}T\frac{\partial v_i}{\partial x_i} + \frac{\sigma_{ij}}{\rho}\frac{\partial v_i}{\partial x_j} = 0, \tag{52}$$

$$\begin{aligned} \frac{\partial \sigma_{ij}}{\partial t} &+ v_k \frac{\partial \sigma_{ij}}{\partial x_k} + \sigma_{ij}\frac{\partial v_k}{\partial x_k} + \frac{2}{5}\left(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_i}\right) \\ &- \frac{4}{15}\frac{\partial q_r}{\partial x_r}\delta_{ij} + \sigma_{ik}\frac{\partial v_j}{\partial x_k} + \sigma_{jk}\frac{\partial v_i}{\partial x_k} - \frac{2}{3}\sigma_{kr}\frac{\partial v_k}{\partial x_r}\delta_{ij} \\ &+ \rho\frac{k}{m}T\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\frac{\partial v_r}{\partial x_r}\delta_{ij}\right) = -\nu\sigma_{ij}, \end{aligned} \tag{53}$$

$$\begin{aligned} \frac{\partial q_i}{\partial t} &+ v_j\frac{\partial q_i}{\partial x_j} + \frac{7}{5}q_i\frac{\partial v_k}{\partial x_k} + \frac{2}{5}q_j\frac{\partial v_j}{\partial x_i} + \frac{7}{5}q_j\frac{\partial v_i}{\partial x_j} \\ &+ \left(\frac{p}{\rho}\delta_{ij} - \frac{\sigma_{ij}}{\rho}\right)\frac{\partial \sigma_{jk}}{\partial x_k} + \frac{p}{\rho}\sigma_{ij}\left(\frac{5}{2T}\frac{\partial T}{\partial x_k} - \frac{1}{\rho}\frac{\partial \rho}{\partial x_k}\right) \\ &+ \frac{5k}{2m}p\frac{\partial T}{\partial x_i} = -\frac{2}{3}\nu q_i. \end{aligned} \tag{54}$$

This system of equations together with Poisson equation (2) is a closed system for the determination of ρ, v_i, p_{ij}, and q_i. In order to obtain (52) and (54) we have decomposed the pressure tensor into a pressure isotropic contribution p plus a stress tensor σ_{ij}, i.e., $p_{ij} = p\delta_{ij} + \sigma_{ij}$. We have written down two evolution equations, the first one refers to its trace and the second one to σ_{ij}. Equation (54) refers to the evolution equation for the heat flux vector. The master equations for σ_{ij} and q_i involve constitutive terms which were taken as functions of the thirteen basic fields according to Grad's method (see Appendix). The constitutive terms refer to a third-order moment p_{ijk}, a contracted

fourth-order moment $q_{ij} = p_{ikkj}$ and two production terms P_{ij} and P_i. According to kinetic theory of gases their expressions in a linearized theory read (see e.g. [18])

$$p_{ijk} = \frac{2}{5}(q_i\delta_{jk} + q_j\delta_{ik} + q_k\delta_{ij}), \qquad P_{ij} = -\nu\sigma_{ij}, \quad (55)$$

$$q_{ij} = \frac{5p^2}{2\rho}\delta_{ij} + \frac{7p}{2\rho}\sigma_{ij}, \qquad P_i = -\frac{2}{3}\nu q_i. \quad (56)$$

Here ν is a collision frequency which, according to the kinetic theory of gases, is related with the shear viscosity coefficient and pressure by $\nu = p/\mu_v$.

A. Static universe

As in the previous section we search for solutions of the system (2) and (50)–(54) in the form of plane waves. At equilibrium the density, temperature and gravitational potential assume constant values while the velocity, stress and heat flux vanish. The perturbations out of equilibrium refer to plane waves of small amplitudes with frequency ω and wave number k propagating in the x direction.

$$\rho = \rho_0 + \delta\rho e^{i(\mathsf{k}x - \omega t)}, \qquad v_x = \delta v_x e^{i(\mathsf{k}x - \omega t)}, \quad (57)$$

$$T = T_0 + \delta T e^{i(\mathsf{k}x - \omega t)}, \qquad \delta\sigma_{xx} = \delta\sigma e^{i(\mathsf{k}x - \omega t)}, \quad (58)$$

$$\sigma_{ij} = \delta\sigma_{xx}\mathrm{diag}\left[1, -\frac{1}{2}, -\frac{1}{2}\right], \qquad q_x = \delta q e^{i(\mathsf{k}x - \omega t)} \quad (59)$$

$$\phi = \phi_0 + \delta\phi e^{i(\mathsf{k}x - \omega t)}. \quad (60)$$

Above we have also that the constant values of density, temperature, and gravitational potential satisfy (50)–(54) but not the Poisson equation (2), so that we have to impose Jeans swindle which says that the (2) is valid only for the perturbations.

From the insertion of the representations (57)–(60) into the (50)–(54) and (2) we get a system of algebraic equations for the amplitudes and in order to have a solution for this system the determinant of the coefficients of the amplitudes must vanish. This implies the dispersion relation:

$$\omega_*^5 + \frac{i}{\mu_*}\omega^4 + \left(1 - \frac{78}{25}\mathsf{k}_*^2 - \frac{6}{25\mu_*^2}\right)\omega_*^3$$
$$+ \frac{i}{\mu_*}\left(1 - \frac{48}{25}\mathsf{k}_*^2\right)\omega_*^2 - \frac{6}{25}\left[\frac{1 - \mathsf{k}_*^2}{\mu_*^2}\right.$$
$$\left. + \frac{11\mathsf{k}_*^2}{2}\left(1 - \frac{9\mathsf{k}_*^2}{11}\right)\right]\omega_* - \frac{3i\mathsf{k}_*^2}{25\mu_*}(5 - 3\mathsf{k}_*^2) = 0. \quad (61)$$

Here again by considering a vanishing shear viscosity $\mu_* = 0$ we get Jeans solution (12).

If we consider big wavelengths $\lambda_J/\lambda < 1$ it follows from (61) five values for the dimensionless frequency

$$\omega_{*1} = -i\left[\frac{5\mu_*}{2}\left(\frac{\lambda_J}{\lambda}\right)^2 + \cdots\right], \quad (62)$$

$$\omega_{*2} = -i\left[1 - \frac{3(1 - 3\mu_*)}{2(3 - 5\mu_*)}\left(\frac{\lambda_J}{\lambda}\right)^2 - \cdots\right], \quad (63)$$

$$\omega_{*3} = -i\left[\frac{2}{5\mu_*} - \frac{9\mu_*}{10}\left(\frac{\lambda_J}{\lambda}\right)^2 - \cdots\right], \quad (64)$$

$$\omega_{*4} = -i\left[\frac{3}{5\mu_*} - \frac{4\mu_*(33 - 50\mu_*^2)}{5(9 - 25\mu_*^2)}\left(\frac{\lambda_J}{\lambda}\right)^2 - \cdots\right], \quad (65)$$

$$\omega_{*5} = i\left[1 - \frac{3(1 + 3\mu_*)}{2(3 + 5\mu_*)}\left(\frac{\lambda_J}{\lambda}\right)^2 - \cdots\right]. \quad (66)$$

All dimensionless frequencies are pure imaginary with ω_{*1} to ω_{*4} implying decaying perturbations in time whereas ω_{*5} grows with time and is identify with the Jeans instability. All frequencies show up a dependence on dissipative phenomena through the dimensionless shear viscosity coefficient μ_*.

For small wavelengths $\lambda_J/\lambda > 1$ it follows from (61) one or two sets of complex conjugate solutions for the dimensionless frequency which correspond to harmonic waves in time and three or one solutions which refer to the decay of perturbations in time, respectively.

As part of our numerical analysis, we investigate the behavior of the real part of $\exp(-i\omega t)$ as a function of time t (61) for the ratio $\xi_J = 2$ (small wavelengths) for different values of the dimensionless shear viscosity coefficient. The solutions are harmonic waves in time and the one without the shear viscosity ($\mu_* = 0$) represents a nondamped wave which corresponds to the Jeans solution as was pointed out previously. The other two solutions with $\mu_* \neq 0$ represent damped harmonic waves and we infer from this figure that the increase of the shear viscosity coefficient increase the wave damping. For large values of the shear viscosity coefficient a strong damping occurs so that the harmonic character of the wave is lost. It was found that the solutions of (61) for the ratio $\xi_J = 1/2$ (large wavelengths) represents the growth of the perturbations with time, showing up the Jeans instability. One can infer from such analysis that the Jeans solution ($\mu_* = 0$) grows more rapid with respect to time than those two where the shear viscosity coefficients do not vanish. This behavior can be explained on the basis of an energy loss due to the dissipative effects so that the growth of the perturbations are damped. Note that the growth with time of the perturbation by increasing the shear viscosity coefficient becomes less accentuated. A similar behavior as that described above can also be obtained from the five-field theory of subsection II A, but here we have used field equations of hyperbolic type rather than of parabolic type so that the heat paradox is avoided.

B. Expanding universe

As in subsection II C we analyze the solution of the thirteen field equations (50)–(54) coupled with Poisson equation (2) in form of plane waves of small amplitudes in an expanding Universe. The background values for the fields of mass density ρ_0, temperature T_0, velocity v_i and gravitational potential ϕ_0 are given by (22), whereas those values for the fields of stress and heat flux are considered to be zero, i.e., $\sigma_{ij}^0 = q_i^0 = 0$. The background values for the fields are superposed by longitudinal plane waves of time dependent small amplitudes, namely $Y = Y_0(t) + \sum_q \delta Y_{(q)} e^{iqx}$. In our case, $Y_0(t) \equiv \{\rho_0, v_{x0}, T_0, \phi_0\}$ provided $q_{x0} = 0 = \sigma_{xx}^0 = 0$, while the perturbed fields are given by $\delta Y_{(q)} = \{\delta\rho(t), \delta v(t), \delta T(t), \delta q(t), \delta\sigma(t)\}$, where we used that the shear tensor has some symmetries, namely only the diagonal components are nonzeros and the trace of such tensor vanishes, $2\sigma_{yy} = 2\sigma_{zz} = -\sigma_{xx}$, as a result we only need to introduce just one function called $\delta\sigma(t)$. Replacing the latter proposals into the balance equations (50)–(54) and (2) and linearization it follows a coupled system of differential equations for the amplitudes:

$$\dot{\delta}_{\rho(q)} = -iqu_{(q)}, \tag{67}$$

$$\dot{u}_{(q)} + 2Hu_{(q)} + \frac{iq}{a^2}\left(\frac{kT_0}{m}(\delta_{\rho(q)} + \delta_{T(q)}) + \delta\phi_{(q)}\right)$$
$$= -\frac{iq}{a^2\rho_0}\delta\sigma_{(q)}, \tag{68}$$

$$\frac{3}{2}\dot{\delta}_{T(q)} + \frac{iq}{a\rho_0}\frac{m}{kT_0}\delta q_{(q)} + iqu_{(q)} = 0, \tag{69}$$

$$\dot{\delta\sigma}_{(q)} + 5H\delta\sigma_{(q)} + \frac{iq8}{15a}\delta q_{(q)} + \nu\delta\sigma_{(q)} + \frac{iqp_0}{3}u_{(q)} = 0, \tag{70}$$

$$\delta\phi_{(q)} = -\frac{4\pi G\rho_0 a^2}{q^2}\delta_{\rho(q)}, \tag{71}$$

$$\dot{\delta q}_{(q)} + 6H\delta q_{(q)} + \frac{iqkT_0}{ma}\delta\sigma_{(q)} + \frac{iq5kT_0}{2ma}\delta_{T(q)}$$
$$+ \frac{2\nu}{3}\delta q_{(q)} = 0. \tag{72}$$

Here also the background solution satisfies Eqs. (50)–(54) and (2) identically without the use of Jeans swindle. The above coupled system can be written in a simpler form by introducing the dimensionless time $\tau = t\sqrt{6\pi G\rho_{i0}}$ and the density, temperature, stress, and heat contrasts defined as $\delta_{\rho(q)} = \delta\rho_{(q)}/\rho_0$, $\delta_{T(q)} = \delta T_{(q)}/T_0$, $\delta_{\sigma(q)} = \delta\sigma_{(q)}/\sigma_0$, and $\delta_{q(q)} = \delta q_{(q)}/q_0$. The functions used to arrange dimensionless heat flux and shear tensor are given by

$$\sigma_0 = \frac{kT_0\rho_0}{m}, \qquad q_0 = \frac{\alpha_0 kT_0\rho_0 a}{iqm}. \tag{73}$$

Furthermore, we recall that the collision frequency is related with the pressure and the coefficient of shear viscosity by $\nu = p/\mu_v$ while the latter is given in terms of the cosmic scale factor by (37) and $a = a_0(6\pi G\rho_{i0}t^2)^{\frac{1}{3}}$. Hence we have that

$$\nu = \frac{p_0}{\mu_v} = \frac{3}{5}\frac{\sqrt{4\pi G\rho_0}}{\mu_*\tau^{\frac{2}{3}(5-2\alpha)}} \tag{74}$$

where $\alpha_0 = 6\pi G\rho_{i0}$. From Eq. (68) we obtain that $\dot{\delta}_{\rho(q)} = -iqu_{(q)}$ and we can rewrite the coupled system of equations (68)–(72) in terms of the dimensionless time τ as

$$\tau^2\delta''_{\rho(q)} + \frac{4}{3}\tau\delta'_{\rho(q)} - \frac{2}{3}\delta_{\rho(q)}$$
$$+ \frac{2}{5\tau^{\frac{2}{3}}}\xi_J^2(\delta_{\rho(q)} + \delta_{T(q)} + \delta_{\sigma(q)}) = 0, \tag{75}$$

$$\tau\left(\delta'_{T(q)} - \frac{2}{3}\delta'_{\rho(q)}\right) + \frac{4}{9}\delta_{\sigma(q)} + \frac{2}{3}\tau\delta_{q(q)} = 0, \tag{76}$$
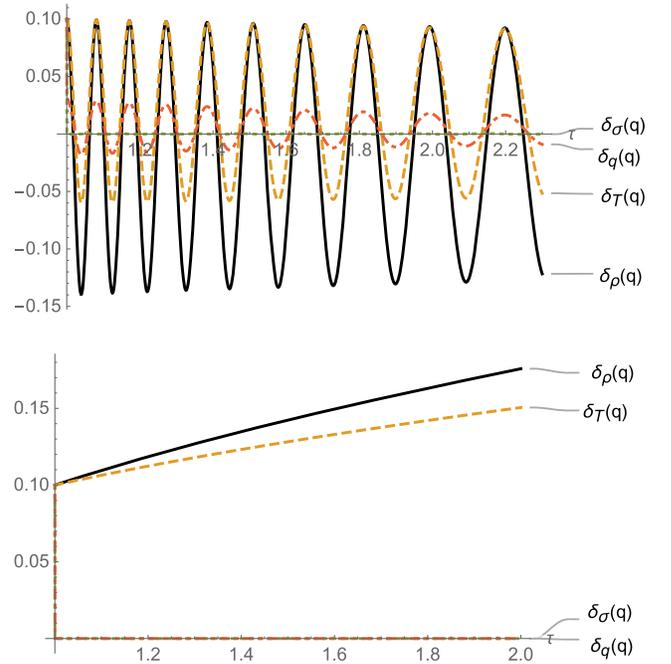
FIG. 4. Upper level: Time evolution of the density (straight line), temperature (dashed line), stress (dotted line), and heat flux (dot-dashed line) as damped harmonic waves for small wavelengths $\xi_J = 100$ and dimensionless shear viscosity $\mu_* = 10^{-5}$. Lower level: Time evolution of the density (straight line), temperature (dashed line), stress (dotted line), and heat flux (dot-dashed line) for large wavelengths $\xi_J = 0.5$ and dimensionless shear viscosity $\mu_* = 10^{-5}$.

$$\tau\delta'_{\sigma(\mathsf{q})} - \frac{4}{3}\delta_{\sigma(\mathsf{q})} + \frac{8}{15}\tau\delta_{q(\mathsf{q})} - \frac{4}{3}\tau\delta'_{\rho(\mathsf{q})}$$
$$= -\frac{3}{5\mu_*}\sqrt{\frac{2}{3}}\tau^{-(7-4\alpha)/3}\delta_{\sigma(\mathsf{q})}, \qquad (77)$$

$$\tau\delta'_{q(\mathsf{q})} + \frac{4}{3}\delta_{q(\mathsf{q})} - \frac{2}{5}\xi_J^2\tau^{-\frac{5}{3}}\delta_{\sigma(\mathsf{q})} - \xi_J^2\delta_{T(\mathsf{q})}$$
$$= -\frac{3}{5\mu_*}\sqrt{\frac{2}{3}}\tau^{-(7-4\alpha)/3}\delta_{q(\mathsf{q})}, \qquad (78)$$

where as was previously introduced the primes refer to a derivation with respect to τ. Equations (75)–(78) represent a coupled system of differential equations for the determination of the mass density δ_ρ, temperature δ_T, stress δ_σ and heat flux δ_q contrasts.

The coupled system of differential equations (75)–(78) was solved numerically with the following initial conditions $\delta_{\rho(\mathsf{q})}(1) = \delta'_{\rho(\mathsf{q})}(1) = \delta_{T(\mathsf{q})}(1) = \delta_{\sigma(\mathsf{q})}(1) = \delta_{q(\mathsf{q})}(1) = 10^{-1}$ and for the case of Maxwellian particle ($\alpha = 1$). In the upper panel of Fig. 4 are plotted the time evolution of the density, temperature, stress, and heat flux contrasts, which behave as damped harmonic waves for small wavelengths. For large wavelengths the same contrasts are shown as function of the time in the lower panel of this figure. It is interesting to note that the density and temperature contrasts grow with time, while the stress and heat flux contrasts heavily decay with time. The growth of the density contrast is associated with Jeans instability.

IV. SUMMARY

We have explored the gravitational amplification of inhomogeneities within the context of five and thirteen (thermodynamic) field theories for static and expanding universes. We provided two simple physical pictures of how the thermal effects along with viscous ones do affect the growth of instabilities. In this scheme, we have started by calculating the dispersion relation in both models, obtaining that the changes introduced by viscosity and the heat flux considerably modified the propagating modes. In the first part, we only considered static universes so the Jeans instability only involved two elements: pressure and gravity. We then studied Jeans instability mechanism for a universe dominated by pressureless matter, generalizing earlier results on the gravitational instability in the presence of heat flux and viscosity. Besides, we calculated the Jeans mass for stellar configurations (mostly molecular clouds) and their behaviors with the number density, temperature, and viscosity. The outcome of such analysis confirmed that astrophysical configurations with viscosity need greater Jeans mass to collapse.

We devoted our effects to extract the main physical insight about the coupled system of contrast density and contrast temperature by focusing in the model of hard sphere and Maxwellian particles. In the small wavelengths regime, increasing the amount of shear viscosity into the system forces the harmonic perturbations to damp faster whereas in the opposite limit larger values of shear viscosity lead to smaller values of density and temperature contrasts. In the limit of large wavelength ($\lambda_J < \lambda_0$), our analytical results indicate that viscous effects are subleading ones provided they generate an additional growing modes proportional to $\tau^{5/3}$ which becomes important for intermediate values of viscosity parameter, whereas in the limit of very small viscosity they can be neglected and the standard behavior of contrast density is recovered ($\propto \tau^{2/3}$) for a matter dominated universe.

In order to further understand the Jeans instability with dissipative effects, we have directed our attention to the thermodynamic thirteen-field model. The aforesaid scheme represents a hyperbolic system of differential equation characterized by two related parameters, the shear viscosity and the collision frequency. We found that dispersion relation becomes a polynomial in the frequency with two order higher in relation to the five-field theory, indicating that the effects associated with the shear viscosity and heat flux become nontrivial. Regarding the dynamical evolution of the density, temperature, stress, and heat flux contrasts for a universe dominated by pressureless matter, we obtain also damped harmonic waves for small wavelengths. In the case of large wavelengths, the density and temperature contrasts grow with time (due to the Jeans mechanism) while the stress and heat flux contrasts decay with time.

Even though, we were able to make some progress in the analysis of Jeans instability with dissipative effects, showing that thermal and viscous effects contribute to reduce the ability of the system to aggregate matter in a cooperative way and therefore the structure formation may be less effective. It would be interesting to see whether analytic/numerical results for the growth of perturbations can be obtained by inspecting the full Boltzmann equation for one baryonlike specie plus self-interacting dark matter. In this way, one could address the issue of warm dark matter coupled to baryons and by doing so one could take into account thermal along with viscous effects as a consequence of the microscopic dynamic of the system. We leave the investigation of this question for future work.

APPENDIX: BALANCE EQUATIONS AND CONSTITUTIVE EQUATIONS WITHIN THE FRAMEWORK OF THE BOLTZMANN EQUATION

In this work we have analyzed the Jeans instability of a viscous and heat conducting ideal nonrelativistic gas through the use of the balance equations of ordinary and extended thermodynamics. Although the balance equations can be determined from a phenomenological theory, the constitutive equations adopted in this work were taken from a kinetic theory based on the Boltzmann equation. In the following we shall sketch the procedure used in the kinetic theory in order to obtain the balance and constitutive equations. For more details the reader should consult a book on the Boltzmann equation (see, e.g., [18]).

The starting point is the Boltzmann equation which is the space-time evolution of the one-particle distribution function $f(\mathbf{x}, \boldsymbol{\xi}, t)$ in the phase space spanned by the space $\mathbf{x}$ and velocity $\boldsymbol{\xi}$ coordinates of the gas particles. In the presence of a gravitational field ϕ the Boltzmann equation reads [18].

$$\frac{\partial f}{\partial t} + \xi_i \frac{\partial f}{\partial x_i} - \frac{\partial \phi}{\partial x_i} \frac{\partial f}{\partial \xi_i} = \mathcal{Q}(f, f). \qquad (A1)$$

The right-hand side of the above equation takes into account the binary elastic collisions between the particles and $\mathcal{Q}(f, f)$ denotes the collision operator of the Boltzmann equation. If (ξ, ξ_1) and (ξ', ξ_1') denote the pre- and postcollisional velocities of two particles at a binary collision the collision operator reads [18].

$$\mathcal{Q}(f, f) = \int [f(\mathbf{x}, \boldsymbol{\xi_1'}, t) f(\mathbf{x}, \boldsymbol{\xi'}, t)$$
$$- f(\mathbf{x}, \boldsymbol{\xi_1}, t) f(\mathbf{x}, \boldsymbol{\xi}, t)] g b \, db \, d\epsilon \, d\boldsymbol{\xi_1}. \qquad (A2)$$

At this point, it is illustrative to show that the above expression of the collision operator already contained the scattering cross section associated with the collisional process amongst classical particles. To do so, one can notice that for simple scattering (elastic), an "event" is defined as the deflection of one particle into a range $d\Omega$ of solid angles about some observation direction $\bar{\Omega}$. Using polar coordinates, one can write the differential solid angle as $d\Omega = \sin\chi \, d\chi \, d\epsilon$, where ϵ denotes an azimuthal angle and χ is the scattering angle which defines the scattering process. Notice that, in general, for a particular interaction potential $V(r)$ between the particles, the scattering angle χ depends on relative velocity $g = |\xi_1 - \xi|$ and impact parameter b (miss distance). If there is a one-to-one relation between b and χ, the particles that will be deflected in the referred range of angles have impact parameters between $b(\chi)$ and $b(\chi) + db(\chi)$. By simple geometry, we can write $\sigma d\Omega = b(\chi) db(\chi) d\epsilon$, so the differential cross section of the scattering process is given by

$$\sigma(\chi, g) = \frac{b(\chi)}{\sin\chi} \left| \frac{db(\chi)}{d\chi} \right|.$$

The latter equation tells us that the collision operator encodes the information of the scattering process in the term $g b(\chi) db(\chi) d\epsilon = g\sigma(\chi, g) d\Omega$. Further, the physical information about the interacting potential amongst the particles is hidden in the generic expression of the scattering angle χ (see [18]). We would like to remark that we will be dealing with classical particles which are treated as hard spheres provided is useful for obtaining an explicit expressions to describe classical gases. It is a tractable model and also describes in good detail collisions at high energies, where the attractive wells are negligible compared with the kinetic energy. In this model, particles of radius $r_0 = d/2$ interact with a central potential that is infinitely hard when particles meet at a distance d. When the interactions are described by the hard sphere of diameter d, one can simplify the scattering section provided the impact parameter and the deflection angle are related by the next expression: $b = d\cos(\chi/2)$; then the scattering section is given by $\sigma_{\text{total}} = d^2/4$ [18]. As expected, total scattering cross section is proportional to the projected area of the sphere. Let us end this remark on the scattering section by mentioning why the classical approach is possible. Since collisions occur at atomic distance, their rigorous analysis requires quantum mechanics. Specifically, this is so whenever the distance of closest approach the scattering section (of the order of $\sqrt{\sigma_{\text{total}}}$) is comparable to or less than the Broglie wavelength for the relative momentum $\hbar/p$. Putting $p \simeq \sqrt{\mu_{\text{red}} kT}$ with $\mu_{\text{red}} = m_1 m_2/(m_1 + m_2)$, quantum effects dominate when $\sigma_{\text{total}} < \hbar^2/(\mu_{\text{red}} kT)$. As long as the latter condition holds, classical dynamics can be used in the calculation of the scattering section as we have done before, and then the overall collisional effects which appear in the collision operator of the Boltzmann equation are obtained on firm grounds. For instance, n-n collisions, $\mu_{\text{red}} > m_{\text{H}} = 1.7 \times 10^{-27}$ kg, and $T = 3000$ K this requires $\sigma_{\text{total}} < 10^{-22}$ m^2.

Now, we show how the macroscopic fields in kinetic theory are given as mean values with respect to the one-particle distribution function. The mass density ρ, velocity $\mathbf{v}$, specific internal energy ϵ, pressure tensor p_{ij}, and heat flux vector q_i are defined by

$$\rho = \int m f d^3\xi, \qquad v_i = \frac{1}{\rho} \int m \xi_i f d^3\xi, \qquad (A3)$$

$$\epsilon = \frac{1}{2\rho} \int m \xi^2 f d^3\xi, \qquad (A4)$$

$$p_{ij} = \int m \xi_i \xi_j f d^3\xi, \qquad q_i = \int \frac{m}{2} \xi^2 \xi_i f d^3\xi, \qquad (A5)$$

where m is the particle mass.

For the case of ordinary thermodynamics the balance equations are obtained by multiplying the Boltzmann equation (A1) with the mass m, momentum $m\xi_i$, and energy $m\xi^2/2$ of a particle and integration over all values of the velocity $d^3\xi$. Following this methodology we arrive at the balance equations of mass $(1)_1$ and of velocity $(1)_2$. For the determination of the balance equation for the specific internal energy one has to subtract from the balance equation for the total energy the corresponding balance equation for the kinetic energy of the particles $\rho v^2/2$. Note that the collision operator does not furnish any contribution to these balance equations, since mass, momentum, and energy are conserved in an elastic collision.

The Maxwellian distribution function

$$f_0 = \frac{m^{\frac{1}{2}}\rho}{(2\pi kT)^{\frac{3}{2}}} \exp\left[\frac{m(\xi - \mathbf{v})^2}{2kT}\right], \quad (A6)$$

refers to the equilibrium solution of the Boltzmann equation (A1). It is a function of the mass density ρ, velocity $\mathbf{v}$ and temperature T of the gas. Above k is the Boltzmann constant. From the knowledge of the equilibrium distribution function it is easy to determine the pressure and the specific internal energy in terms of the mass density and temperature, namely, $p = \rho kT/m$ and $\varepsilon = 3kT/2m$.

The non-equilibrium distribution function is obtained from the Boltzmann equation through the Chapman-Enskog method (see, e.g., [18]). In this method the distribution function is written as a Maxwellian distribution function f_0 plus a small deviation, namely $f = f_0(1 + \varphi)$. In order to determine the deviation φ, the Maxwellian distribution is inserted into the left-hand side of the Boltzmann equation and the time derivatives of the mass density, velocity, and temperature are eliminated by the use of the Euler equations. The expression for the distribution function $f = f_0(1 + \varphi)$ is inserted on the right-hand side of the Boltzmann equation and the products of the deviations are neglected. Hence it is found that the deviation is a function of the velocity and temperature gradients, namely

$$f = f_0 \left\{ 1 - \frac{3\mu_v}{2p} \left[\left(\frac{m\xi^2}{2kT} - \frac{5}{2} \right) \frac{\xi_i}{T} \frac{\partial T}{\partial x_i} \right. \right.$$
$$\left. \left. + \frac{m}{3kT} \xi_i \xi_j \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \frac{\partial v_k}{\partial x_k} \delta_{ij} \right) \right] \right\}. \quad (A7)$$

Here the shear viscosity coefficient is

$$\mu_v = \frac{5}{16} \frac{1}{\Omega^{(2,2)}} \sqrt{\frac{mkT}{\pi}}, \quad (A8)$$

which is given in terms of the collision integral

$$\Omega^{(2,2)} = \int_0^\infty \int_0^\infty e^{-\gamma^2} \gamma^7 (1 - \cos^2\chi) b \, db \, d\gamma. \quad (A9)$$

Above $\gamma = \sqrt{m/2kT}g$ and χ is the scattering angle. For a potential of hard spheres $b = d\cos(\chi/2)$ where d is the diameter of the particle, so that $\Omega^{(2,2)} = d^2$ and the shear viscosity coefficient becomes $\mu_v = \frac{5}{16}\frac{1}{d^2}\sqrt{\frac{mkT}{\pi}}$.

If we substitute the nonequilibrium distribution function (A7) into the definitions of the pressure tensor and heat flux vector (A5) and integrate over all particle velocities we get the Navier-Stokes and Fourier constitutive equations (4), where the relation between the thermal conductivity and shear viscosity coefficients is given by $\lambda_c = 15k\mu_v/4m$ for all spherically symmetric interaction potentials. Furthermore, the shear viscosity coefficient does not depend on the mass density, but only on the temperature. This dependence can be written as $\mu_v \propto T^\alpha$ where the value of α depends on the interaction potential, for example $\alpha = 1/2$ for hard sphere potential and $\alpha = 1$ for soft sphere potential (Maxwellian particles).

In the case of extended thermodynamics we base on Grad's 13-moment method and apart from the balance equations of the mass density and velocity, we introduce the balance equations for the pressure tensor and for the heat flux vector. These balance equations are obtained from the multiplication of the Boltzmann equation by $m\xi_i$ and $m\xi^2\xi_i/2$ and integration over all values of the velocity $d^3\xi$, respectively. They read

$$\frac{\partial p_{ij}}{\partial t} + \frac{\partial(p_{ijk} + p_{ij}v_k)}{\partial x_k} + p_{ki}\frac{\partial v_j}{\partial x_k} + p_{kj}\frac{\partial v_i}{\partial x_k} = P_{ij}, \quad (A10)$$

$$\frac{\partial q_i}{\partial t} + \frac{\partial(q_{ij} + q_i v_j)}{\partial x_j} + p_{ijk}\frac{\partial v_j}{\partial x_k} + q_j\frac{\partial v_i}{\partial x_j}$$
$$- \frac{p_{ki}}{\rho}\frac{\partial p_{kj}}{\partial x_j} - \frac{1}{2}\frac{p_{rr}}{\rho}\frac{\partial p_{ij}}{\partial x_j} = Q_i. \quad (A11)$$

In the above equations we have introduced new moments of the distribution function, namely

$$p_{ijk} = \int m\xi_i\xi_j\xi_k f \, d^3\xi, \qquad q_{ij} = \int \frac{m}{2}\xi^2\xi_i f \, d^3\xi, \quad (A12)$$

and the production terms

$$P_{ij} = \int m\xi_i\xi_j \mathcal{Q}(f,f) \, d^3\xi, \qquad Q_i = \int \frac{m}{2}\xi^2\xi_i \mathcal{Q}(f,f) \, d^3\xi. \quad (A13)$$

In order to close the system of equations we have to express the new moments of the distribution function and production terms as functions of the 13-moments $\rho, v_i, T, \sigma_{ij}, q_i$, where σ_{ij} is the symmetric and traceless part of the pressure tensor p_{ij}. This is attained by using

Grad's distribution function which is given in terms of these quantities, namely (see, e.g., [18])

$$f = f_0 \left\{ 1 + \frac{\rho}{2p^2} \left[\sigma_{ij} \xi_i \xi_j + \frac{4}{5} q_i \xi_i \left(\frac{m\xi^2}{2kT} - \frac{5}{2} \right) \right] \right\}. \quad \text{(A14)}$$

Insertion of the Grad's distribution function (A14) into the definition of the new moments (A12) and

production terms (A13) and integrating over all the velocities $d^3\xi$ we arrive at the linearized constitutive equations (56). Once these constitutive equations are known, the insertion of them into the balance equations (A10) and (A11) implies Eqs. (53) and (54), once we decompose the pressure tensor p_{ij} in (A10) in its trace $p_{rr} = 3p$ and its symmetric traceless part σ_{ij}.

[1] G. Bertone, *Particle Dark Matter: Observations, Models and Searches*, 1st ed. (Cambridge University Press, Cambridge, England, 2010).

[2] V. Gorini, S. Matarrese, U. Moschella, and M. Colpi, *Dark Matter and Dark Energy: A Challenge for Modern Cosmology*, 1st ed. (Springer, New York, 2011).

[3] J. H. Jeans, Phil. Trans. R. Soc. A **199**, 1 (1902).

[4] J. H. Jeans, *Astronomy and Cosmogony* (Cambridge University Press, Cambridge, England, 1928).

[5] W. B. Bonnor, Mon. Not. R. Astron. Soc. **117**, 104 (1957).

[6] N. Carlevaro and G. Montani, Int. J. Mod. Phys. D **18**, 1257 (2009).

[7] N. Carlevaro and G. Montani, arXiv:0912.3641.

[8] M. G. Corona-Galindo and H. Dehnen, Astrophys. Space Sci. **153**, 87 (1989).

[9] M. S. Janaki, N. Chakrabarti, and D. Banerjee, Phys. Plasmas **18**, 012901 (2011).

[10] S. A. Trigger, A. I. Ernshkovich, G. J. F. Van Heijst, and P. P. J. M. Schram, Phys. Rev. E **69**, 066403 (2004).

[11] G. M. Kremer, AIP Conf. Proc. **1786**, 160002 (2016).

[12] G. M. Kremer and R. André, Int. J. Mod. Phys. D **25**, 1650012 (2016).

[13] R. André and G. M. Kremer, in *Proceedings of the 2nd Argentinian-Brazilian Meeting on Gravitation, Relativistic Astrophysics and Cosmology*, edited by G. S. Vila, F. L. Vieyro, and J. C. Fabris, AAA Workshop Series Vol. 7 (Asociación Argentina de Astronomía, La Plata, 2015), p. 69.

[14] E. V. Arbuzova, A. D. Dolgov, and L. Reverberi, Phys. Lett. B **739**, 279 (2014).

[15] S. Capozziello, M. De Laurentis, S. D. Odintsov, and A. Stabile, Phys. Rev. D **83**, 064004 (2011).

[16] S. Capozziello, M. De Laurentis, I. De Martino, M. Formisano, and S. D. Odintsov, Phys. Rev. D **85**, 044022 (2012).

[17] S. Capozziello and M. De Laurentis, Ann. Phys. (Amsterdam) **524**, 545578 (2012).

[18] G. M. Kremer, *An Introduction to the Boltzmann Equation and Transport Processes in Gases* (Springer, Berlin, 2010).

[19] I. Müller and T. Ruggeri, *Rational Extended Thermodynaics*, 2nd ed. (Springer, New York, 1998).

[20] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-uniform Gases*, 3rd ed. (Cambridge University Press, Cambridge, England, 1970).

[21] P. Coles and F. Lucchin, *Cosmology. The Origin and Evolution of Cosmic Structure*, 2nd ed. (John Wiley, New York, 2002).

[22] T. Padmanabhan, *Structure Formation in the Universe* (Cambridge University Press, Cambridge, England, 1992).

[23] G. Mou, F. Yuan, D. Bu, M. Sun, and M. Su, Astrophys. J. **790**, 109 (2014).

[24] J. Roman-Duval, J. M. Jackson, M. Heyer, H. J. Rathborne, and R. Simon, Astrophys. J. **723**, 492 (2010).

[25] S. Hocuk and M. Spaans, Astron. Astrophys. **510**, A110 (2010).

[26] K. C. Sahu, S. Casertano, M. Livio, R. L. Gilliland, N. Panagia, M. D. Albrow, and M. Potter, Nature (London) **411**, 1022 (2001).

[27] H. Velten, D. J. Schwarz, J. C. Fabris, and W. Zimdahl, Phys. Rev. D **88**, 103522 (2013).

[28] G. Acquaviva, A. John, and A. Pénin, Phys. Rev. D **94**, 043517 (2016).